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Effect of Dust Ion Collision on Dust Ion Acoustic Solitary Waves for Nonextensive Plasmas in the Framework of Damped Korteweg–de Vries–Burgers Equation

https://doi.org/10.1515/zna-2018-0519
Received November 25, 2018; accepted April 23, 2019; previously published online May 22, 2019

Abstract: Analytical solitary wave solution of the dust ion acoustic waves (DIAWs) is studied in the framework of the damped Korteweg–de Vries–Burgers (DKdVB) equation in an unmagnetised collisional dusty plasma consisting of negatively charged dust grain, positively charged ions, q-nonextensive electrons, and neutral particles. Using Reductive Perturbation Technique, the DKdVB equation is obtained for DIAWs. The effects of different physical parameters such as dust ion collision frequency parameter (ν_{id0}), viscosity coefficient (η_{10}), the entropic index (q), the speed of the travelling wave (M_0), and the ratio between the unperturbed densities of the electrons and ions (μ) on the analytical solution of DIAWs are observed. The results of the present article may have applications in laboratory and space plasmas.

Keywords: Damped Korteweg–de Vries–Burgers Equation; Dust Ion Acoustic Waves; Dusty Plasmas; Reductive Perturbation Technique; Solitary Wave.

1 Introduction

From the last three decades, research on dusty plasma is one of the most rapidly growing fields in plasma physics. Nonlinear phenomena like solitons, shock waves, and vortices in dusty plasma have been studied theoretically and experimentally by several researchers [1–9]. To study nonlinear waves in dusty plasma, several authors have derived Korteweg–de Vries–Burgers (KdVB) equation by Reduction Perturbation Technique (RPT). Sukla and Mamun [10] derived KdVB equation by RPT, and they studied the properties of solitons and shock waves in strongly coupled unmagnetised dusty plasmas. Solitons and shock waves would be the fundamental nonlinear coherent structures in the dusty plasma, and that was studied by Xie et al. [11] and Verheest [12]. In the dusty plasma, it has been observed that if dissipation is weak, then the balance between nonlinear and dispersion effects can form the symmetrical solitary waves, and the strong dissipation effects produce shocks. Dust acoustic solitary structures in the dusty plasma also have been investigated widely by many researchers [13–15]. Rao et al. [16] observed the shock waves in coupled dusty plasma with Boltzman distribution of ions. There are two types of system depending on the range of the interparticle forces such as extensive system and nonextensive system. The extensive system holds for the system with short-range interparticle forces, and the nonextensive property holds for the system in which long-range interparticle forces (Newtonian gravitational forces and Coulomb electric forces) are present. Roy et al. [17] studied on ion-acoustic shocks in quantum electron–positron–ion plasmas and investigated the nonlinear propagation of quantum ion-acoustic waves in three-component quantum electron–positron–ion plasma. They explained the existence of shock waves and transition of oscillatory to monotonic shocks when either equilibrium electron-to-ion density ratio or kinetic viscosity coefficient exceeds its critical value. Misra et al. [18] showed derivation of KdVB equation with RPT and solved numerically and showed that the perturbations with negative potential may propagate as solitary waves and shocks in plasma with positively charged dusts, whereas solitary waves and shocks with both positive and negative potential may exist when dusts are negatively charged. Das et al. [19] studied the effect of dust ion collision on the dust ion acoustic waves (DIAWs) in the framework of damped Zakharov–Kuznetsov equation. They have shown that the system exhibits quasi-periodic behaviour in absence of dust ion collision and the system became chaotic and when the dust ion collision is taken into consideration. Dev and Deka [20] investigated nonlinear propagation of dust ion acoustic shock waves in dusty pair ion plasma with dust charge fluctuation due to nonthermal positive and negative species in the framework of modified complex Burger equation with complex

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nonlinear coefficient. Roy et al. [21] studied the effect of ion temperature on ion-acoustic solitary waves in plasma with a q-nonextensive electron velocity distribution. They have shown the existence of compressive solitary waves in two-component plasma using Sagdeep pseudo potential approach. Until today, very few works have been done to study the effect of dust ion collision on DIAWs. However, no work has been reported to show the effects of dust ion collision on DIAWs in the framework of damped Korteweg–de Vries–Burgers (DKdVB) equation. Moreover, until today, no work on analytical solitary wave solution of DKdVB equation is reported.

