CONSTRAINT ON THE MULTI-COMPONENT CKP HIERARCHY AND RECURSION OPERATORS

Appendix I

$$
\begin{aligned}
A_{11} &= E_{11}\partial^2+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial+2qrE_{11}+2r^{\intercal}q^{\intercal}E_{11}+2\alpha_x\beta+\alpha\beta_x-2\beta_x^{\intercal}\alpha^{\intercal}-\beta^{\intercal}\alpha_x^{\intercal}\\
&+q\partial^{-1}(C_1^*L^*(r))+L(q)\partial^{-1}(C_1^*r)+r^{\intercal}\partial^{-1}(C_1^*L^*(q^{\intercal}))+L(r^{\intercal})\partial^{-1}(C_1^*q^{\intercal})+A\partial^{-1}\beta\\
&+B\partial^{-1}\alpha^{\intercal}+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}R_{rq}+\frac{3}{2}E_{11}R_{rq}+\frac{1}{2}E_{11}\partial^{-1}R_{rq}\partial-\frac{1}{2}E_{11}\partial^{-1}R_{r_xq}\\
&-E_{11}\partial^{-1}qr\partial^{-1}R_{rq}-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}R_{rq}+E_{11}\partial^{-2}R_{rqrq}+E_{11}\partial^{-2}R_{rr^{\intercal}q^{\intercal}q}\\
&+q\partial^{-2}rR_{(\int r\alpha)\beta q}-q\partial^{-2}rR_{(\int r\beta^{\intercal})\alpha^{\intercal}q}+q\partial^{-2}rR_{C_1^*rq}+r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\alpha)\beta q}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}R_{C_1^*rq}-r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\beta^{\intercal})\alpha^{\intercal}q},\\
A_{12} &= E_{11}qR_q+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}qR_q+\frac{1}{2}E_{11}\partial^{-1}q_xR_q-\frac{1}{2}E_{11}\partial^{-1}qR_q\partial+\frac{1}{2}E_{11}qR_q\\
&-E_{11}\partial^{-1}qr\partial^{-1}qR_q-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}qR_q+E_{11}\partial^{-2}qR_{qrq}+E_{11}\partial^{-2}qR_{r^{\intercal}q^{\intercal}q}\\
&+q\partial^{-1}E_{11}\partial R_q-q\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_q+q\partial^{-1}\alpha\beta R_q+q\partial^{-2}R_{L(\alpha)\beta q}+q\partial^{-2}R_{qC_1^*rq}\\
&+q\partial^{-2}R_{r^{\intercal}C_1^*q^{\intercal}q}-q\partial^{-2}R_{L(\beta^{\intercal})\alpha^{\intercal}q}+q\partial^{-2}rq\partial^{-1}R_{\alpha\beta q}-q\partial^{-2}rq\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}\\
&+q\partial^{-1}(\int rq)R_{E_{11}q}+q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta q}-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}R_{\alpha\beta q}-r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)R_{E_{11}q}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}R_{\alpha\beta q}-r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}+L(q)\partial^{-1}E_{11}R_q\\
&+L(q)\partial^{-2}R_{\alpha\beta q}-L(q)\partial^{-2}R_{\beta^{\intercal}\alpha^{\intercal}q},\\
A_{13} &= E_{11}r^{\intercal}R_q+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}r^{\intercal}R_q+\frac{1}{2}E_{11}\partial^{-1}r_x^{\intercal}R_q-\frac{1}{2}E_{11}\partial^{-1}r^{\intercal}R_q\partial\\
&+\frac{1}{2}E_{11}r^{\intercal}R_q-E_{11}\partial^{-1}qr\partial^{-1}r^{\intercal}R_q-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}r^{\intercal}R_q+E_{11}\partial^{-2}r^{\intercal}R_{qrq}\\
&+E_{11}\partial^{-2}r^{\intercal}R_{r^{\intercal}q^{\intercal}q}+q\partial^{-2}rr^{\intercal}\partial^{-1}R_{\alpha\beta q}-q\partial^{-2}rr^{\intercal}\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}+q\partial^{-1}(\int rr^{\intercal})R_{E_{11}q}\\
&+q\partial^{-1}(\int rr^{\intercal})\partial^{-1}R_{\alpha\beta q}-q\partial^{-1}(\int rr^{\intercal})\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}+r^{\intercal}\partial^{-1}E_{11}\partial R_q-r^{\intercal}\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_q\\
&+r^{\intercal}\partial^{-1}\alpha\beta R_q+r^{\intercal}\partial^{-2}R_{L(\alpha)\beta q}-r^{\intercal}\partial^{-2}R_{L(\beta^{\intercal})\alpha^{\intercal}q}+r^{\intercal}\partial^{-2}R_{qC_1^*rq}+r^{\intercal}\partial^{-2}R_{r^{\intercal}C_1^*q^{\intercal}q}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}R_{\alpha\beta q}-r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})R_{E_{11}q}
\end{aligned}
$$

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\begin{aligned}
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\alpha\beta q}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+L(r^\intercal)\partial^{-1}E_{11}R_q\\
&+L(r^\intercal)\partial^{-2}R_{\alpha\beta q}-L(r^\intercal)\partial^{-2}R_{\beta^\intercal\alpha^\intercal q},\\
A_{14} &= \frac{3}{2}E_{11}R_{q^\intercal q}+(\alpha\beta-\beta^\intercal\alpha^\intercal)\partial^{-1}R_{q^\intercal q}+\frac{1}{2}E_{11}\partial^{-1}R_{q^\intercal q}\partial-\frac{1}{2}E_{11}\partial^{-1}R_{q_x^\intercal q}\\
&-E_{11}\partial^{-1}qr\partial^{-1}R_{q^\intercal q}-E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{q^\intercal q}+E_{11}\partial^{-2}R_{q^\intercal qrq}+E_{11}\partial^{-2}R_{q^\intercal r^\intercal q^\intercal q}\\
&+q\partial^{-2}rR_{(\int q^\intercal\alpha)\beta q}-q\partial^{-2}rR_{(\int q^\intercal\beta^\intercal)\alpha^\intercal q}+q\partial^{-2}rR_{C_1^*q^\intercal q}+r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\alpha)\beta q}\\
&-r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\beta^\intercal)\alpha^\intercal q}+r^\intercal\partial^{-2}q^\intercal R_{C_1^*q^\intercal q},\\
A_{21} &= q\partial^{-1}R_{\beta rq}+q\partial^{-2}r\partial R_{\beta q}+q\partial^{-2}rq\partial^{-1}rR_{\beta q}+q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\beta q}+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta q}\\
&+q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\beta q}+r^\intercal\partial^{-1}R_{\beta q^\intercal q}+r^\intercal\partial^{-2}q^\intercal\partial R_{\beta q}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\beta q}\\
&+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\beta q}+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\beta q}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\beta q}\\
&+L(q)\partial^{-2}rR_{\beta q}+L(r^\intercal)\partial^{-2}q^\intercal R_{\beta q},\\
A_{22} &= q\partial^{-1}\alpha R_{rq}+q\partial^{-1}(\int rL(\alpha))R_q+q\partial^{-1}(\int rq)(\int r\alpha)R_q\\
&+q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\alpha)R_q+r^\intercal\partial^{-1}\alpha R_{q^\intercal q}+r^\intercal\partial^{-1}(\int q^\intercal L(\alpha))R_q\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\alpha)R_q+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\alpha)R_q\\
&+L(q)\partial^{-1}(\int r\alpha)R_q+L(r^\intercal)\partial^{-1}(\int q^\intercal\alpha)R_q+A\partial^{-1}R_q,\\
A_{23} &= -q\partial^{-1}\beta^\intercal R_{rq}-q\partial^{-1}(\int rL(\beta^\intercal))R_q-q\partial^{-1}(\int rq)(\int r\beta^\intercal)R_q\\
&-q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\beta^\intercal)R_q-r^\intercal\partial^{-1}\beta^\intercal R_{q^\intercal q}-r^\intercal\partial^{-1}(\int q^\intercal L(\beta^\intercal))R_q\\
&-r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\beta^\intercal)R_q-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\beta^\intercal)R_q\\
&-L(q)\partial^{-1}(\int r\beta^\intercal)R_q-L(r^\intercal)\partial^{-1}(\int q^\intercal\beta^\intercal)R_q+B\partial^{-1}R_q\\
A_{24} &= -q\partial^{-1}R_{\alpha^\intercal rq}-q\partial^{-2}r\partial R_{\alpha^\intercal q}-q\partial^{-2}rq\partial^{-1}rR_{\alpha^\intercal q}-q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal q}\\
&-q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^\intercal q}-q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal q}-r^\intercal\partial^{-1}R_{\alpha^\intercal q^\intercal q}\\
&-r^\intercal\partial^{-2}q^\intercal\partial R_{\alpha^\intercal q}-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\alpha^\intercal q}-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal q}
\end{aligned}
$$

