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# Landau Quantised Modification of Rayleigh–Taylor Instability in Dense Plasmas

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**Abstract:** Effects of Landau quantisation and exchange-correlation potential on Rayleigh–Taylor instability (RTI)/gravitational instability are investigated in inhomogeneous dense plasmas. Quantum hydrodynamic model is used for the electrons, while the ions are assumed to be cold and classical. RTI is modified with the inclusion of Landau quantisation related to plasma density, ambient magnetic field, exchange speed, and modified Fermi speed. Owing to the exchange-correlation effects, gravitational instability increases, whereas the Landau quantisation effects contribute in the opposite way for quantisation factor $\eta < 1$. Since the exchange-correlation potential is a function of density, by controlling the number density and magnetic field one can control RTI.

**Keywords:** Exchange-Correlation Potential; Gravitational Instability; Landau Quantization; Quantum Plasmas.

## 1 Introduction

Plasmas are enriched by heterogeneous phenomena, typically the challenge of inertial confinement fusion [1], growth of interface perturbations [2], tunnelling of aerodynamic wind [3], launching shock pulses into the foil of metal [4], material mixing, Doppler broadening of gamma rays [5], striking the interstellar clouds by blast waves [6], symmetry-breaking by supernova explosion [7], etc. These phenomena are rooted in the promising mechanism of Rayleigh–Taylor instability (RTI) or simply mixing instability. RTI is akin to falling water out of a glass, mixing the vinaigrette on shaking, flapping of the flags, and mushroom cloud from atomic explosion [7]. RTI explains the broad infrared emission of Fe-II, Ni-II, Ar-II, and Co-II, indicating the mixing from low-velocity to high-velocity cores. The production of hard X-rays is an indirect proof of mixing. Hence, understanding RTI helps in explaining the physical mechanism of fundamental research and technology, for example, nuclear weapons design [8].

There are many factors that may affect RTI, such as plasma density inhomogeneity [9], thickness scale of the perturbed interface, mass ablation [10], temperature-gradient-dependent magnetic field [11], inhomogeneous magnetic field [12], Weibel instability, resonant absorption, motion of superthermal electrons [13], stationary ponderomotive force [14], etc. All these are entirely studied in classical plasmas, therefore it is a need to introduce non-ideal effects such as Landau quantisation, exchange and correlation potential, etc. in quantum plasmas. Quantum plasma has emerged as a rapidly growing research area. Quantum plasma exists in dense astrophysical environments, particularly in the interior of Jupiter, white dwarfs, and neutron stars, as well as in metals and semiconductors. It is well known that quantum plasma has the properties of high particle number density and low temperature compared to classical plasmas, and therefore are associated with a de Broglie length longer than the inter-particle distance. Such plasmas are characterised by the Fermi pressure associated with degeneracy, where all quantum states are fully occupied below a certain level, tunnelling potential, exchange-correlation potential, and Landau quantisation [15, 16]. A recognised fact of moving either orbital-like gyro or spinning electrons is the magnetic field induction and the associated moment along the axis of gyration. Magnetic moment creates magnetism in the plasma. The external magnetic field alters the spinning. There are two magnetic effects due to the strong magnetic field: first is the Landau quantisation or Landau diamagnetism, which arises from the quantisation of the orbital-like gyro motion of charged particles; the second is the Pauli paramagnetism due to the spin of electrons. Landau quantisation effect and the tunnelling potential [17] are of a purely quantum nature. The external magnetic field enhances the total energy of the plasma system through Landau quantisation. The free electrons exhibit Landau diamagnetism at $T_{Fe} > T_e$, while the fixed electrons produce Pauli paramagnetism.

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Orbital quantisation modifies the thermodynamic properties of the plasma at equilibrium.

