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Soliton and Shock Profiles in Electron-positron-ion Degenerate Plasmas for Both Nonrelativistic and Ultra-Relativistic Limits

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Abstract: An attempt has been taken to find a general equation for degenerate pressure of Chandrasekhar and constants, by using which one can study nonrelativistic as well as ultra-relativistic cases instead of two different equations and constants. Using the general equation, ion-acoustic solitary and shock waves have been studied and compared, numerically and graphically, the two cases in same situation of electron-positron-ion plasmas. Korteweg–de Vries (KdV) and KdV–Barger equations have been derived as well as their solution to study the soliton and shock profiles, respectively.

Keywords: Chandrasekhar limits; Degenerate plasma; Shocks; Solitons.

1 Introduction

The propagation of the ion-acioustic (IA) waves is very important from the view of its vital role in unnderstanding the electrostatic disturbances in space and laboratory plasma. On the other hand, it is a common idea that electron-positron plasmas have presumably appeared in the early universe [1, 2] and are frequently encountered in active galactic nuclei [3] and pulsar magnetospheress [4, 5]. This electron-positron plasma is usually characterised as a fully ionised gas consisting of electrons and positrons of equal mass. The electron-positron plasmas are generated naturally by pair production in high-energy process in the vicinity of several astrophysical objects and laboratory experiments with finite life time [6]. Due to the long life time of positrons, most of the plasmas (astrophysical and laboratory) usually contains ions, in addition to the electrons and positrons [7]. It has also been shown that over a wide range of parameters, annihilation of electrons and positrons, which is the analog of recombination in plasma composed of ions and electrons, is relatively unimportant in classical [8], as well as in dense quantum plasmas [9] to study the collective plasma oscillations. Recently, there has been a great deal of interest in studying linear and nonlinear wave motions in such plasmas [10, 11]. The nonlinear studies have been focused in the nonlinear self-consistent structures [10–12] such as envelope solition and shokes. Soliton is self-reinforcing solitary wave which preserves its shape and speed, even after collisions with other solitary wave. When the dissipation is weak at the characteristic dynamical timescales of the system, it arises because of the balance between the effects of nonlinearity and dispersion. Korteweg–de Vries (KdV) equation is used to study the both solion and solitary waves. The properties of wave motion in an electron-positron-ion (e-p-i) plasma should be different from those in two-component electron-positron plasmas. Rizzato [13] and Berezhiani et al. [14] have investigated envelope solitons of electromagnetic waves in three-component e-p-i plasmas.

The ultradense degenerate electron-positron plasmas with ions are delivered to be found in compact astrophysical bodies like neutron stars and the inner layers of white dwarfs [9, 15–17] and also in the laser-matter interaction experiments [18, 19]. Therefore, the study of influence of quantum effects on dense e-p-i plasmas become important. A dense plasma is usually characterised as cold and degenerate such as the encountered in metals and semiconductors. However, it has been remarked that a hot fusion plasma such as that found in dense stellar objects e.g. neutrons stars and white dwarfs, may also be considered as quantum degenerate plasmas [20]. Quantum plasmas obey the Fermi–Dirac distribution leading to the Fermi pressure and new forces arising due to Bohm potential [20] play a vital role. In such environments, the electron-positron annihilation rate is higher then naturally expected. Thus, the electron and positron density are very high. The degenerate electron number density in such a compact object is so high (Table 1) that the electron Fermi energy is comparable to the electron mass energy and the electron speed is comparable to the speed of light in vacuum [17, 21, 22].

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Table 1: A table for approximate mass density ρ_0 (in gm cm^{-3}) corresponding n_0 (in gm cm^{-3}) and temperature T (in Kelvin) for white dwarfs and neutron stars [17].

Objects	ρ_0	n_0	T
White dwarfs	10^6–10^8	$3\times(10^{29}$–$10^{31})$	10^5–10^6
Neutron stars	10^{12}–10^{14}	$3\times(10^{35}$–$10^{37})$	10^{11}–10^{12}

We note that to estimate the plasma number density (n_0) we have used $n_0=z_in_{i0}=\rho_0/2m_p$.

