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Reliable Sampled-Data Control of Fuzzy Markovian Systems with Partly Known Transition Probabilities

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Abstract: This article presents a fuzzy dynamic reliable sampled-data control design for nonlinear Markovian jump systems, where the nonlinear plant is represented by a Takagi–Sugeno fuzzy model and the transition probability matrix for Markov process is permitted to be partially known. In addition, a generalised as well as more practical consideration of the real-world actuator fault model which consists of both linear and nonlinear fault terms is proposed to the above-addressed system. Then, based on the construction of an appropriate Lyapunov–Krasovskii functional and the employment of convex combination technique together with free-weighting matrices method, some sufficient conditions that promising the robust stochastic stability of system under consideration and the existence of the proposed controller are derived in terms of linear matrix inequalities, which can be easily solved by any of the available standard numerical softwares. Finally, a numerical example is provided to illustrate the validity of the proposed methodology.

Keywords: Fuzzy Dynamic Reliable Controller; Markovian Jump Systems; Stochastic Stability.

1 Introduction

In real-world problems, most of dynamical systems often encounter random changes in variable structures and parameters, which can be commonly modelled as Markovian jump systems (MJSs). These changes may be due to component failures and repairs, changing subsystems' interconnections, and sudden environmental disturbances. To mention few examples of MJSs are as follows: manufacturing systems, solar thermal receivers, communication systems, electrical power systems, and robot manipulators. Generally, MJS is modelled by a set of systems with the transitions between the systems determined by a Markov process that takes values over a finite set. So far in the literature, most of the researchers have only considered the environment that the transition probabilities of MJSs are completely known and a great number of fruitful results such as stability and stabilisation analysis [1, 2], H_∞ control [3], and dissipativity control [4] has been reported for this kind of systems. Nevertheless, in practice, the transition probabilities of MJSs may not be exactly known; thus, it is necessary and important to further consider a more general jump system with partly known transition probabilities. Based on this hypothesis, very recently, the problems of the stability and stabilisation for continuous-time and discrete-time MJSs have been investigated in [5–11].

On the other side, it is commonly known that most of the physical systems and the industrial processes are described by nonlinear models, so it is important to design a suitable controller for nonlinear systems to preserve their stability [12–14]. At the same time, the general description of Takagi–Sugeno (TS) fuzzy systems provides an efficient way to describe many nonlinear systems as an average weighted sum of linear subsystems. This particular form offers a general framework to represent the nonlinear plant and provides an effective platform to facilitate the stability analysis and the controller synthesis. Due to these facts, TS fuzzy model-based technique has been intensively employed to deal the nonlinear MJSs and also many important results have been reported. For instance, see [15, 16]. In [17], a robust state feedback fuzzy controller was designed for a class of uncertain Markovian jump nonlinear systems with partially known transition probabilities. The problem of H_∞ fuzzy control for a class of nonlinear singular MJSs

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with partly known transition rates was investigated in [18]. In addition, the problem of global exponential stability of Markovian jump fuzzy cellular neural networks with uncertain transition rates was studied in [6].

Due to the rapid development of modern computer technologies, most of the continuous plants are controlled by the digital controllers, which are called as sampled-data control systems. Besides, it is difficult to measure the continuous information about the states of real plants and also quite complicate to analyze the continuous information of control signal in all time. Thus, the sampled-data control technology has grew up more superior than other control approaches. Recently, much attentions have been paid to the analysis of the sampled-data control systems. For examples, see [19–22]. The problem of dissipative control for TS fuzzy systems under time-varying sampling with a known upper bound on the sampling intervals was reported in [23]. The sampled-data H_∞ control design of uncertain active suspension systems via fuzzy model approach was proposed in [24].

In many practical control systems, the actuators aging, zero shift, electromagnetic interference, and nonlinear amplication in different frequency fields are the main sources for actuator faults. Moreover, unexpected faults and failures may occur in the actuators that result substantial damages in the control systems. Therefore, a high degree of fault tolerance control is essential for providing overall better performance of the control systems. On the other hand, maintenance or repairs in the highly automated industrial systems cannot be always achieved. In such a situation, fault-tolerant or reliable control can assure to retain the desired closed-loop system performances while tolerating such failures. In [25], a robust fault-tolerant sampled-data H_∞ control design for uncertain offshore steel jacket platform systems with linear fractional uncertainties was proposed. Li et al. [26] designed a sampled-data controller for interval type-2 fuzzy systems with actuator faults. The robust reliable stabilisation problem for a class of uncertain TS fuzzy systems with time-varying delays was investigated in [27]. The reliable control problem for a class of uncertain switched cascade nonlinear systems in the presence of structural uncertainties was studied in [28]. The delay-dependent robust fault detection problem for continuous-time MJSs with partly unknown transition rates and time-varying delay was discussed in [29]. However, to the best of our knowledge, the problem of sampled-data control for TS fuzzy MJSs with partly known transition probabilities and mixed actuator failures has not been reported in the literature that motivates us to consider this interesting problem.

Based on the above-mentioned discussions, in this article, a control design method for reliable sampled-data controller of TS fuzzy MJSs with partly known transition probabilities and mixed actuator failures is presented. In particular, the novelty of this article lies in the fact that based on fuzzy logic rules, a more realistic actuator model consisting linear and nonlinear faults/failures is considered, which can be visualised in many real-world situations. The main contributions of this paper include the followings:

- By adopting a generalised actuator fault model containing both linear and nonlinear terms, the reliable controller with sampled-data is designed for a class of uncertain TS fuzzy systems.
- The free-connection weighting matrix method is proposed to study the stability of MJSs through considering the relationship among the transition rates of various subsystems.
- Based on these matters, some improved and computationally more efficient delay-dependent stabilisability criteria are obtained for the uncertain TS fuzzy systems.

Finally, two simulation examples are provided to illustrate the effectiveness of the developed results.

Notations: The superscripts "T" and "(–1)" stand for matrix transposition and matrix inverse, respectively; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively; the notation $P>0(\geq 0)$ means that P is real, symmetric, and positive definite (positive semidefinite); I is the identity matrix of appropriate dimension; the notation $\mathbb{E}[\cdot]$ stands for the expectation operator; and "$*$" is used to represent a term that is induced by symmetry.

2 Problem Formulation and Preliminaries

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, where Ω is the sample space, $\mathcal{F}$ the algebra of events, and $\mathbb{P}$ the probability measure defined on $\mathcal{F}$. In this article, we consider a class of uncertain continuous-time MJSs over the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, which can be described by the following TS fuzzy model:

$$\begin{aligned}&\text{Plant rule } l\text{:}\quad \text{IF } \theta_1(t) \text{ is } \Gamma_1^l,\ \theta_2(t) \text{ is } \Gamma_2^l \cdots\\ &\qquad \text{and } \theta_p(t) \text{ is } \Gamma_p^l \text{ THEN}\\ &\dot{x}(t)=(A_l(r(t))+\Delta A_l(r(t),\,t))x(t)+(B_l(r(t))\\ &\qquad +\Delta B_l(r(t),\,t))u^f(t),\end{aligned} \tag{1}$$

where $\theta_s(t)$, Γ_s^l ($l=1, 2, \ldots, r$, $s=1, 2 \ldots, p$) are the premise variables and the fuzzy sets, respectively; r is the number of IF-THEN fuzzy rules; $x(t) \in \mathbb{R}^n$ is the system state; and $u^f(t)$ is the control input with mixed actuator faults. The system mode $r(t)(t \geq 0)$ is a continuous-time Markov process on the above-specified probability space, which takes values in a finite state space $\mathbb{N}=\{1, 2, \ldots, \mathcal{N}\}$. The set $\mathbb{N}$ comprises the operation modes of the system and the system mode $r(t)$ has the following mode transition probabilities:

