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Analytical Solitons for Langmuir Waves in Plasma Physics with Cubic Nonlinearity and Perturbations

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Abstract: We presented an analytical study on dynamics of solitons for Langmuir waves in plasma physics. The mathematical model is given by the perturbed nonlinear Schrödinger equation with full nonlinearity and Kerr law nonlinearity. There are three techniques of integrability were employed to extract exact solutions along with the integrability conditions. The topological 1-soliton solutions, singular 1-soliton solutions, and plane wave solution were reported by Ricatti equation expansion approach and then the bright 1-soliton solution, singular 1-soliton solution, periodic singular solutions, and plane wave solution were derived with the help of trial solution method. Finally, based on the G'/G-expansion scheme, we obtained the hyperbolic function travelling wave solution, trigonometric function travelling wave solution, and plane wave solution.

Keywords: Integrability; Nonlinear Schrödinger Equation; Solitons.

1 Introduction

The nonlinear dynamics of solitons for Langmuir waves in the context of plasma physics is ruled by the perturbed nonlinear Schrödinger equation (NLSE) [1–3]. The perturbation with full nonlinearity and cubic nonlinearity are taken into account. In the previous works, many techniques of integrability were applied to extract to exact solitons, which include multiple-scale perturbation analysis and Lie symmetry approach [1–3]. In this study, three tools of integration carried out to the soliton solutions along with the constraint conditions. They are Ricatti equation expansion approach, trial solution method, and G'/G-expansion scheme [4–12].

The dimensionless form of NLSE is given by [1–3]

$$iq_t + aq_{xx} + bF(|q|^2)q = iR. \tag{1}$$

In (1), x is a spatial variable, while t is the temporal variable and the dependent variable $q(x, t)$ is the Langmuir wave profile. In (1), F is a real-valued algebraic function and it is necessary to have the smoothness of the complex function $F(|q|^2)q: C \to C$. Considering the complex plane C as a two-dimensional linear space R^2, the function $F(|q|^2)q$ is k times continuously differentiable, so that

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k((-n, n)\times(-m, m); R^2). \tag{2}$$

As it appears, (1) is not integrable, in general. There are two special cases of the function F that will be studied in this article. They are commonly referred to as the cubic NLSE and the second one is commonly known as the power-law NLSE. The soliton solutions of (1) are called Langmuir solitons and, in the form of cavitons, were observed in 1974. These cavitons are local regions from which plasma is ousted by the electromagnetic field. In the presence of strong magnetic field, cavitons in a moving plasma were observed. Ion acoustic solitons have been detected earlier in 1970–1971. The perturbation term is

$$R = -i\alpha|q|^2 q_{xx} - i\beta(|q|^2)_x q_x + i\gamma|q_x|^2 q - i\xi(|q|^2)_{xx} q \tag{3}$$

where α, β, γ, and ξ are constants. The first three perturbation terms arise in the study of interaction between Langmuir waves and ion acoustic waves in plasmas, provided the velocity of the Langmuir waves is small as compared to the sound velocity. These perturbation terms may be regarded as combination of nonlinear and spatial dispersion. The coefficient of ξ arises when electromagnetic solitons are studied in the context of relativistic plasmas.

2 Soliton Solutions

In order to solve (1), we use the following wave transformation [13–21]

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$$q(x,\,t)=U(\tau)e^{i\Phi(x,t)} \tag{4}$$

where $U(\tau)$ represents the shape of the pulse and

$$\tau=B(x-vt), \tag{5}$$

$$\Phi(x,\,t)=-\kappa x+\omega t+\theta. \tag{6}$$

In (4), the function $\Phi(x, t)$ is the phase component of the soliton. Then, in (6), κ is the soliton frequency, while ω is the wave number of the soliton and θ is the phase constant. Finally, in (5), v is the velocity of the soliton. Substituting (4) into (1) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v=-2\kappa a, \tag{7}$$

and

$$\gamma=\alpha+\beta, \tag{8}$$

while the real part gives

$$+aB^2U''-(\omega+a\kappa^2)U+b\,F(U^2)U-\kappa^2(\gamma-\alpha)U^3 -B^2(\alpha+2\xi)U^2U''-B^2(2\beta+2\xi-\gamma)U(U')^2=0. \tag{9}$$

