

Appendix A.

$$\begin{aligned}
\psi_1^{[0]} &= \frac{1}{\sqrt{\rho}} e^{-\frac{1}{2}it(\rho^2+6r\rho^4)} (-i + 2ix\rho - 2t(\rho^2 + 12r\rho^4))/\sqrt{\rho}, \\
\psi_2^{[0]} &= \frac{1}{\sqrt{\rho}} e^{\frac{1}{2}it(\rho^2+6r\rho^4)} (1 + 2x\rho + 2it(\rho^2 + 12r\rho^4)), \\
\psi_1^{[1]} &= \frac{1}{12\rho^{3/2}} e^{-\frac{1}{2}it(\rho^2+6r\rho^4)} (3 + 6x\rho + 30it\rho^2 - 12x^2\rho^2 - 24itx\rho^3 + 8x^3\rho^3 \\
&\quad + 744irt\rho^4 + 12t^2\rho^4 + 24itx^2\rho^4 - 288irtx\rho^5 - 24t^2x\rho^5 + 288rt^2\rho^6 - 8it^3\rho^6 \\
&\quad + 288irtx^2\rho^6 - 576rt^2x\rho^7 + 1728r^2t^2\rho^8 - 288irt^3\rho^8 - 3456r^2x\rho^9 \\
&\quad - 3456ir^2t^3\rho^{10} - 13824ir^3t^3\rho^{12} + 24i\rho^2b_1 - 24\rho^2d_1), \\
\psi_2^{[1]} &= \frac{1}{12\rho^{3/2}} e^{\frac{1}{2}it(\rho^2+6r\rho^4)} (3i - 6ix\rho + 30t\rho^2 - 12ix^2\rho^2 + 24tx\rho^3 - 8ix^3\rho^3 \\
&\quad + 744rt\rho^4 + 12it^2\rho^4 + 24tx^2\rho^4 + 288rtx\rho^5 + 24it^2x\rho^5 + 288irt^2\rho^6 - 8t^3\rho^6 \\
&\quad + 288rtx^2\rho^6 + 576irt^2x\rho^7 + 1728ir^2t^2\rho^8 - 288rt^3\rho^8 + 3456ir^2x\rho^9 \\
&\quad - 3456r^2t^3\rho^{10} - 13824r^3t^3\rho^{12} + 24\rho^2b_1 + 24i\rho^2d_1), \\
\psi_1^{[2]} &= \frac{1}{480\rho^{5/2}} e^{-\frac{1}{2}it(\rho^2+6r\rho^4)} (45i + 30ix\rho + 210t\rho^2 + 120ix^2\rho^2 - 1200tx\rho^3 \\
&\quad - 240ix^3\rho^3 + 29400rt\rho^4 - 1080it^2\rho^4 + 1680tx^2\rho^4 + 80ix^4\rho^4 - 29760rtx\rho^5 \\
&\quad + 2640it^2x\rho^5 - 320tx^3\rho^5 - 32ix^5\rho^5 - 41280irt^2\rho^6 - 1200t^3\rho^6 \\
&\quad + 35520rtx^2\rho^6 - 480it^2x^2\rho^6 + 160tx^4\rho^6 + 94080irt^2x\rho^7 + 320t^3x\rho^7 \\
&\quad - 3840rtx^3\rho^7 + 320it^2x^3\rho^7 - 339840ir^2t^2\rho^8 - 58560rt^3\rho^8 \\
&\quad - 11520irt^2x^2\rho^8 - 320t^3x^2\rho^8 + 1920rtx^4\rho^8 + 748800ir^2t^2x\rho^9 \\
&\quad + 11520rt^3x\rho^9 - 160it^4x\rho^9 + 7680irt^2x^3\rho^9 - 887040r^2t^3\rho^{10} \\
&\quad + 3840irt^4\rho^{10} + 32t^5\rho^{10} - 69120ir^2t^2x^2\rho^{10} - 11520rt^3x^2\rho^{10} \\
&\quad + 138240r^2t^3x\rho^{11} - 7680irt^4x\rho^{11} + 46080ir^2t^2x^3\rho^{11} - 4285440r^3t^3\rho^{12} \\
&\quad + 69120ir^2t^4\rho^{12} - 138240r^2t^3x^2\rho^{12} + 552960r^3t^3x\rho^{13} - 138240ir^2t^4x\rho^{13} \\
&\quad + 552960ir^3t^4\rho^{14} + 46080r^2t^5\rho^{14} - 552960r^3t^3x^2\rho^{14} - 1105920ir^3t^4x\rho^{15} \\
&\quad + 1658880ir^4t^4\rho^{16} + 552960r^3t^5\rho^{16} - 3317760ir^4t^4x\rho^{17} + 3317760r^4t^5\rho^{18}
\end{aligned}$$