Cao et al. [18] studied RTI using quantum magnetohydrodynamic equations and solved the second-order differential equation under fixed boundary conditions. They pointed out that the magnetic field has a stabilizing effect on RTI in a similar manner as in classical plasmas but is significantly affected by quantum effects. Ali et al. [19] investigated the RTI in an inhomogeneous, dense magnetoplasma and found that the density gradient modifies the growth rate of RTI. Modestov et al. [20] studied the influence of magnetic field on RTI in metal quantum plasmas. They observed that the paramagnetic effects in a quantum plasma make RTI weaker; however, for the case of ferromagnetic effects with perturbations of long and moderate wavelengths, certain stabilisation always takes place due to the nonlinear character of quantum plasma magnetisation. In 2012, Wang et al. [1] discussed the stabilisation of the RTI due to density gradients, magnetic fields, and quantum effects in ideal, incompressible quantum magnetoplasmas.

In this article, we present the RT wave instability in a non-uniform quantum magnetoplasma by assuming fluid-streaming due to the diamagnetic drift, gravitational drift, and additional diamagnetic-type drifts due to exchange-correlation potential under conditions $\omega_{ci}^2 \gg (\omega - \mathbf{V}_{0i}.\mathbf{k})^2$, where $\omega_{ci} = eB_0/m_i c$ is the ion cyclotron frequency. Both electrons and ions are magnetised, but ions are treated as classical whereas quantum effects are included for electrons in the quantum hydrodynamic model. It is observed that the growth rate and the real wave frequency are significantly modified with the Fermi distribution including the Landau quantisation effects [21]. The Landau quantisation effects stabilise the RTI for the quantisation factor $\eta_e = \frac{\hbar\omega_{ce}}{E_{Fe}} < 1$, where $\omega_{ce} = \frac{eB_0}{m_e c}$ is the cyclotron frequency and $E_{Fe} = k_B T_{Fe0}$ is the Fermi energy of the plasma species. The rest of the article goes as follows: In Section 2, we solve the quantum hydrodynamic fluid equations using the plasma approximation. Plasmas for which the de Broglie wavelength has influence over the inter-particle distance are found in stellar and interstellar media. The dispersion of RTI is derived in Section 3. Section 4 describes the growth rate and numerical results, and Section 5 presents the discussion. The summary of the work is given in Section 6.

## 2 Mathematical Model

A dense quantum magnetoplasma consisting of ions and electrons is assumed. The quasi-neutrality condition for equilibrium is given as $n_{e0} = n_{i0} = n_0$. An external uniform magnetic field is applied in the $z$-direction, that is, $\mathbf{B}_0 = B_0\hat{z}$. At equilibrium, the gravitational field and the density gradient act in opposite directions: the density gradient is supposed to be in the negative $x$-direction and gravitational field in positive $x$-direction i.e. $\nabla n_{0i} = -|\nabla n_{0i}|\hat{x}$, $\mathbf{g} = g\hat{x}$. The propagation vector of the instability wave and the electric field are considered to be in the $y$-direction, that is, $\mathbf{E}_1 = (0, E_y, 0)$ and $\mathbf{k} = (0, k_y, 0)$. As dense magnetoplasma environments have strong magnetic fields, Landau quantisation effects cannot be ignored. Whether RTI is increased or squeezed depends upon the strength of quantisation, that is, the value of $\eta$, which cannot exceed unity.

To investigate the effects of Landau quantisation (Landau diamagnetism) due to the quantisation of the gyro-like orbital motion of the electrons in a strong magnetic field on the RTI in a quantum plasma with a density gradient, the following set of fluid equations is used in the quantum hydrodynamic (QHD) model:

Momentum equation

$$\begin{aligned} m_j n_j \left[\frac{\partial}{\partial t} + (\mathbf{v}_j.\nabla)\right]\mathbf{v}_j &= q_j n_j \left[\mathbf{E} + (\mathbf{v}_j \times \mathbf{B}_0)/c\right] \\ &- \nabla P_j + m_j n_j \mathbf{g} + \frac{\hbar^2}{4m_j}\nabla\left(\nabla^2 n_j\right) \\ &- V_{j,xc}\nabla n_j. \end{aligned} \tag{1}$$

