

Erratum

Ingo Steinbach*

Erratum to: Quantum-Phase-Field Concept of Matter: Emergent Gravity in the Dynamic Universe

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The work “Quantum-phase-field concept of matter: Emergent gravity in the dynamic universe”, published in [1], outlines a framework to describe physical matter from the solution of a one-dimensional non-linear wave equation. Unfortunately, a central result, the presented analytic form of this solution, (13) in [1] is incorrect. The corrected form can be found on arXiv [2]. In the present Erratum, besides the correct solution, some background information about these types of non-linear wave equations and their solutions is added. We start from a functional f^{DW} , the famous “double-well potential”, according to Landau’s theory of phase transitions [3] with positive constants r and u , expanded in temperature T around the critical temperature T_c . $\tilde{\phi}$ is the “order parameter” in the original Ginzburg-Landau theory. In this theory the potential function is expanded to the fourth order, keeping only even contributions of the order parameter $\tilde{\phi}$:

$$f^{DW} = r(T - T_c)\tilde{\phi}^2 + \frac{u}{2}\tilde{\phi}^4. \quad (1)$$

The potential has a minimum at $\tilde{\phi} = 0$ for $T > T_c$ and two minima at $\tilde{\phi}_m = \pm\sqrt{\frac{r(T_c - T)}{u}}$ for $T < T_c$. These minima describe the disordered and ordered state of the system, respectively. Here only the ordered state $T < T_c$ is discussed and to be consistent with conventions in the

phase-field literature the order parameter is normalized to $0 \leq \phi \leq 1$, $\phi = \frac{1}{2}\left(\frac{\tilde{\phi}}{|\tilde{\phi}_m|} + 1\right)$, see Figure 1.

$$f^{DW} = \frac{\gamma}{4}\phi^2(1 - \phi)^2. \quad (2)$$

Also, we will understand $\phi = \phi(s, t)$ as a field variable in space s and time t and define the Hamiltonian H as an integral over the potential density and the Ginsburg gradient operators accounting for fluctuations:

$$H = U \int_{-\infty}^{+\infty} ds \left[\epsilon \left(\frac{\partial \phi}{\partial s} \right)^2 + f^{DW} \right]. \quad (3)$$

U is a constant with dimension of energy, ϵ is a constant with dimension of length and γ an inverse length.

The well-known minimum solution $\frac{\delta}{\delta \phi} G = 0$ is for the boundary conditions $\phi(-\infty) = 0$ and $\phi(+\infty) = 1$:

$$\phi(s, t) = \frac{1}{2} \left[1 + \tanh \left\{ \frac{3(s + vt)}{\eta} \right\} \right], \quad (4)$$

with $\eta = \sqrt{72 \frac{\epsilon}{\gamma}}$ for a traveling wave with speed v . Note that for $v \neq 0$ a symmetry breaking contribution has to be added to the potential (2), which is omitted here to keep the focus on the type of potential. Details can be

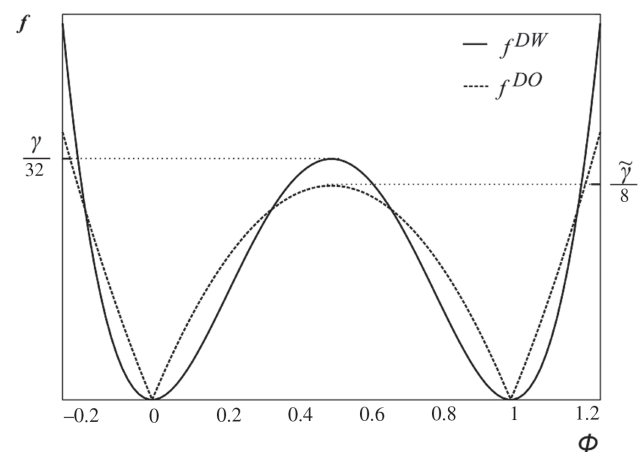


Figure 1: The “double-well” vs. the “double-obstacle” potential.

*Corresponding author: Ingo Steinbach, Ruhr-University Bochum, ICAMS, Universitaetsstrasse 150, 44801 Bochum, Germany, E-mail: ingo.steinbach@rub.de

found in appendix of [4]. For the boundary conditions $\phi(-\infty) = \phi(+\infty) = 0$ we have, besides the trivial solution $\phi \equiv 0$:

$$\phi(s, t) = \frac{1}{2} \left[\tanh \left\{ \frac{3(s - s_1 + vt)}{\eta} \right\} - \tanh \left\{ \frac{3(s - s_2 - vt)}{\eta} \right\} \right], \quad (5)$$

where the waves, peaked at s_1 and s_2 at $t = 0$, respectively, $s_1 < s_2$, travel with opposite speed, see Figure 2.