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\begin{aligned}
&-r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\alpha^\intercal q}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal q}-L(q)\partial^{-2}rR_{\alpha^\intercal q}\\
&-L(r^\intercal)\partial^{-2}q^\intercal R_{\alpha^\intercal q},
\end{aligned}
$$

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\begin{aligned}
B_{11} &= (\alpha\beta-\beta^\intercal\alpha^\intercal)\partial^{-1}R_{rr}+\frac{3}{2}E_{11}R_{rr}+\frac{1}{2}E_{11}\partial^{-1}R_{rr}\partial-\frac{1}{2}E_{11}\partial^{-1}R_{r_xr}\\
&\quad -E_{11}\partial^{-1}qr\partial^{-1}R_{rr}-E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{rr}+E_{11}\partial^{-2}R_{rqrr}+E_{11}\partial^{-2}R_{rr^\intercal q^\intercal r}\\
&\quad +q\partial^{-2}rR_{(\int r\alpha)\beta r}-q\partial^{-2}rR_{(\int r\beta^\intercal)\alpha^\intercal r}+q\partial^{-2}rR_{C_1^*rr}+r^\intercal\partial^{-2}q^\intercal R_{(\int r\alpha)\beta r}\\
&\quad +r^\intercal\partial^{-2}q^\intercal R_{C_1^*rr}-r^\intercal\partial^{-2}q^\intercal R_{(\int r\beta^\intercal)\alpha^\intercal r},\\
B_{12} &= E_{11}\partial^2+(\alpha\beta-\beta^\intercal\alpha^\intercal)\partial+2qrE_{11}+2r^\intercal q^\intercal E_{11}+2\alpha_x\beta+\alpha\beta_x-2\beta_x^\intercal\alpha^\intercal-\beta^\intercal\alpha_x^\intercal\\
&\quad +q\partial^{-1}(C_1^*L^*(r))+L(q)\partial^{-1}(C_1^*r)+r^\intercal\partial^{-1}(C_1^*L^*(q^\intercal))+L(r^\intercal)\partial^{-1}(C_1^*q^\intercal)\\
&\quad +A\partial^{-1}\beta+B\partial^{-1}\alpha^\intercal+E_{11}qR_r+(\alpha\beta-\beta^\intercal\alpha^\intercal)\partial^{-1}qR_r+\frac{1}{2}E_{11}\partial^{-1}q_xR_r\\
&\quad -\frac{1}{2}E_{11}\partial^{-1}qR_r\partial+\frac{1}{2}E_{11}qR_r-E_{11}\partial^{-1}qr\partial^{-1}qR_r-E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}qR_r\\
&\quad +E_{11}\partial^{-2}qR_{qrr}+E_{11}\partial^{-2}qR_{r^\intercal q^\intercal r}+q\partial^{-1}E_{11}\partial R_r-q\partial^{-1}\beta^\intercal\alpha^\intercal R_r+q\partial^{-1}\alpha\beta R_r\\
&\quad +q\partial^{-2}R_{L(\alpha)\beta r}+q\partial^{-2}R_{qC_1^*rr}+q\partial^{-2}R_{r^\intercal C_1^*q^\intercal r}-q\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal r}\\
&\quad +q\partial^{-2}rq\partial^{-1}R_{\alpha\beta q}-q\partial^{-2}rq\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+q\partial^{-1}(\int rq)R_{E_{11}r}\\
&\quad +q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta r}-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\alpha\beta r}\\
&\quad -r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+r^\intercal\partial^{-1}(\int q^\intercal q)R_{E_{11}r}+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\alpha\beta r}\\
&\quad -r^\intercal\partial^{-1}(\int q^\intercal r)\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+L(q)\partial^{-1}E_{11}R_r+L(q)\partial^{-2}R_{\alpha\beta r}\\
&\quad -L(q)\partial^{-2}R_{\beta^\intercal\alpha^\intercal r},\\
B_{13} &= E_{11}r^\intercal R_r+(\alpha\beta-\beta^\intercal\alpha^\intercal)\partial^{-1}r^\intercal R_r+\frac{1}{2}E_{11}\partial^{-1}r_x^\intercal R_r-\frac{1}{2}E_{11}\partial^{-1}r^\intercal R_r\partial\\
&\quad +\frac{1}{2}E_{11}r^\intercal R_r-E_{11}\partial^{-1}qr\partial^{-1}r^\intercal R_r-E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}r^\intercal R_r\\
&\quad +E_{11}\partial^{-2}r^\intercal R_{qrr}+E_{11}\partial^{-2}r^\intercal R_{r^\intercal q^\intercal r}+q\partial^{-2}rr^\intercal\partial^{-1}R_{\alpha\beta r}\\
&\quad -q\partial^{-2}rr^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+q\partial^{-1}(\int rr^\intercal)R_{E_{11}r}+q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\alpha\beta r}\\
&\quad -q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+r^\intercal\partial^{-1}E_{11}\partial R_r-r^\intercal\partial^{-1}\beta^\intercal\alpha^\intercal R_r+r^\intercal\partial^{-1}\alpha\beta R_r
\end{aligned}
$$

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\begin{aligned}
&+r^\intercal\partial^{-2}R_{L(\alpha)\beta r}-r^\intercal\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal r}+r^\intercal\partial^{-2}R_{qC_1^*rr}+r^\intercal\partial^{-2}R_{r^\intercal C_1^*q^\intercal r}\\
&+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\alpha\beta r}-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)R_{E_{11}r}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\alpha\beta r}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+L(r^\intercal)\partial^{-1}E_{11}R_r\\
&+L(r^\intercal)\partial^{-2}R_{\alpha\beta r}-L(r^\intercal)\partial^{-2}R_{\beta^\intercal\alpha^\intercal r},\\
B_{14} &= \frac{3}{2}E_{11}R_{q^\intercal r}+(\alpha\beta-\beta^\intercal\alpha^\intercal)\partial^{-1}R_{q^\intercal r}+\frac{1}{2}E_{11}\partial^{-1}R_{q^\intercal r}\partial\\
&-\frac{1}{2}E_{11}\partial^{-1}R_{q_x^\intercal r}-E_{11}\partial^{-1}qr\partial^{-1}R_{q^\intercal r}-E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{q^\intercal r}\\
&+E_{11}\partial^{-2}R_{q^\intercal qrr}+E_{11}\partial^{-2}R_{q^\intercal r^\intercal q^\intercal r}+q\partial^{-2}rR_{(\int q^\intercal\alpha)\beta r}\\
&-q\partial^{-2}rR_{(\int q^\intercal\beta^\intercal)\alpha^\intercal r}+q\partial^{-2}rR_{C_1^*q^\intercal r}+r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\alpha)\beta r}\\
&-r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\beta^\intercal)\alpha^\intercal r}+r^\intercal\partial^{-2}q^\intercal R_{C_1^*q^\intercal r}\\
B_{21} &= q\partial^{-1}R_{\beta rr}+q\partial^{-2}r\partial R_{\beta r}+q\partial^{-2}rq\partial^{-1}rR_{\beta r}+q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\beta r}\\
&+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta r}+q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\beta r}+r^\intercal\partial^{-1}R_{\beta q^\intercal r}\\
&+r^\intercal\partial^{-2}q^\intercal\partial R_{\beta r}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\beta r}+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\beta r}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r)\partial^{-1}rR_{\beta r}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\beta r}+L(q)\partial^{-2}rR_{\beta q}\\
&+L(r^\intercal)\partial^{-2}q^\intercal R_{\beta r},\\
B_{22} &= q\partial^{-1}\alpha R_{rr}+q\partial^{-1}(\int rL(\alpha))R_r+q\partial^{-1}(\int rq)(\int r\alpha)R_r\\
&+q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\alpha)R_r+r^\intercal\partial^{-1}\alpha R_{q^\intercal r}+r^\intercal\partial^{-1}(\int q^\intercal L(\alpha))R_r\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\alpha)R_r+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\alpha)R_r\\
&+L(q)\partial^{-1}(\int r\alpha)R_r+L(r^\intercal)\partial^{-1}(\int q^\intercal\alpha)R_r+A\partial^{-1}R_r,\\
B_{23} &= -q\partial^{-1}\beta^\intercal R_{rr}-q\partial^{-1}(\int rL(\beta^\intercal))R_r-q\partial^{-1}(\int rq)(\int r\beta^\intercal)R_r\\
&-q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\beta^\intercal)R_r-r^\intercal\partial^{-1}\beta^\intercal R_{q^\intercal r}-r^\intercal\partial^{-1}(\int q^\intercal L(\beta^\intercal))R_r\\
&-r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\beta^\intercal)R_r-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\beta^\intercal)R_r\\
&-L(q)\partial^{-1}(\int r\beta^\intercal)R_r-L(r^\intercal)\partial^{-1}(\int q^\intercal\beta^\intercal)R_r+B\partial^{-1}R_r
\end{aligned}
$$