Continuity equation

$$\frac{\partial n_j}{\partial t} + \nabla.(n_j \mathbf{v}_j) = 0, \tag{2}$$

where the exchange-correlations potential $V_{j,xc} = \frac{0.985e^2}{\epsilon} n_j^{1/3}\left[1 + \frac{0.034}{a_{Bj}n_j^{1/3}}\ln\left(1 + 18.37 a_{Bj} n_j^{1/3}\right)\right]$ is included to analyze the complete picture of the quantum plasma [15, 22, 23]. Furthermore, $\mathbf{E} = \mathbf{E}_1$, $n_j = n_{0j} + (\mathbf{r}.\nabla)n_{0j} + n_{1j}$, $P_j = m_j v_{Fj}^{'2} n_j$, and $\mathbf{v}_j = \mathbf{V}_{0j} + \mathbf{v}_{1j}$, where $\mathbf{V}_{0j}$ is the fluid streaming due to the diamagnetic drift, gravitational drift, and diamagnetic-type drift due to exchange-correlation potential given by

$$\begin{aligned} \mathbf{V}_{0j} = &-\frac{ck_B T_j}{q_j B_0}\hat{z} \times \left(\frac{\nabla n_{0j}}{n_{0j}}\right) + \frac{m_j c}{q_j}\frac{\hat{z} \times \mathbf{g}}{B_0} \\ &- \frac{cV_{j,xc}}{q_j B_0}\hat{z} \times \left(\frac{\nabla n_{0j}}{n_{0j}}\right). \end{aligned} \tag{3}$$

Here, $\nabla.\mathbf{V}_{0j} = 0$, $\frac{\partial}{\partial t}\mathbf{V}_{0j} = 0$, $\nabla^2 n_{0j} = 0$, and $\frac{\partial n_{0j}}{\partial t} = 0$. The effective Fermi speed modified by Landau quantisation for the $j^{\text{th}}$ species is $v_{Fj}^{\prime 2} = \frac{3}{5} v_{Fj0}^2 \left\{ \delta_j + \frac{5}{12}\pi^2 \left( \frac{T_j^2}{T_{Fj0}^2} \right) \gamma_j \right\}$. The coefficients of Landau quantisation are $\delta_j = \left[ \frac{5\eta_j}{6} + \frac{5\eta_j}{3}\left(1-\eta_j\right)^{3/2} + \left(1-\eta_j\right)^{5/2} \right]$ and $\gamma_j = \left[ \frac{\eta_j}{4} + \left(1-\eta_j\right)^{1/2} + \frac{\eta_j}{2}\left(1-\eta_j\right)^{-1/2} \right]$. $v_{Fj0} = \sqrt{\frac{2k_B T_{Fj0}}{m_j}}$ is the Fermi speed, $k_B$ is the Boltmann constant, and $T_{Fj0} = \frac{1}{2}\left(3\pi^2 n_{0j}\right)^{2/3} \frac{\hbar^2}{k_B m_j}$ is the Fermi temperature of the plasma species [17]. Since electrons have quantum nature, they exhibit Fermi temperature, Bohm potential, and exchange-correlation potential. However, because of the smaller mass of electrons, gravitational force may be ignored, and the linearised equation of motion for electrons is

$$
\begin{aligned}
m_e n_{0e} \left( \frac{\partial}{\partial t} + \mathbf{V}_{0e}.\nabla \right) \mathbf{v}_{1e} &= -en_{0e}(\mathbf{E}_1 + \mathbf{v}_{e1} \times \mathbf{B}_0/c) \\
&\quad - en_{1e}(\mathbf{V}_{0e} \times \mathbf{B}_0/c) - \nabla P_e \\
&\quad + \frac{\hbar^2}{4m_e} \nabla\left(\nabla^2 n_{1e}\right) - V_{e,xc} \nabla n_{1e}.
\end{aligned} \tag{4}
$$

The equation of continuity for electrons is

$$
\frac{\partial n_{1e}}{\partial t} + \nabla.(n_{0e}\mathbf{v}_{1e}) + \nabla.(n_{1e}\mathbf{V}_{0e}) = 0. \tag{5}
$$

For the plasma ions, quantum effects can be neglected while the gravitational force is taken care of:

$$
\begin{aligned}
m_i n_{0i} \left( \frac{\partial}{\partial t} + \mathbf{V}_{0i}\,.\,\nabla \right) \mathbf{v}_{i1} &= en_{i0}(\mathbf{E} + \mathbf{v}_{i1} \times \mathbf{B}_0/c) \\
&\quad + en_{1i}(\mathbf{V}_{0i} \times \mathbf{B}_0/c) - k_B T_i \nabla n_{i1} + m_i n_{i1}\mathbf{g}.
\end{aligned} \tag{6}
$$