$$\text{Prob}(r(t+\Delta t)=j|r(t)=i)=\begin{cases}\pi_{ij}\Delta(t)+o(\Delta(t)), \text{ if } i\neq j,\\ 1+\pi_{ii}\Delta(t)+o(\Delta(t)), \text{ if } i=j,\end{cases}$$

where $\Delta(t)>0$ and $\lim\limits_{\Delta(t)\to 0}\dfrac{O(\Delta(t))}{\Delta(t)}=0$; $\pi_{ij} \geq 0$ is the transition rate from mode i at time t to mode j at time $t+\Delta(t)$ if $i\neq j$ and $\pi_{ii}=-\sum\limits_{j=1,i\neq j}^{\mathcal{N}}\pi_{ij}$; $A_l(r(t))$ and $B_l(r(t))$ denote the system matrices of l-th rule, which are known and real-valued constant matrices of appropriate dimensions. $\Delta A_l(r(t), t)$ and $\Delta B_l(r(t), t)$ are the uncertain matrices with appropriate dimensions representing time-varying parameter uncertainties and satisfying the following condition:

$$[\Delta A_l(r(t), t)\Delta B_l(r(t), t)]=M_l(r(t))F_l(r(t), t)[N_{al}(r(t))N_{bl}(r(t))],$$

where $F_l(r(t), t)$ is the unknown time-varying matrix satisfying $F_l^T(r(t), t)F_l(r(t), t)\leq I$, $\forall t\geq 0$, $r(t)\in\mathbb{N}$; $N_{al}(r(t))$ and $N_{bl}(r(t))$ are known constant matrices of appropriate dimensions.

It is noteworthy that most of the traditional works on MJSs are investigated with completely known transition probabilities, see [2–4, 6, 7, 16, 21]. However, in practice, it is not quite easy to measure all the transition probabilities because of their cost is too expensive. Therefore, the studies on MJSs with partly known or incomplete transition probabilities have attracted much attention among the research communities. Based on this scenario, for instance, the transition rate matrix Π for system (1) with $\mathcal{N}$ operation modes is expressed as

$$\Pi=\begin{bmatrix}\pi_{11} & ? & \pi_{13} & \ldots & \pi_{1\mathcal{N}}\\ \pi_{21} & ? & ? & \ldots & ?\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ ? & \pi_{\mathcal{N}2} & \pi_{\mathcal{N}3} & \ldots & \pi_{\mathcal{N}\mathcal{N}}\end{bmatrix},$$

where "?" represents the unknown transition rate. For the simplicity of presentation, we introduce the following notation: For all $i\in\mathbb{N}$, let $\mathbb{T}=\mathbb{T}_{\mathbb{K}}^i+\mathbb{T}_{\mathbb{UK}}^i$ where

$$\mathbb{T}_{\mathbb{K}}^i=\{j\text{:if } \pi_{ij} \text{ is known}\} \text{ and } \mathbb{T}_{\mathbb{UK}}^i=\{j\text{:if } \pi_{ij} \text{ is unknown}\}.$$

Moreover, if $\mathbb{T}_{\mathbb{K}}^i\neq\varnothing$, then $\mathbb{T}_{\mathbb{K}}^i$ can also be rewritten as $\mathbb{T}_{\mathbb{K}}^i=\{k_1^i, \ldots, k_m^i\}$, $1\leq m\leq\mathcal{N}$, where k_m^i represents the m-th known element with index k_i^i in the i-th row of matrix Π.

Now, we will present the reliable controller design for the above-considered fuzzy-model-based uncertain MJSs (1). It should ne noted that most of the existing fault-tolerant control results are mainly focused with the linear multiplicative fault matrix. However, the actuator faults need not be linear multiplicative faults all the time because it sometimes may couple with nonlinearity due to some mechanical reasons such as the dead zone and relay. Therefore, it is necessary and important to consider the nonlinearity in the actuator faults. Taking this fact into account, in this article, the actuator fault model of the control input can be chosen in the following form:

$$u^f(t)=\Xi_1 u(t)+\Phi(u(t)), \tag{2}$$

where $0<\Xi_1=diag\{e_1, e_2, \ldots, e_m\}\leq I$ is the actuator fault matrix, $u(t)$ the state feedback control input and is given by $u(t)=K_m(r(t))x(t)$, where $K_m(r(t))$ are the state feedback gain matrices to be determined and the vector function $\Phi(u(t))=[\Phi_1(u(t)), \Phi_2(u(t)), \ldots, \Phi_m(u(t))]^T$ is the nonlinear fault term of the true control input $u(t)$, which is assumed to satisfy the following condition

$$\Phi^T(u(t))\Phi(u(t))\leq u^T(t)\Xi_2 u(t), \tag{3}$$

where $\Xi_2=diag\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$.

By implementing the fuzzy model approach together with the sampled-data technique in the control design part, the reliable controller (2) can be further written as

Control rule m: IF $\theta_1(t_k)$ is Γ_1^m, $\theta_2(t_k)$ is Γ_2^m, $\cdots$, $\theta_p(t_k)$ is Γ_p^m THEN

$$u^f(t)=\Xi_1 K_m(r(t))x(t_k)+\Phi(u(t_k)), \ t_k\leq t<t_{k+1}, \tag{4}$$

where t_k is the sampling instant satisfying $0<t_1<t_2<\cdots<t_k<\cdots$ and $x(t_k)$ is the state vector of m-th subsystem at instant t_k. Let $\tau(t)=t-t_k$, $t_k\leq t\leq t_{k+1}$ be a piecewise linear function with derivative $\dot{\tau}(t)=1$ for $t\neq t_k$. It is clear that $0\leq\tau(t)<t_{k+1}-t_k<\tau$, where τ is the maximum bound of distance between any two sampling instants. Then, (4) can be rewritten as

$$u^f(t)=\Xi_1 K_m(r(t))x(t-\tau(t))+\Phi(u(t-\tau(t))).$$

Moreover, the defuzzified output of the fuzzy-model-based sampled-data controller can be expressed in the following form:

$$u^f(t)=\sum_{m=1}^{r}h_m(\theta(t_k))[\Xi_1 K_m(r(t))x(t-\tau(t))+\Phi(u(t-\tau(t)))]. \tag{5}$$

Using the center-average defuzzifer, product inference, and singleton fuzzifier, (1) can be inferred as

$$\dot{x}(t)=\sum_{l=1}^{r} h_l(\theta(t))[(A_l(r(t))+\Delta A_l(r(t),t))x(t) \\ +(B_l(r(t))+\Delta B_l(r(t),t))u^f(t)], \quad (6)$$

where $h_l(\theta(t))=\omega_l(\theta(t))/\sum_{l=1}^{r}\omega_l(\theta(t))$ and $\sum_{l=1}^{r}h_l(\theta(t))=1$; $\omega_l(\theta(t))=\prod_{s=1}^{p}\Gamma_s^l(\theta_s(t))$ in which $\Gamma_s^l(\theta_s(t))$ is the grade of membership of $\theta_s(t)$ in Γ_s^l.