In this present article, our aim is to derive the analytical solitary wave solution of the DKdVB equation for small values of the coefficients of Burger term and damping term in the dusty plasma for q-nonextensive electrons distribution and to find the effects of the different values of the special parameters such as μ, q, M_0, ν_{id0}, η_{10}, and τ on the amplitude and the width of the solitary waves.

The remaining part of the article is composed as follows: In Section 2, we have considered the basic equations. Nonlinear analyses are discussed in Section 3. In Section 4, we have obtained numerical simulation for different values of the parameters and present discussion. The conclusions are presented in the Section 5.

2 Basic Equations

In this work, an unmagnetised collisional dusty plasma has been considered that contains cold inertial ions, stationary dusts with negative charge, and q-nonextensive electrons. The normalised ion fluid equations that include the equation of continuity, equation of momentum balance, and Poisson equation, governing the DIAWs, are given by

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \phi}{\partial x}+\eta\frac{\partial^2 u}{\partial x^2}-\nu_{id}u, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=(1-\mu)n_e-n+\mu, \tag{3}$$

$$n_e=n_{e0}\{1+(q-1)\phi\}^{\frac{q+1}{2(q-1)}} \tag{4}$$

where n is the number density of ions normalised to its equilibrium value n_0, u is the ion fluid velocity normalised to ion acoustic speed $C_s=\sqrt{\left(\frac{k_BT_e}{m_i}\right)}$, with T_e as electron temperature, k_B as Boltzmann constant, and m_i as mass of ions. The electrostatic wave potential ϕ is normalised to $\frac{k_BT_e}{e}$, with e as magnitude of electron charge. The space variable x is normalised to the Debye length $\lambda_D=\left(\frac{T_e}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, and time t is normalised to $\omega_{pi}^{-1}=\left(\frac{m_i}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, with ω_{pi} as ion–plasma frequency. Here, ν_{id} is the dust ion collisional frequency and $\mu=\frac{n_{0e}}{n_{0i}}$, where n_{0e} and n_{0i} are the unperturbed number densities of electrons and ions, respectively.

In order to describe q-nonextensive electron, we have considered the following distribution function as [22]:

$$f_e(v)=C_q\left\{1+(q-1)\left[\frac{m_ev^2}{2k_BT_e}-\frac{e\phi}{k_BT_e}\right]\right\}^{\frac{1}{q-1}},$$

where ϕ is the electrostatic potential, and other variables or parameters have their usual meaning. It is highly significant to take that $f_e(v)$ as it is the prominent distribution that maximises the Tsallis entropy and keeps the law of thermodynamics. Then, the constant of normalisation is given by

$$C_q=n_{e0}\frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q}-\frac{1}{2}\right)}\sqrt{\frac{m_e(1-q)}{2\pi k_BT_e}}\quad \text{for } -1<q<1$$

and

$$C_q=n_{e0}\frac{1+q}{2}\frac{\Gamma(\frac{1}{1-q}+\frac{1}{2})}{\Gamma(\frac{1}{1-q})}\sqrt{\frac{m_e(1-q)}{2\pi k_BT_e}}\quad \text{for } q>1$$

Integrating the distribution function $f_e(v)$ over the velocity space, one can obtain the q-nonextensive electron number density as

$$n_e=n_{e0}\left\{1+(q-1)\frac{e\phi}{k_BT_e}\right\}^{\frac{q+1}{2(q-1)}}$$

Thus, the normalised q-nonextensive electron number density [22] takes the form:

$$n_e=n_{e0}\{1+(q-1)\phi\}^{\frac{q+1}{2(q-1)}} \tag{5}$$

3 Nonlinear Analysis

The RPT as [23] is used to derive the DKdVB equation in unmagnetised collisional dusty plasma to study the nonlinear wave propagation of DIAWs. The independent variables are stretched as [24]

$$\begin{cases}\xi=\varepsilon^{1/2}(x-vt)\\ \tau=\varepsilon^{3/2}t\end{cases} \tag{6}$$

where ε is the strength of nonlinearity, and v is the phase velocity of the DIAWs to be determined from the lowest order of ε. The expansions of the dependent variables $n, u, \phi, \eta, \nu_{id}$ are as follows:

$$\begin{cases} n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \cdots, \\ u = 0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots, \\ \phi = 0 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots, \\ \eta = \varepsilon^{1/2} \eta_0, \\ \nu_{id} \sim \varepsilon^{3/2} \nu_{id0}. \end{cases} \tag{7}$$

Substituting the above expansions (7) along with stretching coordinates (6) into (1) to (3) and equating the coefficients of the lowest order of ε, the dispersion relation is obtained as

$$v = \frac{1}{\sqrt{a(1-\mu)}}, \tag{8}$$

with $a = \frac{q+1}{2}$.