The continuity equation for ions is

$$
\frac{\partial n_{1i}}{\partial t} + \nabla.(n_{0i}\mathbf{v}_{1i}) + \nabla.(n_{1i}\mathbf{V}_{0i}) = 0. \tag{7}
$$

Since all quantities vary sinusoidally, $n_{1i} = n_{1i}\exp[i(\mathbf{k}.\mathbf{r} - \omega t)]$, $\mathbf{v}_{1i} = \mathbf{v}_{1i}\exp[i(\mathbf{k}.\mathbf{r} - \omega t)]$, and $\mathbf{E}_1 = \mathbf{E}_1\exp[i(\mathbf{k}.\mathbf{r} - \omega t)]$, and then (6) becomes

$$
\begin{aligned}
-i\omega m_i n_{0i}\mathbf{v}_{1i} + im_i n_{0i}(\mathbf{V}_{0i}\,.\,\mathbf{k})\mathbf{v}_{1i} &= en_{0i}\mathbf{E}_1 + en_{1i}\mathbf{V}_{0i} \times \mathbf{B}_0/c + en_{0i}\mathbf{v}_{1i} \\
&\quad \times \mathbf{B}_0/c - ik_B T_i \mathbf{k} n_{1i} + m_i n_{1i}\mathbf{g}.
\end{aligned} \tag{8}
$$

According to the assumed geometry, $\mathbf{g} = g\widehat{x}$, $\mathbf{B}_0 = B_0\widehat{z}$, $\nabla n_{0i} = -|\nabla n_{0i}|\widehat{x}$, $\mathbf{E}_1 = E_1\widehat{y}$, and $\mathbf{k} = k_y\widehat{y}$, and $v_{1xi}$ and $v_{1yi}$ are give below:

$$
v_{1xi} = \frac{eE_{1y}}{\omega_{ci} m_i} - i\frac{v_{ti}^2 k_y}{\omega_{ci}}\frac{n_{1i}}{n_{0i}} \tag{9}
$$

and

$$
\begin{aligned}
v_{1yi} &= \frac{-ie\left(\omega - V_{0yi}k_y\right)}{\omega_{ci}^2 m_i} E_{1y} \\
&\quad - \left( V_{0yi} + \frac{g}{\omega_{ci}} + \frac{v_{ti}^2 k_y\left(\omega - V_{0yi}k_y\right)}{\omega_{ci}^2} \right)\frac{n_{1i}}{n_{0i}},
\end{aligned} \tag{10}
$$

where $v_{ti}^2 = \frac{k_B T_i}{m_i}$ is the thermal motion of ions. Here, $\omega_{ci}^2 \gg \left(\omega - V_{0yi}k_y\right)^2$, and $V_{0xi} = 0$. Now the linearised equation of continuity for ions with $\frac{\partial n_{0i}}{\partial t} = 0$, and $\nabla.\mathbf{V}_{0i} = 0$ gives

$$
-i\left(\omega - V_{0yi}k_y\right)n_{1i} + n_{0i}ik_y v_{1yi} - v_{1xi}|\nabla_x n_{0i}| = 0. \tag{11}
$$

Eliminating $v_{1xi}$ and $v_{1yi}$ from (9) to (11) by inserting the values of $v_{1xi}$ and $v_{1yi}$ from (9) to (10), respectively, into above equation, we get

$$
\begin{aligned}
&\frac{e}{\omega_{ci} m_i}\left( k_y \frac{\left(\omega - V_{0yi}k_y\right)}{\omega_{ci}} - \kappa_{ni} \right) E_{1y} \\
&\quad - i\left[ \omega + k_y\frac{g}{\omega_{ci}} + \frac{v_{ti}^2 k_y^2\left(\omega - V_{0yi}k_y\right)}{\omega_{ci}^2} - \frac{v_{ti}^2 k_y}{\omega_{ci}}\kappa_{ni} \right]\frac{n_{1i}}{n_{0i}} \\
&\quad = 0,
\end{aligned} \tag{12}
$$