By considering the fault control law (2) together with the sampled-data control law (5), the closed-loop system of (1) can be written as follows:

$$\dot{x}(t)=\sum_{l=1}^{r}\sum_{m=1}^{r} h_l(\theta(t))h_m(\theta(t_k))[(A_{li}+\Delta A_{li}(t))x(t) \\ +(B_{li}+\Delta B_{li}(t))\Xi_1 K_{mi}x(t-\tau(t)) \\ +(B_{li}+\Delta B_{li}(t))\Phi(u(t-\tau(t)))], \quad (7)$$

where $A_{li}=A_l(r(t))$, $\Delta A_{li}(t)=\Delta A_l(r(t),t)$, $B_{li}=B_l(r(t))$, $\Delta B_{li}(t)=\Delta B_l(r(t),t)$, and $K_{mi}=K_m(r(t))$.

Remark 2.1: It should be mentioned that the reliable or fault-tolerant controller under consideration (2) is more general because it contains all the possible faults of actuators which are given as follows:

(i) if $\Xi_1=1$ and $\Phi(\cdot)=0$, then the actuator is healthy one;
(ii) if $\Xi_1<1$ and $\Phi(\cdot)=0$, then the actuator may loss its effectiveness;
(iii) if $\Xi_1<1$ and $\Phi(\cdot)$ is constant, then the actuator partially strucks and it can be adjustable; and
(iv) if $\Xi_1<1$ and $\Phi(\cdot)$ is time varying, then the actuator may loss its effectiveness and may go slightly out of the control.

The following definitions and lemmas are more useful to prove our main results.

Definition 2.2: [17] The fuzzy MJS (7) is said to be stochastically stable if for any initial condition x_0 and $r_0\in\mathbb{N}$, the following inequality holds:

$$\mathbb{E}\left[\int_0^\infty \|x(t)\|^2 \mathrm{d}t | x_0, r_o\right]<\infty.$$

Lemma 2.3: [27] Given constant matrices Σ_1, Σ_2, and Σ_3, where $\Sigma_1=\Sigma_1^T$ and $0<\Sigma_2=\Sigma_2^T$. Then $\Sigma_1+\Sigma_3^T\Sigma_2^{-1}\Sigma_3<0$ if and only if $\begin{bmatrix}\Sigma_1 & \Sigma_3^T\\ \Sigma_3 & -\Sigma_2\end{bmatrix}<0$ or equivalently $\begin{bmatrix}-\Sigma_2 & \Sigma_3\\ \Sigma_3^T & \Sigma_1\end{bmatrix}<0$.

Lemma 2.4: [25] For given positive definite matrix R_1, any matrix W with appropriate dimension, two vector functions $\xi(t)$, $x(t)$ and two scalars τ_1, τ_2 satisfying $\tau_2>\tau_1$ such that the integration concerned is well defined, then we have

$$-\int_{\tau_1}^{\tau_2}\dot{x}_1^T(s)R_1\dot{x}(s)\mathrm{d}s\le(\tau_2-\tau_1)\xi^T(t)WR_1^{-1}W^T\xi(t) \\ +2\xi(t)W\int_{\tau_1}^{\tau_2}\dot{x}(s)\mathrm{d}s.$$

Lemma 2.5: [25] Given matrices Q, H, and E of appropriate dimensions, where $Q=Q^T$, then $Q+HF(t)E+E^TF^T(t)H^T<0$ for all $F(t)$ satisfying $F^T(t)F(t)\le I$, if and only if there exists a scalar $\cdots>0$ such that $Q+\cdots HH^T+\cdots^{-1}E^TE<0$.

3 Robust Reliable Sampled-Data Control Design

In this section, we aim to design a mode-dependent robust reliable sampled-data state feedback controller for the fuzzy MJS (6) with partially known transition probabilities such that the resulting closed-loop system (7) is stochastically stable. First, we consider the case in which the matrices A_{li} and B_{li} are fixed, that is $\Delta A_{li}(t)=0$ and $\Delta B_{li}(t)=0$. Then, the corresponding nominal closed-loop system is obtained as

$$\dot{x}(t)=\sum_{l=1}^{r}\sum_{m=1}^{r} h_l(\theta(t))h_m(\theta(t_k))[A_{li}x(t)+B_{li}\Xi_1K_{mi}x(t-\tau(t)) \\ +B_{li}\Phi(u(t-\tau(t)))]. \quad (8)$$

In the following theorem, we will present sufficient conditions which ensures that the nominal system (8) is stochastically stable with the partially known transition probabilities based on the construction of an appropriate Lyapunov functional and the utilisation of free-connection matrix approach. Further, we will propose a design method of robust sampled-data state feedback controller for the nominal and uncertain MJSs (7) and (8).

Theorem 3.1: For given positive scalars $\tau>0$, $\rho>0$ and the matrices Ξ_1, Ξ_2, K_{mi}, the nominal closed-loop system (8) with partly known transition rate matrix is stochastically stable if there exist matrices $P_i>0$, $Q_i>0$, $R>0$, $Z>0$ and any matrices $W_i=W_i^T$, $M_i=M_i^T$, L_b, N_b $(b=1,2,3)$ with appropriate dimensions such that the following linear matrix inequalities hold for $l,m=1,2,\ldots,r$ and $i\in\mathbb{N}$:

$$\begin{bmatrix}[\Omega^{lm}]_{4\times4} & \Gamma^{lm}Z & \sqrt{\tau}\bar{N}^T\\ * & -Z & 0\\ * & * & -Z\end{bmatrix}<0,\quad \begin{bmatrix}[\Omega^{lm}]_{4\times4} & \Gamma^{lm}Z & \sqrt{\tau}\bar{L}^T\\ * & -Z & 0\\ * & * & -Z\end{bmatrix}<0, \quad (9)$$

$$\sum_{j\in\mathbb{T}_k^i}\pi_{ij}(Q_j-M_i)\le R, \tag{10}$$

$$P_j-W_i\le 0,\ j\in\mathbb{T}_{uk}^i,\ j\ne i, \tag{11}$$

$$Q_j-M_i\le 0,\ j\in\mathbb{T}_{uk}^i,\ j\ne i, \tag{12}$$

$$P_j-W_i\ge 0,\ j\in\mathbb{T}_{uk}^i,\ j=i, \tag{13}$$

$$Q_j-M_i\ge 0,\ j\in\mathbb{T}_{uk}^i,\ j=i, \tag{14}$$

where

$$\begin{aligned}
\Omega_{1,1}^{lm}&=\sum_{j\in\mathbb{T}_k^i}\pi_{ij}[P_j-W_i]+Q_i+\tau R+P_iA_{li}+A_{li}^TP_i+2L_1,\\
\Omega_{1,2}^{lm}&=P_iB_{li}\Xi_1K_{mi}+N_1-L_1+L_2^T-L_3^T,\\
\Omega_{1,3}^{lm}&=-N_1+L_3^T,\ \Omega_{1,4}^{lm}=P_iB_{li},\ \Omega_{2,2}^{lm}=2N_2-2L_2\\
&\quad+\rho K_{mi}^T\Xi_2K_{mi},\ \Omega_{2,3}^{lm}=-N_2+N_3^T,\\
\Omega_{3,3}^{lm}&=-Q_i-2N_3,\ \Omega_{4,4}^{lm}=-\rho I,\\
\Gamma^{lm}&=\left[\sqrt{\tau}A_{li}\quad \sqrt{\tau}B_{li}\Xi_1K_{mi}\quad 0\quad \sqrt{\tau}B_{li}\right]^T\\
\bar{N}&=[N_1\quad N_2\quad N_3\quad 0],\\
\bar{L}&=[L_1\quad L_2\quad L_3\quad 0].
\end{aligned}$$