$$\begin{aligned}
& + 7962624r^5t^5\rho^{20} - 240\rho^2(i - 2ix\rho + 2t\rho^2 + 24rt\rho^4)^2b_1 + 960i\rho^3b_2 \\
& + 240i\rho^2d_1 - 960ix\rho^3d_1 + 960t\rho^4d_1 + 960ix^2\rho^4d_1 - 1920tx\rho^5d_1 + 11520rt\rho^6d_1 \\
& - 960it^2\rho^6d_1 - 23040rtx\rho^7d_1 - 23040irt^2\rho^8d_1 - 138240ir^2t^2\rho^{10}d_1 - 960\rho^3d_2),
\end{aligned}$$

$$\begin{aligned}
\psi_2^{[2]} = & \frac{1}{480\rho^{5/2}}e^{\frac{1}{2}it(\rho^2+6r\rho^4)}(-45 + 30x\rho - 210it\rho^2 - 120x^2\rho^2 - 1200itx\rho^3 \\
& - 240x^3\rho^3 - 29400irt\rho^4 + 1080t^2\rho^4 - 1680itx^2\rho^4 - 80x^4\rho^4 - 29760irtx\rho^5 \\
& + 2640t^2x\rho^5 - 320itx^3\rho^5 - 32x^5\rho^5 + 41280rt^2\rho^6 + 1200it^3\rho^6 \\
& - 35520irtx^2\rho^6 + 480t^2x^2\rho^6 - 160itx^4\rho^6 + 94080rt^2x\rho^7 + 320it^3x\rho^7 \\
& - 3840irtx^3\rho^7 + 320t^2x^3\rho^7 + 339840r^2t^2\rho^8 + 58560irt^3\rho^8 - 80t^4\rho^8 \\
& + 11520rt^2x^2\rho^8 + 320it^3x^2\rho^8 - 1920irtx^4\rho^8 + 748800r^2t^2x\rho^9 \\
& + 11520irt^3x\rho^9 - 160t^4x\rho^9 + 7680rt^2x^3\rho^9 + 887040ir^2t^3\rho^{10} \\
& - 3840rt^4\rho^{10} - 32it^5\rho^{10} + 69120r^2t^2x^2\rho^{10} + 11520irt^3x^2\rho^{10} \\
& + 138240ir^2t^3x\rho^{11} - 7680rt^4x\rho^{11} + 46080r^2t^2x^3\rho^{11} + 4285440ir^3t^3\rho^{12} \\
& - 69120r^2t^4\rho^{12} - 1920irt^5\rho^{12} + 138240ir^2t^3x^2\rho^{12} + 552960ir^3t^3x\rho^{13} \\
& - 138240r^2t^4x\rho^{13} - 552960r^3t^4\rho^{14} - 46080ir^2t^5\rho^{14} + 552960ir^3t^3x^2\rho^{14} \\
& - 1105920r^3t^4x\rho^{15} - 1658880r^4t^4\rho^{16} - 552960ir^3t^5\rho^{16} - 3317760r^4t^4x\rho^{17} \\
& - 3317760ir^4t^5\rho^{18} - 7962624ir^5t^5\rho^{20} - 240i\rho^2(1 + 2x\rho + 2it\rho^2 + 24irt\rho^4)^2b_1 \\
& + 960\rho^3b_2 + 240\rho^2d_1 + 960x\rho^3d_1 + 960it\rho^4d_1 + 960x^2\rho^4d_1 + 1920itx\rho^5d_1 \\
& + 11520irt\rho^6d_1 - 960t^2\rho^6d_1 + 23040irtx\rho^7d_1 - 23040rt^2\rho^8d_1 \\
& - 138240r^2t^2\rho^{10}d_1 + 960i\rho^3d_2),
\end{aligned}$$

Appendix B.