where $\kappa_{ni} = \frac{|\nabla_x n_{0i}|}{n_0}$ is the inverse scale length of inhomogeneity due to the density gradient in ions. Rewriting the equation of motion (see (4)) for electron, after using the Fourier transformation $\frac{\partial}{\partial t} = -i\omega$, $\nabla = i\mathbf{k}$ for the perturbed quantities, we have

$$
\begin{aligned}
-in_{0e}(\omega - \mathbf{V}_{0e}.\mathbf{k})\mathbf{v}_{1e} &= -\frac{en_{0e}}{m_e}\mathbf{E}_1 - n_{0e}\mathbf{v}_{e1} \times \widehat{z}\omega_{ce} \\
&\quad - n_{1e}\omega_{ce}\mathbf{V}_{0e} \times \widehat{z} - i\mathbf{k}n_{e1}V_{FB,xc}^2,
\end{aligned} \tag{13}
$$

where $\omega_{ce} = \frac{eB}{m_e c}$, $V_{FB,xc}^2 = v_{Fe}^{\prime 2} + v_B^2 + v_{e,xc}^2$, $v_B^2 = \frac{\hbar^2}{4m_e^2}k_y^2$, and $v_{e,xc}^2 = \frac{V_{e,xc}}{m_e}$. As the drift is along $\mathbf{k}$, using $\mathbf{V}_{0e}.\mathbf{k} = V_{0ye}k_y$ in the above equation, we get

$$
\begin{aligned}
\left(\omega - V_{0ye}k_y\right)\mathbf{v}_{1e} &= -i\frac{e}{m_e}\mathbf{E}_1 - i\mathbf{v}_{e1} \times \widehat{z}\omega_{ce} \\
&\quad - i\frac{n_{1e}}{n_{0e}}\omega_{ce}\mathbf{V}_{0e} \times \widehat{z} + \mathbf{k}\frac{n_{e1}}{n_{0e}}V_{FB,xc}^2.
\end{aligned} \tag{14}
$$

For the given geometry $\mathbf{E}_1 = (0, E_y, 0)$ and $\mathbf{k} = (0, k_y, 0)$, the $x$- and $y$-components give the following expressions for the perturbed velocity components of electrons:

$$v_{xe} = \frac{e}{m_e}\frac{E_{y1}}{\omega_{ce}} + ik_y\frac{n_{e1}}{n_{0e}}\frac{V_{FB,xc}^2}{\omega_{ce}} \tag{15}$$

and

$$v_{ye} = i\frac{e}{m_e}\left(\frac{\omega - V_{0ye}k_y}{\omega_{ce}^2}\right)E_{y1} - \frac{n_{1e}}{n_{0e}}V_{0ye} - k_y\frac{n_{e1}}{n_{0e}}V_{FB,xc}^2\left(\frac{\omega - V_{0ye}k_y}{\omega_{ce}^2}\right). \tag{16}$$

So $v_{ye} \approx -\frac{n_{1e}}{n_{0e}}V_{0ye}$ for $\omega_{ce}^2 \gg (\omega - V_{0ye}k_y)^2$. The equation of continuity, i.e. (5), for electrons with constant $\mathbf{V}_{0e}$, $\mathbf{k} = k_y\hat{y}$, $\nabla n_{e0} = -|\nabla_x n_{0e}|\hat{x}$ is

$$-i\omega n_{1e} + n_{0e}ik_y v_{ye} + V_{0e}ik_y n_{e1} - v_{xe}|\nabla_x n_{0e}| = 0. \tag{17}$$

On substituting $v_{xe}$ from (15) to $v_{ye} \approx -\frac{n_{1e}}{n_{0e}}V_{0ye}$ into (16), we get

$$E_{y1} = -i\frac{m_e}{e}\left(\frac{\omega\omega_{ce}}{\kappa_{ne}} + k_y V_{FB,xc}^2\right)\frac{n_{1e}}{n_{0e}}, \tag{18}$$

where $\kappa_{ne} = \frac{|\nabla_x n_{0e}|}{n_0}$ is inverse scale length of inhomogeneity due to the density gradient in electrons.