Taking the coefficients of the next higher order of ε, we obtain the DKdVB equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + C\frac{\partial^2 \phi_1}{\partial \xi^2} + D\phi_1 = 0, \tag{9}$$

where $A = \left(\frac{3}{2v} - v^3(1-\mu)b\right)$, $B = \frac{v^3}{2}$, $C = -\frac{\eta_{10}}{2}$, and $D = \frac{\nu_{id0}}{2}$ with $b = \frac{(q+1)(3-q)}{8}$.

In absence of C and D, i.e. for $C = 0$ and $D = 0$, (9) takes the form of well-known KdV equation with the solitary wave solution

$$\phi_1 = \phi_m \mathrm{sech}^2\left(\frac{\xi - M_0\tau}{W}\right), \tag{10}$$

where amplitude of the solitary waves $\phi_m = \frac{3M_0}{A}$ and width of the solitary waves $W = 2\sqrt{\frac{B}{M_0}}$, with M_0 is the speed of the ion-acoustic solitary waves or Mach number.

It is well established for the KdV equation that

$$I = \int_{-\infty}^{\infty} \phi_1^2 \mathrm{d}\xi \tag{11}$$

is a conserved quantity [25].

For small values of C and D, let us assume that amplitude, width, and velocity of the ion acoustic waves are dependent on τ [26–29], and the slow time-dependent solution of (9) is of the form

$$\phi_1 = \phi_m(\tau)\mathrm{sech}^2\left(\frac{\xi - M(\tau)\tau}{W(\tau)}\right), \tag{12}$$

where the amplitude $\phi_m(\tau) = \frac{3M(\tau)}{A}$, width $W(\tau) = 2\sqrt{B/M(\tau)}$, and velocity $M(\tau)$ have to be determined.

Differentiating (11) with respect to τ and using (9), one can obtain

$$\frac{\mathrm{d}I}{\mathrm{d}\tau} + 2DI = 2C\int_{-\infty}^{\infty}\left(\frac{\partial \phi_1}{\partial \xi}\right)^2 \mathrm{d}\xi, \tag{13}$$

$$\Rightarrow \frac{\mathrm{d}I}{\mathrm{d}\tau} + 2DI = 2C \times \frac{24}{5}\frac{M^{5/2}(\tau)}{A^2\sqrt{B}}. \tag{14}$$

where

$$\int_{-\infty}^{\infty}\left(\frac{\partial \phi_1}{\partial \xi}\right)^2 \mathrm{d}\xi = \frac{24}{5}\frac{M^{5/2}(\tau)}{A^2\sqrt{B}} \tag{15}$$

and

$$I = \int_{-\infty}^{\infty} \phi_1^2 \mathrm{d}\xi,$$

$$I = \int_{-\infty}^{\infty} {\phi_m}^2(\tau)\mathrm{sech}^4\left(\frac{\xi - M(\tau)\tau}{W(\tau)}\right)\mathrm{d}\xi,$$

$$I = \frac{24\sqrt{B}}{A^2}M^{3/2}(\tau). \tag{16}$$

Substituting (16) and (15) into (14), we obtain

$$\frac{\mathrm{d}M(\tau)}{\mathrm{d}\tau} + PM(\tau) = QM^2(\tau), \tag{17}$$

which is the Bernoulli's equation, where $P = \frac{4}{3}D$ and $Q = \frac{4}{15}\frac{C}{B}$.

The solution of (17) is

$$M(\tau) = \frac{PM_0}{M_0 Q(1 - e^{P\tau}) + Pe^{P\tau}}$$

Therefore, the slow time dependence form of the ion-acoustic solitary wave solution of the DKdVB (9) is given by (12) where $M(\tau) = \frac{PM_0}{M_0 Q(1-e^{P\tau}) + Pe^{P\tau}}$ and $M(0) = M_0$ for $\tau = 0$.

