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Unexpected Behavior on Nonlinear Tunneling of Chirped Ultrashort Soliton Pulse in Non-Kerr Media with Raman Effect

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Abstract: In this manuscript, the ultrashort soliton pulse propagation through nonlinear tunneling in cubic quintic media is investigated. The effect of chirping on propagation characteristics of the soliton pulse is analytically investigated using similarity transformation. In particular, we investigate the propagation dynamics of ultrashort soliton pulse through dispersion barrier for both chirp and chirp-free soliton. By investigating the obtained soliton solution, we found that chirping has strong influence on soliton dynamics such as pulse compression with amplification. These two important dynamics of chirped soliton in cubic quintic media open new possibilities to improve the solitonic communication system. Moreover, we surprisingly observe that a dispersion well is formed for the chirped case whereas a barrier is formed for the chirp-free case, which has certain applications in the construction of logic gate devices to achieve ultrafast switching.

Keywords: Chirped Soliton; Nonlinear Tunneling; Similarity Transformation; Soliton Compression; Soliton Switching; Symbolic Computation.

1 Introduction

The nonlinear Schrödinger (NLS) equation is one of the most important and universal model in modern nonlinear science. Consequently, NLS equation is used to describe the soliton propagation in various nonlinear media. Because the solitons have been theoretically demonstrated [1, 2] and experimentally observed [3] in optical fibers, optical solitons have been attractive in optical communication systems [4]. In particular, it is well known that temporal optical solitons have been the objects of extensively theoretical and experimental studies recently because of their potential applications in long distance optical fiber communication and all-optical ultrafast switching devices. The type of waveguides used in optical communication systems are generally Kerr type [5]. However, as the intensity of the incident light field becomes stronger, non-Kerr nonlinearity effect comes into play, and because of this additional effect, the physical features and the stability of NLS soliton can change [6]. The influences of non-Kerr nonlinearity on optical soliton propagation are described by the NLS equation with higher-order nonlinear terms [7, 8]. The corresponding refractive index profile for cubic quintic nonlinear media is $n = n_0 + (3\chi^{(3)}/8n_0)I + (5\chi^{(5)}/16n_0)I^2$, in which n_0, $\chi^{(3)}$, and $\chi^{(5)}$ are the linear refractive indexes of the medium, the third-order susceptibility, and the fifth-order susceptibility, respectively. The polarizations induced through these susceptibilities give the cubic and quintic (non-Kerr) terms in an NLS equation.

Very recently, Peng et al. [9] discussed the (3+1)-dimensional spatiotemporal optical solitons in both the dispersive medium with cubic-quintic nonlinearity and the saturable medium. Chirped and chirp-free self-similar solution is obtained for the cubic quintic NLS equation (CQNLSE) with variable coefficients by Dai et al. [10]. In passive optical fibers, the influence of quintic nonlinearity on optical wave breaking has been investigated [11]. The generation of rogue waves induced by cubic quintic and second-order dispersion in metamaterial has been reported [12]. Bright and dark solitary wave solutions are derived for higher order nonlinear Schrödinger (HNLS) with quintic non-Kerr nonlinearities in the study of Choudhuri and Porsezian [13]. Ultrashort pulse propagation in twin core nonlinear fiber with cubic quintic effect has been investigated by Qi et al. [14]. Interaction behaviors of soliton in cubic quintic nonlinear media with Raman effect have been investigated by Wang et al. [15]. Moreover, the variable coefficients of NLSEs have attracted a great deal of interest in dispersion-managed optical fibers [16, 17]. As seen from previous studies, the HNLS equation with variable coefficients in the anomalous group velocity dispersion (GVD) regime of the inhomogeneous optical fibers has been investigated from different points of view (e.g. artificially induced inhomogeneity, randomly induced

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imperfections, and soliton control and management), and some analytic soliton solutions for special parameter relations have been obtained [18].