It is also well established that the special form of the potential (1) or (2) is not fixed from basic principles, besides that, between the two minima there should be a potential barrier to separate the minima with a given activation energy $\propto U$. Only close to the critical point, i.e. where the activation energy $U \rightarrow 0$, a rigorous renormalization group treatment may be applied to show that higher order contributions will become irrelevant [5]. Aside from the critical point, no argument exists to truncate the Landau expansion of the potential to the fourth order or to select the given form. In the “multi-phase-field theory” [6] there arises, however, an additional constraint for the potential. In this theory a set of fields ϕ^I , $I = 1 \dots N$ is defined which form junctions by the condition $\sum_{I=1}^N \phi^I = 1$. The functional is replaced by $H = \sum_{I=1}^N H^I$ where each term H^I has the special form (3). In the center of the junction, where $\phi^I = \phi^J = \frac{1}{N}$ for all I, J , the maximum of the potential f_m^{DW} can be evaluated:

$$\sum_{I=1}^N f_m^{DW} = \frac{\gamma}{4} N \left(\frac{1}{N} \right)^2 \left(1 - \frac{1}{N} \right)^2 \approx \frac{\gamma}{4} \frac{1}{N} \quad \text{for } N \gg 1, \quad (6)$$

i.e. for $N > 3$ the energy of the junction decreases with the order N and approaches 0 for large N . This must

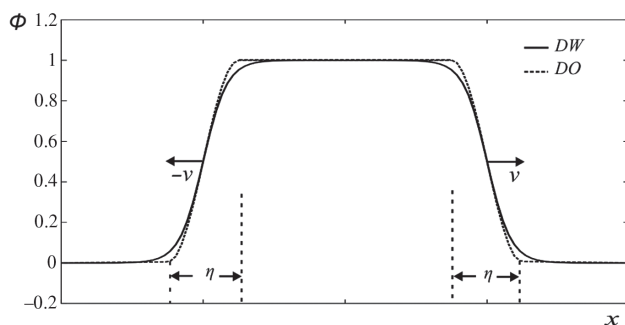


Figure 2: Traveling wave solution for the double-well and double obstacle potential. Note, that for the double-obstacle potential the width η is sharply defined whereas the for the double-well potential the 95 % decay is evaluated for the width.

be termed “unphysical”, as junctions between objects loose their penalty and the system would return to the disordered state. To remedy this problem, the so-called “double-obstacle potential” is introduced [6]:

$$f^{DO} = \frac{\tilde{\gamma}}{2} |\phi(1 - \phi)|. \quad (7)$$

It has the same topology as (2) (see Fig. 1) but a maximum power of 2. Further on it has the advantage that it defines a linear wave aside from the breakpoints. We calculate the maximum potential of the junction f_m^{DO} :

$$\sum_{I=1}^N f_m^{DO} = \frac{\tilde{\gamma}}{2} N \frac{1}{N} \left(1 - \frac{1}{N} \right) \approx \frac{\tilde{\gamma}}{2} \quad \text{for } N \gg 1, \quad (8)$$

i.e. the energy of the junction increases with the order N and approaches a constant $\propto \tilde{\gamma}$ for large N , as it should. The main drawback of this potential is the non-analytical form with the absolute signs. The “non-linearity” of (2) is hidden in the breakpoints at $\phi = 0$ and $\phi = 1$. Only a piece-wise solution is possible for the boundary conditions $\phi(-\infty) = 0$ and $\phi(+\infty) = 1$, $\eta = \pi \sqrt{\frac{\epsilon}{\tilde{\gamma}}}$,

$$\phi = \begin{cases} 0 & \text{for } s < -vt - \frac{\eta}{2}, \\ \frac{1}{2} + \frac{1}{2} \sin \left(\frac{\pi(s + vt)}{\eta} \right) & \text{for } -vt - \frac{\eta}{2} \leq s < -vt + \frac{\eta}{2}, \\ 1 & \text{for } s > -vt + \frac{\eta}{2}. \end{cases} \quad (9)$$

For $\phi(-\infty) = \phi(+\infty) = 0$, one finds (see Fig. 2):

$$\phi = \begin{cases} 0 & \text{for } s < s_1 - vt - \frac{\eta}{2}, \\ \frac{1}{2} + \frac{1}{2} \sin \left(\frac{\pi(s - s_1 + vt)}{\eta} \right) & \text{for } s_1 - vt - \frac{\eta}{2} \leq s < s_1 - vt + \frac{\eta}{2}, \\ 1 & \text{for } s_1 - vt + \frac{\eta}{2} \leq s < s_2 + vt - \frac{\eta}{2}, \\ \frac{1}{2} - \frac{1}{2} \sin \left(\frac{\pi(s - s_2 - vt)}{\eta} \right) & \text{for } s_2 + vt - \frac{\eta}{2} \leq s < s_2 + vt + \frac{\eta}{2}, \\ 0 & \text{for } s \geq s_2 + vt + \frac{\eta}{2}. \end{cases} \quad (10)$$

The last (10) replaces (13) in [1].

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