Several authors have theoretically investigated the collective effects in dense unmagnetised and magnetised e-p-i quantum plasmas under the assumption of low-phase velocity in comparison with electron or positron Fermi velocity [23–25]. It is found that Bohm potential leads to the wave dispersion due to quantum correlation of density fluctuations associated with wave-like nature of the charge carriers. In these studies, the authors have focused on the lower order quantum corrections appearing in the well-known classical modes. Nowadays, a number of authors have become interested to study the properties of matter under extreme conditions [26–29]. Recently, a number of theoretical investigations have also been mate of the nonlinear propagation of electrostatic waves in degenerate plasma [30–32]. However, these investigations are based on the equation of state valid for the nonrelativistic limit. Some investigations have made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma based on the degenerate electron equation of state valid for ultra-relativistic limit [33–36].

Many scientist have considered unmegnetised nonrelativistic degenerate plasmas [30, 31] and ultra-relativistic degenerate plasmas [33, 35] while studying the nonlinear propagation of electrostatic waves. In this article, an attempt has been taken to develop a general equation [as shown in (6)] with the constants parameters [denoted in (7–9)], which can be used nonrelativistic and ultra-relativistic degenerate plasmas simultaneously. Earlier, Haider and Mamun [37] developed a general form of such a case and using the equation they studied nonlinear propagation of IA solitary wave with degenerate electrons. In this work, a more simplified general form of equation of state, to express the degenerate pressure for ultra-relativistic and nonrelativistic cases, has developed and applied in studying the ion-acoustic solitons and shokes in a degenerate dense plasma system in the absence of the magnetic field but containing inertial ion fluid and inertialess (comparing to ions) ultra-relativistic or nonrelativistic electrons and positrons. Standard KdV and KdV–Barger equation have derived with their constants to study soliton and shock profile, respectively, which might be relevant to interstellar compact object like white dwarf and neutron stars.

This article is organised as follows. A general equation for degenerate pressure of Chandrasekhar has been developed in Section 2. The basic governing equations in unmagnetised e-p-i plasma system are given in Section 3. The Solitary waves equation is derived and its solution in Section 4. The Shock waves equation is also derived as well as its solution in Section 5. A numerical analysis is discussed in Section 6 and finally, a brief discussion containing comperision is presented in Section 7.

2 General Equation for Degenerate Pressure of Chandrasekhar

According to the Chandrasekhar [26–28], the pressure for the nonrelativistic and ultra-relativistic degenerate electrons/fermions fluid can be expressed by the following equations, respectively as

$$P_N = K_1\rho^{5/3}, \tag{1}$$

$$P_U = K_2\rho^{4/3}. \tag{2}$$

Where, the values of K_1 and K_2 and ρ are given below

$$K_1 = \frac{2}{20}\left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{m(\mu H)^{5/3}}, \tag{3}$$

$$K_2 = \left(\frac{3}{\pi}\right)^{1/3}\frac{hc}{8(\mu H)^{4/3}}, \tag{4}$$

$$\rho = n\mu H, \tag{5}$$

with h is the Plank's constant, c is the speed of light, m is the mass of electron, n is the number density of plasma particle, μ is the molecular weight, and H is the mass of proton.

Now, generalising (1) and (2), we can write the following equation that can be used both nonrelativistic and ultra-relativistic limits

$$P = \frac{K}{\gamma}n^{\gamma}, \tag{6}$$

where $\gamma=5/3$ for nonrelativistic case and $\gamma=4/3$ for ultra-relativistic case and constant K is given by

$$K = \frac{1}{18(\gamma-1)}\left(\frac{3\Lambda^3}{\pi}\right)^{(\gamma-1)}E, \tag{7}$$

with

$$\Lambda=\left(\frac{h}{mc}\right), \tag{8}$$

$$E=mc^2. \tag{9}$$

3 Basic Equations

The nonlinear propagation of IA solitary and shock waves have been considered in a ultra cold degenerate plasma containing inertial ion fluid with inertia less electron and positron following ultra-relativistic or nonrelativistic limits. The one-dimensional nonlinear dynamics of electrostatic purturbations are governed by the following equations

$$\frac{\partial n_i}{\partial t}+\frac{\partial(n_i u_i)}{\partial x}=0, \tag{10}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial\psi}{\partial x}+\eta'\frac{\partial^2 u_i}{\partial x^2}, \tag{11}$$

$$0=-j\frac{\partial\psi}{\partial x}-\frac{\beta}{\gamma n_s}\frac{\partial n_s^\gamma}{\partial x}, \tag{12}$$