From (8) and (9), we have

$$+aB^2U''-(\omega+a\kappa^2)U+b\,F(U^2)U-\kappa^2\,\beta\,U^3 -B^2(\alpha+2\xi)U^2U''-B^2(\beta+2\xi-\alpha)U(U')^2=0. \tag{10}$$

The Kerr law of nonlinearity originates from the fact that a light wave in an optical fibre faces nonlinear responses from nonharmonic motion of electrons bound in molecules, caused by an external electric field. Even though the nonlinear responses are extremely weak, their effects appear in various ways over long distance of propagation that is measured in terms of light wavelength. The origin of nonlinear response is related to the nonharmonic motion of bound electrons under the influence of an applied field. As a result, the induced polarisation is not linear in the electric field but involves higher order terms in electric field amplitude.

The Kerr law nonlinearity is the case when $F(u)=u$, so that (1) reduces to

$$iq_t+aq_{xx}+b(|q|^2)q=\alpha|q|^2q_{xx} +\beta(|q|^2)_x q_x-\gamma|q_x|^2q+\xi(|q|^2)_{xx}q, \tag{11}$$

and (10) simplifies to

$$aB^2U''-(\omega+a\kappa^2)U+(b-\kappa^2\beta)U^3-B^2(\alpha+2\xi)U^2U'' -B^2(\beta+2\xi-\alpha)U(U')^2=0. \tag{12}$$

2.1 Application of the Riccati Equation Expansion Approach

In this section, the Ricatti equation expansion approach will be shown in detail to obtain the singular solutions, singular and dark soliton solutions to (1). According to the homogeneous balance method, (12) has the solution in the form

$$U(\tau)=A_0+A_1\varphi(\tau), \tag{13}$$

where $\varphi(\tau)$ satisfies the Riccati equation

$$\varphi'(\tau)=f+l\varphi^2(\tau) \tag{14}$$

where f and l are all nonzero real-valued constants that are independent on τ. Equation (14) is the well-known Riccati equation, which admits the following explicit solutions:

$$\varphi(\tau)=\frac{\sqrt{fl}}{l}\tan(\sqrt{fl}\tau) \tag{15}$$

$$\varphi(\tau)=-\frac{\sqrt{fl}}{l}\cot(\sqrt{fl}\tau) \tag{16}$$

$$\varphi(\tau)=\frac{\sqrt{fl}}{l}\{\tan(2\sqrt{fl}\tau)\pm\sec(2\sqrt{fl}\tau)\} \tag{17}$$

when $fl>0$,

$$\varphi(\tau)=-\frac{\sqrt{-fl}}{l}\tanh(\sqrt{-fl}\tau) \tag{18}$$

$$\varphi(\tau)=-\frac{\sqrt{-fl}}{l}\coth(\sqrt{-fl}\tau) \tag{19}$$

when $fl<0$, and

$$\varphi(\tau)=-\frac{1}{l\tau} \tag{20}$$

when $f=0$.