$$\begin{aligned}
q[2] = & (e^{it(\rho^2+6r\rho^4)})\rho(45 + 360it\rho^2 - 180x^2\rho^2 + 6624it\rho^4 - 468t^2\rho^4 + 576itx^2\rho^4 - 144x^4\rho^4 \\
& - 13536rt^2\rho^6 - 192it^3\rho^6 + 16128irtx^2\rho^6 - 1440t^2x^2\rho^6 - 384itx^4\rho^6 + 64x^6\rho^6 \\
& - 58175r^2t^2\rho^8 - 16128irt^3\rho^8 - 528t^4\rho^8 - 43776rt^2x^2\rho^8 - 768it^3x^2\rho^8 \\
& - 4608irtx^4\rho^8 + 192t^2x^4\rho^8 - 304128ir^2t^3\rho^{10} - 22272rt^4\rho^{10} - 384it^5\rho^{10} \\
& - 317952r^2t^2x^2\rho^{10} - 27648irt^3x^2\rho^{10} + 192t^4x^2\rho^{10} + 4608rt^2x^4\rho^{10} \\
& - 1658880ir^3t^3\rho^{12} - 345600r^2t^4\rho^{12} - 23040irt^5\rho^{12} + 64t^6\rho^{12} - 331776ir^2t^3x^2\rho^{12} \\
& + 9216rt^4x^2\rho^{12} + 27648r^2t^2x^4\rho^{12} - 2322432r^3t^4\rho^{14} - 552960ir^2t^5\rho^{14} \\
& + 4608rt^6\rho^{14} - 1327104ir^3t^3x^2\rho^{14} + 165888r^2t^4x^2\rho^{14} \\
& - 5640192r^4t^t\rho^{16} - 6635520ir^3t^5\rho^{16} + 138240r^2t^6\rho^{16} + 1327104r^3t^4x^2\rho^{16} \\
& - 39813120ir^4t^5\rho^{18} + 2211840r^3t^6\rho^{18} + 3981312r^4t^4x^2\rho^{18} - 95551488ir^5t^5\rho^{20} \\
& + 19906560r^4t^6\rho^{20} + 95551488r^5t^6\rho^{22} + 191102976r^6t^6\rho^{24} + 48\rho^2(-12i\rho^4(t \\
& + 12rt\rho^2)^2 + 4\rho^6(t + 12rt\rho^2)^3 + 3i(1 + 4x^2\rho^2) - 3t(\rho^2 + 4(-5r + x^2)\rho^4 \\
& + 48rx^2\rho^6))b_1 + 144\rho^4b_1^2 + 48x\rho^3(9 + 4x^2\rho^2 - 12\rho^4(t + 12rt\rho^2)^2 \\
& + 24it(\rho^2 + 12r\rho^4))d_1 + 144\rho^4d_1^2)/(9 + 108x^2\rho^2 + 396t^2\rho^4 + 48x^4\rho^4 \\
& + 16416rt^2\rho^6 - 288t^2x^2\rho^6 + 64x^6\rho^6 + 176832r^2t^2\rho^8 + 432t^4\rho^8 - 16128rt^2x^2\rho^8 \\
& + 192t^2x^4\rho^8 + 23808rt^4\rho^{10} - 152064r^2t^2x^2\rho^{10} + 192t^4x^2\rho^{10} + 4608rt^2x^4\rho^{10} \\
& + 483840r^2t^4\rho^{12} + 64t^6\rho^{12} + 9216rt^4x^2\rho^{12} + 27648r^2t^2x^4\rho^{12} + 4313088r^3t^4\rho^{14} \\
& + 4608rt^6\rho^{14} + 165888r^2t^4x^2\rho^{14} + 14266368r^4t^4\rho^{16} + 138240r^2t^6\rho^{16} \\
& + 1327104r^3t^4x^2\rho^{16} + 2211840r^3t^6\rho^{18} + 3981312r^4t^4x^2\rho^{18} \\
& + 19906560r^4t^6\rho^{20} + 95551488r^5t^6\rho^{22} + 191102976r^6t^6\rho^{24} + 48t\rho^4(9 \\
& - 12x^2\rho^2 + 4t^2\rho^4 + 1728r^2t^2\rho^8 + 6912r^3t^2\rho^{10} + 12r\rho^2(17 - 12x^2\rho^2 + 12t^2\rho^4))b_1 \\
& + 144\rho^4b_1^2 + 48x\rho^3(4x^2\rho^2 - 3(1 + 4\rho^4(t + 12rt\rho^2)^2))d_1 + 144\rho^4d_1^2).
\end{aligned}$$