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Nonreciprocal Transmission of Electromagnetic Waves Using an Electric–Gyrotropic Dispersive Medium

<https://doi.org/10.1515/zna-2019-0120>

Received April 12, 2019; accepted September 11, 2019; previously published online December 20, 2019

Abstract: A nonreciprocal transmission structure is designed using a one-dimension multilayer medium, which consists of two asymmetric structure filled with the electric–gyrotropic dispersive media. The total transmission coefficients have been deduced using the transfer matrix method. Numerical results further provided evidence for the occurrence of the nonreciprocal surface electromagnetic waves. These states are affected by the thickness of layers, incident angles, and the externally applied magnetic fields. Given that the electric–gyrotropic media are inherently dispersive, our investigations will contribute to the practical application of nonreciprocal structures.

Keywords: Dispersive Mediums; Gyrotropic; Nonreciprocal.

1 Introduction

In recent investigations, it has been reported that the interaction of surface electromagnetic (EM) waves (SEWs) at an interface yields significant new information relevant to material research, such as the enhanced transmission of light [1], super-resolution imaging [2], biosensing [3], mode converter [4], and so on. A well-known SEW is surface plasmon polaritons (SPPs) [5, 6]. They can be found at the interface between two media with a negative permittivity and a normal media. After the application of a transverse magnetic (TM) excitation wave, these SPPs can

be captured and observed using special coupling devices [7] such as prisms and grating couplings. Other familiar types of interface modes include the optical Tamm states (OTSs) [8–10] that are formed at the interface between two photonic structures with overlapping band gaps. Similar to OTSs, another category of confined EM modes, called Tamm plasmon polaritons, is formed at the boundary between a metal layer and a Bragg mirror [11–13]. Although the mechanisms vary among the different SEWs, their transmission properties are quite similar at the interface.

One of the novel properties of SEWs is nonreciprocity, which occurs when the structure is in the presence of magneto-optical materials (MOMs) [14–18]. In MOMs, the nonreciprocity of EM parameters is associated with a break in time reversal symmetry due to an external static magnetic field or spontaneous magnetisation [19, 20]. The nonreciprocity of SPPs was initially observed many years ago, and this phenomenon is currently applied in numerous optical devices. Nevertheless, SPPs only propagate parallel to an interface and decay exponentially in an orthogonal direction away from the surface because they possess an imaginary propagation constant. Therefore, the nonreciprocal behaviour of SPPs is inconspicuous, even if MOMs subjected to an external magnetic field [7]. However, in contrast to SPPs, OTSs are designed to address these limitations using multilayers, and strong nonreciprocal OTSs can be generated via one-way resonant optical tunnelling at the interface between two different magnetic–photonic materials [21, 22]. In particular, an important advantage of OTSs is their low attenuation. Moreover, they can exhibit both TE and TM polarisation [7]. Therefore, when optical elements are designed with functionality that depends on the nonreciprocity of surface waves, OTSs are a viable alternative.

The nonreciprocity of OTSs is determined by the asymmetric gyro-parameters of the MOMs and can be enhanced and optimised by the selection of appropriate asymmetric gyro-parameters [12, 16]. Therefore, the EM parameters of a magneto-optical medium are not fixed and are dependent on the external magnetic field and the frequency of the incident waves [23]. In addition, the propagating properties of nonreciprocal OTSs are influenced by the external magnetic field and the frequency of the incident

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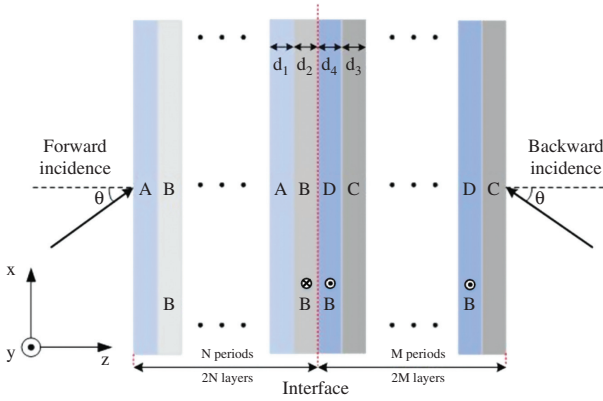


Figure 1: A simplified schematic description of a mathematical model based on ABAB...ABDC...DCDC. It consists of two different semi-infinite one-dimensional photonic crystals in which A and C are dielectric media. B and D are gyrotropic materials.

waves. In this investigation, we consider the asymmetric multilayer composite structure consisting of a dispersive electric–gyrotropic medium. Under the influence of different external magnetic fields and incident frequencies, the different nonreciprocal behaviours can be demonstrated.