Proof: To achieve the desired result, we first construct a stochastic Lyapunov functional candidate in the following form:

$$V_a(x(t),\,t,\,i)=\sum_{a=1}^{4}V_a(x(t),\,t,\,i), \tag{15}$$

where

$$\begin{aligned}
V_1(x(t),\,t,\,i)&=x^T(t)P_ix(t),\\
V_2(x(t),\,t,\,i)&=\int_{t-\tau}^{t}x^T(s)Q_ix(s)\mathrm{d}s,\\
V_3(x(t),\,t,\,i)&=\int_{-\tau}^{0}\int_{t+\theta}^{t}x^T(s)Rx(s)\mathrm{d}s\mathrm{d}\theta,\\
V_4(x(t),\,t,\,i)&=\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned}$$

Now, by calculating the derivatives of $V_a(x(t),\,t,\,i)$ $(a=1,2,3,4)$ along the trajectories of nominal closed-loop system (8), we can get

$$\pounds V_1(x(t),\,t,\,i)=2x^T(t)P_i\dot{x}(t)+vx^T(t)\sum_{j=1}^{\mathcal{N}}\pi_{ij}P_jx(t), \tag{16}$$

$$\begin{aligned}\pounds V_2(x(t),\,t,\,i)&=x^T(t)Q_ix(t)-x^T(t-\tau)Q_ix(t-\tau)\\&\quad+\int_{t-\tau}^{t}x^T(s)\sum_{j=1}^{\mathcal{N}}\pi_{ij}Q_jx(s)\mathrm{d}s,\end{aligned} \tag{17}$$

$$\pounds V_3(x(t),\,t,\,i)=\tau x^T(t)Rx(t)-\int_{t-\tau}^{t}x^T(s)Rx(s)\mathrm{d}s, \tag{18}$$

$$\pounds V_4(x(t),\,t,\,i)=\tau\dot{x}^T(t)Z\dot{x}(t)-\int_{t-\tau}^{t}\dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s. \tag{19}$$

Considering the circumstance that the information of transition probabilities is not completely known, for any arbitrary matrices $W_i=W_i^T$ and $M_i=M_i^T$, the following zero equations hold due to $\sum_{j=1}^{\mathcal{N}}\pi_{ij}=0$.

$$-x^T(t)\sum_{j=1}^{\mathcal{N}}\pi_{ij}W_ix(t)=0 \text{ and} \tag{20}$$

$$-\int_{t-\tau}^{t}x^T(s)\sum_{j=1}^{\mathcal{N}}\pi_{ij}M_ix(s)\mathrm{d}s=0. \tag{21}$$

Using Lemma 2.4 and the time delay interval to the integral term in (19), for any arbitrary matrices $N=[N_1^T\quad N_2^T\quad N_3^T\quad 0]^T$ and $L=[L_1^T\quad L_2^T\quad L_3^T\quad 0]^T$, the following inequalities hold:

$$\begin{aligned}&-\int_{t-\tau}^{t-\tau(t)}\dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s\le[\tau-\tau(t)]\eta^T(t)NZ^{-1}N^T\eta(t)\\&\quad+2\eta^T(t)N[x(t-\tau(t))-x(t-\tau)],\end{aligned} \tag{22}$$

$$\begin{aligned}&-\int_{t-\tau(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s\le\tau(t)\eta^T(t)LZ^{-1}L^T\eta(t)+2\eta^T(t)L[x(t)\\&\quad-x(t-\tau(t))],\end{aligned} \tag{23}$$

where $\eta^T(t)=[x^T(t)\quad x^T(t-\tau(t))\quad x^T(t-\tau)\quad \Phi^T(u(t-\tau(t)))]$. From (3), for any scalar $\rho>0$, we can have

$$\rho(u^T(t-\tau(t))\Xi_2u(t-\tau(t))-\Phi^T(u(t-\tau(t))\Phi(u(t-\tau(t))))\ge 0. \tag{24}$$

Combining (16)–(24), we can get

$$\begin{aligned}
\pounds V(x(t),\,t,\,i)&\le 2x^T(t)P_i\sum_{l=1}^{r}\sum_{m=1}^{r}h_l(\theta(t))h_m(\theta(t_k))[A_{li}x(t)\\
&\quad+B_{li}\Xi_1K_{mi}x(t-\tau(t))+B_{li}\Phi(u(t-\tau(t)))]+x^T(t)\sum_{j\in\mathbb{T}_k^i}\pi_{ij}[P_j-W_i]x(t)\\
&\quad+x^T(t)\sum_{j\in\mathbb{T}_{uk}^i}\pi_{ij}[P_j-W_i]x(t)+x^T(t)[Q_i+\tau R]x(t)\\
&\quad+\tau\dot{x}^T(t)Z\dot{x}(t)-x^T(t-\tau)Q_ix(t-\tau)\\
&\quad+\int_{t-\tau}^{t}x^T(s)\left[\sum_{j\in\mathbb{T}_k^i}\pi_{ij}(Q_j-M_i)-R\right]x(s)\mathrm{d}s\\
&\quad+\int_{t-\tau}^{t}x^T(s)\left[\sum_{j\in\mathbb{T}_{uk}^i}\pi_{ij}(Q_j-M_i)\right]x(s)\mathrm{d}s\\
&\quad+[\tau-\tau(t)]\eta^T(t)NZ^{-1}N\eta(t)+2\eta^T(t)N[x(t-\tau(t))\\
&\quad-x(t-\tau)]+\tau(t)\eta^T(t)LZ^{-1}L\eta(t)+2\eta^T(t)L[x(t)-x(t-\tau(t))]
\end{aligned}$$

$$
\begin{aligned}
&+\rho x^T(t-\tau(t))K_{mi}^T\Xi_2 K_{mi}x(t-\tau(t))\\
&-\rho\Phi^T(u(t-\tau(t))\Phi(u(t-\tau(t)).\\
\le&\sum_{l=1}^{r}\sum_{m=1}^{r}h_l(\theta(t))h_m(\theta(t_k))\eta^T(t)[[\Omega^{lm}]_{4\times4}+\tau\dot{x}^T(t)Z\dot{x}(t)\\
&+[\tau-\tau(t)](t)NZ^{-1}N+\tau(t)LZ^{-1}L]\eta(t)\\
&+x^T(t)\sum_{j\in\mathbb{T}_{uk}^i}\pi_{ij}[P_j-W_i]x(t)\\
&+\int_{t-\tau}^{t}x^T(s)\left[\sum_{j\in\mathbb{T}_k^i}\pi_{ij}(Q_j-M_i)-R\right]x(s)\mathrm{d}s\\
&+\int_{t-\tau}^{t}x^T(s)\left[\sum_{j\in\mathbb{T}_{uk}^i}\pi_{ij}(Q_j-M_i)\right]x(s)\mathrm{d}s.
\end{aligned}
$$

Further, it is noted that $\pi_{ii}=-\sum_{j=1}^{\mathcal{N}}\pi_{ij}$ and $\pi_{ij}\ge 0$ for all $j\neq i$ which implies $\pi_{ii}<0$ for all $i\in\mathbb{N}$.

Then, it follows from Schur complement Lemma 2.3 that if $i\in\mathbb{T}_{\mathbb{K}}$, the inequalities (9)–(14) lead to obtain

$$
\pounds V(x(t),t,i)<0. \tag{25}
$$

Moreover, if $i\in\mathbb{T}_{\mathbb{UK}}^i$, the inequalities (9)–(14) also imply that the inequality (25) holds. Then, it is easy to obtain $\mathbb{E}\left[\int_0^\infty\|x(t)\|^2\mathrm{d}t|x_0,r_o\right]<\infty$, which means that the nominal closed-loop system (8) is stochastically stable according to Definition 2.2. Thus, the proof is completed. ■

Next, let us concentrate on the design of reliable sampled-data control law that guarantees the stochastic stability of nominal closed-loop system (8) with partly known transition rates and nonlinear actuator faults. Now, the following theorem is proposed to design the mode-dependent stabilising state feedback reliable sampled-data controller based on the conditions obtained in Theorem 3.1.