Substituting ansatz (13) along with (14) into (12), collecting the coefficients of φ, and solving the resulting system, we find

$$B=\pm\frac{A_1}{2l}\sqrt{\frac{-2l(b-\kappa^2\beta)}{al-A_1^2f(\beta+4\xi)}}, \tag{21}$$

$$\omega=-\frac{al\kappa^2+A_1^2f(b-\kappa^2\beta)}{l}, \tag{22}$$

$$A_0=0, \tag{23}$$

$$\alpha=-6\xi-\beta, \tag{24}$$

where l, κ, f, and A_1 are arbitrary constants, which immediately prompts the constraint

$$l(\kappa^2\beta-b)(al-A_1^2 f(b-\kappa^2\beta))>0. \tag{25}$$

Substituting (21)–(25) into (13) and inserting the result into the transformation (3), we get the exact solutions of (1) as follows:

Topological 1-soliton solutions:

$$q(x,t)=\mp A_1\sqrt{-\frac{f}{l}}\tanh\left[A_1\sqrt{\frac{f(b-\kappa^2\beta)}{2al-2A_1^2 f(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2 f(b-\kappa^2\beta)}{l}\right)t+\theta\right\}}, \tag{26}$$

and

$$q(x,t)=\mp A_1\sqrt{-\frac{f}{l}}\coth\left[A_1\sqrt{\frac{f(b-\kappa^2\beta)}{2al-2A_1^2 f(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2 f(b-\kappa^2\beta)}{l}\right)t+\theta\right\}}. \tag{27}$$

Singular 1-soliton solutions:

$$q(x,t)=\mp A_1\sqrt{\frac{f}{l}}\tan\left[A_1\sqrt{\frac{f(\kappa^2\beta-b)}{2al-2A_1^2 f(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2 f(b-\kappa^2\beta)}{l}\right)t+\theta\right\}}, \tag{28}$$

$$q(x,t)=\mp A_1\sqrt{\frac{f}{l}}\left\{\tan\left[2A_1\sqrt{\frac{f(\kappa^2\beta-b)}{2al-2A_1^2 f(\beta+4\xi)}}(x+2a\kappa t)\right]\pm\sec\left[2A_1\sqrt{\frac{f(\kappa^2\beta-b)}{2al-2A_1^2 f(\beta+4\xi)}}(x+2a\kappa t)\right]\right\}\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2 f(b-\kappa^2\beta)}{l}\right)t+\theta\right\}}, \tag{29}$$

and

$$q(x,t)=\pm A_1\sqrt{\frac{f}{l}}\cot\left[A_1\sqrt{\frac{f(\kappa^2\beta-b)}{2al-2A_1^2 f(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2 f(b-\kappa^2\beta)}{l}\right)t+\theta\right\}}. \tag{30}$$

Plane wave solution:

$$q(x,t)=\pm\frac{2a}{\sqrt{2a(\kappa^2\beta-b)}(x+2a\kappa t)}e^{i\{-\kappa x-a\kappa^2 t+\theta\}}. \tag{31}$$

These solutions will be defined subjected to the constraint conditions (8) and (24).

On the other hand, when

$$\alpha+2\xi=0, \tag{32}$$

and

$$\beta+2\xi-\alpha=0, \tag{33}$$

then we have

$$\alpha=-2\xi,\quad \beta=-4\xi. \tag{34}$$

By the constraint conditions (8) and (34), we get

$$aB^2U''-(\omega+a\kappa^2)U+(b+4\kappa^2\xi)U^3=0, \tag{35}$$

and

$$\gamma=-6\xi. \tag{36}$$

Substituting ansatz (13) along with (14) into (35), collecting the coefficients of φ, and solving the resulting system, we find

$$B=\pm\frac{A_1}{2al}\sqrt{-2a(b+4\kappa^2\xi)}, \tag{37}$$

$$\omega=-\frac{al\kappa^2+A_1^2 f(b+4\kappa^2\xi)}{l}, \tag{38}$$

$$A_0=0, \tag{39}$$

where l, κ, f, and A_1 are arbitrary constants, which immediately prompts the constraint

$$a(b+4\kappa^2\xi)<0. \tag{40}$$

Substituting (37)–(39) into (13) and inserting the result into the transformation (3), we get the exact solutions of (1) as follows:

Topological 1-soliton solutions:

$$q(x,t)=\mp A_1\sqrt{-\frac{f}{l}}\tanh\left[A_1\sqrt{\frac{f(b+4\kappa^2\xi)}{2al}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2 f(b+4\kappa^2\xi)}{l}\right)t+\theta\right\}}, \tag{41}$$

and

$$q(x,t)=\mp A_1\sqrt{-\frac{f}{l}}\coth\left[A_1\sqrt{\frac{f(b+4\kappa^2\xi)}{2al}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2f(b+4\kappa^2\xi)}{l}\right)t+\theta\right\}}. \quad (42)$$

Singular 1-soliton solutions:

$$q(x,t)=\pm A_1\sqrt{\frac{f}{l}}\tan\left[A_1\sqrt{-\frac{f(b+4\kappa^2\xi)}{2al}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2f(b+4\kappa^2\xi)}{l}\right)t+\theta\right\}}, \quad (43)$$

$$q(x,t)=\pm A_1\sqrt{\frac{f}{l}}\left\{\tan\left[2A_1\sqrt{-\frac{f(b+4\kappa^2\xi)}{2al}}(x+2a\kappa t)\right]\pm\sec\left[2A_1\sqrt{-\frac{f(b+4\kappa^2\xi)}{2al}}(x+2a\kappa t)\right]\right\}\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2f(b+4\kappa^2\xi)}{l}\right)t+\theta\right\}}, \quad (44)$$

and

$$q(x,t)=\mp A_1\sqrt{\frac{f}{l}}\cot\left[A_1\sqrt{-\frac{f(b+4\kappa^2\xi)}{2al}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(\frac{al\kappa^2+A_1^2f(b+4\kappa^2\xi)}{l}\right)t+\theta\right\}}. \quad (45)$$

Plane wave solution:

$$q(x,t)=\pm\frac{2a}{\sqrt{-2a(b+4\kappa^2\xi)(x+2a\kappa t)}}e^{i\{-\kappa x-a\kappa^2t+\theta\}}. \quad (46)$$

These solutions will be defined subjected to the constraint conditions (34) and (36).

2.2 Application of the Trial Solution Method

Equation (12) has the following trial equation

$$(U')^2=A_0+A_1U+A_2U^2+A_3U^3+A_4U^4. \quad (47)$$

Thus, we have

$$U''=\frac{A_1}{2}+A_2U+\frac{3A_3}{2}U^2+2A_4U^3. \quad (48)$$

Substituting (47) and (48) into (12), collecting the coefficients of U, and solving the resulting system we find

$$A_4=\frac{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}{2aB^2}, \quad (49)$$

$$\omega=-a\kappa^2+aB^2A_2-2A_0B^2(\beta+4\xi), \quad (50)$$

$$\alpha=-6\xi-\beta, \quad (51)$$

$$A_1=A_3=0, \quad (52)$$

where A_0, κ, B, and A_2 are arbitrary constants.

Then (47) becomes

$$(U')^2=A_0+A_2U^2+\left(\frac{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}{2aB^2}\right)U^4. \quad (53)$$

The integral form of (53) is

$$\pm(\tau-\tau_0)=\int\frac{\mathrm{d}U}{\sqrt{A_0+A_2U^2+\left(\frac{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}{2aB^2}\right)U^4}}. \quad (54)$$

If we set $A_0=0$ in (54) and integrating with resect to U, we get the following exact solution of (1):

Bright 1-soliton solution:

$$q(x,t)=\pm\sqrt{-\frac{2aA_2B^2}{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}}\operatorname{sech}[\sqrt{A_2}(x+2a\kappa t)]\, e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}. \quad (55)$$

Singular 1-soliton solution:

$$q(x,t)=\pm\sqrt{\frac{2aA_2B^2}{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}}\operatorname{csch}[\sqrt{A_2}(x+2a\kappa t)]\, e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}. \quad (56)$$