In this report, we design an original structure, which is represented by ABAB...ABDC...DCDC. As shown in Figure 1, the structure is composed of two periodically layered electric–gyrotropic medium, and the dotted line represents their interface. In the cells on the left (AB), A and B are a normal dielectric and a gyrotropic material, respectively. In the cells on the right (DC), C and D are a normal dielectric and a gyrotropic material, respectively. The thickness of layers A, B, C, and D are denoted by d_1 , d_2 , d_3 , and d_4 , respectively. An external static magnetic field is necessary for the phenomenon of nonreciprocity. The static magnetic fields added to the model at the interface have opposite directions and the same value. The background of the model is air, and the wave vector $k(\omega)$ is along the xz plane with an incidence angle (θ).

2 Model and Theoretical Analysis

To simplify, we use ε_{rA} and ε_{rC} to represent the relative permittivities of the isotropic dielectric A and C, respectively. Layers B and D are gyrotropic materials, and their permittivities are ε_{rB} and ε_{rD} . The gyrotropic layers' (B and D) dielectric tensors are functions of frequency ω and the external static magnetic field B because of the Voigt geometry and can be expressed as [23]:

$$\tilde{\varepsilon}_r = \begin{pmatrix} \varepsilon_{r1} & 0 & i\varepsilon_{rg} \\ 0 & \varepsilon_{r2} & 0 \\ -i\varepsilon_{rg} & 0 & \varepsilon_{r1} \end{pmatrix} \quad (1)$$

with the constitutive parameters defined by $\varepsilon_{r1} = 1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{le}^2}$, $\varepsilon_{r2} = 1 - \left(\frac{\omega_{ep}}{\omega}\right)^2$ and $\varepsilon_{rg} = -\frac{\omega_{le}\omega_{ep}^2}{\omega(\omega^2 - \omega_{le}^2)}$. $\omega_{ep} = \left(\frac{ne^2}{m\varepsilon_0}\right)^{\frac{1}{2}}$ is the plasma frequency, which is related to the electronic charge, the electronic mass, the electron density, and the free-space permittivity. $\omega_{le} = \frac{eB}{m}$ is the gyrofrequency, which is related to the electronic charge, the electronic mass, and the external static magnetic field. In addition, it is important to note that the direction of the static magnetic field is along the $+y$ direction in layer B, and the sign of ω_{le} is positive, while the direction of the static magnetic field is along the $-y$ direction in layer D, and the sign of ω_{le} is negative.

To investigate the nonreciprocal properties, an incident TM mode plane wave in the xz plane (Fig. 1) is assumed. The plane wave can be expressed as $H_y(x, z) = H_{y0}e^{i(k_x x + k_z z)}$, where k_x and k_z represent the component of wave vector along the x axis and z axis, respectively. Based on the transfer matrix method [16, 20], the fields in a layer can be written as follows:

$$\begin{aligned} H_y(x, z) &= H_{y0}^+ \exp[i(k_x x + k_z z)] \\ &\quad + H_{y0}^- \exp[i(k_x x - k_z z)] \\ &= H_y^+ + H_y^- \end{aligned} \quad (2)$$

Based on the classical Maxwell equations, the EM fields are continuous at the interface of two adjacent layers (m-layer and n-layer) and can be written as follows:

$$\begin{bmatrix} H_{my}^+ \\ H_{my}^- \end{bmatrix} = T_{mn} \begin{bmatrix} H_{ny}^+ \\ H_{ny}^- \end{bmatrix} \quad (3)$$

where T_{mn} is the transformation matrix of the fields from the m-layer to n-layer, which is defined as

$$T_{mn} = \frac{1}{2a_m} \begin{bmatrix} a_m - b_m + a_n + b_n & a_m - b_m - a_n + b_n \\ a_m + b_m - a_n - b_n & a_m + b_m + a_n - b_n \end{bmatrix} \quad (4)$$