$$
\begin{aligned}
B_{24} &= -q\partial^{-1}R_{\alpha^\intercal rr} - q\partial^{-2}r\partial R_{\alpha^\intercal r} - q\partial^{-2}rq\partial^{-1}rR_{\alpha^\intercal r} - q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal r}\\
&\quad -q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^\intercal r} - q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal r} - r^\intercal\partial^{-1}R_{\alpha^\intercal q^\intercal r}\\
&\quad -r^\intercal\partial^{-2}q^\intercal\partial R_{\alpha^\intercal r} - r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\alpha^\intercal r} - r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal r}\\
&\quad -r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\alpha^\intercal r} - r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal r}\\
&\quad -L(q)\partial^{-2}rR_{\alpha^\intercal r} - L(r^\intercal)\partial^{-2}q^\intercal R_{\alpha^\intercal r},
\end{aligned}
$$

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\begin{aligned}
C_{11} &= (\alpha\beta - \beta^\intercal\alpha^\intercal)\partial^{-1}R_{r\alpha} + E_{11}R_{r\alpha} + \frac{1}{2}E_{11}\partial^{-1}R_{r\alpha}\partial - \frac{1}{2}E_{11}\partial^{-1}R_{r_x\alpha}\\
&\quad +\frac{1}{2}E_{11}R_{r\alpha} - E_{11}\partial^{-1}qr\partial^{-1}R_{r\alpha} - E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{r\alpha} + E_{11}\partial^{-2}R_{rqr\alpha}\\
&\quad +E_{11}\partial^{-2}R_{rr^\intercal q^\intercal\alpha} + q\partial^{-2}rR_{(\int r\alpha)\beta\alpha} - q\partial^{-2}rR_{(\int r\beta^\intercal)\alpha^\intercal\alpha} + q\partial^{-2}rR_{C_1^*r\alpha}\\
&\quad +r^\intercal\partial^{-2}q^\intercal R_{(\int r\alpha)\beta\alpha} + r^\intercal\partial^{-2}q^\intercal R_{C_1^*r\alpha} - r^\intercal\partial^{-2}q^\intercal R_{(\int r\beta^\intercal)\alpha^\intercal\alpha},\\
C_{12} &= E_{11}qR_\alpha + (\alpha\beta - \beta^\intercal\alpha^\intercal)\partial^{-1}qR_\alpha + \frac{1}{2}E_{11}\partial^{-1}q_xR_\alpha - \frac{1}{2}E_{11}\partial^{-1}qR_\alpha\partial\\
&\quad +\frac{1}{2}E_{11}qR_\alpha - E_{11}\partial^{-1}qr\partial^{-1}qR_\alpha - E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}qR_\alpha\\
&\quad +E_{11}\partial^{-2}qR_{qr\alpha} + E_{11}\partial^{-2}qR_{r^\intercal q^\intercal\alpha} + q\partial^{-1}E_{11}\partial R_\alpha - q\partial^{-1}\beta^\intercal\alpha^\intercal R_\alpha\\
&\quad +q\partial^{-1}\alpha\beta R_\alpha + q\partial^{-2}R_{L(\alpha)\beta\alpha} + q\partial^{-2}R_{qC_1^*r\alpha} + q\partial^{-2}R_{r^\intercal C_1^*q^\intercal\alpha}\\
&\quad -q\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal\alpha} + q\partial^{-2}rq\partial^{-1}R_{\alpha\beta\alpha} - q\partial^{-2}rq\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}\\
&\quad +q\partial^{-1}(\int rq)R_{E_{11}\alpha} + q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta\alpha} - q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}\\
&\quad +r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\alpha\beta\alpha} - r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha} + r^\intercal\partial^{-1}(\int q^\intercal q)R_{E_{11}\alpha}\\
&\quad +r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\alpha\beta\alpha} - r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha} + L(q)\partial^{-1}E_{11}R_\alpha\\
&\quad +L(q)\partial^{-2}R_{\alpha\beta\alpha} - L(q)\partial^{-2}R_{\beta^\intercal\alpha^\intercal\alpha},\\
C_{13} &= E_{11}r^\intercal R_\alpha + (\alpha\beta - \beta^\intercal\alpha^\intercal)\partial^{-1}r^\intercal R_\alpha + \frac{1}{2}E_{11}\partial^{-1}r_x^\intercal R_\alpha\\
&\quad -\frac{1}{2}E_{11}\partial^{-1}r^\intercal R_\alpha\partial + \frac{1}{2}E_{11}r^\intercal R_\alpha - E_{11}\partial^{-1}qr\partial^{-1}r^\intercal R_\alpha\\
&\quad -E_{11}\partial^{-1}r^\intercal q^\intercal\partial^{-1}r^\intercal R_\alpha + E_{11}\partial^{-2}r^\intercal R_{qr\alpha} + E_{11}\partial^{-2}r^\intercal R_{r^\intercal q^\intercal\alpha} + q\partial^{-2}rr^\intercal\partial^{-1}R_{\alpha\beta\alpha}\\
&\quad -q\partial^{-2}rr^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha} + q\partial^{-1}(\int rr^\intercal)R_{E_{11}\alpha} + q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\alpha\beta\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&-q\partial^{-1}(\int rr^{\intercal})\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\alpha}+r^{\intercal}\partial^{-1}E_{11}\partial R_{\alpha}-r^{\intercal}\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_{\alpha}+r^{\intercal}\partial^{-1}\alpha\beta R_{\alpha}\\
&+r^{\intercal}\partial^{-2}R_{L(\alpha)\beta\alpha}-r^{\intercal}\partial^{-2}R_{L(\beta^{\intercal})\alpha^{\intercal}\alpha}+r^{\intercal}\partial^{-2}R_{qC_1^*r\alpha}+r^{\intercal}\partial^{-2}R_{r^{\intercal}C_1^*q^{\intercal}\alpha}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}R_{\alpha\beta\alpha}-r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\alpha}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})R_{E_{11}\alpha}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}R_{\alpha\beta\alpha}-r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\alpha}+L(r^{\intercal})\partial^{-1}E_{11}R_{\alpha}\\
&+L(r^{\intercal})\partial^{-2}R_{\alpha\beta\alpha}-L(r^{\intercal})\partial^{-2}R_{\beta^{\intercal}\alpha^{\intercal}\alpha},\\
C_{14} &= \frac{3}{2}E_{11}R_{q^{\intercal}\alpha}+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}R_{q^{\intercal}\alpha}+\frac{1}{2}E_{11}\partial^{-1}R_{q^{\intercal}\alpha}\partial\\
&-\frac{1}{2}E_{11}\partial^{-1}R_{q_x^{\intercal}\alpha}-E_{11}\partial^{-1}qr\partial^{-1}R_{q^{\intercal}\alpha}-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}R_{q^{\intercal}\alpha}\\
&+E_{11}\partial^{-2}R_{q^{\intercal}qr\alpha}+E_{11}\partial^{-2}R_{q^{\intercal}r^{\intercal}q^{\intercal}\alpha}+q\partial^{-2}rR_{(\int q^{\intercal}\alpha)\beta\alpha}\\
&-q\partial^{-2}rR_{(\int q^{\intercal}\beta^{\intercal})\alpha^{\intercal}\alpha}+q\partial^{-2}rR_{C_1^*q^{\intercal}\alpha}+r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int q^{\intercal}\alpha)\beta\alpha}\\
&-r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int q^{\intercal}\beta^{\intercal})\alpha^{\intercal}\alpha}+r^{\intercal}\partial^{-2}q^{\intercal}R_{C_1^*q^{\intercal}\alpha}\\
C_{21} &= E_{11}\partial^2+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial+2qrE_{11}+2r^{\intercal}q^{\intercal}E_{11}+2\alpha_x\beta+\alpha\beta_x-2\beta_x^{\intercal}\alpha^{\intercal}-\beta^{\intercal}\alpha_x^{\intercal}\\
&+q\partial^{-1}(C_1^*L^*(r))+L(q)\partial^{-1}(C_1^*r)+r^{\intercal}\partial^{-1}(C_1^*L^*(q^{\intercal}))+L(r^{\intercal})\partial^{-1}(C_1^*q^{\intercal})+A\partial^{-1}\beta\\
&+B\partial^{-1}\alpha^{\intercal}+q\partial^{-1}R_{\beta r\alpha}+q\partial^{-2}r\partial R_{\beta\alpha}+q\partial^{-2}rq\partial^{-1}rR_{\beta\alpha}+q\partial^{-2}rr^{\intercal}\partial^{-1}q^{\intercal}R_{\beta\alpha}\\
&+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta\alpha}+q\partial^{-1}(\int rr^{\intercal})\partial^{-1}q^{\intercal}R_{\beta\alpha}+r^{\intercal}\partial^{-1}R_{\beta q^{\intercal}\alpha}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}\partial R_{\beta\alpha}+r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}rR_{\beta\alpha}+r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}q^{\intercal}R_{\beta\alpha}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}rR_{\beta\alpha}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}q^{\intercal}R_{\beta\alpha}+L(q)\partial^{-2}rR_{\beta\alpha}\\
&+L(r^{\intercal})\partial^{-2}q^{\intercal}R_{\beta\alpha},\\
C_{22} &= q\partial^{-1}\alpha R_{r\alpha}+q\partial^{-1}(\int rL(\alpha))R_{\alpha}+q\partial^{-1}(\int rq)(\int r\alpha)R_{\alpha}\\
&+q\partial^{-1}(\int rr^{\intercal})(\int q^{\intercal}\alpha)R_{\alpha}+r^{\intercal}\partial^{-1}\alpha R_{q^{\intercal}\alpha}+r^{\intercal}\partial^{-1}(\int q^{\intercal}L(\alpha))R_{\alpha}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}\alpha)(\int r\alpha)R_{\alpha}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})(\int q^{\intercal}\alpha)R_{\alpha}\\
&+L(q)\partial^{-1}(\int r\alpha)R_{\alpha}+L(r^{\intercal})\partial^{-1}(\int q^{\intercal}\alpha)R_{\alpha}+A\partial^{-1}R_{\alpha},\\
C_{23} &= -q\partial^{-1}\beta^{\intercal}R_{r\alpha}-q\partial^{-1}(\int rL(\beta^{\intercal}))R_{\alpha}-q\partial^{-1}(\int rq)(\int r\beta^{\intercal})R_{\alpha}\\
&-q\partial^{-1}(\int rr^{\intercal})(\int q^{\intercal}\beta^{\intercal})R_{\alpha}-r^{\intercal}\partial^{-1}\beta^{\intercal}R_{q^{\intercal}\alpha}-r^{\intercal}\partial^{-1}(\int q^{\intercal}L(\beta^{\intercal}))R_{\alpha}
\end{aligned}
$$