Periodic singular solutions:

$$q(x,t)=\pm\sqrt{-\frac{2aA_2B^2}{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}}\sec[\sqrt{-A_2}(x+2a\kappa t)]\, e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}, \quad (57)$$

and

$$q(x,t)=\pm\sqrt{-\frac{2aA_2B^2}{\kappa^2\beta-b+A_2B^2(\beta+4\xi)}}\csc[\sqrt{-A_2}(x+2a\kappa t)]\, e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}. \quad (58)$$

Plane wave solution:

$$q(x,t)=\pm\frac{2a}{\sqrt{2a(\kappa^2\beta-b)(x+2a\kappa t)}}e^{i\{-\kappa x-a\kappa^2t+\theta\}}. \quad (59)$$

These solutions will be defined subjected to the constraint conditions (8) and (51).

Similarly, (35) has the following trial equation

$$(U')^2=A_0+A_1U+A_2U^2+A_3U^3+A_4U^4. \tag{60}$$

Thus, we have

$$U''=\frac{A_1}{2}+A_2U+\frac{3A_3}{2}U^2+2A_4U^3. \tag{61}$$

Substituting (60) and (61) into (35), collecting the coefficients of U, and solving the resulting system, we find

$$A_4=-\frac{4\kappa^2\xi+b}{2aB^2}, \tag{62}$$

$$\omega=-a\kappa^2+aB^2A_2, \tag{63}$$

$$A_1=A_3=0, \tag{64}$$

where A_0, κ, B, and A_2 are arbitrary constants.

Then (60) becomes

$$(U')^2=A_0+A_2U^2-\left(\frac{4\kappa^2\xi+b}{2aB^2}\right)U^4. \tag{65}$$

The integral form of (65) is

$$\pm(\tau-\tau_0)=\int\frac{\mathrm{d}U}{\sqrt{A_0+A_2U^2-\left(\frac{4\kappa^2\xi+b}{2aB^2}\right)U^4}}. \tag{66}$$

If we set $A_0=0$ in (66) and integrating with resect to U, we get the following exact solution of (1):

Bright 1-soliton solution:

$$q(x,\,t)=\pm\sqrt{\frac{2aA_2B^2}{b+4\kappa^2\xi}}\operatorname{sech}[\sqrt{A_2}(x+2a\kappa t)]e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}. \tag{67}$$

Singular 1-soliton solution:

$$q(x,\,t)=\pm\sqrt{-\frac{2aA_2B^2}{b+4\kappa^2\xi}}\operatorname{csch}[\sqrt{A_2}(x+2a\kappa t)]e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}. \tag{68}$$

Periodic singular solutions:

$$q(x,\,t)=\pm\sqrt{\frac{2aA_2B^2}{b+4\kappa^2\xi}}\sec[\sqrt{-A_2}(x+2a\kappa t)]e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}, \tag{69}$$

and

$$q(x,\,t)=\pm\sqrt{\frac{2aA_2B^2}{b+4\kappa^2\xi}}\csc[\sqrt{-A_2}(x+2a\kappa t)]e^{i\{-\kappa x-a(\kappa^2-B^2A_2)t+\theta\}}. \tag{70}$$

Plane wave solution:

$$q(x,\,t)=\pm\frac{2a}{\sqrt{-2a(b+4\kappa^2\xi)(x+2a\kappa t)}}e^{i\{-\kappa x-a\kappa^2t+\theta\}}. \tag{71}$$

These solutions will be defined subjected to the constraint conditions (34) and (36).

2.3 *G′G*-Expansion Approach

In this section, the G'/G-expansion method [4] will be shown in detail to obtain the singular solutions, singular and dark soliton solutions to (1). According to the homogeneous balance method, (12) has the solutions in the form

$$U(\tau)=A_0+A_1\left(\frac{G'(\tau)}{G(\tau)}\right), \tag{72}$$

where $G(\tau)$ satisfies the second-order linear ordinary differential equation

$$G''(\tau)+\lambda G'(\tau)+\mu G(\tau)=0, \tag{73}$$

where λ and μ are real constants to be determined.