In the transformation matrix, $a_{m(n)} = \frac{-i\varepsilon_{rg}k_x}{\varepsilon'}$ and $b_{m(n)} = \frac{\varepsilon_{r2}k_{m(n)z}}{\varepsilon'}$, here $\varepsilon' = \varepsilon_{r2}^2 - \varepsilon_{r1}\varepsilon_{r2}$, $k_z(m)$, $k_z(n)$ correspond to the wave vectors in the z direction of the m-layer and n-layer, which are defined as $k_{m(n)z} = \sqrt{\left(\frac{\omega}{c}\right)^2 \mu_{rm(n)} \left(\varepsilon_{r1m(n)} - \frac{\varepsilon_{rgm(n)}^2}{\varepsilon_{r1m(n)}}\right) - k_x^2}$, and c is the speed of light in vacuum. In the model, layers A and C are isotropic dielectrics, and there are no off-diagonal elements in their permittivity. Therefore, the EM parameters can be simplified to $\varepsilon_1 = \varepsilon_2$ and $\varepsilon_g = 0$.

Then, the full transfer matrix is given by

$$T = T_{0m}(P_m T_{mn})^N (P_{m'} T_{m'n'})^M T_{n'0} \quad (5)$$

where N and M represent the periods of the right part and the left part, respectively. P_m and $P_{m'}$ represent the phase-shift factors in one layer from the front boundary to the rear boundary, which is defined as follows:

$$P_m = \begin{vmatrix} e^{-ik_{mz}d_m} & 0 \\ 0 & e^{ik_{mz}d_m} \end{vmatrix} \quad (6)$$

where d_m is thickness of m th layer. In matrix (5), T_{0m} and $T_{n'0}$ characterise the transfer matrices for transmission through air to medium m and through medium n' to air, respectively. Thus, the transmission and reflection of the electric fields can be determined as follows (5):

$$\begin{bmatrix} H_{\text{in}}^+ \\ H_{\text{in}}^- \end{bmatrix} = T \begin{bmatrix} H_{\text{out}}^+ \\ H_{\text{out}}^- \end{bmatrix} \quad (7)$$

According to the transfer matrix method, the total transmission coefficient can be calculated as follows:

$$t = \frac{H_{\text{out}}^+}{H_{\text{in}}^+} = \frac{1}{|T(1, 1)|} \quad (8)$$

3 Results and Analysis

In order to demonstrate the nonreciprocity in the designed structure, the transmission properties were numerically calculated for an incident TM mode plane wave traveling from the left boundary of the structure. The structure parameters used in the calculations are as follows: $M = 2$, $N = 3$, $d_1 = 6.5$ mm, $d_2 = 5$ mm, $d_3 = 20$ mm, $d_4 = 22$ mm, and an incidence angle of $\theta = \pi/6$. Layers A and C are normal materials and assume $\varepsilon_{rA} = 2.56$ and $\varepsilon_{rC} = 11.5$ [24], respectively. For the anisotropy layers B and D , the permittivities depend on (1). In this case, B and D are a kind of dispersive electric–gyrotropic medium and assume $n = 8 \times 10^{17} \text{ m}^{-3}$, $\omega_{ep} = 5.040 \times 10^{10} \text{ s}^{-1}$, and $\omega_{le} = \pm 1.757 \times 10^{11} \text{ Bs}^{-1}$ [23, 25, 26]. The value of the external magnetic field was initially set as $B = 0.02T$. Therefore, according to formula (1), the dispersive and gyrotropic properties of electric–gyrotropic medium are significant in the microwave regime.

Figure 2 shows the transmission coefficients for different incident frequencies, where the solid and dotted lines represent forward and backward waves, respectively.