Optical pulse compression finds important applications in optical fibers. The pulse compressors based on the nonlinear effects in optical fibers can be classified into two major categories: fiber grating or prism pair and soliton effect compressors [19]. The grating pair provides anomalous GVD required for the compression of positively chirped pulses, whereas the soliton effect compressor consists of only a piece of fiber whose length is suitably chosen. These methods are convenient with sources that inherently produce chirped pulses. Recently, it was shown that the chirped pulses find applications in pulse compression or amplification and are particularly useful in designing fiber-optic amplifiers, optical pulse compressors, and solitary wave-based communication links [20]. Very recently, Cao et al. [21] have confirmed that the chirping enhances the supercontinuum generation and improves the spectral flatness. The chirped pulses have been investigated analytically [22] as well as numerically [23]. In the context of a nonlinear chirp, femtosecond pulse propagation with non-Kerr nonlinear terms and cubic-quintic-septic nonlinearities was investigated [24]. Senthil Nathan et al. [25] claimed the pedestal-free soliton compression through chirped soliton in non-Kerr media. The effects of higher-order terms on chirped chiral soliton propagation have been extensively studied by Vyas et al. [26]. Chirped and chirp-free nonautonomous solitons have been discussed with the dispersion and nonlinearity management [27]. Very recently, the amplification of unchirped soliton pulse in dispersion decreasing fiber has been reported [28].

The concept of soliton tunneling is a new and exciting area in the optical soliton-based communication system. The tunneling process is the transmission of solitons across regions in which considerable changes in the variable coefficients are found. It should be emphasized that the nonlinear soliton tunneling effect is a subject of constantly renewed interest. In the femtosecond nonlinear fiber optics, the most intriguing enigma of optical solitons is connected to the so-called soliton spectral tunneling effect [29]. This effect is characterized in the spectral domain by the passage of a femtosecond soliton through a potential barrierlike spectral inhomogeneity of GVD, including the forbidden band of positive GVD. Recently, the tunneling effects of solitons governed by various NLS equations were extensively investigated. Wang et al. [30] studied the tunneling effects of spatial similaritons passing through the nonlinear barrier (or well). Dai et al. [31] discussed the tunneling effects of bright and dark similaritons in a two-dimensional (planar) waveguide. Zhong and Belić [32] confirmed that a one-dimensional optical pulse can be compressed by the nonlinear barrier (or well) or dispersion barrier (well) with an exponential decay of coefficients. The tunneling effects of bright and dark similaritons passing through the nonlinear barrier (or well) governed by the generalized coupled NLS equations in the birefringent fiber have been reported by Dai et al. [33]. Dai et al. [34] and Zhu [35] discussed nonlinear tunneling for controllable rogue waves in (1+1) and (2+1) dimensions, respectively. Recently, enigmas of optical and matter-wave soliton nonlinear tunneling have been revealed by Belyaeva et al. [36]. The nonlinear tunneling of optical solitons through strong nonlinear organic thin films and polymeric waveguides has also been investigated and exhibits jumplike nonadiabatic fission reactions, eventually resulting in soliton “fission reactions” [37]. The nonlinear tunneling of femtosecond soliton in birefringence fiber has been studied [38].

However, we notice that there has not been a discussion about the effect of chirping on soliton tunneling in cubic quintic media with Raman effect. This may provide a physical motivation for our work.

The paper is organized as follows. In Section 2, the theoretical model is introduced and the respective similarity transformation is presented. Section 3 presents the one-soliton solution. For specific choices of variable coefficients, the dynamical behavior of chirped soliton solution is discussed in the aspect of nonlinear tunneling through barrier or well in Section 4. Finally, concluding remarks are presented in Section 5.

2 Theoretical Model and Similarity Transformation

In this paper, we will investigate the inhomogeneous cubic-quintic NLS equations with Raman effect, which describe the effects of quintic nonlinearity on the ultrashort optical pulse propagation in non-Kerr media. In particular, we have to focus our investigation on the effect of chirping on soliton tunneling:

$$iQ_z+\frac{D(z)}{2}Q_{tt}+R(z)\left|Q\right|^2Q+\Omega(z)\left|Q\right|^4Q+i\alpha(z)Q\frac{\partial(\left|Q\right|^2)}{\partial t}=0, \quad (1)$$

where $Q(z, t)$ is the complex envelope of the electric field, and z and t are the normalized distance and time, respectively. $D(z)$ represents the GVD, and $R(z)$ and $\Omega(z)$ are the cubic and quintic nonlinearity functions, respectively. $\alpha(z)$ represents the Raman effect, responsible for the self-frequency shift [39]. The polarizations induced through the third- and fifth-order susceptibilities give the cubic and quintic (non-Kerr) terms in an NLS equation. The nonlinearity that arises because of the fifth-order susceptibility can be obtained in many optical materials such as semiconductors, semiconductor-doped glasses, AlGaAs, polydiacetylene toluene sulfonate, chalcogenide glasses, and some transparent organic materials.