$$\frac{\partial^2\psi}{\partial x^2}=(\mu_p+1)n_e-\mu_p n_p-n_i. \tag{13}$$

Where n_i is the ion number density normalised by its equilibrium value n_{i0}, u_i is the ion fluid speed normalised by $C_i=(m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the rest mass of electron (ion), and c being the speed of light in vacuum. ψ is the electrostatic wave potential normalised by $m_e c^2/e$ with e being the magnitude of the charge of an electron, m_e is the mass of electron. The time variable (t) is normalised by $\omega_{pi}^{-1}=(m_i/4\pi n_{i0}e^2)^{1/2}$ and the space variables is normalised by $\lambda_i=(z_i m_e c^2/4\pi n_{i0}e^2)^{1/2}$. In (12), n_s represents n_e (n_p) is the electron (positron) number density normalised by their equilibrium values n_{e0} (n_{p0}), $j=+1$ for positron and $j=-1$ for electron and also $\beta=(K/E)n_0^{\gamma-1}$. We have considered the viscuss term, i.e. coefficient of viscosity (η') is equals to zero at the time of studying solitary waves.

Now, using equilibrium charge neutrality condition $n_{e0}=n_{i0}+n_{p0}$, one can write $\mu_e=1+\mu_p$ where $\mu_e=n_{e0}/n_{i0}$ and $\mu_p=n_{p0}/n_{i0}$.

Rearranging (12), the value of n_e and n_p can be express, respectively, as

$$n_e=\left(1+\frac{\gamma-1}{\beta}\psi\right)^{\frac{1}{\gamma-1}}, \tag{14}$$

$$n_p=\left(1-\frac{\gamma-1}{\beta}\psi\right)^{\frac{1}{\gamma-1}}. \tag{15}$$

4 Solitary Wave

Introducing independent variables through the stretched coordinates [38–45], to follow the reductive perturbation technique [38] to construct a weakly nonlinear theory for the electrostatic waves with a small but finite amplitude, as

$$\xi=\epsilon^{1/2}(x-V_0 t),$$
$$\tau=\epsilon^{3/2}t,$$

where ϵ is a small parameter measuring the weakness of the dispersion and V_0 is the unknown wave phase speed (to be determined later) is normalised by the ion-acoustic speed (C_i). It may be noted here that ξ is normalised by the Debye radius (λ_i), τ is normalised by the ion plasma period (ω_{pi}^{-1}). The perturbed quantities can be expanded about their equilibrium values in powers of ϵ as [38–45]

$$n_i=1+\epsilon n_i^{(1)}+\epsilon^2 n_i^{(2)}+\cdots, \tag{16}$$

$$u_i=\epsilon u_i^{(1)}+\epsilon^2 u_i^{(2)}+\cdots, \tag{17}$$

$$\psi=\epsilon\psi^{(1)}+\epsilon^2\psi^{(2)}+\cdots. \tag{18}$$

Using the stretched coordinates and (16)–(18) in (10)–(13); and equating the coefficients of $\epsilon^{3/2}$ from the continuity and momentum equation and coefficients of ϵ from Poisson's equation, one can obtain the first-order continuity, momentum, and Poisson's equation as

$$u_i^{(1)}=\frac{1}{V_0}\psi^{(1)}, \tag{19}$$

$$n_i^{(1)}=\frac{1}{V_0^2}\psi^{(1)}, \tag{20}$$

$$n_i^{(1)}=\left(\frac{1+2\mu_p}{\beta}\right)\psi^{(1)}. \tag{21}$$

Comparing (20) and (21), the linear dispersion relation can be written as

$$V_0 = \sqrt{\frac{\beta}{1+2\mu_p}}. \tag{22}$$

To the next higher order of ϵ, i.e. equating the coefficients of $\epsilon^{5/2}$ from continuity and momentum equation and coefficients of ϵ^2 from Poisson's equation, one can write, respectively,

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}[n_i^{(1)} u_i^{(1)}] + \frac{\partial u_i^{(2)}}{\partial \xi} = 0, \tag{23}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_0 \frac{\partial u_i^{(2)}}{\partial \xi} + u_i \frac{\partial u_i^{(1)}}{\partial \xi} + \frac{\partial \psi^{(2)}}{\partial \xi} = 0, \tag{24}$$

$$\frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \left(\frac{1+2\mu_p}{\beta}\right)\psi^{(2)} - \frac{2-\gamma}{2\beta}[\psi^{(1)}]^2 + n_i^{(2)} = 0. \tag{25}$$