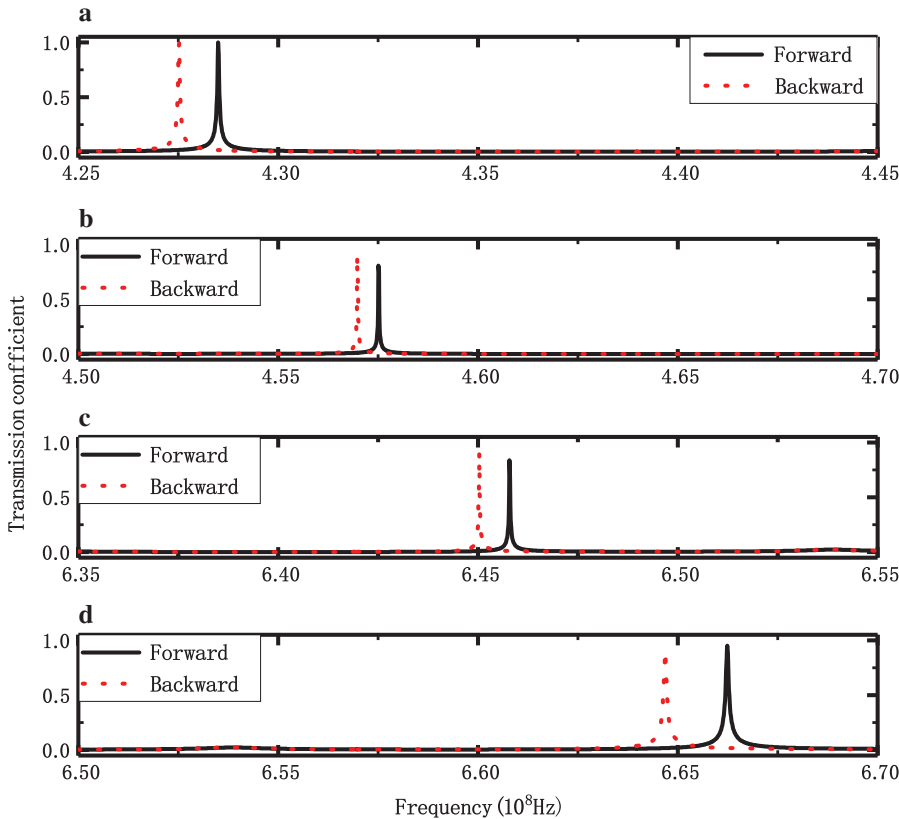


Figure 2: Transmittance of the structure when $N = 3$ and $M = 2$. The solid line represents the forward direction, and the dotted line represents the backward direction. (a) Backward 4.2751×10^8 Hz, forward 4.2850×10^8 Hz; (b) backward 4.5698×10^8 Hz, forward 4.5751×10^8 Hz; (c) backward 6.450×10^8 Hz, forward 6.4579×10^8 Hz; (d) backward 6.6468×10^8 Hz, forward 6.6624×10^8 Hz.

It can be seen that there are four pairs of different nonreciprocal transmission peaks in the range from 4.25×10^8 to 6.7×10^8 Hz at a specific incident angle. It should also be noted that the results are different from the pair of nonreciprocal transmission peaks when fixed EM parameters are used for the nondispersive gyrotropic media [12, 19]. It is more instructive to design multichannel isolators for a specific incident angle in practice. Furthermore, the nonreciprocal frequency intervals and transmission peaks in Figure 2a are larger than those of Figure 2b–d. As such, the forward frequency of 4.2850×10^8 Hz and the backward frequency of 4.2751×10^8 Hz are the desired nonreciprocal frequencies for the proposed structure. Thus, the first pair of peaks was chosen for further investigation.

To further verify the nonreciprocal properties of Figure 2a, the magnetic field distributions are described in Figure 3 for 4.2850×10^8 and 4.2751×10^8 Hz, respectively. When the wave at 4.2850×10^8 Hz is incident

from the left boundary, Figure 3a indicates that the wave propagates completely through the multilayers, which is in accordance with the results shown in Figure 2a. However, the incident wave at the same frequency travelling from the right boundary is totally reflected and forms a standing wave in the incident region (Fig. 3b). Similar results are shown for a frequency of 4.2751×10^8 Hz in Figure 3c and d, where the incident magnetic field from the right boundary is totally transmitted through the multilayers (Fig. 3b). Moreover, the incident magnetic field from the left boundary is totally reflected to form a standing wave in the incident region (Fig. 3d).