$$
\begin{aligned}
& -r^{\intercal}\partial^{-1}(\int q^{\intercal}q)(\int r\beta^{\intercal})R_{\alpha}-r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})(\int q^{\intercal}\beta^{\intercal})R_{\alpha}\\
& -L(q)\partial^{-1}(\int r\beta^{\intercal})R_{\alpha}-L(r^{\intercal})\partial^{-1}(\int q^{\intercal}\beta^{\intercal})R_{\alpha}+B\partial^{-1}R_{\alpha}\\
C_{24} = & -q\partial^{-1}R_{\alpha^{\intercal}r\alpha}-q\partial^{-2}r\partial R_{\alpha^{\intercal}\alpha}-q\partial^{-2}rq\partial^{-1}rR_{\alpha^{\intercal}\alpha}-q\partial^{-2}rr^{\intercal}\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\alpha}\\
& -q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^{\intercal}\alpha}-q\partial^{-1}(\int rr^{\intercal})\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\alpha}-r^{\intercal}\partial^{-1}R_{\alpha^{\intercal}q^{\intercal}\alpha}\\
& -r^{\intercal}\partial^{-2}q^{\intercal}\partial R_{\alpha^{\intercal}\alpha}-r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}rR_{\alpha^{\intercal}\alpha}-r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\alpha}\\
& -r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}rR_{\alpha^{\intercal}\alpha}-r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\alpha}\\
& -L(q)\partial^{-2}rR_{\alpha^{\intercal}\alpha}-L(r^{\intercal})\partial^{-2}q^{\intercal}R_{\alpha^{\intercal}\alpha},
\end{aligned}
$$