Now, using (23)–(25), KdV equation can be readily obtained as

$$\frac{\partial \psi^{(1)}}{\partial \tau} + AB\psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + B\frac{\partial^3 \psi^{(1)}}{\partial \xi^3} = 0, \tag{26}$$

where

$$A = \frac{3}{V_0^4} - \left(\frac{2-\gamma}{\beta^2}\right), \tag{27}$$

$$B = \frac{V_0^3}{2}. \tag{28}$$

Now, transforming the independent variables ζ and τ' to $\zeta = \xi - U_0 \tau$ and $\tau' = \tau$ and imposing the appropriate boundary conditions (viz. $\psi^{(1)} \to 0$, $\partial\psi^{(1)}/\partial\zeta \to 0$, $\partial^2\psi^{(1)}/\partial\zeta^2 \to 0$ at $\zeta \to \pm\infty$), one can express the stationary solution [46, 47] of the KdV equation (26) as

$$\psi^{(1)} = \psi_m \,\mathrm{sech}^2(\zeta/\Delta). \tag{29}$$

This is to note here that U_0 is a constant speed, normalised by C_i, which describe the speed of the solitary structure. This is always a positive quantity and most of cases upper range is ≤ 1 (i.e. generally $0 < U_0 \le 1$).

The value of B is negligible compared to A, so we can write the amplitude (ψ_m) and the width (Δ) of the solitary wave denoted in equation (29) as

$$\psi_m = \frac{3U_0}{A}, \tag{30}$$

$$\Delta = \sqrt{\frac{4B}{U_0}}. \tag{31}$$

It is obvious from (30) and (31) that U_0 is a linear function of amplitude and inverse function of width, which mean that the profile of the faster solitary structures will be taller and narrower then slower one.

5 Shock Wave

Introducing the stretched coordinates in reductive perturbation method [38, 48, 49] to obtain KdV–Barger equation, as

$$\xi = \epsilon^{1/2}(x - V_0 t),$$
$$\tau = \epsilon^{3/2} t,$$
$$\eta' = \epsilon^{1/2}\eta_0,$$

and expanding the perturbed quantities about their equilibrium values in powers of ϵ as in (16)–(18) and equating the coefficients of the lowest order of ϵ from the continuity, momentum, and Poisson's equation, one can obtain the first order of these equations as such as (19)–(21) and the linear dispersion relation as

$$V_0 = \sqrt{\frac{\beta}{1+2\mu_p}}, \tag{32}$$

which is same as the dispersion relation as solitary waves as in (22).

To the next higher order of ϵ, i.e. equating the coefficients of $\epsilon^{5/2}$ from continuity and momentum equation and coefficients of ϵ^2 from Poisson's equation, one can write, respectively,

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}[n_i^{(1)} u_i^{(1)}] + \frac{\partial u_i^{(2)}}{\partial \xi} = 0, \tag{33}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_0 \frac{\partial u_i^{(2)}}{\partial \xi} + u_i \frac{\partial u_i^{(1)}}{\partial \xi} + \frac{\partial \psi^{(2)}}{\partial \xi} - \eta_0 \frac{\partial^2 u_i^{(1)}}{\partial \xi^2} = 0, \tag{34}$$

$$\frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \left(\frac{1+2\mu_p}{\beta}\right)\psi^{(2)} - \frac{2-\gamma}{2\beta^2}[\psi^{(1)}]^2 + n_i^{(2)} = 0. \tag{35}$$

Now, using (33)–(35), one can readily obtain the KdV–Barger equation as

$$\frac{\partial \psi^{(1)}}{\partial \tau} + AB\psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + B\frac{\partial^3 \psi^{(1)}}{\partial \xi^3} = C\frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \tag{36}$$

where

$$A = \frac{3}{V_0^4} - \left(\frac{2-\gamma}{\beta^2}\right), \tag{37}$$

$$B = \frac{V_0^3}{2}, \tag{38}$$

$$C = \frac{\eta_0}{2}. \tag{39}$$

Transforming the independent variables ζ and τ' to $\zeta = \xi - U_0\tau$ and $\tau' = \tau$, and imposing the appropriate boundary conditions as in the solitary waves, one can express the stationary solution [48, 49] of the KdV–Barger equation (36) as

$$\psi^{(1)} = \psi_m[1 - \tanh(\zeta/\Delta)], \tag{40}$$

where the amplitude (ψ_m as in equation (30) and the width Δ) are given by

$$\psi_m = \frac{3U_0}{A}, \tag{41}$$

$$\Delta = \sqrt{\frac{2C}{U_0}}. \tag{42}$$

It is found from (41) and (42) that U_0 is a linear function of amplitude and inverse function of width, as same as in the case of SWs, which mean that the profile of the faster shock waves will be taller and narrower then slower one.