The nonreciprocal transmission depends on the nonreciprocal OTSs in the interface, which are influenced by the thickness of the gyrotropic media. The transmission coefficients in the range 4.25×10^8 to 4.45×10^8 Hz for the different thickness values of $d_1 = 6, 6.5, 7$, and 7.5 mm are shown in Figure 4. In this case, the other conditions are

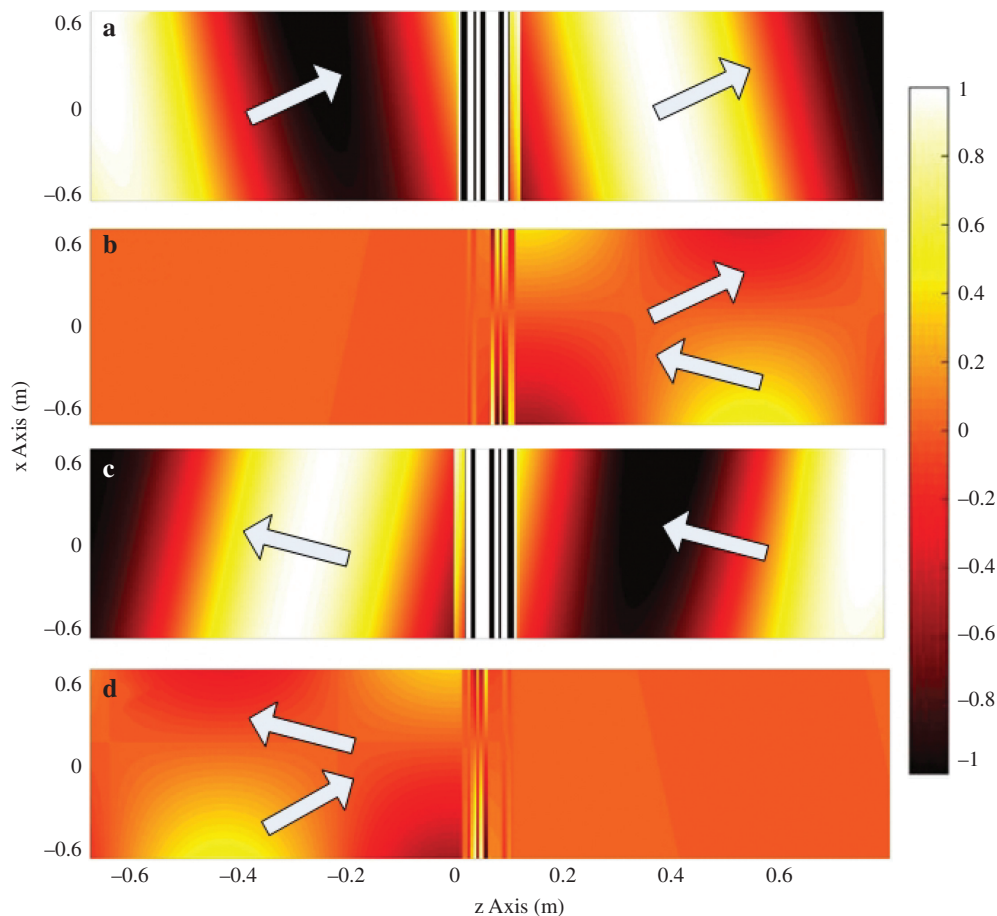


Figure 3: The distributions of magnetic fields H_y along xz plane. Same magnetic field values ($B = 0.02T$) are obtained for two opposite directions for the finite structure. (a) For front illumination, the frequency is 4.2850×10^8 Hz, and the incidence angle is $\pi/6$. (b) For back illumination, the frequency is 4.2850×10^8 Hz, and the incidence angle is $\pi/6$. (c) For back illumination, the frequency is 4.275×10^8 Hz, and the incidence angle is $\pi/6$. (d) For front illumination, the frequency is 4.275×10^8 Hz, and the incidence angle is $\pi/6$.