Our first goal is to transform (1) into the constant coefficient CQNLSE with Raman effect, which is popularly known as the Kundu-Eckhaus equation [40]:

$$iU_Z+a_1U_{TT}+a_2|U|^2U+a_3|U|^4U+ia_4U\frac{\partial(|U|^2)}{\partial T}=0, \tag{2}$$

where a_1, a_2, a_3, and a_4 are real constants.

The last two nonlinear terms in (2) describe the propagation of ultrashort femtosecond pulses in an optical fiber. For example, the higher power nonlinearity and higher-order dispersion terms must be taken into account when the intensity of the incident light gets stronger and the pulse width becomes narrower. Both the theoretical and the experimental studies show the use of the nonlinear dependency of the refractive index on field intensity in NLS equation, possessing remarkable behavior on soliton dynamics. To get the soliton solutions of (1) with those of (2), we will construct the transformation [41] as follows:

$$Q(z,t)=A(z)U(Z(z),T(z,t))\exp[i\phi(z,t)], \tag{3}$$

where $A(z)$ is the amplitude, and the effective propagation distance $Z(z)$ and the similarity variable $T(z, t)$ are both to be determined. The substitution of (3) into (1) leads to (2) with the following set of partial differential equations:

$$A_z+D(z)_zA\phi_{tt}=0, \tag{4}$$

$$T_z+2D(z)_zT_t\phi_t=0, \tag{5}$$

$$\phi_z+D(z)(\phi_t)^2=0, \tag{6}$$

$$T_{tt}=0, \tag{7}$$

$$D(z)(T_t)^2-a_1Z_z=0, \tag{8}$$

$$R(z)(A)^2-a_2Z_z=0, \tag{9}$$

$$\Omega(z)(A)^4-a_3Z_z=0, \tag{10}$$

$$G(z)(A)^2T_t-a_4Z_z=0. \tag{11}$$

With the help of symbolic computation, one obtains the following expressions for the similarity variable, width, central position, amplitude, effective propagation distance, and phase of the pulse:

$$T=\frac{t-t_c(z)}{W(z)}, \tag{12}$$

$$W(z)=W_0(1-c_0d(z)), \tag{13}$$

$$t_c(z)=t_0-(c_0t_0+b_0)d(z), \tag{14}$$

$$A(Z)=\frac{\sqrt{a_2D(z)}}{\sqrt{a_1R(z)W(Z)}}, \tag{15}$$

$$Z(z)=\frac{d(z)}{a_1W_0W(z)}, \tag{16}$$

$$\phi(z,t)=-\frac{c_0}{4(1-c_0d(z))}t^2-\frac{b_0}{2(1-c_0d(z))}t-\frac{b_0^2d(z)}{4(1-c_0d(z))}. \tag{17}$$

Further, the necessary and sufficient condition for the existence of the one-soliton solution is given as follows:

$$\Omega(z)=\frac{a_1a_3R(z)^2W(z)^2}{a_2^2D(z)}, \tag{18}$$

$$\alpha(z)=\frac{a_4a_3R(z)W(z)}{a_2}. \tag{19}$$

$W(z)$ and $t_c(z)$ are the width and the central position of the soliton, respectively.

3 Soliton Solution

It has been proven that the exact soliton solution of (2) can exist under the condition $2a_3=-\rho a_4$. In this case, we obtain the one-soliton solution of (2) as follows:

$$Q(z,t)=A(z)\varphi(Z,T)\exp[i\phi(z,t)], \tag{20}$$

where

$$\varphi(Z,T)=\frac{m}{2}Sech\left(\frac{\delta+\xi+\xi^*}{2}\right)$$
$$\exp\left\{i\frac{a_1k^2d(z)}{a_2W_0W(z)}-\frac{\delta}{2}+i\int\rho\frac{mm^*\exp(\xi+\xi)}{(1+\exp(\delta+\xi+\xi^*))^2}\mathrm{dt}\right\},$$

$$\mathrm{Exp}(\delta)=\frac{a_2 mm^*}{2a_1(k+k^*)^2},$$

$$\xi=k\frac{t-t_c(z)}{W(z)}+i\frac{a_1k^2d(z)}{a_2W_0W(z)}.$$

ρ is the arbitrary real constant, and m is the arbitrary constant. By choosing the suitable parameters approximate to the real situation, it is possible to explain the various soliton controls and dispersion-nonlinearity management. Therefore, the analytic one-soliton solution (5) is of practical importance to characterize the propagating features of the chirp and chirp-free ultrashort soliton pulses through the potential barrier with inhomogeneous effects in cubic quintic (CQ) nonlinear medium. Very recently, bright and dark soliton solutions for (2) have been obtained through bilinear BT [42]. Very recently, rogue wave solutions have been obtained for coupled cubic quintic NLS equation with variable coefficients [43].