Theorem 3.2: The nominal closed-loop system (8) with partly known transition rate matrix is stochastically stable, if there exist $X_i>0$, $\hat{Q}_i>0$, $\hat{R}_i>0$, $V>0$ and any matrices $\hat{L}_b$, $\hat{N}_b$ $(b=1,2,3)$, $\hat{W}_i$, $\hat{M}_i$, Y_{mi} such that the following LMI conditions hold for l, $m=1,2,\dots,r$ and $i\in\mathbb{N}$:

$$
\begin{aligned}
\tilde{\Xi}_1&=\begin{bmatrix}[\Theta^{lm}]_{6\times6} & \sqrt{\tau}\tilde{N} & \Lambda_{1i}\\ * & V-2X_i & 0\\ * & * & -\Xi_{1i}\end{bmatrix}<0,\\
\tilde{\Xi}_2&=\begin{bmatrix}[\Theta^{lm}]_{6\times6} & \sqrt{\tau}\tilde{L} & \Lambda_{1i}\\ * & V-2X_i & 0\\ * & * & -\Xi_{1i}\end{bmatrix}<0,\ i\in\mathbb{T}_k^i
\end{aligned}\tag{26}
$$

$$
\begin{aligned}
\tilde{\Xi}_3&=\begin{bmatrix}[\Psi^{lm}]_{6\times6} & \sqrt{\tau}\tilde{N} & \Lambda_{2i}\\ * & V-2X_i & 0\\ * & * & -\Xi_{2i}\end{bmatrix}<0,\\
\tilde{\Xi}_4&=\begin{bmatrix}[\Psi^{lm}]_{6\times6} & \sqrt{\tau}\tilde{L} & \Lambda_{2i}\\ * & V-2X_i & 0\\ * & * & -\Xi_{2i}\end{bmatrix}<0,\ i\in\mathbb{T}_{uk}^i
\end{aligned}\tag{27}
$$

$$
\sum_{j\in\mathbb{T}_k^i}\pi_{ij}(\hat{Q}_{ij}-\hat{M}_i)\le\hat{R}_i, \tag{28}
$$

$$
\begin{bmatrix}-\hat{W}_i & X_i\\ * & -X_j\end{bmatrix}\le 0,\ j\in\mathbb{T}_{uk}^i,\ j\neq i, \tag{29}
$$

$$
\hat{Q}_{ij}-\hat{M}_i\le 0,\ j\in\mathbb{T}_{uk}^i,\ j\neq i, \tag{30}
$$

$$
X_j-\hat{W}_j\ge 0,\ j\in\mathbb{T}_{uk}^i,\ j=i, \tag{31}
$$

$$
\hat{Q}_j-\hat{M}_j\ge 0,\ j\in\mathbb{T}_{uk}^i,\ j=i, \tag{32}
$$

$$
\Theta_{1,1}^{lm}=\hat{Q}_i+\tau\hat{R}_i+A_{li}X_i+X_iA_{li}^T+2\hat{L}_1-\sum_{j\in\mathbb{T}_k^i}\pi_{ij}\hat{W}_i+\pi_{ii}X_i,
$$

$$
\Psi_{1,1}^{lm}=\hat{Q}_i+\tau\hat{R}_i+A_{li}X_i+X_iA_{li}^T+2\hat{L}_1-\sum_{j\in\mathbb{T}_k^i}\pi_{ij}\hat{W}_i,
$$

$$
\Theta_{1,2}^{lm}=\Psi_{1,2}^{lm}=B_{li}E_1Y_{mi}+\hat{N}_1-\hat{L}_{i1}+\hat{L}_{i2}^T-L_{i3}^T,
$$

$$
\Theta_{1,3}^{lm}=\Psi_{1,3}^{lm}=-\hat{N}_{i1}+L_{i3}^T,\ \Theta_{1,4}^{lm}=\Psi_{1,4}^{lm}=B_{li},\ \Theta_{1,5}^{lm}=\Psi_{1,5}^{lm}=\sqrt{\tau}X_iA_{li}^T,
$$

$$
\Theta_{2,2}^{lm}=\Psi_{2,2}^{lm}=2\hat{N}_{i2}-2\hat{L}_{i2},
$$

$$
\Theta_{2,3}^{lm}=\Psi_{2,3}^{lm}=-\hat{N}_{i2}+\hat{N}_{i3}^T,\ \Theta_{2,5}^{lm}=\Psi_{2,5}^{lm}=\sqrt{\tau}Y_{mi}^T\Xi_1^TB_{li}^T,
$$

$$
\Theta_{2,6}^{lm}=\Psi_{2,6}^{lm}=\sqrt{\rho}Y_{mi}^T,\ \Theta_{3,3}^{lm}=\Psi_{3,3}^{lm}=-\hat{Q}_i-2\hat{N}_3,
$$

$$
\Theta_{4,4}^{lm}=\Psi_{4,4}^{lm}=-\rho I,\ \Theta_{4,5}^{lm}=\Psi_{4,5}^{lm}=\sqrt{\tau}B_{li}^T,\ \Theta_{5,5}^{lm}=\Psi_{5,5}^{lm}=V-2X_i,
$$

$$
\Theta_{6,6}^{lm}=\Psi_{6,6}^{lm}=\Xi_2^{-1},
$$

$$
\tilde{N}=[\hat{N}_1\quad \hat{N}_2\quad \hat{N}_3\quad 0],\ \tilde{L}=[\hat{L}_1\quad \hat{L}_2\quad \hat{L}_3\quad 0]
$$

$$
\Lambda_{1i}=\left[\sqrt{\pi_{ik_1^i}}X_i\quad \sqrt{\pi_{ik_2^i}}X_i\quad \dots\quad \sqrt{\pi_{ik_{r-1}^i}}X_i\quad \sqrt{\pi_{ik_{r+1}^i}}X_i\quad ..\quad \sqrt{\pi_{ik_m^i}}X_i\right]
$$

$$
\Xi_{1i}=diag\{X_{k_1^i},X_{k_2^i},\dots,X_{k_{r-1}^i},X_{k_{r+1}^i},\dots,X_{k_m^i}\},
$$

$$
\Lambda_{2i}=\left[\sqrt{\pi_{ik_1^i}}X_i\quad \sqrt{\pi_{ik_2^i}}X_i\quad \dots\quad \sqrt{\pi_{ik_m^i}}X_i\right],
$$

$$
\Xi_{2i}=diag\{X_{k_1^i},X_{k_2^i},\dots,X_{k_m^i}\}.
$$

Moreover, the stabilisation control gain matrices are given by $K_{mi}=Y_{mi}X_i^{-1}$, $m=1,2,\dots,r$ and $i\in|\mathbb{N}$.