Substituting (72) along with (73) into (12) leads to

$$\begin{aligned}&aB^2\left\{2A_1\left(\frac{G'}{G}\right)^3+3A_1\lambda\left(\frac{G'}{G}\right)^2+(2A_1\mu+A_1\lambda^2)\left(\frac{G'}{G}\right)+\lambda\mu A_1\right\}\\&-(\omega+a\kappa^2)\left\{A_0+A_1\left(\frac{G'}{G}\right)\right\}+(b-\kappa^2\beta)\left\{A_0+A_1\left(\frac{G'}{G}\right)\right\}^3\\&-B^2(\alpha+2\xi)\left\{A_0+A_1\left(\frac{G'}{G}\right)\right\}^2\left\{2A_1\left(\frac{G'}{G}\right)^3+3A_1\lambda\left(\frac{G'}{G}\right)^2\right.\\&\left.+(2A_1\mu+A_1\lambda^2)\left(\frac{G'}{G}\right)+\lambda\mu A_1\right\}-B^2(\beta+2\xi-\alpha)\left\{A_0+A_1\left(\frac{G'}{G}\right)\right\}\\&\left\{A_1\left(\frac{G'}{G}\right)^2+\lambda A_1\left(\frac{G'}{G}\right)+A_1\mu\right\}^2=0.\end{aligned} \tag{74}$$

Then, equating the coefficient of each power of G'/G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$B=A_1\sqrt{-\frac{2(b-\kappa^2\beta)}{4a+A_1^2(\lambda^2-4\mu)(\beta+4\xi)}},\tag{75}$$

$$\omega=-a\kappa^2+\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta),\tag{76}$$

$$A_0=\frac{\lambda A_1}{2},\tag{77}$$

$$\alpha=-6\xi-\beta,\tag{78}$$

where κ, λ, μ, and A_1 are arbitrary constants, which immediately prompts the constraint

$$(\kappa^2\beta-b)(4a+A_1^2(\lambda^2-4\mu)(\beta+4\xi))>0.\tag{79}$$

Substituting the solution set (75)–(78) into (72), the solution formulae of (12) can be written as

$$U(\tau)=A_1\left\{\frac{\lambda}{2}+\frac{G'(\tau)}{G(\tau)}\right\}.\tag{80}$$

Substituting the general solutions of second-order linear ODE into (80) gives three types of travelling wave solutions.

Case I: When $\Delta=\lambda^2-4\mu>0$, we obtain the hyperbolic function travelling wave solution

$$q(x,t)=\frac{A_1\sqrt{\lambda^2-4\mu}}{2}\times\left\{\frac{C_1\sinh\left[A_1\sqrt{\frac{(\lambda^2-4\mu)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]+C_2\cosh\left[A_1\sqrt{\frac{(\lambda^2-4\mu)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]}{C_1\cosh\left[A_1\sqrt{\frac{(\lambda^2-4\mu)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]+C_2\sinh\left[A_1\sqrt{\frac{(\lambda^2-4\mu)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]}\right\}\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta)\right)t+\theta\right\}},\tag{81}$$

where C_1 and C_2 are arbitrary constants.

On the other hand, assuming $C_1\neq0$ and $C_2=0$, the topological 1-soliton solution of (1) can be written as

$$q(x,t)=\frac{A_1\sqrt{\lambda^2-4\mu}}{2}\tanh\left[A_1\sqrt{\frac{(\lambda^2-4\mu)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta)\right)t+\theta\right\}}.\tag{82}$$

Next, assuming $C_1=0$ and $C_2\neq0$, then we obtain singular 1-soliton solution for (1) as

$$q(x,t)=\frac{A_1\sqrt{\lambda^2-4\mu}}{2}\coth\left[A_1\sqrt{\frac{(\lambda^2-4\mu)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta)\right)t+\theta\right\}}.\tag{83}$$