4 Results and Discussion

With certain dispersion and nonlinearity managements, the optical soliton transmission in the optical fibers can be controlled [44–46]. Consequently, by choosing the specific choice of dispersion and nonlinearity profiles of the inhomogeneous optical fibers, we will discuss how to achieve the switchable soliton and soliton amplification and compression effectively through chirping. The meaning of the results lies in two aspects: first, we will confirm that a switchable chirped soliton in the anomalous dispersion regime of the inhomogeneous optical non-Kerr fiber exists. Second, by choosing the variable coefficient profiles, we will show the achievement of soliton amplification with compression via the nonlinear tunneling of chirped soliton through the dispersion barrier on the exponential background. To our knowledge, this kind of soliton control has not been reported so far.

4.1 Effect of Chirping on Soliton Tunneling without the Exponential Background

To further understand the previously mentioned problem, we calculate variable coefficients with a dispersion barrier in which dispersion and nonlinearity have the following form [47, 48]:

$$D(z)=d_0\exp(-rz)+hSech(\mu(z-z_0)),$$

$$R(z)=R_0\exp(-rz). \qquad (21)$$

In this expression, a positive sign indicates the dispersion potential barrier. Here, we have to consider the dispersion barrier or well with exponential decay; h indicates the barrier's amplitude, μ is the parameter relating to the barrier's width, z_0 denotes the position of the barrier, and r is a decaying parameter.

Here, we compare the dynamics of soliton tunneling with chirp and chirp-free soliton for the same sign of barrier's height ($h>0$). Note that chirped soliton gets compressed width while passing through the barrier even in the absence of the exponential background ($r=0$). Without the chirping, when solitons pass through the dispersion barrier, they amplify and form the peak and then attenuate and recover its original shape, as shown in Figure 1a. For the dispersion well, the reverse case occurs. By contrast, the barrier will disappear and form a dip in the presence of chirping, as depicted in Figure 1b. This feature is interesting, and this is the first time that it is reported. It underlines an important feature on the effect of the chirp parameter. From this result, we conclude that the tunneling behavior of the soliton is completely affected in the presence of chirping. Moreover, this chirp in the soliton leads to efficient compression as well as amplification and, thus, is particularly useful in the design of optical fiber amplifiers and optical pulse compressors in the solitonic communication system.

4.2 Effect of Chirping on Soliton Tunneling with Exponential Background

Next, we consider the compression problem of the soliton pulse when it passes through the barriers of dispersion or nonlinearity with a special form. In this case, we consider the exponential decay, and the decaying parameter r should not be equal to zero; i.e. $r\neq 0$. d_0 and R_0 are constant parameters; i.e. $d_0>0$ and $R_0>0$.

Figure 2a provides the soliton pulse width management technique through the soliton tunneling with exponential decay in the absence of chirping. The main features of the soliton pulse consist of the fact that the soliton's propagation direction is changed (phase shift), the amplitude is invariant, and the pulse width gradually becomes narrower and narrower during propagation. In the presence of chirping, soliton pulse is amplified exponentially during the propagation with more effective compression after crossing the dispersion barrier, as shown in Figure 2c. Furthermore, we would like to stress