4 Effects of Parameters

The effects of the parameters, i.e. ion collision frequency parameter (ν_{id0}), the entropic index (q), time (τ), ratio of unperturbed number densities of electrons and ions (μ), viscosity coefficient (η_{10}), and Mach number (M_0) on the solitary wave solution of the DKdVB (9), have been studied in this section.

In Figure 1, the soliton solution of the DKdVB equation is plotted from (12) for different values of the dust ion

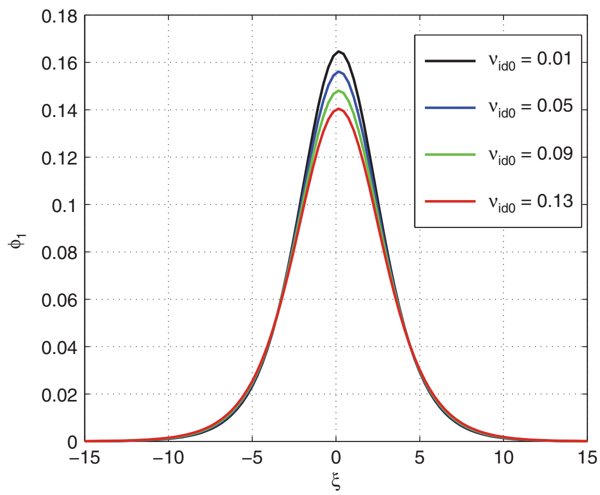

Figure 1: Variation of solitary wave from (12) for the different values of ν_{id0} with $M_0 = 0.1$, $q = 0.6$, $\tau = 2$, $\eta_{10} = 0.1$, and $\mu = 0.5$.

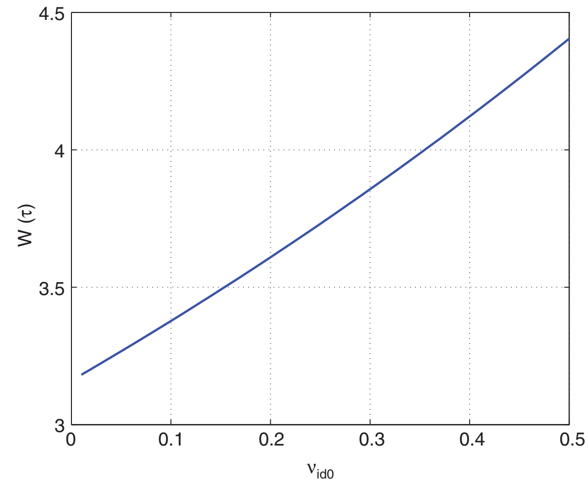

Figure 3: Variation of width of the solitary wave from (12) with respect to ν_{id0} and all other parameters are the same as Figure 1.

collision frequency parameter (ν_{id0}). The values of other parameters are $\mu = 0.5$, $q = 0.6$, $\tau = 2$, $\eta_{10} = 0.1$, $M_0 = 0.1$. It is observed that the solution produces solitary waves, and the amplitude of the solitary waves decreases as the value of the parameter ν_{id0} increases and the width of the solitary waves increases for increasing value of ν_{id0}. To show the variation of the amplitude and width of the solitary wave with respect to the dust ion collision frequency, Figures 2 and 3 are plotted.

In Figure 2, the amplitude of the solitary wave is plotted with respect to the collision frequency parameter (ν_{id0}), and it is obvious that the amplitude of the solitary wave decreases as the dust ion collision frequency parameter (ν_{id0}) increases gradually for positive nonzero values, and the other parameters remain the same as those in Figure 1, and for $\nu_{id0} = 0$, the solitary wave solution does not exist.

In Figure 3, the width of the solitary wave is plotted, and it is obvious that the width of the solitary wave increases as the dust ion collision frequency parameter (ν_{id0}) increases gradually for positive nonzero values and the other parameters remain the same as those values in Figure 1, and for $\nu_{id0} = 0$, the solitary wave does not exist. That is why the figure does not show any amplitude and width of the solitary wave in case of $\nu_{id0} = 0$. In absence of dust ion collision, (9) reduces to planar KdVB equation, and its solution does not produce solitary wave.