Proof: We aim to design the fuzzy dynamic-based reliable sampled-data controller (5) that stabilises the nominal closed-loop system (8). In order to achieve this, pre- and postmultiply (9) by $diag\{X_i, X_i, X_i, I, X_i, I\}$ and (10)–(14) by X_i and its transpose, respectively. Now, introducing the following new variables $X_i=P_i^{-1}$, $V=Z^{-1}$, $\hat{Q}_i=X_iQ_iX_i$, $\hat{R}_i=X_iRX_i$, $\hat{Q}_{ij}=X_iQ_jX_i$, $\hat{N}_i=X_iNX_i$, $\hat{L}_i=X_iLX_i$, $\hat{W}_i=X_iW_iX_i$, $\hat{M}_i=X_iM_iX_i$ and using the Schur complement lemma, we obtain the inequalities (26)–(32). Therefore, if the LMIs (26)–(32) hold, we can conclude that the nominal closed-loop system (8) is stochastically stable. ■

Now, we are in a position to give the result on robust fuzzy controller for the uncertain fuzzy MJS (7) because the parameter uncertainties are not considered in the previous theorem. Similar to Theorem 3.2, we have the following theorem that presents an LMI-based method for the design of a mode-dependent robust fuzzy sampled-data controller for the uncertain fuzzy MJS (7) with partly known transition rate matrix.

Theorem 3.3: Consider the uncertain fuzzy MJS (7) with partly known transition rate matrix. If there exist matrices $X_i>0$, $\hat{Q}_i>0$, $\hat{R}>0$, $V>0$, any matrices $\hat{L}$, $\hat{N}$, $\hat{W}_i$, $\hat{M}_i$, Y_{mi} and positive scalars β_b $(b=1, 2, 3)$ such that the following LMI conditions hold for $l, m=1, 2, \ldots, r$ and $i\in\mathbb{N}$

$$\begin{bmatrix} \tilde{\Xi}_v & \beta_1\bar{M}_{ali}^T & \bar{N}_{ali}^T & \beta_2\bar{M}_{ali}^T & \bar{N}_{dmli}^T & \beta_3\bar{M}_{ali}^T & \bar{N}_{bmli}^T \\ * & -\beta_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\beta_1 I & 0 & 0 & 0 & 0 \\ * & * & * & -\beta_2 I & 0 & 0 & 0 \\ * & * & * & * & -\beta_2 I & 0 & 0 \\ * & * & * & * & * & -\beta_3 I & 0 \\ * & * & * & * & * & * & -\beta_3 I \end{bmatrix}<0,$$
$$v=1, 2,\ i\in\mathbb{T}_k^i, \tag{33}$$

$$\begin{bmatrix} \tilde{\Xi}_v & \beta_1\bar{M}_{ali}^T & \bar{N}_{ali}^T & \beta_2\bar{M}_{ali}^T & \bar{N}_{dmli}^T & \beta_3\bar{M}_{ali}^T & \bar{N}_{bmli}^T \\ * & -\beta_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\beta_1 I & 0 & 0 & 0 & 0 \\ * & * & * & -\beta_2 I & 0 & 0 & 0 \\ * & * & * & * & -\beta_2 I & 0 & 0 \\ * & * & * & * & * & -\beta_3 I & 0 \\ * & * & * & * & * & * & -\beta_3 I \end{bmatrix}<0,$$
$$v=3, 4,\ i\in\mathbb{T}_{uk}^i, \tag{34}$$

$$\sum_{j\in\mathbb{T}_k^i}\pi_{ij}(\hat{Q}_{ij}-\hat{M}_i)\leq\hat{R}_i, \tag{35}$$

$$\begin{bmatrix} -\hat{W}_i & X_i \\ * & -X_j \end{bmatrix}\leq 0,\ j\in\mathbb{T}_{uk}^i,\ j\neq i, \tag{36}$$

$$\hat{Q}_{ij}-\hat{M}_i\leq 0,\ j\in\mathbb{T}_{uk}^i,\ j\neq i, \tag{37}$$

$$X_j-\hat{W}_j\geq 0,\ j\in\mathbb{T}_{uk}^i,\ j=i, \tag{38}$$

$$\hat{Q}_j-\hat{M}_j\geq 0,\ j\in\mathbb{T}_{uk}^i,\ j=i, \tag{39}$$

where $\tilde{\Xi}_v(v=1, 2, 3, 4)$ are given in Theorem 3.2 with $\bar{M}_{ali}^T=[M_{li}^T\ \ 0_{4n}\ \ \sqrt{\tau}M_{li}^T]^T$, $\bar{N}_{ali}=[X_i^TN_{ali}^T\ \ 0_{5n}]$, $\bar{N}_{dmli}=[0\ \ Y_{mi}^TN_{bli}^T\ \ 0_{4n}]$, $\bar{N}_{bmli}=[0_{3n}\ \ Y_{mi}^TN_{bli}^T\ \ 0_{2n}]$, then the closed-loop system (7) is robustly stochastically stable. In addition, if the above LMIs are feasible, then the state feedback control gains matrices can be estimated as $K_{mi}=Y_{mi}X_i^{-1}$, $m=1, 2, \ldots, r$ and $i\in\mathbb{N}$.

Proof: The proof of this theorem immediately follows from Theorem 3.2, by replacing the matrices A_{li} and B_{li} with $A_{li}+M_{li}F_{li}(t)N_{ali}$ and $B_{li}+M_{li}F_{li}(t)N_{bli}$ in the proof of Theorem 3.2 and for any scalars $\beta_b>0$ $(b=1, 2, 3)$, using Lemma 2.5, it is easy to get the LMIs (33)–(39). Hence, the uncertain fuzzy-model-based MJS (1) can be stabilised with the proposed sampled-data controller (5). ■

Remark 3.4: It should be highly pointed out that the stability results based on partially known transition probability rates are too difficult to obtain compared to the results of completely known transition probability rates and their solutions are more conservative because of the lack of information. However, the solutions to this issue when the transition rates are partially known are more applicable than the known one. In this article, Theorem 3.3 presents the sufficient conditions for stochastic stability of considered fuzzy-model-based MJSs (1) and existence of reliable sampled-data controller (5). Moreover, the result obtained in this theorem is delay-dependent stability condition. Using the technique of free-weighting matrices would reduce the conservativeness. The obtained conditions are formulated in terms of LMIs, which could be easily solved using the LMI toolbox in MATLAB. Moreover, the conditions in Theorem 3.3 do not require the complete knowledge of the transition probabilities during the jump process, which means that they are more powerful and desirable. The main advantage of this article is that the proposed criterion has fewer variables, however, does not increase any conservatism, which has been proved theoretically.

4 Numerical Example

To validate the established results in the previous section, an application oriented example is considered and its simulations are presented.

Example 4.1: In this example, a single link robot arm [17] is considered and its motion is governed by the following differential equation:

$$\ddot{\theta}(t)=\frac{-MgL}{J}\sin(\theta(t))-\frac{D(t)}{J}\dot{\theta}(t)+\frac{1}{J}u^f(t), \tag{40}$$

where $\theta(t)$ is the angle position of the arm, $u^f(t)$ the control input, M the mass of the payload, J the moment of inertia, g the acceleration of gravity, L the length of the arm, and $D(t)$ the coefficient of viscous friction. The values of parameters g and L are given as $g=9.81$ and $L=0.5$. For this example, it is assumed that the parameter $D(t)=D=2$ is time invariant and the parameters M and J have two different modes that are shown in Table 1. Let $x_1(t)=\theta(t)$ and $x_2(t)=\dot{\theta}(t)$. Under the condition $-179.4270<\theta(t)<179.4270$, the nonlinear term $\sin(\theta(t))$ in (40) can be represented as follows:

$$\sin(x_1(t))=h_1(x_1(t))\cdot x_1(t)+h_2(x_1(t))\cdot\beta\cdot x_1(t), \tag{41}$$

where $\beta=10^{-2}/\pi$ and $h_1(x_1(t))$, $h_2(x_1(t))\in$ [0, 1] with $h_1(x_1(t))+h_2(x_1(t))=1$. By solving the (41), the corresponding membership functions $h_1(x_1(t))$ and $h_2(x_1(t))$ are obtained as

$$h_1(x_1(t))=\begin{cases}\dfrac{\sin(x_1(t))-\beta x_1(t)}{x_1(t).(1-\beta)}, & x_1(t)\neq 0,\\ 1, & x_1(t)=0,\end{cases},$$

$$h_2(x_1(t))=\begin{cases}\dfrac{x_1(t)-\sin(x_1(t))}{x_1(t).(1-\beta)}, & x_1(t)\neq 0,\\ 0 & x_1(t)=0.\end{cases}$$