$$
\begin{aligned}
D_{11} = & (\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}R_{r\beta}+E_{11}R_{r\beta}+\frac{1}{2}E_{11}\partial^{-1}R_{r\beta}\partial-\frac{1}{2}E_{11}\partial^{-1}R_{r_x\beta}\\
& +\frac{1}{2}E_{11}R_{r\beta}-E_{11}\partial^{-1}qr\partial^{-1}R_{r\beta}-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}R_{r\beta}+E_{11}\partial^{-2}R_{rqr\beta}\\
& +E_{11}\partial^{-2}R_{rr^{\intercal}q^{\intercal}\beta}+q\partial^{-2}rR_{(\int r\alpha)\beta\beta}-q\partial^{-2}rR_{(\int r\beta^{\intercal})\alpha^{\intercal}\beta}+q\partial^{-2}rR_{C_1^*r\beta}\\
& +r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\alpha)\beta\beta}+r^{\intercal}\partial^{-2}q^{\intercal}R_{C_1^*r\beta}-r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\beta^{\intercal})\alpha^{\intercal}\beta},\\
D_{12} = & E_{11}qR_{\beta}+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}qR_{\beta}+\frac{1}{2}E_{11}\partial^{-1}q_xR_{\beta}-\frac{1}{2}E_{11}\partial^{-1}qR_{\beta}\partial\\
& +\frac{1}{2}E_{11}qR_{\beta}-E_{11}\partial^{-1}qr\partial^{-1}qR_{\beta}-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}qR_{\beta}+E_{11}\partial^{-2}qR_{qr\beta}\\
& +E_{11}\partial^{-2}qR_{r^{\intercal}q^{\intercal}\beta}+q\partial^{-1}E_{11}\partial R_{\beta}-q\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_{\beta}+q\partial^{-1}\alpha\beta R_{\beta}\\
& +q\partial^{-2}R_{L(\alpha)\beta\beta}+q\partial^{-2}R_{qC_1^*r\beta}+q\partial^{-2}R_{r^{\intercal}C_1^*q^{\intercal}\beta}-q\partial^{-2}R_{L(\beta^{\intercal})\alpha^{\intercal}\beta}\\
& +q\partial^{-2}rq\partial^{-1}R_{\alpha\beta\beta}-q\partial^{-2}rq\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+q\partial^{-1}(\int rq)R_{E_{11}\beta}\\
& +q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta\beta}-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}R_{\alpha\beta\beta}\\
& -r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)R_{E_{11}\beta}+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}R_{\alpha\beta\beta}\\
& -r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+L(q)\partial^{-1}E_{11}R_{\beta}+L(q)\partial^{-2}R_{\alpha\beta\beta}\\
& -L(q)\partial^{-2}R_{\beta^{\intercal}\alpha^{\intercal}\beta},\\
D_{13} = & E_{11}r^{\intercal}R_{\beta}+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}r^{\intercal}R_{\beta}+\frac{1}{2}E_{11}\partial^{-1}r_x^{\intercal}R_{\beta}-\frac{1}{2}E_{11}\partial^{-1}r^{\intercal}R_{\beta}\partial
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}E_{11}r^{\intercal}R_{\beta}-E_{11}\partial^{-1}qr\partial^{-1}r^{\intercal}R_{\beta}-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}r^{\intercal}R_{\beta}+E_{11}\partial^{-2}r^{\intercal}R_{qr\beta}\\
&+E_{11}\partial^{-2}r^{\intercal}R_{r^{\intercal}q^{\intercal}\beta}+q\partial^{-2}rr^{\intercal}\partial^{-1}R_{\alpha\beta\beta}+L(r^{\intercal})\partial^{-2}R_{\alpha\beta\beta}-L(r^{\intercal})\partial^{-2}R_{\beta^{\intercal}\alpha^{\intercal}\beta}\\
&-q\partial^{-2}rr^{\intercal}\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+q\partial^{-1}(\int rr^{\intercal})R_{E_{11}\beta}+q\partial^{-1}(\int rr^{\intercal})\partial^{-1}R_{\alpha\beta\beta}\\
&-q\partial^{-1}(\int rr^{\intercal})\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+r^{\intercal}\partial^{-1}E_{11}\partial R_{\beta}-r^{\intercal}\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_{\beta}+r^{\intercal}\partial^{-1}\alpha\beta R_{\beta}\\
&+r^{\intercal}\partial^{-2}R_{L(\alpha)\beta\beta}-r^{\intercal}\partial^{-2}R_{L(\beta^{\intercal})\alpha^{\intercal}\beta}+r^{\intercal}\partial^{-2}R_{qC_1^*r\beta}+r^{\intercal}\partial^{-2}R_{r^{\intercal}C_1^*q^{\intercal}\beta}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}R_{\alpha\beta\beta}-r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})R_{E_{11}\beta}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}R_{\alpha\beta\beta}-r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}\beta}+L(r^{\intercal})\partial^{-1}E_{11}R_{\beta},\\
D_{14} &= \frac{3}{2}E_{11}R_{q^{\intercal}\beta}+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial^{-1}R_{q^{\intercal}\beta}+\frac{1}{2}E_{11}\partial^{-1}R_{q^{\intercal}\beta}\partial-\frac{1}{2}E_{11}\partial^{-1}R_{q_x^{\intercal}\beta}\\
&-E_{11}\partial^{-1}qr\partial^{-1}R_{q^{\intercal}\beta}-E_{11}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}R_{q^{\intercal}\beta}+E_{11}\partial^{-2}R_{q^{\intercal}qr\beta}+E_{11}\partial^{-2}R_{q^{\intercal}r^{\intercal}q^{\intercal}\beta}\\
&+q\partial^{-2}rR_{(\int q^{\intercal}\alpha)\beta\beta}-q\partial^{-2}rR_{(\int q^{\intercal}\beta^{\intercal})\alpha^{\intercal}\beta}+q\partial^{-2}rR_{C_1^*q^{\intercal}\beta}+r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int q^{\intercal}\alpha)\beta\beta}\\
&-r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int q^{\intercal}\beta^{\intercal})\alpha^{\intercal}\beta}+r^{\intercal}\partial^{-2}q^{\intercal}R_{C_1^*q^{\intercal}\beta}\\
D_{21} &= q\partial^{-1}R_{\beta r\beta}+q\partial^{-2}r\partial R_{\beta\beta}+q\partial^{-2}rq\partial^{-1}rR_{\beta\beta}+q\partial^{-2}rr^{\intercal}\partial^{-1}q^{\intercal}R_{\beta\beta}\\
&+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta\beta}+q\partial^{-1}(\int rr^{\intercal})\partial^{-1}q^{\intercal}R_{\beta\beta}+r^{\intercal}\partial^{-1}R_{\beta q^{\intercal}\beta}\\
&+r^{\intercal}\partial^{-2}q^{\intercal}\partial R_{\beta\beta}+r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}rR_{\beta\beta}+r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}q^{\intercal}R_{\beta\beta}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}rR_{\beta\beta}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}q^{\intercal}R_{\beta\beta}+L(q)\partial^{-2}rR_{\beta\beta}\\
&+L(r^{\intercal})\partial^{-2}q^{\intercal}R_{\beta\beta},\\
D_{22} &= E_{11}\partial^2+(\alpha\beta-\beta^{\intercal}\alpha^{\intercal})\partial+2qrE_{11}+2r^{\intercal}q^{\intercal}E_{11}+2\alpha_x\beta+\alpha\beta_x-2\beta_x^{\intercal}\alpha^{\intercal}-\beta^{\intercal}\alpha_x^{\intercal}\\
&+q\partial^{-1}(C_1^*L^*(r))+L(q)\partial^{-1}(C_1^*r)+r^{\intercal}\partial^{-1}(C_1^*L^*(q^{\intercal}))+L(r^{\intercal})\partial^{-1}(C_1^*q^{\intercal})+A\partial^{-1}\beta\\
&+B\partial^{-1}\alpha^{\intercal}+q\partial^{-1}\alpha R_{r\beta}+q\partial^{-1}(\int rL(\alpha))R_{\beta}+q\partial^{-1}(\int rq)(\int r\alpha)R_{\beta}\\
&+q\partial^{-1}(\int rr^{\intercal})(\int q^{\intercal}\alpha)R_{\beta}+r^{\intercal}\partial^{-1}\alpha R_{q^{\intercal}\beta}+r^{\intercal}\partial^{-1}(\int q^{\intercal}L(\alpha))R_{\beta}\\
&+r^{\intercal}\partial^{-1}(\int q^{\intercal}q)(\int r\alpha)R_{\beta}+r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})(\int q^{\intercal}\alpha)R_{\beta}\\
&+L(q)\partial^{-1}(\int r\alpha)R_{\beta}+L(r^{\intercal})\partial^{-1}(\int q^{\intercal}\alpha)R_{\beta}+A\partial^{-1}R_{\beta},\\
D_{23} &= -q\partial^{-1}\beta^{\intercal}R_{r\beta}-q\partial^{-1}(\int rL(\beta^{\intercal}))R_{\beta}-q\partial^{-1}(\int rq)(\int r\beta^{\intercal})R_{\beta}
\end{aligned}
$$

$$
\begin{aligned}
&-q\partial^{-1}(\int rr^{\intercal})(\int q^{\intercal}\beta^{\intercal})R_{\beta}-r^{\intercal}\partial^{-1}\beta^{\intercal}R_{q^{\intercal}\beta}-r^{\intercal}\partial^{-1}(\int q^{\intercal}L(\beta^{\intercal}))R_{\beta}\\
&-r^{\intercal}\partial^{-1}(\int q^{\intercal}q)(\int r\beta^{\intercal})R_{\beta}-r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})(\int q^{\intercal}\beta^{\intercal})R_{\beta}\\
&-L(q)\partial^{-1}(\int r\beta^{\intercal})R_{\beta}-L(r^{\intercal})\partial^{-1}(\int q^{\intercal}\beta^{\intercal})R_{\beta}+B\partial^{-1}R_{\beta}\\
D_{24} \;=\;& -q\partial^{-1}R_{\alpha^{\intercal}r\beta}-q\partial^{-2}r\partial R_{\alpha^{\intercal}\beta}-q\partial^{-2}rq\partial^{-1}rR_{\alpha^{\intercal}\beta}-q\partial^{-2}rr^{\intercal}\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\beta}\\
&-q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^{\intercal}\beta}-q\partial^{-1}(\int rr^{\intercal})\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\beta}-r^{\intercal}\partial^{-1}R_{\alpha^{\intercal}q^{\intercal}\beta}\\
&-r^{\intercal}\partial^{-2}q^{\intercal}\partial R_{\alpha^{\intercal}\beta}-r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}rR_{\alpha^{\intercal}\beta}-r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\beta}\\
&-r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}rR_{\alpha^{\intercal}\beta}-r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}\beta}\\
&-L(q)\partial^{-2}rR_{\alpha^{\intercal}\beta}-L(r^{\intercal})\partial^{-2}q^{\intercal}R_{\alpha^{\intercal}\beta},
\end{aligned}
$$

with $A=\alpha_{xx}+2qr\alpha+2r^{\intercal}q^{\intercal}\alpha+q(\int L^{*}(r)\alpha)+r^{\intercal}(\int L^{*}(q^{\intercal})\alpha)+L(q)(\int r\alpha)+L(r^{\intercal})(\int q^{\intercal}\alpha)$ and $B=-\beta_{xx}^{\intercal}-2qr\beta^{\intercal}-2r^{\intercal}q^{\intercal}\beta^{\intercal}-q(\int L^{*}(r)\beta^{\intercal})-r^{\intercal}(\int L^{*}(q^{\intercal})\beta^{\intercal})-L(q)(\int r\beta^{\intercal})-L(r^{\intercal})(\int q^{\intercal}\beta^{\intercal})$.