In Figure 4, the solution of the DKdVB equation is plotted from (12) for different values of the nonextensive parameter (q). The other parameters are $\mu = 0.5$,

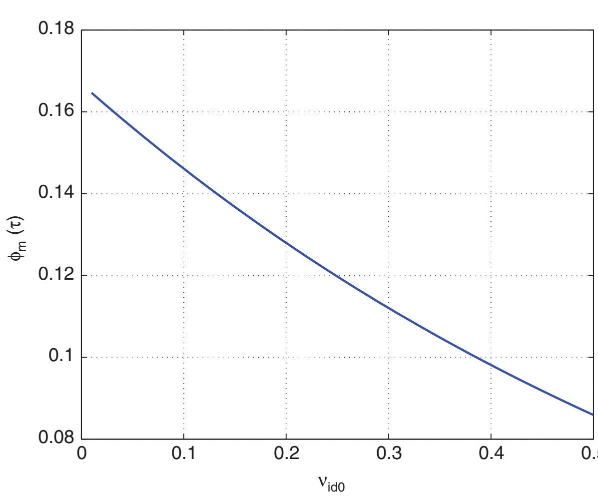

Figure 2: Variation of the amplitude of the solitary wave from (12) with respect to ν_{id0} and all other parameters are the same as Figure 1.

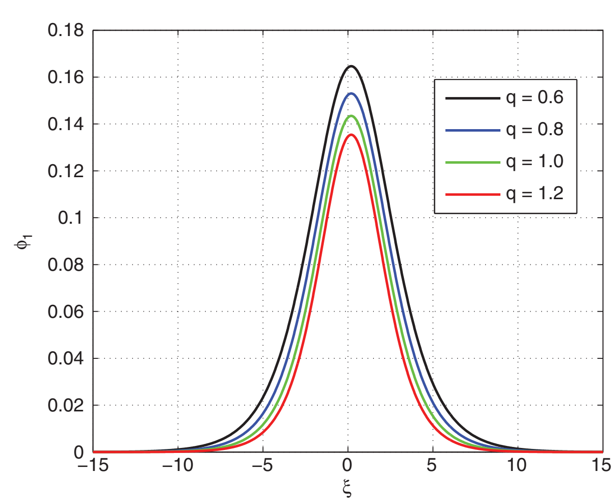

Figure 4: Variation of solitary wave from (12) for the different values of q with $M_0 = 0.1$, $\tau = 2$, $\eta_{10} = 0.1$, $\nu_{id0} = 0.01$, and $\mu = 0.5$.

$\tau = 2, \eta_{10} = 0.1, M_0 = 0.1$ and $\nu_{id0} = .01$. The figure shows that the amplitude and width of the solitary waves decrease as the value of the nonextensive parameter q increases for fixed dust ion collision and other parameters.

In Figure 5, the solution is plotted from (12) for different values of the parameter μ, where $\mu = \frac{n_{0e}}{n_{0i}}$, i.e. ratio of unperturbed number densities of electrons and ions, and the other parameters are $q = 0.6, \tau = 2, \eta_{10} = 0.1, M_0 = 0.1, \nu_{id0} = 0.01$. It is observed that both the amplitude and width of the solitary waves decrease as the value of the ratio of the parameter number of unperturbed electrons and ions increases for fixed dust ion collision.

In Figure 6, the solitary wave solution is depicted from (12) for different values of the viscosity coefficient parameter of plasma (η_{10}). The other parameters are $\mu = 0.5, q = 0.6, \tau = 2, \nu_{id0} = 0.01, M_0 = 0.1$. The viscosity coefficient (η_{10}) plays an important role on the dust ion-acoustic solitary waves (DIASWs). The variation of the amplitude and width of the solitary waves with respect to (η_{10}) is obvious from the next two consecutive figures.

Figure 7 represents the variation of the amplitude of the solitary wave solution of DKdVB equation with respect to the coefficient of viscosity η_{10}. It is seen that the amplitude of the solitary waves decreases monotonically if the values of the viscosity coefficient η_{10} increase gradually.