# 3 Dispersion Relation

To calculate the modified dispersion relation of RTI in a quantum plasma, we eliminate $E_{y1}$, $n_{1e}$, and $n_{1i}$ from (17) to (18) by assuming $n_{1e} \approx n_{1i}$, and we have

$$A\omega^2 + B\omega - C = 0, \tag{19}$$

where $A = \frac{k_y}{\omega_{ci}\kappa_{ne}}$, $B = \left\{-\left(V_{0yi}\frac{k_y^2}{\omega_{ci}} + \kappa_{ni}\right)\frac{1}{\kappa_{ne}} + \left(\frac{k_y V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2 k_y}{\omega_{ci}}\right)\frac{k_y}{\omega_{ci}} + 1\right\}$, and $C = \left\{\left(V_{0yi}\frac{k_y^2}{\omega_{ci}} + \kappa_{ni}\right)\left(\frac{k_y V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2 k_y}{\omega_{ci}}\right) - k_y\frac{g}{\omega_{ci}}\right\}$ are quadratic coefficients. The quadratic solution is

$$\omega = \frac{1}{2}\left\{V_{0yi}k_y - \left(\frac{k_y V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2 k_y}{\omega_{ci}}\right)\kappa_{ne} + (\kappa_{ni} - \kappa_{ne})\frac{\omega_{ci}}{k_y}\right\} \pm i\sqrt{\kappa_{ne}g - \omega_{ci}\kappa_{ne}\left(V_{0yi}\frac{k_y^2}{\omega_{ci}} + \kappa_{ni}\right)\left(\frac{V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2}{\omega_{ci}}\right)}. \tag{20}$$

Equation (20) is further solved for $B^2 \ll 4AC$, which gives the modified dispersion relation of RTI for a quantum plasma including the gradient of magnetic effects. The quantum effects include Landau quantisation, Fermi pressure, exchange-correlation potential, and tunnelling potential.

# 4 Growth Rate

Let $\omega = \omega + i\gamma$ in (20), where $\gamma$ represents the growth rate of RTI. On comparing the real and imaginary parts in (20), the RT phase speed becomes

$$\omega = \frac{1}{2}\left\{V_{0yi}k_y - \left(\frac{k_y V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2 k_y}{\omega_{ci}}\right)\kappa_{ne} + (\kappa_{ni} - \kappa_{ne})\frac{\omega_{ci}}{k_y}\right\} \tag{21}$$

and the growth rate is

$$\gamma = \sqrt{\kappa_{ne}g - \omega_{ci}\kappa_{ne}\left(V_{0yi}\frac{k_y^2}{\omega_{ci}} + \kappa_{ni}\right)\left(\frac{V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2}{\omega_{ci}}\right)}. \tag{22}$$

Equation (22) is valid for $g > \omega_{ci}\left(V_{0yi}\frac{k_y^2}{\omega_{ci}} + \kappa_{ni}\right)\left(\frac{V_{FB,xc}^2}{\omega_{ce}} + \frac{v_{ti}^2}{\omega_{ci}}\right)$.

# 5 Results and Discussion

Equation (22) presents the analytical expression for the growth rate $\gamma$ of RTI for dense plasmas. It is seen that the quantisation effects have a large impact on the growth rate of RTI. In this section, a graphical analysis of the growth rate of instability is presented for typical parameters in cgs units: $B_0 = (1 - 100) \times 10^9$G, $n_{0e} = (1 \times 10^{24} - 1 \times 10^{27})$ cm$^{-3}$, $n_{0i} = n_{0e}$, $T_{Fe0} = \frac{1}{2}(3\pi^2 n_{0e})^{2/3}\frac{\hbar^2}{k_B m_e} > T_e = 1 \times 10^6$ K, $T_i = 10^4$ K, $k \approx 1 \times 10^8$ cm$^{-1}$, $g \approx 1 \times 10^{13}$ cm/s$^2$, $\kappa_{ni} = 1 \times 10^{-2}$ cm$^{-1}$, $\kappa_{ne} = 1 \times 10^2$ cm$^{-1}$.