Appendix II

$$
\begin{aligned}
A_{11} \;=\;& E_{22}\partial^{2}+(\beta^{\intercal}\alpha^{\intercal}-\alpha\beta)\partial+2qrE_{22}+2r^{\intercal}q^{\intercal}E_{22}-2\alpha_{x}\beta-\alpha\beta_{x}+2\beta_{x}^{\intercal}\alpha^{\intercal}+\beta^{\intercal}\alpha_{x}^{\intercal}\\
&+q\partial^{-1}(C_{2}^{*}L^{*}(r))+L(q)\partial^{-1}(C_{2}^{*}r)+r^{\intercal}\partial^{-1}(C_{2}^{*}L^{*}(q^{\intercal}))+L(r^{\intercal})\partial^{-1}(C_{2}^{*}q^{\intercal})-A\partial^{-1}\beta\\
&-B\partial^{-1}\alpha^{\intercal}+(\beta^{\intercal}\alpha^{\intercal}-\alpha\beta)\partial^{-1}R_{rq}+E_{22}R_{rq}+\frac{1}{2}E_{22}\partial^{-1}R_{rq}\partial-\frac{1}{2}E_{22}\partial^{-1}R_{r_{x}q}\\
&+\frac{1}{2}E_{22}R_{rq}-E_{22}\partial^{-1}qr\partial^{-1}R_{rq}-E_{22}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}R_{rq}+E_{22}\partial^{-2}R_{rqrq}\\
&E_{22}\partial^{-2}R_{rr^{\intercal}q^{\intercal}q}-q\partial^{-2}rR_{(\int r\alpha)\beta q}+q\partial^{-2}rR_{(\int r\beta^{\intercal})\alpha^{\intercal}q}+q\partial^{-2}rR_{C_{2}^{*}rq}\\
&-r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\alpha)\beta q}+r^{\intercal}\partial^{-2}q^{\intercal}R_{C_{2}^{*}rq}+r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\beta^{\intercal})\alpha^{\intercal}q},\\
A_{12} \;=\;& E_{22}qR_{q}+(\beta^{\intercal}\alpha^{\intercal}-\alpha\beta)\partial^{-1}qR_{q}+\frac{1}{2}E_{22}\partial^{-1}q_{x}R_{q}-\frac{1}{2}E_{22}\partial^{-1}qR_{q}\partial\\
&+\frac{1}{2}E_{22}qR_{q}-E_{22}\partial^{-1}qr\partial^{-1}qR_{q}-E_{22}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}qR_{q}+E_{22}\partial^{-2}qR_{qrq}\\
&+E_{22}\partial^{-2}qR_{r^{\intercal}q^{\intercal}q}+q\partial^{-1}E_{22}\partial R_{q}+q\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_{q}-q\partial^{-1}\alpha\beta R_{q}\\
&-q\partial^{-2}R_{L(\alpha)\beta q}+q\partial^{-2}R_{qC_{2}^{*}rq}+q\partial^{-2}R_{r^{\intercal}C_{2}^{*}q^{\intercal}q}+q\partial^{-2}R_{L(\beta^{\intercal})\alpha^{\intercal}q}\\
&-q\partial^{-2}rq\partial^{-1}R_{\alpha\beta q}+q\partial^{-2}rq\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}+q\partial^{-1}(\int rq)R_{E_{22}q}+q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta q}\\
&-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}-r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}R_{\alpha\beta q}+r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}R_{\beta^{\intercal}\alpha^{\intercal}q}
\end{aligned}
$$

$$
\begin{aligned}
&+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\alpha\beta q}-r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+L(q)\partial^{-1}E_{22}R_q\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)R_{E_{22}q}-L(q)\partial^{-2}R_{\alpha\beta q}+L(q)\partial^{-2}R_{\beta^\intercal\alpha^\intercal q},\\
A_{13} &= E_{22}r^\intercal R_q+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}r^\intercal R_q+\frac{1}{2}E_{22}\partial^{-1}r_x^\intercal R_q\\
&-\frac{1}{2}E_{22}\partial^{-1}r^\intercal R_q\partial+\frac{1}{2}E_{22}r^\intercal R_q-E_{22}\partial^{-1}qr\partial^{-1}r^\intercal R_q\\
&-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}r^\intercal R_q+E_{22}\partial^{-2}r^\intercal R_{qrq}+E_{22}\partial^{-2}r^\intercal R_{r^\intercal q^\intercal q}-q\partial^{-2}rr^\intercal\partial^{-1}R_{\alpha\beta q}\\
&+q\partial^{-2}rr^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+q\partial^{-1}(\int rr^\intercal)R_{E_{22}q}+q\partial^{-1}(\int rr^\intercal)R_{\alpha\beta q}\\
&-q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+r^\intercal\partial^{-1}E_{22}\partial R_q+r^\intercal\partial^{-1}\beta^\intercal\alpha^\intercal R_q-r^\intercal\partial^{-1}\alpha\beta R_q\\
&-r^\intercal\partial^{-2}R_{L(\alpha)\beta q}+r^\intercal\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal q}+r^\intercal\partial^{-2}R_{qC_2^*rq}+r^\intercal\partial^{-1}R_{r^\intercal C_2^*q^\intercal q}\\
&-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\alpha\beta q}+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)R_{E_{22}q}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\alpha\beta q}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal q}+L(r^\intercal)\partial^{-1}E_{22}R_q\\
&-L(r^\intercal)\partial^{-2}R_{\alpha\beta q}+L(r^\intercal)\partial^{-2}R_{\beta^\intercal\alpha^\intercal q},\\
A_{14} &= \frac{3}{2}E_{22}R_{q^\intercal q}+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}R_{q^\intercal q}+\frac{1}{2}E_{22}\partial^{-1}R_{q^\intercal q}\partial\\
&-\frac{1}{2}E_{22}\partial^{-1}R_{q_x^\intercal q}-E_{22}\partial^{-1}qr\partial^{-1}R_{q^\intercal q}-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{q^\intercal q}\\
&+E_{22}\partial^{-2}R_{q^\intercal qrq}+E_{22}\partial^{-2}R_{q^\intercal r^\intercal q^\intercal q}-q\partial^{-2}rR_{(\int q^\intercal\alpha)\beta q}\\
&+q\partial^{-2}rR_{(\int q^\intercal\beta^\intercal)\alpha^\intercal q}+q\partial^{-2}rR_{C_2^*q^\intercal q}-r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\alpha)\beta q}\\
&r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\beta^\intercal)\alpha^\intercal q}+r^\intercal\partial^{-2}q^\intercal R_{C_2^*q^\intercal q}\\
A_{21} &= -q\partial^{-1}R_{\beta rq}-q\partial^{-2}r\alpha R_{\beta q}-q\partial^{-2}rq\partial^{-1}rR_{\beta q}-q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\beta q}\\
&+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta q}+q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\beta q}-r^\intercal\partial^{-1}R_{\beta q^\intercal q}-r^\intercal\partial^{-2}q^\intercal\partial R_{\beta q}\\
&-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\beta q}-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\beta q}+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\beta q}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\beta q}-L(q)\partial^{-2}rR_{\beta q}-L(r^\intercal)\partial^{-2}q^\intercal R_{\beta q},\\
A_{22} &= -q\partial^{-1}\alpha R_{rq}-q\partial^{-1}(\int rL(\alpha))R_q+q\partial^{-1}(\int rq)(\int r\alpha)R_q\\
&+q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\alpha)R_q-r^\intercal\partial^{-1}\alpha R_{q^\intercal q}-r^\intercal\partial^{-1}(\int q^\intercal L(\alpha))R_q
\end{aligned}
$$