It is evident from the above-mentioned membership functions that when $x_1(t)$ is about 0 rad, then $h_1(x_1(t))=1$, $h_2(x_1(t))=0$ and when $x_1(t)$ is about π rad or $-\pi$ rad, then $h_1(x_1(t))=0$, $h_2(x_1(t))=1$. Thus, the state space representation of single link robot arm (40) can be expressed by the following two rule TS fuzzy system [17]:

Plant rule 1: IF $x_1(t)$ is about 0 rad, THEN

$$\dot{x}(t)=(A_{1i}+\Delta A_{1i}(t))x(t)+(B_{1i}+\Delta B_{1i}(t))u^f(t),$$

Plant rule 2: IF $x_1(t)$ is about π rad or $-\pi$ rad, THEN

$$\dot{x}(t)=(A_{2i}+\Delta A_{2i}(t))x(t)+(B_{2i}+\Delta B_{2i}(t))u^f(t), \tag{42}$$

Table 1: Modes of the parameters *M* and *J*.

Mode	Parameter *M*	Parameter *J*
1	1	1
2	5	5

where

$$A_{11}=\begin{bmatrix}0 & 1\\ -gL & -D\end{bmatrix},\ A_{21}=\begin{bmatrix}0 & 1\\ -\beta gL & -D\end{bmatrix},$$

$$A_{12}=\begin{bmatrix}0 & 1\\ -gL & -0.8D\end{bmatrix},\ A_{22}=\begin{bmatrix}0 & 1\\ -\beta gL & -0.8D\end{bmatrix},$$

$$A_{13}=\begin{bmatrix}0 & 1\\ -gL & -0.5D\end{bmatrix},\ A_{23}=\begin{bmatrix}0 & 1\\ -\beta gL & -0.5D\end{bmatrix},$$

$$A_{14}=\begin{bmatrix}0 & 1\\ -gL & -0.4D\end{bmatrix},\ A_{24}=\begin{bmatrix}0 & 1\\ -\beta gL & -0.4D\end{bmatrix},$$

$$B_{11}=B_{21}=[0\ \ 1],\ B_{12}=B_{22}=[0\ \ 0.8],$$

$$B_{13}=B_{23}=[0\ \ 0.5],\ B_{14}=B_{24}=[0\ \ 0.4],$$

The uncertain parameters are given by $M_{li}=\begin{bmatrix}0\\1\end{bmatrix}$, $N_{a1i}=N_{a2i}=[0\ \ 0.1D]$, $N_{b1i}=N_{b2i}=0$ $(i=1,\ 2)$. To illustrate the efficiency of proposed method, we consider two different transition probability matrices that are given as follows:

$$\Pi_1=\begin{bmatrix}-0.5 & ? & 0.3 & ?\\ ? & -0.6 & ? & 0.3\\ 0.2 & 0.3 & ? & ?\\ ? & ? & 0.3 & ?\end{bmatrix}\text{and }\Pi_2=\begin{bmatrix}-0.7 & ? & 0.1 & 0.1\\ ? & -0.4 & 0.1 & ?\\ 0.5 & 0.3 & ? & ?\\ ? & ? & 0.4 & ?\end{bmatrix}.$$

Now, our purpose is to design the reliable sampled-data state feedback controller (5) such that the closed-loop system of (42) is robustly stochastically stable. Choosing the values $\tau=0.1$, $\Xi_1=1.3$, $\Xi_2=1.4$, $\Phi(u(t))=0.4\sin(u(t))$, and $F_i(t)=-0.5\sin(t)$ together with the transition probability matrix Π_1 and solving the LMIs in Theorem 3.3 with aid of Matlab LMI Tool box, we can get a set of feasible solutions which is not given here due to the page constraint. Moreover, the corresponding gain matrices are calculated as follows:

$$K_{11}=[-22.5132\ \ -12.0164],\ K_{12}=[-27.7232\ \ -17.2512],$$

$$K_{13}=[-26.7009\ \ -25.0982],\ K_{14}=[-21.3342\ \ -20.7044],$$

$$K_{21}=[-24.8219\ \ -14.1022],\ K_{22}=[-25.9873\ \ -19.0517],$$

$$K_{23}=[-26.9832\ \ -27.8102],\ K_{24}=[-19.3515\ \ -20.7146].$$

Moreover, Figures 1 and 2, respectively, represent the state responses of the closed-loop system (42) in the absence of nonlinear faults and in the presence of nonlinear faults based on the above-calculated gain matrices and the initial condition [0.5 −0.5]. From these figures, it can be strongly concluded that the performance of the closed-loop system (42) with the proposed controller (5) is satisfactory. Figure 3 depicts the state responses of the unforced system (42) in which the performance is poor

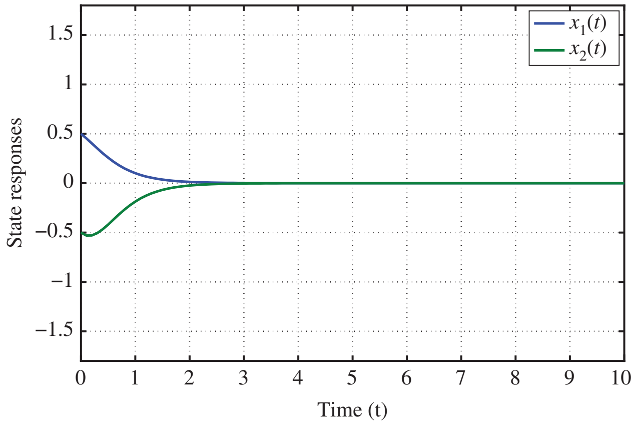

Figure 1: State responses of the closed-loop system (42) without nonlinear fault.

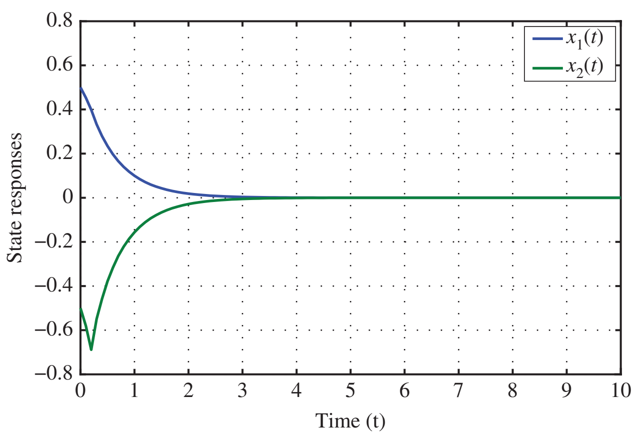

Figure 2: State responses of the closed-loop system (42) with nonlinear fault.