Figure 8 shows the variation of the width of the solitary wave solution of DKdVB equation with respect to the

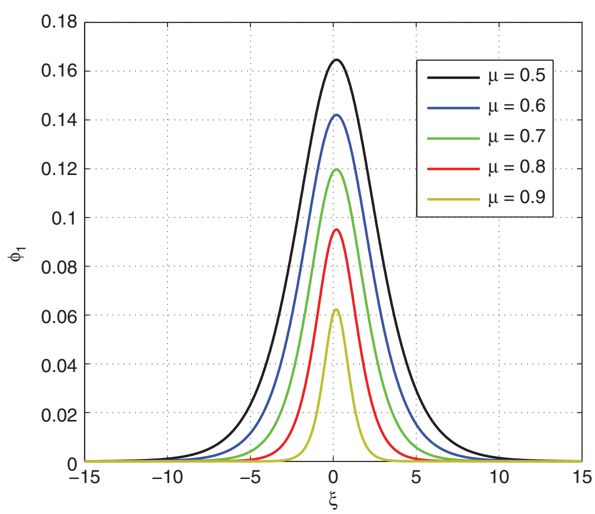

Figure 5: Variation of solitary wave from (12) for the different values of μ with $q = 0.6$, $M_0 = 0.1$, $\tau = 2$, $\eta_{10} = 0.1$, and $\nu_{id0} = 0.01$.

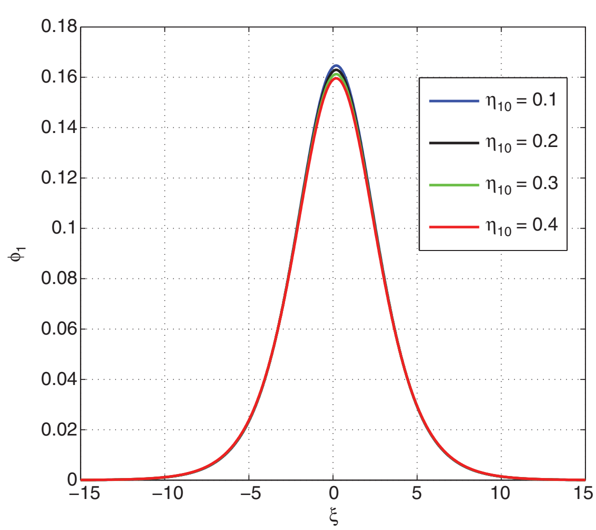

Figure 6: Variation of solitary wave from (12) for the different values of η_{10} with $q = 0.6$, $\tau = 2$, $M_0 = 0.1$, $\nu_{id0} = 0.01$, and $\mu = 0.5$.

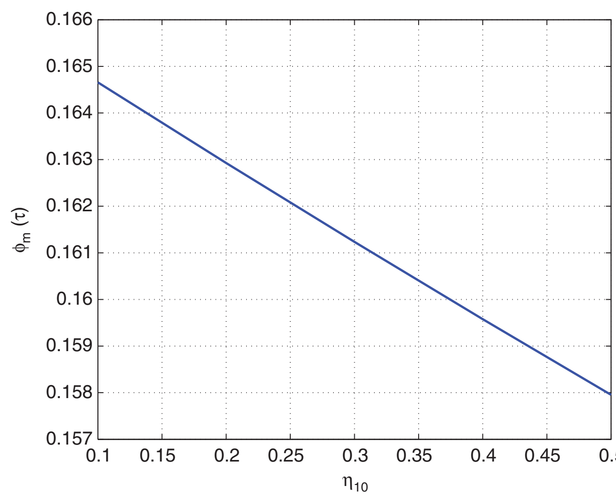

Figure 7: Variation of the amplitude of the solitary wave from (12) with respect to η_{10} for $q = 0.6$, $\tau = 2$, M=0.1, $\nu_{id0} = 0.01$, and $\mu = 0.5$.

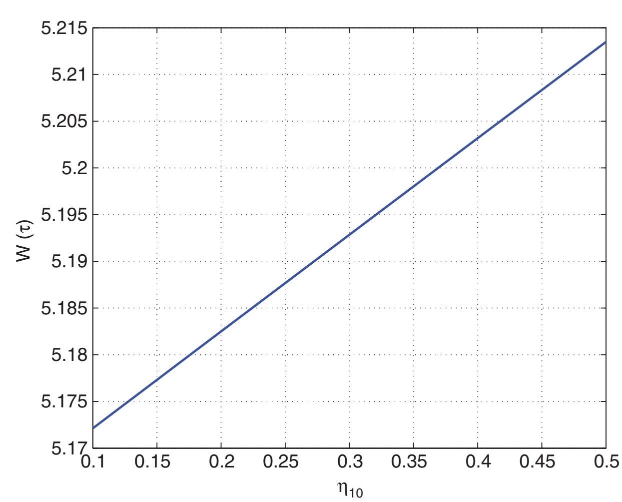

Figure 8: Variation of width of the solitary wave from (12) with respect to η_{10} for $q = 0.6$, $\tau = 2$, $M_0 = 0.1$, $\nu_{id0} = 0.01$, and $\mu = 0.5$.