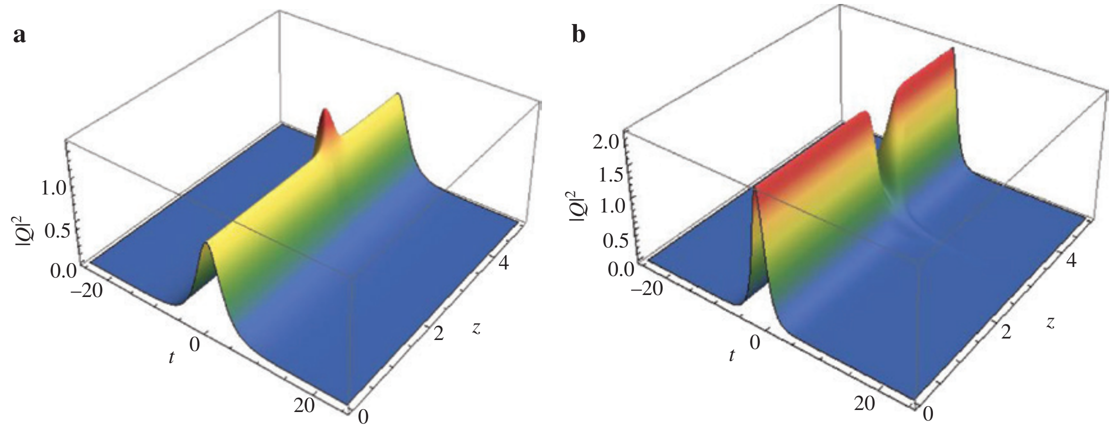

Figure 1: (a) Soliton propagation through dispersion barrier with $a_1 = a_2 = 1$, $z_0 = 3$, $\mu = 4$, $r = 0$, $h = 1$, and $c_0 = 0$. (b) $c_0 = 1.5$, and remaining parameters are the same as in panel (a).

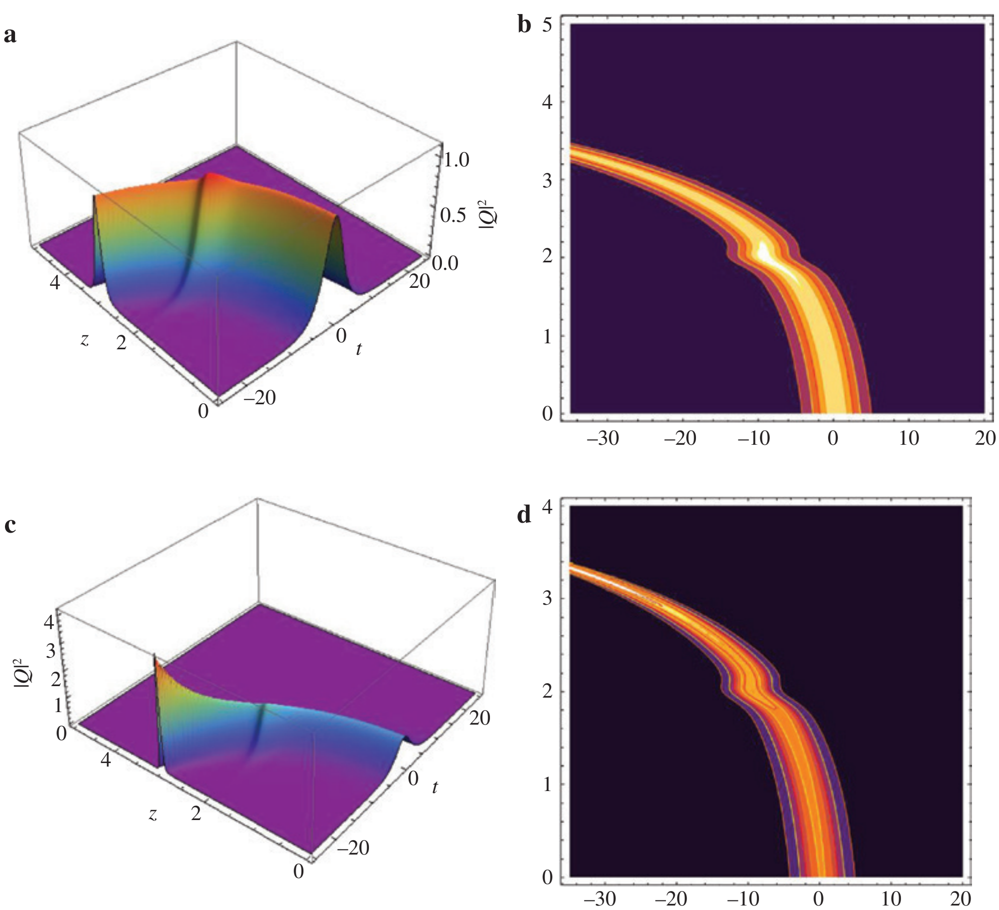

Figure 2: (a) Soliton propagation through dispersion barrier without chirping. Soliton propagation through dispersion barrier with $a_1 = a_2 = 1$, $z_0 = 2$, $\mu = 4$, $r = 0$, $h = 1$, and $c_0 = 0$. (b) Corresponding density plot. (c) Soliton propagation through dispersion barrier with chirping. Soliton propagation through dispersion barrier with $a_1 = a_2 = 1$, $z_0 = 2$, $\mu = 4$, $r = 0$, $h = 1$, and $c_0 = 1.3$. (d) Corresponding density plot.