Figures 1–5 show a considerable impact over the normalised growth rate $\gamma/\omega_{pi}$ (Figs. 1 and 2) and $\gamma/\omega_{ci}$ (Figs. 3–5) due to the contribution of Landau quantisation in quantum plasmas. Figure 1 describes the behaviour of RTI versus $\eta$ at different ambient magnetic fields $B_0$ in the presence of tunnelling potential, exchange-correlation potential, and Landau quantised Fermi statistical pressure. The dashed curve describes the instability at small magnetic field and the solid curve at strong magnetic field.

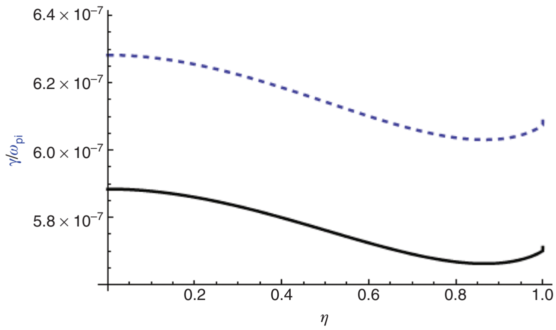


**Figure 1:** Normalised growth rate of RTI versus $\eta$. The dashed curve is for small $B_0 = 10^{11}$(G) and the solid curve is for large $B_0 = 1.1 \times 10^{11}$ (G).

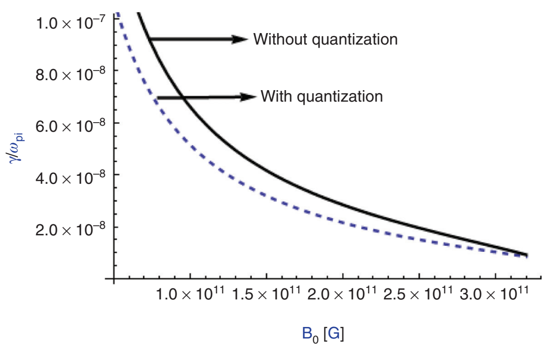


**Figure 2:** Normalised growth rate of RTI versus $B_0$ (G). The solid curve is without quantisation and the dashed curve is with Landau quantisation for $\eta < 1$. Both curves meet at $\eta = 1$.

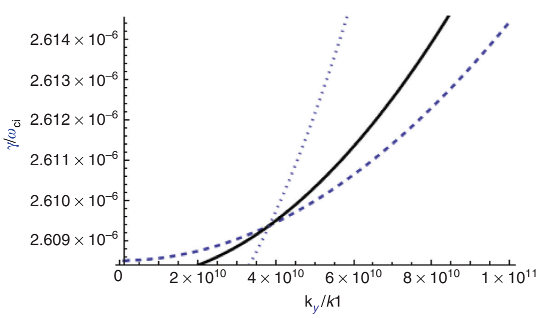


**Figure 3:** Normalised growth rate of RTI versus $k_y/\kappa_i$ in the presence of Landau quantisation at $\eta = 0.201$ (dotted curve), $\eta = 0.588$ (solid curve), and $\eta = 0.934$ (dashed curve).

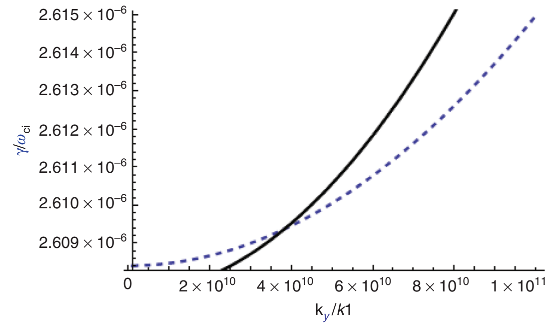


**Figure 4:** Normalised growth rate of RTI versus $k_y/\kappa_i$ with Landau quantisation (dashed curve) and without Landau quantisation (solid curve).

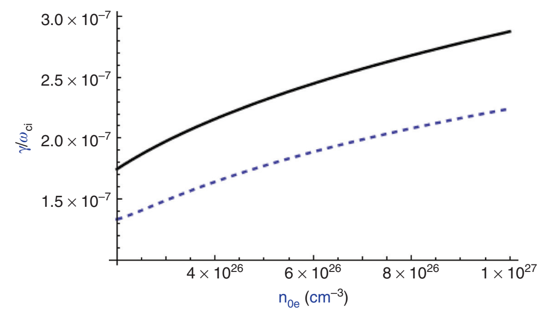


**Figure 5:** Normalised growth rate of RTI versus $n_{0e}$ ($\mathrm{cm}^{-3}$) with Landau quantisation (dashed curve) and without Landau quantisation (solid curve).