viscosity coefficient η_{10}. It is obvious that the width of the solitary waves increases monotonically if the value of the viscosity coefficient η_{10} increases gradually.

In Figure 9, the solitary wave solution from (12) is plotted for different values of the time parameter τ, and the other parameters are $\mu = 0.5$, $q = 0.6$, $\eta_{10} = 0.1$, $\nu_{id0} = 0.01$, $M_0 = 0.1$. It is seen that the amplitude and width of the solitary waves change as the time (τ) is changed. The variation of the amplitude and width of the solitary waves with respect to the time (τ) is shown in the next two figures.

In Figure 10, the variation of the amplitude of the solitary wave solution of DKdVB equation with respect to time (τ) has been depicted. It is seen that the amplitude of the solitary waves decreases as the time τ increases gradually when dust ion collision and other parameters are fixed.

In Figure 11, it is obvious that the width of the solitary waves increases as the time τ increases gradually.

It is clear from Figure 12 that the amplitude of the solitary wave increases and the width of the solitary wave decreases as the value of the Mach number increases.

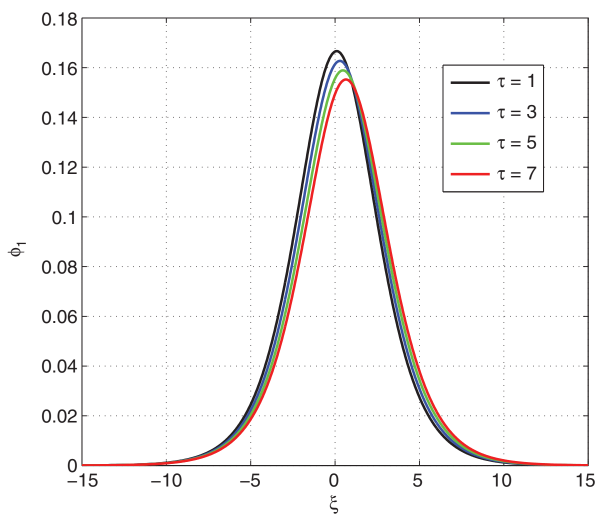

Figure 9: Variation of solitary wave from (12) for the different values of τ with $q = 0.6$, $\eta_{10} = 0.1$, $M_0 = 0.1$, $\nu_{id0} = 0.01$, and $\mu = 0.5$.

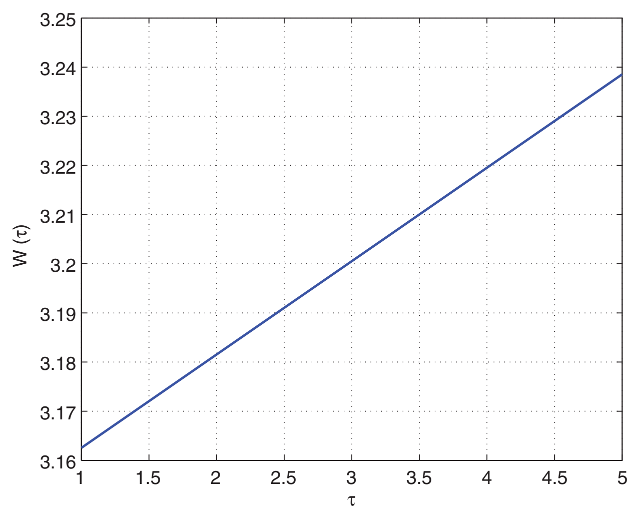

Figure 11: Variation of the width of solitary wave from (12) with respect to τ and the other parameters are $\mu = 0.5$, $q = 0.6$, $\eta_{10} = 0.1$, $\nu_{id0} = 0.01$, $M_0 = 0.1$.