that chirping does not influence on the phase. In optical communication systems, by using this technique, fiber loss can be effectively overcome. Similarly, in Figure 2, it can be seen that after passing the well, the soliton is compressed about $z = z_0$, which indicates that the pulse can be compressed to a desired width and amplitude in a controllable (lossless, increasing, or decreasing) manner by the choice of the chirping parameter.

4.3 Effect of Chirping on Cubic Quintic and Raman Effect

Next we discuss the influence of quintic nonlinearity and Raman effect along with the chirping on the formation of barrier or well. From the variation of cubic quintic and Raman term with propagation distance as shown in Figure 3, one can make the barrier or well to regulate the tunneling without modifying the height and the width. Here, we would like to notice that for unchirped soliton pulse, the Raman effect is invariant along the propagation distance, as depicted in Figure 3b. This presents a potentially important method for controlling the Raman effect through chirping in cubic quintic media.

The chirping parameter is powerful enough to be the dominant factor to control the soliton amplification and the soliton compression, as shown in Figure 4a. During the propagation, the soliton amplitude increases rapidly, and the soliton width is compressed because of the value of chirp ($c_0 \neq 0$). The results have shown that the soliton's width and amplitude are strongly affected by the chirping parameter alone, even for homogeneous medium. In this study, we achieve this behavior for constant values of GVD and nonlinearity coefficients, which means that the medium is homogeneous. Furthermore, in the absence of chirping, soliton width remains unchanged or invariant, which can be clearly perceived in Figure 4b. Intuitively, the chirping parameter plays an important role to regulate the width of solitons (dashed line), as shown in Figure 4b. We can also infer that in the presence of chirping, width function is fully manipulated by the dispersion parameter through (13). Note that the dip is formed at $z = z_0$ because of the choice of $z_0 = 3$.

4.4 Switchable Characteristics of Chirped Soliton in CQ Media

On the exponential background, for both chirp and chirp-free case, no significant change in the dispersion barrier

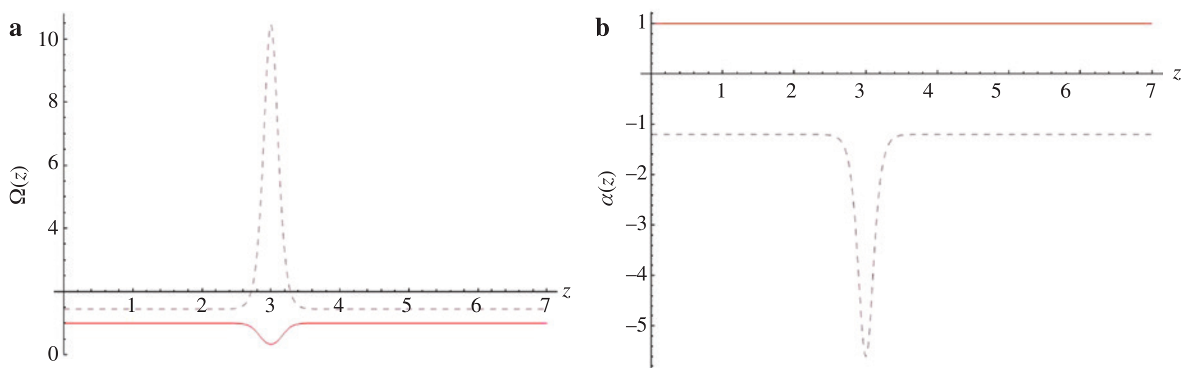

Figure 3: (a) Variation of quintic nonlinearity with propagation distance for chirped (dashed line) and unchirped (solid line). (b) Variation of Raman term with propagation distance for chirped (dashed line) and unchirped (solid line).