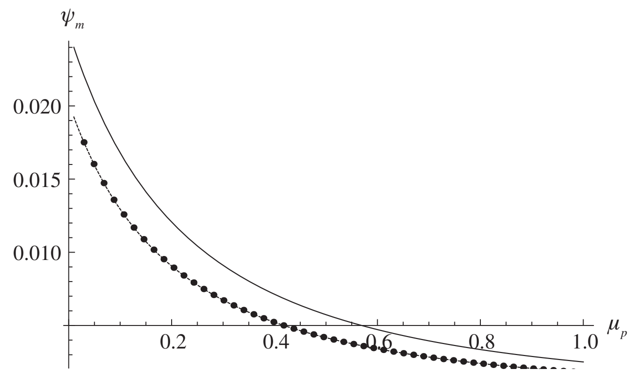

Figure 1: Showing the variation of amplitude of solitary and shock waves with μ_p with $U_0 = 0.1$ and $n_0 = 10^{36}$ for $\gamma = 4/3$ (dotted curve) and $\gamma = 5/3$ (solid curve).

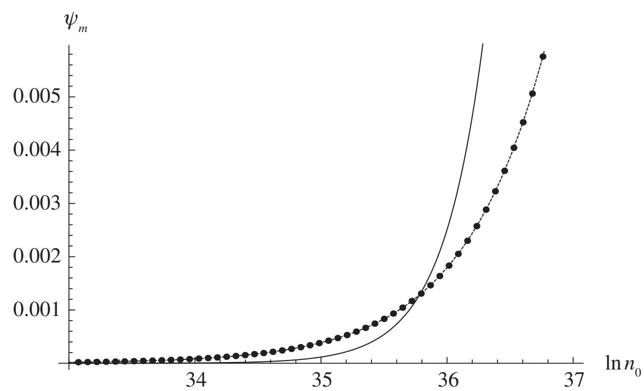

Figure 2: Showing the variation of amplitude of solitary and shock waves with $\ln n_0$ with $U_0 = 0.1$ and $\mu_p = 1$ for $\gamma = 4/3$ (dotted curve) and $\gamma = 5/3$ (solid curve).

6 Numerical Analysis

As (27) and (37) indicate that the value of A in both solitary and shock wave equations is same and in both case the amplitude is $3U_0/A$, so the variation of amplitude of solitary and shock waves with depending parameters are same. Thus, the solitary and shock waves associate with a positive potential ($\psi_m > 0$) if $A > 0$ or a negative potential ($\psi_m < 0$) if $A < 0$. It is clear that $A = 0$ corresponds to $\mu_p = (-3 \pm \sqrt{2})/6$ for ultra-relativistic limit and $\mu_p = -2/3$ or $-1/3$ for nonrelativistic limit. It means that negative solitary and shock waves can be associate at $\mu_p < (-3 \pm \sqrt{2})/6$ for ultra-relativistic case and $\mu_p < -2/3$ or $-1/3$ for nonrelativistic case. However, $\mu_p (= n_{p0}/n_{i0})$ cannot be negative. So, the value of A is always positive and the solitary and shock waves always associated with positive potentials. Figure 1 shows the variation of amplitude (ψ_m) of the solitons and shocks with μ_p for $U_0 = 0.1$ and $n_0 = 10^{36}$ both ultra-relativistic (dotted line) and nonrelativistic (solid line) limits. As the value of μ_p increases, the amplitude decreases for both the cases but the amplitude for nonrelativistic case is always higher than the ultra-relativistic situation. Figure 2 shows the variation of amplitude of the solitary and shock waves with particle number density for both ultra-relativistic and nonrelativistic cases. From this figure, it can be said that there is critical value ($\approx 8 \times 10^{35}$ for $U_0 = 0.1$ and $\mu_p = 1$) below which the amplitude of ultra-relativistic case is higher than for nonrelativistic cases for same value of particle number density and above this critical value the amplitude of nonrelativistic cases is higher.