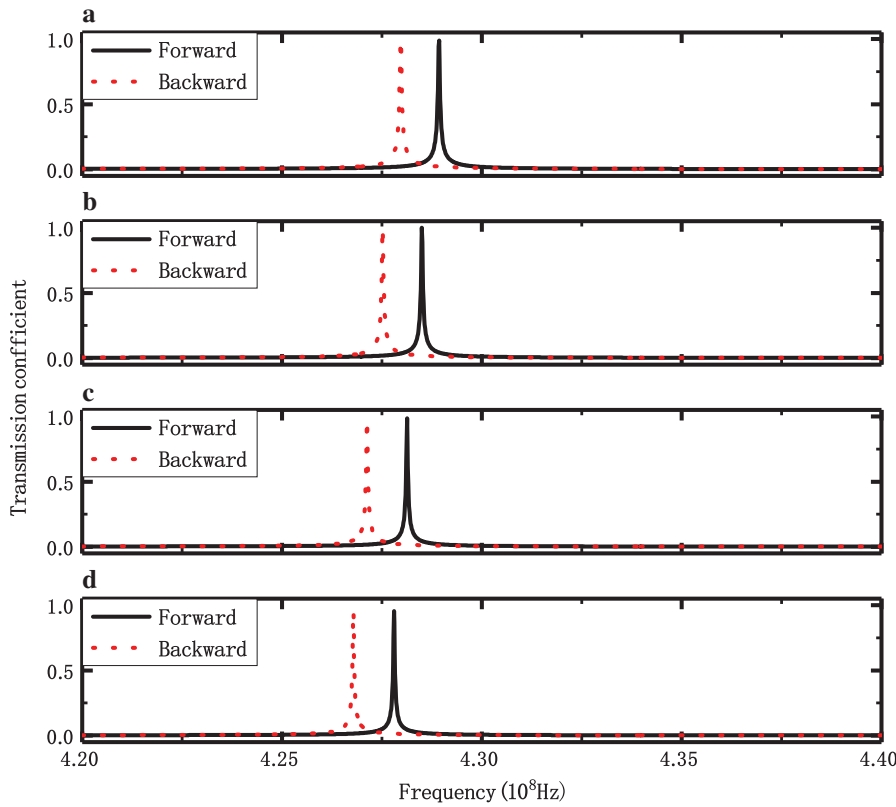


Figure 4: The relation between the transmission peaks and thickness d_1 : (a) $d_1 = 6 \text{ mm}$, (b) $d_1 = 6.5 \text{ mm}$, (c) $d_1 = 7 \text{ mm}$, (d) $d_1 = 7.5 \text{ mm}$.

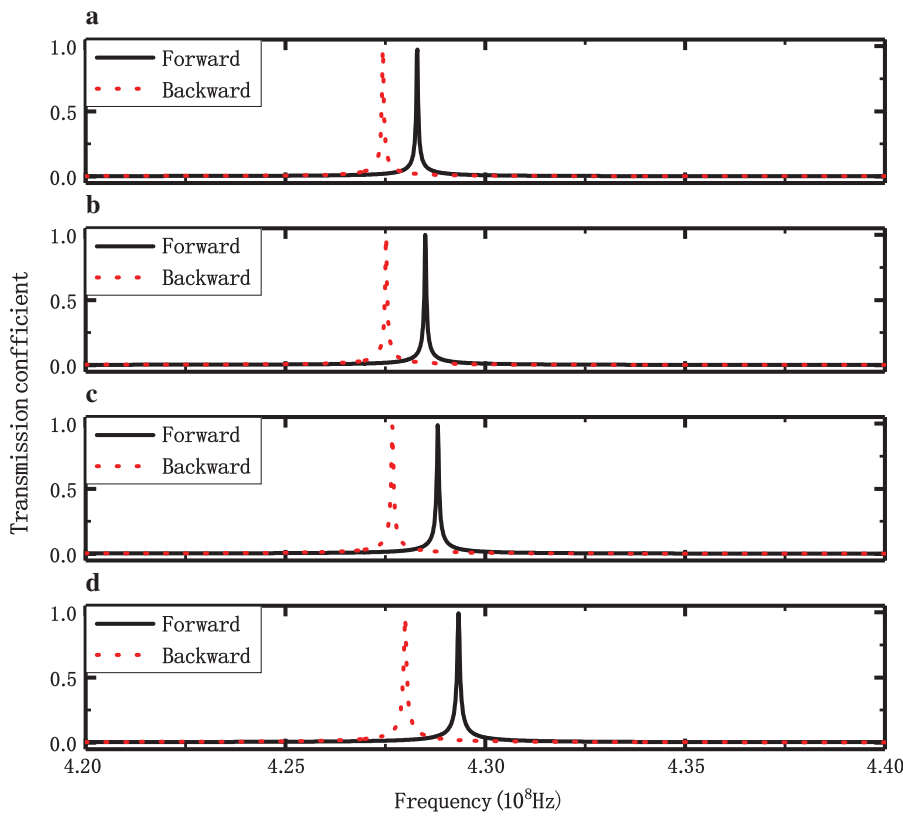


Figure 5: The relation between the transmission peaks and the incidence angle θ : (a) $\theta = \pi/7$, (b) $\theta = \pi/6$, (c) $\theta = \pi/5$, (d) $\theta = \pi/4$.