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\begin{aligned}
&+r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\alpha)R_q + r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\alpha)R_q\\
&-L(q)\partial^{-1}(\int r\alpha)R_q - L(r^\intercal)\partial^{-1}(\int q^\intercal\alpha)R_q - A\partial^{-1}R_q,\\
A_{23} &= q\partial^{-1}\beta^\intercal R_{rq} + q\partial^{-1}(\int rL(\beta^\intercal))R_q - q\partial^{-1}(\int rq)(\int r\beta^\intercal)R_q\\
&-q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\beta^\intercal)R_q + r^\intercal\partial^{-1}\beta^\intercal R_{q^\intercal q} + r^\intercal\partial^{-1}(\int q^\intercal L(\beta^\intercal))R_q\\
&-r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\beta^\intercal)R_q - r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\beta^\intercal)R_q\\
&+L(q)\partial^{-1}(\int r\beta^\intercal)R_q + L(r^\intercal)\partial^{-1}(\int q^\intercal\beta^\intercal)R_q - B\partial^{-1}R_q\\
A_{24} &= q\partial^{-1}R_{\alpha^\intercal rq} + q\partial^{-2}r\partial R_{\alpha^\intercal q} + q\partial^{-2}rq\partial^{-1}rR_{\alpha^\intercal q} + q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal q}\\
&-q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^\intercal q} - q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal q} + r^\intercal\partial^{-1}R_{\alpha^\intercal q^\intercal q}\\
&+r^\intercal\partial^{-2}q^\intercal\partial R_{\alpha^\intercal q} + r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\alpha^\intercal q} + r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal q}\\
&-r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\alpha^\intercal q} - r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal q}\\
&+L(q)\partial^{-2}rR_{\alpha^\intercal q} + L(r^\intercal)\partial^{-2}q^\intercal R_{\alpha^\intercal q},
\end{aligned}
$$

$$
\begin{aligned}
B_{11} &= (\beta^\intercal\alpha^\intercal - \alpha\beta)\partial^{-1}R_{rr} + E_{22}R_{rr} + \frac{1}{2}E_{22}\partial^{-1}R_{rr}\partial - \frac{1}{2}E_{22}\partial^{-1}R_{r_xr}\\
&+\frac{1}{2}E_{22}R_{rr} - E_{22}\partial^{-1}qr\partial^{-1}R_{rr} - E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{rr} + E_{22}\partial^{-2}R_{rqrr}\\
&+E_{22}\partial^{-2}R_{rr^\intercal q^\intercal r} - q\partial^{-2}rR_{(\int r\alpha)\beta r} + q\partial^{-2}rR_{(\int r\beta^\intercal)\alpha^\intercal r} + q\partial^{-2}rR_{C_2^*rr}\\
&-r^\intercal\partial^{-2}q^\intercal R_{(\int r\alpha)\beta r} + r^\intercal\partial^{-2}q^\intercal R_{C_2^*rr} + r^\intercal\partial^{-2}q^\intercal R_{\int r\beta^\intercal\alpha^\intercal r},\\
B_{12} &= E_{22}\partial^2 + (\beta^\intercal\alpha^\intercal - \alpha\beta)\partial + 2qrE_{22} + 2r^\intercal q^\intercal E_{22} - 2\alpha_x\beta - \alpha\beta_x + 2\beta_x^\intercal\alpha^\intercal + \beta^\intercal\alpha_x^\intercal\\
&+q\partial^{-1}(C_2^*L^*(r)) + L(q)\partial^{-1}(C_2^*r) + r^\intercal\partial^{-1}(C_2^*L^*(q^\intercal)) + L(r^\intercal)\partial^{-1}(C_2^*q^\intercal)\\
&-A\partial^{-1}\beta - B\partial^{-1}\alpha^\intercal + E_{22}qR_r + (\beta^\intercal\alpha^\intercal - \alpha\beta)\partial^{-1}qR_r + \frac{1}{2}E_{22}\partial^{-1}q_xR_r\\
&-\frac{1}{2}E_{22}\partial^{-1}qR_r\partial + \frac{1}{2}E_{22}qR_r - E_{22}\partial^{-1}qr\partial^{-1}qR_r - E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}qR_r\\
&+E_{22}\partial^{-2}qR_{qrr} + E_{22}\partial^{-2}qR_{r^\intercal q^\intercal r} + q\partial^{-1}E_{22}\partial R_r + q\partial^{-1}\beta^\intercal\alpha^\intercal R_r - q\partial^{-1}\alpha\beta R_r\\
&-q\partial^{-2}R_{L(\alpha)\beta r} + q\partial^{-2}R_{qC_2^*rr} + q\partial^{-2}R_{r^\intercal C_2^*q^\intercal r} + q\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal r}
\end{aligned}
$$

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\begin{aligned}
&-q\partial^{-2}rq\partial^{-1}R_{\alpha\beta r}+q\partial^{-2}rq\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+q\partial^{-1}(\int rq)R_{E_{22}r}+q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta r}\\
&-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\alpha\beta r}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r)R_{E_{22}r}+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\alpha\beta r}-r^\intercal\partial^{-1}(\int q^\intercal r)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}\\
&+L(q)\partial^{-1}E_{22}R_r-L(q)\partial^{-2}R_{\alpha\beta r}+L(q)\partial^{-2}R_{\beta^\intercal\alpha^\intercal r},\\
B_{13} \;=\;& E_{22}r^\intercal R_r+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}r^\intercal R_r+\frac{1}{2}E_{22}\partial^{-1}r_x^\intercal R_r-L(r^\intercal)\partial^{-2}R_{\alpha\beta r}\\
&-\frac{1}{2}E_{22}\partial^{-1}r^\intercal R_r\partial+\frac{1}{2}E_{22}r^\intercal R_r-E_{22}\partial^{-1}qr\partial^{-1}r^\intercal R_r+L(r^\intercal)\partial^{-2}R_{\beta^\intercal\alpha^\intercal r}\\
&-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}r^\intercal R_r+E_{22}\partial^{-2}r^\intercal R_{qrr}+E_{22}\partial^{-2}r^\intercal R_{r^\intercal q^\intercal r}-q\partial^{-2}rr^\intercal\partial^{-1}R_{\alpha\beta r}\\
&+q\partial^{-2}rr^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+q\partial^{-1}(\int rr^\intercal)R_{E_{22}r}+q\partial^{-1}(\int rr^\intercal)R_{\alpha\beta r}\\
&-q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+r^\intercal\partial^{-1}E_{22}\partial R_r+r^\intercal\partial^{-1}\beta^\intercal\alpha^\intercal R_r-r^\intercal\partial^{-1}\alpha\beta R_r\\
&-r^\intercal\partial^{-2}R_{L(\alpha)\beta r}+r^\intercal\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal r}+r^\intercal\partial^{-2}R_{qC_2^*rr}+r^\intercal\partial^{-1}R_{r^\intercal C_2^*q^\intercal r}\\
&-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\alpha\beta r}+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)R_{E_{22}r}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\alpha\beta r}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal r}+L(r^\intercal)\partial^{-1}E_{22}R_r,\\
B_{14} \;=\;& \frac{3}{2}E_{22}R_{q^\intercal r}+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}R_{q^\intercal r}+\frac{1}{2}E_{22}\partial^{-1}R_{q^\intercal r}\partial\\
&-\frac{1}{2}E_{22}\partial^{-1}R_{q_x^\intercal r}-E_{22}\partial^{-1}qr\partial^{-1}R_{q^\intercal r}-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{q^\intercal r}\\
&+E_{22}\partial^{-2}R_{q^\intercal qrr}+E_{22}\partial^{-2}R_{q^\intercal r^\intercal q^\intercal r}-q\partial^{-2}rR_{(\int q^\intercal\alpha)\beta r}\\
&+q\partial^{-2}rR_{(\int q^\intercal\beta^\intercal)\alpha^\intercal r}+q\partial^{-2}rR_{C_2^*q^\intercal r}-r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\alpha)\beta r}\\
&r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\beta^\intercal)\alpha^\intercal r}+r^\intercal\partial^{-2}q^\intercal R_{C_2^*q^\intercal r}\\
B_{21} \;=\;& -q\partial^{-1}R_{\beta rr}-q\partial^{-2}r\alpha R_{\beta r}-q\partial^{-2}rq\partial^{-1}rR_{\beta r}-q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\beta r}\\
&+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta r}+q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\beta r}-r^\intercal\partial^{-1}R_{\beta q^\intercal r}\\
&-r^\intercal\partial^{-2}q^\intercal\partial R_{\beta r}-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\beta r}-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\beta r}\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\beta r}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\beta r}-L(q)\partial^{-2}rR_{\beta r}\\
&-L(r^\intercal)\partial^{-2}q^\intercal R_{\beta r},
\end{aligned}
$$