Case II: When $\Delta=\lambda^2-4\mu<0$, we obtain the trigonometric function travelling wave solution

$$q(x,t)=\frac{A_1\sqrt{4\mu-\lambda^2}}{2}\times\left\{\frac{-C_1\sin\left[A_1\sqrt{\frac{(4\mu-\lambda^2)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]+C_2\cos\left[A_1\sqrt{\frac{(4\mu-\lambda^2)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]}{C_1\cos\left[A_1\sqrt{\frac{(4\mu-\lambda^2)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]+C_2\sin\left[A_1\sqrt{\frac{(4\mu-\lambda^2)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]}\right\}\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta)\right)t+\theta\right\}},\tag{84}$$

where C_1 and C_2 are arbitrary constants.

Moreover, with the assumption $C_1\neq0$ and $C_2=0$,

$$q(x,t)=-\frac{A_1\sqrt{4\mu-\lambda^2}}{2}\tan\left[A_1\sqrt{\frac{(4\mu-\lambda^2)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta)\right)t+\theta\right\}},\tag{85}$$

and when $C_1=0$, $C_2\neq 0$, the singular periodic solution of (1) will be

$$q(x,t)=\frac{A_1\sqrt{4\mu-\lambda^2}}{2}\cot\left[A_1\sqrt{\frac{(4\mu-\lambda^2)(\kappa^2\beta-b)}{8a+2A_1^2(\lambda^2-4\mu)(\beta+4\xi)}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b-\kappa^2\beta)\right)t+\theta\right\}}. \quad (86)$$

Case III: When $\Delta=\lambda^2-4\mu=0$, we obtain plane wave solution

$$q(x,t)=A_1\left(\frac{C_2}{C_1+C_2\frac{A_1\sqrt{2a(\kappa^2\beta-b)}}{2a}(x+2a\kappa t)}\right)e^{i\{-\kappa x-a\kappa^2 t+\theta\}}, \quad (87)$$

where C_1 and C_2 are arbitrary constants.

Similarly, (35) has the solutions in the form

$$U(\tau)=A_0+A_1\left(\frac{G'(\tau)}{G(\tau)}\right). \quad (88)$$

Substituting (88) along with (73) into (35) leads to

$$aB^2\left\{2A_1\left(\frac{G'}{G}\right)^3+3A_1\lambda\left(\frac{G'}{G}\right)^2+(2A_1\mu+A_1\lambda^2)\left(\frac{G'}{G}\right)+\lambda\mu A_1\right\}-(\omega+a\kappa^2)\left\{A_0+A_1\left(\frac{G'}{G}\right)\right\}+(b+4\kappa^2\xi)\left\{A_0+A_1\left(\frac{G'}{G}\right)\right\}^3=0. \quad (89)$$

Then, equating the coefficient of each power of G'/G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$B=\frac{A_1}{2a}\sqrt{-2a(\beta+4\xi)}, \quad (90)$$

$$\omega=-a\kappa^2+\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi), \quad (91)$$

$$A_0=\frac{\lambda A_1}{2}, \quad (92)$$

where κ, λ, μ, and A_1 are arbitrary constants, which immediately prompts the constraint

$$a(\beta+4\xi)<0. \quad (93)$$

Substituting the solution set (90)–(92) into (88), the solution formulae of (35) can be written as

$$U(\tau)=A_1\left\{\frac{\lambda}{2}+\frac{G'(\tau)}{G(\tau)}\right\}. \quad (94)$$

Substituting the general solutions of second-order linear ODE into (94) gives three types of travelling wave solutions.