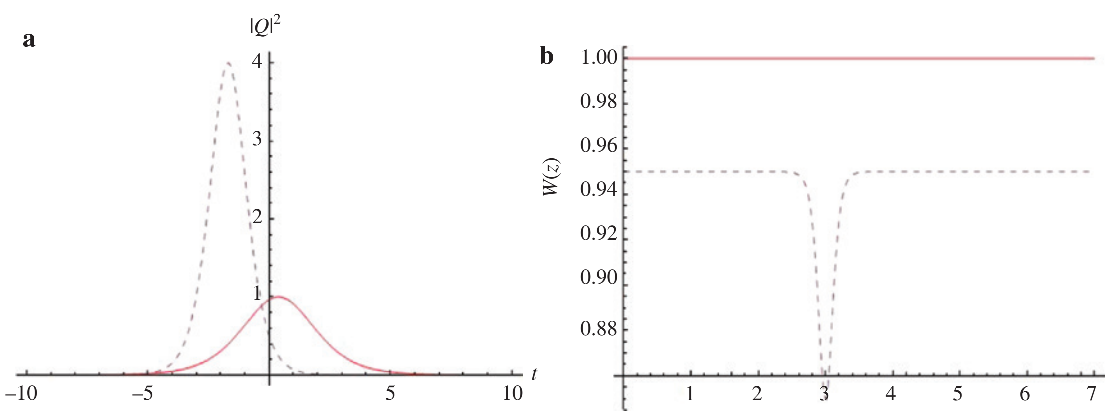

Figure 4: (a) Intensity profile for chirped (dashed line) and unchirped (solid line) soliton. (b) Variation of width function with propagation distance for chirped (dashed line) and unchirped (solid line) soliton.

was found. By contrast, with the absence of the exponential background, we observed the existence of barrier for chirp-free bright soliton propagation through tunneling, whereas well is formed for chirped bright soliton propagation, as depicted in Figure 1a and b. For the first time, this peculiar property of soliton in non-Kerr media is noted. The study related to the effect of chirping on soliton tunneling with Raman effect has never been reported earlier to our knowledge. Moreover, we suggest that this interesting property of soliton can be used to construct logic gate switching devices by controlling chirping. Thus, we can achieve the soliton switching in CQ media by induced chirping via tunneling.

5 Conclusions and Remarks

In this paper, similarity transformation is used to find the soliton solution of the quintic NLS equation with Raman effect. We have explicitly calculated the chirp and chirp-free bright soliton solutions. Using obtained soliton solution, the tunneling behavior of soliton along the propagation direction is discussed. To understand the effect of chirp parameter c_0 on optical solitons in CQ media, we investigate the transmission property. From the previously mentioned results, we conclude that the chirp parameter affects the intensity, the width, and the tunneling behavior of soliton, which is quite different from the chirp-free one. We also report herein the existence of a barrier for chirp-free bright soliton propagation through tunneling, whereas a well is formed for the chirped pulse propagation. We can discuss the same result for the dark soliton pulse. For the limit of length, we neglect them in our present paper. For physical and optical interests, relevant properties of the soliton solution have been analyzed and graphically discussed in Figures 1–4.

The nonlinear tunneling of optical soliton in non-Kerr optical medium exhibits several interesting propagation scenarios discussed as follows:

1. Soliton compression and amplification can be enhanced by chirping, which has been observed in Figure 2c, and the soliton structure is not destroyed during the process. By Comparing Figure 2a with Figure 2c, we found that $c_0 > 0$ has some effect on the unlimited transmission of the compressed amplified soliton in an inhomogeneous fiber. These properties of soliton pulse amplification and compression are beneficial in the amplification noise suppression for the ultrahigh bit rate long-haul optical communication systems.
2. By locating the barrier's position, one can achieve the switchable soliton by chirping in CQ nonlinear media. This peculiar property of chirped soliton in CQ media opens a new window to achieve the switchable soliton via tunneling at desired location. Through our results, one can conclude that the chirp parameter c_0 has a universal influence on soliton dynamics. These interesting propagation scenarios of chirped soliton in non-Kerr media exhibit various possibilities to construct soliton-based logic gate devices. We hope that the previously mentioned phenomena could be observed in future experiments and the results could be useful for soliton control, soliton compression, and amplification to improve the solitonic communication system. The results obtained in this paper may be considered as potential applications in areas such as nonlinear optical switches and all-optical data processing schemes and in the design of pulse compressors and amplifiers. Our results are of special application in ultrafast optical pulse propagation systems, and further research should be an interesting task. The results presented here offer a new scheme for the implementation of soliton compression and splitting and may have promising applications in future all-optical devices based on soliton signals.

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