The variation of width of solitary waves with μ_p and $\ln n_0$ is shown in Figures 3 and 4 for both ultra-relativistic and nonrelativistic cases. Increasing the value of μ_p, the width of the solitary waves decreases which means that positron number density makes the solitary waves narrower. It is also found that the width is higher for nonrelativistic cases than ultra-relativistic cases for the same μ_p. However, increasing the plasma number density the width increases for both the cases. Below the critical value, the width is higher for ultra-relativistic cases than nonrelativistic one, and after the critical value, the rate of increment is higher for nonrelativistic cases then ultra-relativistic one.

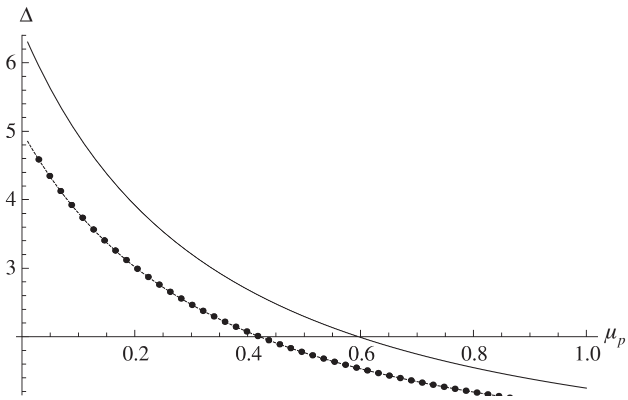

Figure 3: Showing the variation of width of solitary waves with μ_p with $U_0 = 0.1$ and $n_0 = 10^{36}$ for $\gamma = 4/3$ (dotted curve) and $\gamma = 5/3$ (solid curve).

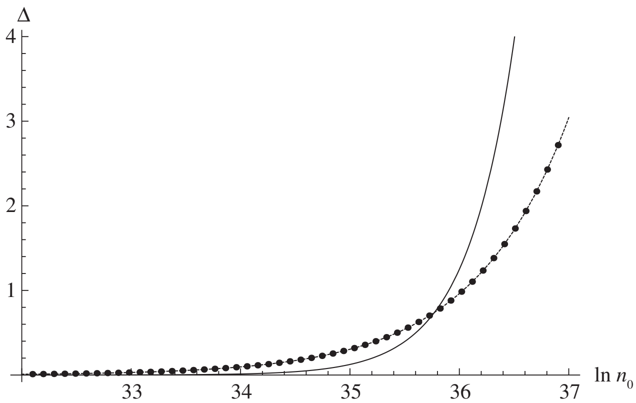

Figure 4: Showing the variation of width of solitary waves with ln n_0 with $U_0 = 0.1$ and $\mu_p = 1$ for $\gamma = 4/3$ (dotted curve) and $\gamma = 5/3$ (solid curve).

7 Discussion

Soliton and shock profiles have been analysed in a collisionless electron-positron-ion plasma containing ultra-cold inertial ion fluid and ultra-cold inertia less degenerate electron and positron fluid by deriving KdV and KdV–Barger equation, respectively, by reductive perturbation method. The findings are summarised as follows:

1. In present plasma system, solitons and shocks are always associated with positive potentials and propagate with same amplitude.
2. Profile of the faster solitons and shocks are more spiky than slower one.
3. There must be a critical value of μ_p and n_0, considering other parameters constant, for which amplitudes of solitons and shocks are same for ultra-relativistic and nonrelativistic cases.
4. The amplitude and width of the solitons increases with increasing n_0 for both ultra-relativistic and non-relativistic cases, which means that higher n_0 leads stronger soliton profile.
5. Below the critical value both the amplitude and width for ultra-relativistic case are higher than nonrelativistic one and above the critical value it is higher for nonrelativistic case.
6. Increasing the value of μ_p, the amplitude and width of solitons decrease for both ultra-relativistic and non-relativistic cases, i.e. higher μ_p makes week soliton profile.
7. For the case of shocks the amplitude increases with n_0 and decreases with μ_p for both the cases as such as solitons but the width is a function of η_0, not n_0 and μ_p. So the width of the shocks is not same as solitons and independent of n_0 and μ_p and increases with increasing η_0 for both ultra-relativistic and nonrelativistic cases.

The ranges of the plasma parameters for astrophysical compact objects like white dwarfs and neutron stars are very wide. The plasma parameters for white dwarfs and neutron stars are shown in Table 1. The plasma parameters used in our present theoretical analysis correspond to white dwarfs. However, our present theoretical analysis can applied to neutron stars. Therefore, our present results may be useful for understanding the localised electrostatic disturbance in astrophysical compact objects, particularly, in white dwarf and neutron stars.

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