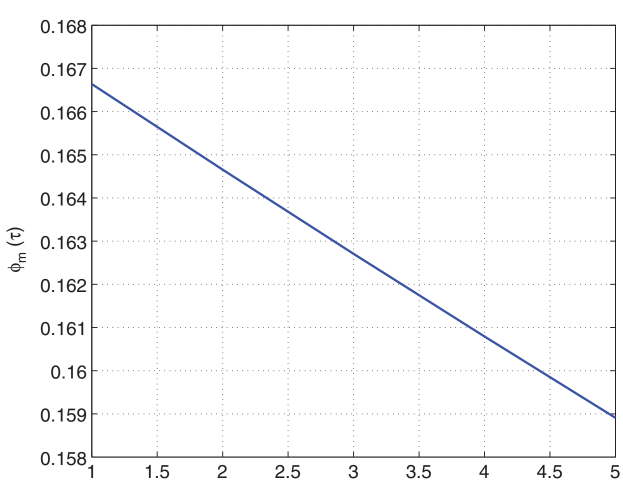

Figure 10: Variation of the amplitude of solitary wave from (12) with respect to τ and the other parameters are $\mu = 0.5$, $q = 0.6$, $\eta_{10} = 0.1$, $\nu_{id0} = 0.01$, $M_0 = 0.1$.

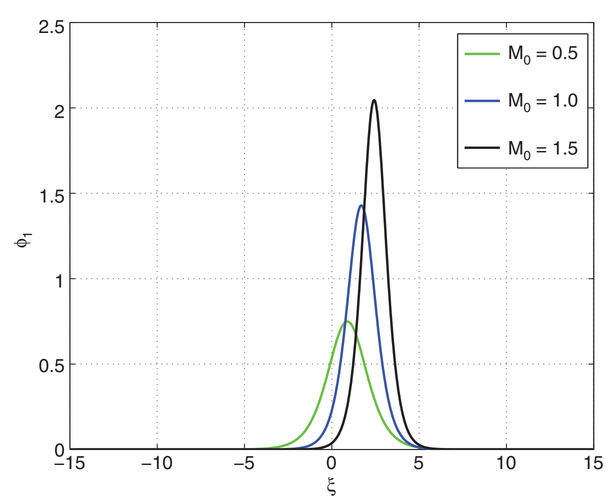

Figure 12: Variation of solitary wave from (12) for the different values of M_0 with $q = 0.6$, $\eta_{10} = 0.1$, $\tau = 2$, $\nu_{id0} = 0.01$, and $\mu = 0.5$.

Thus, Mach number plays an important role on the variation of the amplitude and width of the solitary waves. Hence, we can conclude that the subsonic and supersonic waves exist in the framework of the DKdVB equation.

From the above description, it is seen that solitary wave solution exists for the small values of the parameters involved in the coefficients of the dissipation and damping terms in the DKdVB equation. Although for the planar KdVB equation solitary wave solution does not exist, the solution of the planar KdVB equation exhibits either the transition of a monotonic to oscillatory shock wave or the transition of an oscillatory to monotonic shock wave, depending on the parameters of the system. In this article, it is observed that the proper balancing of the dispersion term with dissipation and damping produces solitary waves.

5 Conclusions

In this article, we have studied the effects of the different parameters on the DIASWs in a dusty plasma with negatively charged ions, nonextensive electron, and stationary dust particles. The RPT is employed to derive the DKdVB equation. This is the first time analytical solution has been derived in the framework of DKdVB equation for the small values of the coefficient of dissipation and damping. The effects of the parameters dust ion collision (ν_{id0}), the entropic index q, Mach number (M), coefficient of viscosity (η_{10}), the ratio between the unperturbed densities of the electrons and ions (μ), and time (τ) on the amplitude and width of the DIASWs solution have been investigated. It is seen that the parameters have played an important role on the shape of the DIASWs in a collisional dusty plasma. The results may be useful in laboratory plasmas as well as space environments where q-nonextensive distributed electrons are present.

Acknowledgements: The authors thank the reviewers for their valuable suggestions and comments, which help to improve the quality of the article. Also, the authors are grateful to Dr. Asit Saha, Sikkim Manipal Institute of Technology, Sikkim Manipal University, Majitar, Rangpo, East-Sikkim 737136, India, for his kind cooperation.

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