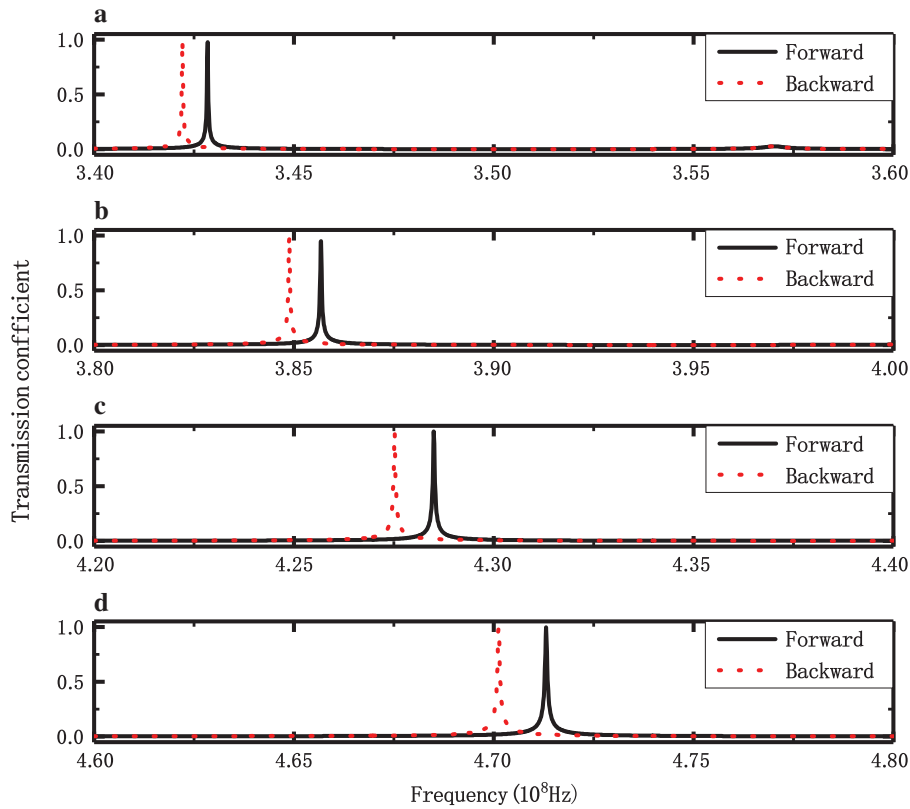


Figure 6: The relation between the transmission peaks and the external magnetic value: (a) $B = 0.016T$, (b) $B = 0.018T$, (c) $B = 0.02T$, (d) $B = 0.022T$.