Case I: When $\Delta=\lambda^2-4\mu>0$, we obtain the hyperbolic function travelling wave solution

$$q(x,t)=\frac{A_1\sqrt{\lambda^2-4\mu}}{2}\times\left\{\frac{C_1\sinh\left[\frac{A_1}{2}\sqrt{-\frac{(\lambda^2-4\mu)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]+C_2\cosh\left[\frac{A_1}{2}\sqrt{-\frac{(\lambda^2-4\mu)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]}{C_1\cosh\left[\frac{A_1}{2}\sqrt{-\frac{(\lambda^2-4\mu)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]+C_2\sinh\left[\frac{A_1}{2}\sqrt{-\frac{(\lambda^2-4\mu)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]}\right\}\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi)\right)t+\theta\right\}}, \quad (95)$$

where C_1 and C_2 are arbitrary constants.

On the other hand, assuming $C_1\neq 0$ and $C_2=0$, the topological 1-soliton solution of (1) can be written as

$$q(x,t)=\frac{A_1\sqrt{\lambda^2-4\mu}}{2}\tanh\left[\frac{A_1}{2}\sqrt{-\frac{(\lambda^2-4\mu)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi)\right)t+\theta\right\}}. \quad (96)$$

Next, assuming $C_1=0$ and $C_2\neq 0$, then we obtain singular 1-soliton solution for (1) as

$$q(x,t)=\frac{A_1\sqrt{\lambda^2-4\mu}}{2}\coth\left[\frac{A_1}{2}\sqrt{-\frac{(\lambda^2-4\mu)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi)\right)t+\theta\right\}}. \quad (97)$$

Case II: When $\Delta=\lambda^2-4\mu<0$, we obtain the trigonometric function travelling wave solution

$$q(x,t)=\frac{A_1\sqrt{4\mu-\lambda^2}}{2}\times\left\{\frac{-C_1\sin\left[\frac{A_1}{2}\sqrt{-\frac{(4\mu-\lambda^2)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]+C_2\cos\left[\frac{A_1}{2}\sqrt{-\frac{(4\mu-\lambda^2)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]}{C_1\cos\left[\frac{A_1}{2}\sqrt{-\frac{(4\mu-\lambda^2)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]+C_2\sin\left[\frac{A_1}{2}\sqrt{-\frac{(4\mu-\lambda^2)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]}\right\}\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi)\right)t+\theta\right\}}, \quad (98)$$

where C_1 and C_2 are arbitrary constants.

In addition, with the assumption $C_1\neq 0$ and $C_2=0$,

$$q(x,t)=-\frac{A_1\sqrt{4\mu-\lambda^2}}{2}\tan\left[\frac{A_1}{2}\sqrt{-\frac{(4\mu-\lambda^2)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi)\right)t+\theta\right\}}, \quad (99)$$

and when $C_1=0$, $C_2\neq 0$, the singular periodic solution of (1) will be

$$q(x,t)=\frac{A_1\sqrt{4\mu-\lambda^2}}{2}\cot\left[\frac{A_1}{2}\sqrt{-\frac{(4\mu-\lambda^2)(b+4\kappa^2\xi)}{2a}}(x+2a\kappa t)\right]\times e^{i\left\{-\kappa x-\left(a\kappa^2-\frac{A_1^2}{4}(\lambda^2-4\mu)(b+4\kappa^2\xi)\right)t+\theta\right\}}. \quad (100)$$

Case III: When $\Delta=\lambda^2-4\mu=0$, we obtain plane wave solution

$$q(x,t)=A_1\left(\frac{C_2}{C_1+C_2\frac{A_1\sqrt{-2a(b+4\kappa^2\xi)}}{2a}(x+2a\kappa t)}\right)e^{i\{-\kappa x-a\kappa^2 t+\theta\}}, \quad (101)$$

where C_1 and C_2 are arbitrary constants.

3 Conclusions

This work studied the perturbed NLSE that describes the propagation of Langmuir waves in plasmas. The perturbations with full nonlinearity and cubic nonlinearity are considered. We use the Ricatti equation expansion approach, trial solution method, and G'/G-expansion scheme to integrate the perturbed NLSE. As a result, many new exact soliton solutions are derived, and the corresponding parameter constraint conditions are exhibited.

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