It can be noticed that the behaviour of RTI $\gamma/\omega_{pi}$ is modified by the Landau quantisation effect. The instability is suppressed at higher magnetic fields in comparison with that at smaller $B_0$ for $\eta < 1$. Physically, the suppressing mechanism of the instability can be explained in terms of the quantisation of energy states on increasing $B_0$ at $\eta < 1$. The electrons from the excited states are accommodated in the induced stable states. As a result, the remaining few particles may contribute to the wave instability. On the other hand, at small $\eta$, both curves give the maximum instability, and on increasing $\eta$, both curves, although at different instability frequencies, decrease in a similar way so that $\eta$ approaches unity. In Figure 2, the upper plot is for the case without quantisation (solid) where $\gamma/\omega_{pi}$ decreases with $B_0$(G) where $\eta < 1$. On the other hand, $\gamma/\omega_{pi}$ also decreases with the increase of $B_0$(G), that is, with the quantisation effects (dashed curve). The dashed graph meets the solid graph at $\eta = 1$, which shows the absence of Landau quantisation. Comparison of the graphs shows that $\gamma/\omega_{pi}$ becomes small with the inclusion of Landau quantisation effects at lager values of $B_0$. This shows the stabilisation of RTI with the inclusion of quantisation effects.

Figure 3 shows $\gamma/\omega_{ci}$ against the dimensionless wave vector $k/\kappa_i$ at different values of $\eta$ (the dotted curve for $\eta = 0.201$, the solid curve for $\eta = 0.588$, and the dashed curve for $\eta = 0.934$). For the dotted curve, the instability is defined over a very small range of wave vectors; however, a uniform increment of the instability is observed. For the solid curve, RTI increases for a wide spectrum wave vectors. The dashed curve describes the widest wave vector spectrum of increasing RTI. Figure 4 depicts the variation of $\gamma/\omega_{ci}$ versus $k/\kappa_i$ with Landau quantisation (dashed curve), while the solid curve is without Landau quantisation. On increasing the normalised wave vector, RTI increases. Both curves intersect at a particular wave vector, where both scenarios have the same instability. Overall, without Landau quantisation, the instability spectrum is small compared to the Landau quantisation wave vector spectrum. Figure 5 shows that $\gamma/\omega_{ci}$ increases with $n_{0e}$ ($\mathrm{cm}^{-3}$) with (dashed curve) and without quantisation (solid curve). However, there is a significant reduction in RTI in the presence of Landau quantisation for the whole range of number densities as compared to the instability without Landau quantisation. This is mainly due to the induction of new states due to the quantisation of gyro-like orbital motion in dense plasmas. The induced states provide the reason for settling down the charged particles in lower states, which reduces the instability.

## 6 Summary

In summary, we studied RTI in quantum plasmas including quantum effects from Fermi pressure, Bohm potential, exchange-correlation potential, and Landau quantisation of the orbital motion of electrons. We found that the magnetic field $B_0$ and hence the Landau quantisation play a role for the stabilisation of RTI for $\eta < 1$. The plasma density and the exchange-correlation potential, which directly depends on the density $n_{0e}$, increase the growth rate of instability. Quantisation effects on RTI disappear at $\eta = 0$, but quantisation has minimum effects at $\eta = 1$, which means the balance of magnetisation energy $\hbar\omega_{ce}$ and Fermi energy $E_{Fe}$. Since $\eta$ depends on both $B_0$ and $n_{0e}$, the range of quantisation effects shifts to larger values of $B_0$(or $n_{0e}$) by increasing the values of $n_{0e}$ (or $B_0$). This study provides a sound explanation of the collapse of a heavy astrophysical body to a more dense body. The conversion of the Red Giant to the Planetary Nebula and of the Red Supergiant to Supernova are good examples of RT/gravitational instability.

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