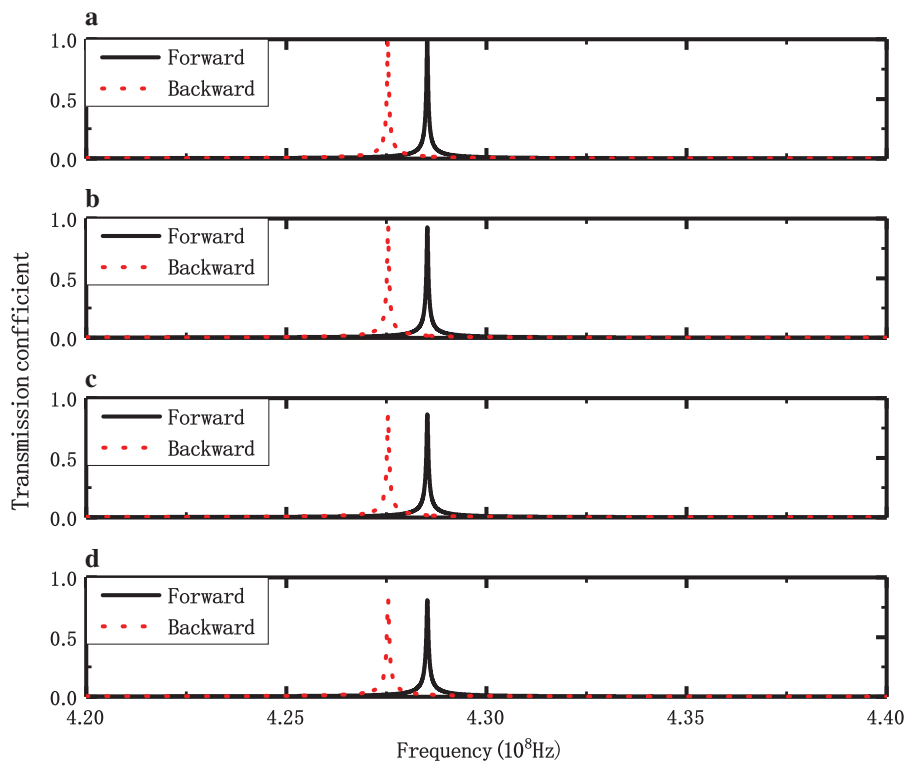


Figure 7: The relation between the nonreciprocal transmission peaks and the loss in the magneto-optical layers, where the other conditions are the same as those in Figure 2a. (a) $\gamma = 0.0$, (b) $\gamma = 0.04$, (c) $\gamma = 0.08$, (d) $\gamma = 0.12$.

the same as those of Figure 2a. It is evident that the nonreciprocal transmission peaks for the forward and backward waves are red-shifted with an increase in the thickness d_1 . In addition, the separation between the two peaks is stable when the thickness d_1 increases.

The angle of incidence also affects the nonreciprocal properties. Figure 5 shows the transmission spectra for different incident angles $\theta = \pi/7, \pi/6, \pi/5$, and $\pi/4$, where the other conditions are the same as those of Figure 2a. In Figure 5, it is evident that the frequencies of the nonreciprocal transmission peaks are blue-shifted with an increase in the incidence angle. In addition, the separation between the backward and the forward peaks increases when there is an increase in the angle of incidence, and the nonreciprocity is more pronounced.

The asymmetric gyrotropic EM parameters in the layers are a basic condition for nonreciprocity of EM waves, and the external static magnetic field can change and enhance the asymmetry. Hence, the value of the magnetic field is one of the key factors that influence the properties of nonreciprocal propagation. Figure 6 shows the transmission spectra for external magnetic fields $B = 0.016T, 0.018T, 0.02T$, and $0.022T$. The other parameters are the same as those used for the results shown in Figure 2a. As the strength of the external magnetic field is increased, the transmission peaks are blue-shifted, and the separation between the two peaks increases. These results indicate that an increase in the strength of the external magnetic fields can enhance and modulate the properties of the nonreciprocal propagating.

In the electric–gyrotropic medium, the dissipation is an intrinsic property due to the collisions of electrons plasma in motion, and the loss effect on the nonreciprocity of EM waves in the electric–gyrotropic layer should be taken into account. Although the loss is relative to the electronic collision frequency, for simplicity, the same fixed loss of the permittivity of the main diagonal element in the electric–gyrotropic medium is considered for the B and D electric–gyrotropic layers. We assume $\varepsilon'_{r1(2)} = \varepsilon_{r1(2)} + i\gamma$, and γ represents the dissipation. Figure 7 indicates the transmission spectra for the cases of different values of the loss, where the other conditions are the same as those in Figure 2a. Figure 7 shows that the nonreciprocal transmission peaks appear decreased when the material loss is introduced from $\gamma = 0$ to $\gamma = 0.12$, and the significant absorption can be observed in the structure. Although the nonreciprocal transmission peaks are suppressed in the case of larger loss, the nonreciprocal properties remain the same as in the nondissipative case, which is due to fewer layers in the designed structure.

4 Conclusions

In this investigation, a nonreciprocal structure $(AB)^N(DC)^M$ consisting of the electric–gyrotropic dispersive media was designed, and the nonreciprocal properties were investigated. For a specific incident angle, four pairs of different nonreciprocal transmission peaks were observed, which is different from those of a one-dimensional nonreciprocal structure with fixed EM parameters. The theoretical and numerical results show that the nonreciprocal transmission peaks are red-shifted with an increase in the thickness of the electric–gyrotropic dispersive media and blue-shifted with an increase in the angle of incidence and the external magnetic fields. Additionally, the increases in the incident angle and the external magnetic field also enhance the nonreciprocal transmission properties. A tunable and multichannel nonreciprocal structure can be exploited to develop novel nonlinear EM devices such as multichannel EM isolators, EM diode, and so on.

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