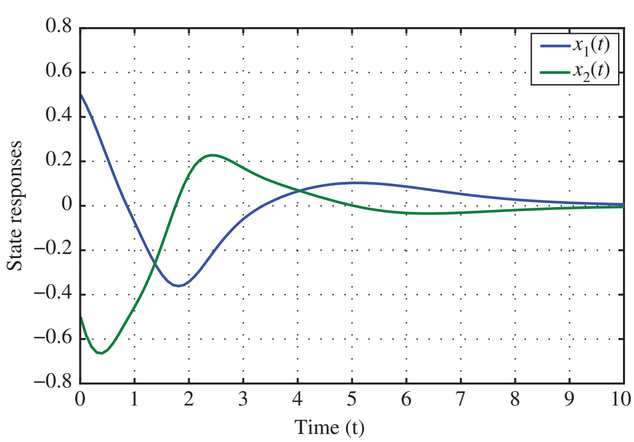

Figure 3: State responses of the unforced system (42).

compared to the performances of closed-loop system in Figures 1 and 2. Furthermore, Figure 4 displays the mode responses of the system (42) with the transition probability matrix Π_1.

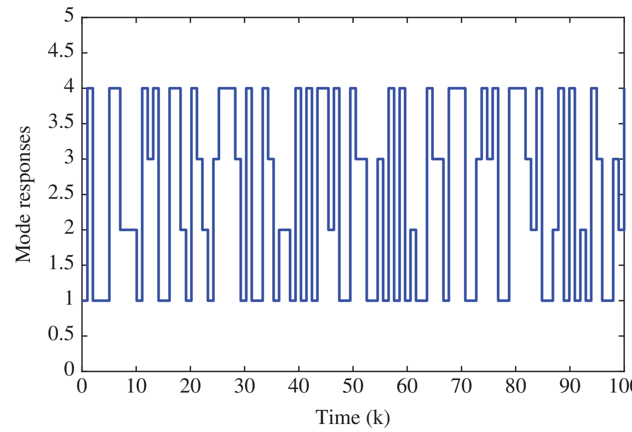

Figure 4: Mode responses with Π_1.

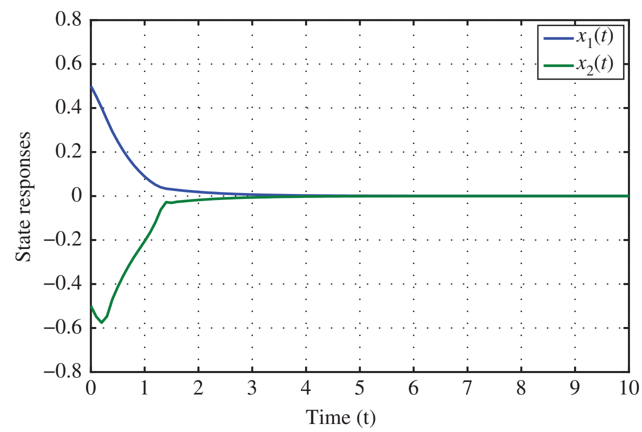

Figure 5: State responses of uncertain closed-loop system (42) without nonlinear fault.

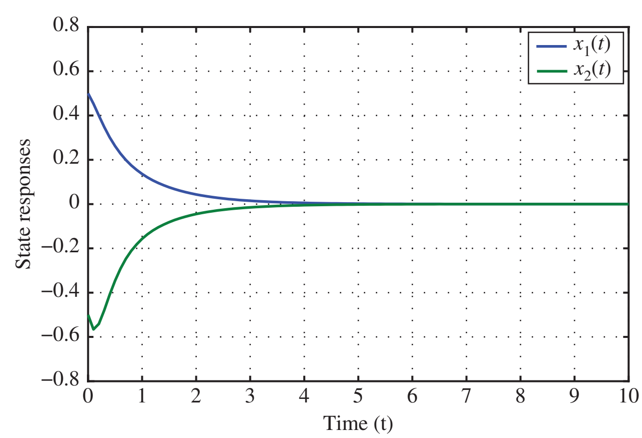

Figure 6: State responses of uncertain closed-loop system (42) with nonlinear fault.

Next, by taking the transition probability matrix Π_2 along with the other parameters as in the previous case and solving the LMIs obtained in Theorem 3.3, we can get the following sampled-data gain matrices:

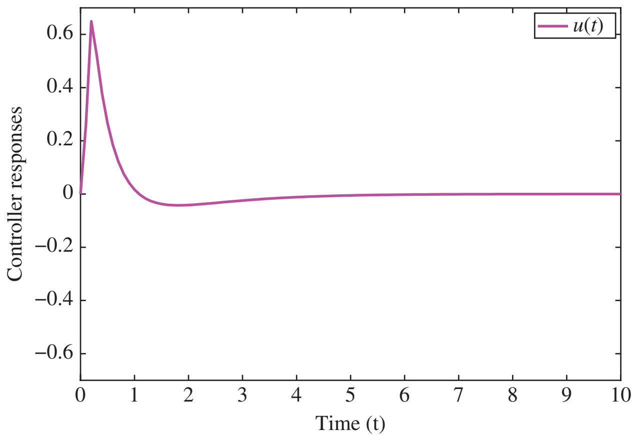

Figure 7: Control responses of system (42).

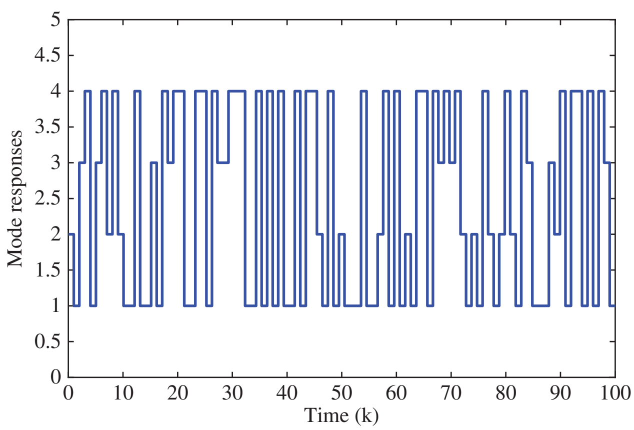

Figure 8: Mode responses with Π_2.

$$K_{11}=[-42.2849\quad -53.5305],\ K_{12}=[-28.6318\quad -20.4412],$$
$$K_{13}=[-41.8705\quad -42.4729],\ K_{14}=[-29.9031\quad -31.7417],$$
$$K_{21}=[-42.2905\quad -53.5238],\ K_{22}=[-28.6258\quad -20.4387],$$
$$K_{23}=[-41.8595\quad -42.4705],\ K_{24}=[-29.8891\quad -31.7402].$$

Based on the above-mentioned gain values, the state responses of the closed-loop system (42) in the absence of nonlinear faults and in the presence of nonlinear faults are plotted in Figures 5 and 6, respectively. It is observed from Figures 5 and 6 that the proposed controller (5) effectively stabilises the system (42) in both the cases. Furthermore, Figures 7 and 8 show the control and the mode responses of the system (42).

5 Conclusion

In this article, we have proposed a mode-dependent robust reliable sampled-data feedback control law in the presence of nonlinear actuator faults for the uncertain fuzzy Markov jump system with incomplete knowledge of transition probabilities. By constructing an appropriate Lyapunov–Krasovskii functional and employing free-weighting matrix technique, a new set of sufficient conditions has been obtained which guarantees the robust stochastic stability of the considered system. Based on such stability conditions, a design method of the robust sampled-data state feedback controller for uncertain fuzzy MJS has been developed in terms of LMIs. Simulation results have been given to illustrate the application of the proposed design method. In addition, our future research works include the mixed H_∞ and passivity based filter design [30] and the extended dissipativity-based control design [31] for the considered uncertain fuzzy Markov jump system with external disturbance and incomplete knowledge of transition probabilities.

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