$$
\begin{aligned}
B_{22} &= -q\partial^{-1}\alpha R_{rr} - q\partial^{-1}(\int rL(\alpha))R_r + q\partial^{-1}(\int rq)(\int r\alpha)R_r \\
&\quad +q\partial^{-1}(\int rr^{\intercal})(\int q^{\intercal}\alpha)R_r - r^{\intercal}\partial^{-1}\alpha R_{q^{\intercal}r} - r^{\intercal}\partial^{-1}(\int q^{\intercal}L(\alpha))R_r \\
&\quad +r^{\intercal}\partial^{-1}(\int q^{\intercal}q)(\int r\alpha)R_r + r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})(\int q^{\intercal}\alpha)R_r \\
&\quad -L(q)\partial^{-1}(\int r\alpha)R_r - L(r^{\intercal})\partial^{-1}(\int q^{\intercal}\alpha)R_r - A\partial^{-1}R_r, \\
B_{23} &= q\partial^{-1}\beta^{\intercal}R_{rr} + q\partial^{-1}(\int rL(\beta^{\intercal}))R_r - q\partial^{-1}(\int rq)(\int r\beta^{\intercal})R_r \\
&\quad -q\partial^{-1}(\int rr^{\intercal})(\int q^{\intercal}\beta^{\intercal})R_r + r^{\intercal}\partial^{-1}\beta^{\intercal}R_{q^{\intercal}r} + r^{\intercal}\partial^{-1}(\int q^{\intercal}L(\beta^{\intercal}))R_r \\
&\quad -r^{\intercal}\partial^{-1}(\int q^{\intercal}q)(\int r\beta^{\intercal})R_r - r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})(\int q^{\intercal}\beta^{\intercal})R_r \\
&\quad +L(q)\partial^{-1}(\int r\beta^{\intercal})R_r + L(r^{\intercal})\partial^{-1}(\int q^{\intercal}\beta^{\intercal})R_r - B\partial^{-1}R_r \\
B_{24} &= q\partial^{-1}R_{\alpha^{\intercal}rr} + q\partial^{-2}r\partial R_{\alpha^{\intercal}r} + q\partial^{-2}rq\partial^{-1}rR_{\alpha^{\intercal}r} + q\partial^{-2}rr^{\intercal}\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}r} \\
&\quad -q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^{\intercal}r} - q\partial^{-1}(\int rr^{\intercal})\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}r} + r^{\intercal}\partial^{-1}R_{\alpha^{\intercal}q^{\intercal}r} \\
&\quad +r^{\intercal}\partial^{-2}q^{\intercal}\partial R_{\alpha^{\intercal}r} + r^{\intercal}\partial^{-2}q^{\intercal}q\partial^{-1}rR_{\alpha^{\intercal}r} + r^{\intercal}\partial^{-2}q^{\intercal}r^{\intercal}\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}r} \\
&\quad -r^{\intercal}\partial^{-1}(\int q^{\intercal}q)\partial^{-1}rR_{\alpha^{\intercal}r} - r^{\intercal}\partial^{-1}(\int q^{\intercal}r^{\intercal})\partial^{-1}q^{\intercal}R_{\alpha^{\intercal}r} \\
&\quad +L(q)\partial^{-2}rR_{\alpha^{\intercal}r} + L(r^{\intercal})\partial^{-2}q^{\intercal}R_{\alpha^{\intercal}r},
\end{aligned}
$$

$$
\begin{aligned}
C_{11} &= (\beta^{\intercal}\alpha^{\intercal} - \alpha\beta)\partial^{-1}R_{r\alpha} + E_{22}R_{r\alpha} + \frac{1}{2}E_{22}\partial^{-1}R_{r\alpha}\partial - \frac{1}{2}E_{22}\partial^{-1}R_{r_x\alpha} \\
&\quad +\frac{1}{2}E_{22}R_{r\alpha} - E_{22}\partial^{-1}qr\partial^{-1}R_{r\alpha} - E_{22}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}R_{r\alpha} + E_{22}\partial^{-2}R_{rqr\alpha} \\
&\quad E_{22}\partial^{-2}R_{rr^{\intercal}q^{\intercal}\alpha} - q\partial^{-2}rR_{\int r\alpha)\beta\alpha} + q\partial^{-2}rR_{(\int r\beta^{\intercal})\alpha^{\intercal}\alpha} + q\partial^{-2}rR_{C_2^*r\alpha} \\
&\quad -r^{\intercal}\partial^{-2}q^{\intercal}R_{(\int r\alpha)\beta\alpha} + r^{\intercal}\partial^{-2}q^{\intercal}R_{C_2^*r\alpha} + r^{\intercal}\partial^{-2}q^{\intercal}R_{\int r\beta^{\intercal}\alpha^{\intercal}\alpha}, \\
C_{12} &= E_{22}qR_{\alpha} + (\beta^{\intercal}\alpha^{\intercal} - \alpha\beta)\partial^{-1}qR_{\alpha} + \frac{1}{2}E_{22}\partial^{-1}q_xR_{\alpha} - \frac{1}{2}E_{22}\partial^{-1}qR_{\alpha}\partial \\
&\quad +\frac{1}{2}E_{22}qR_{\alpha} - E_{22}\partial^{-1}qr\partial^{-1}qR_{\alpha} - E_{22}\partial^{-1}r^{\intercal}q^{\intercal}\partial^{-1}qR_{\alpha} \\
&\quad +E_{22}\partial^{-2}qR_{qr\alpha} + E_{22}\partial^{-2}qR_{r^{\intercal}q^{\intercal}\alpha} + q\partial^{-1}E_{22}\partial R_{\alpha} + q\partial^{-1}\beta^{\intercal}\alpha^{\intercal}R_{\alpha} \\
&\quad -q\partial^{-1}\alpha\beta R_{\alpha} - q\partial^{-2}R_{L(\alpha)\beta\alpha} + q\partial^{-2}R_{qC_2^*r\alpha} + q\partial^{-2}R_{r^{\intercal}C_2^*q^{\intercal}\alpha}
\end{aligned}
$$

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\begin{aligned}
&+q\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal\alpha}-q\partial^{-2}rq\partial^{-1}R_{\alpha\beta\alpha}+q\partial^{-2}rq\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}\\
&+q\partial^{-1}(\int rq)R_{E_{22}\alpha}+q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta\alpha}-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}\\
&-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\alpha\beta\alpha}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}+r^\intercal\partial^{-1}(\int q^\intercal q)R_{E_{22}\alpha}\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\alpha\beta\alpha}-r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}+L(q)\partial^{-1}E_{22}R_\alpha\\
&-L(q)\partial^{-2}R_{\alpha\beta\alpha}+L(q)\partial^{-2}R_{\beta^\intercal\alpha^\intercal\alpha},\\
C_{13} &= E_{22}r^\intercal R_\alpha+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}r^\intercal R_\alpha+\frac{1}{2}E_{22}\partial^{-1}r_x^\intercal R_\alpha-\frac{1}{2}E_{22}\partial^{-1}r^\intercal R_\alpha\partial+\frac{1}{2}E_{22}r^\intercal R_\alpha\\
&-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}r^\intercal R_\alpha+E_{22}\partial^{-2}r^\intercal R_{qr\alpha}+E_{22}\partial^{-2}r^\intercal R_{r^\intercal q^\intercal\alpha}-q\partial^{-2}rr^\intercal\partial^{-1}R_{\alpha\beta\alpha}\\
&+q\partial^{-2}rr^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}+q\partial^{-1}(\int rr^\intercal)R_{E_{22}\alpha}+q\partial^{-1}(\int rr^\intercal)R_{\alpha\beta\alpha}\\
&-q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}+r^\intercal\partial^{-1}E_{22}\partial R_\alpha+r^\intercal\partial^{-1}\beta^\intercal\alpha^\intercal R_\alpha-r^\intercal\partial^{-1}\alpha\beta R_\alpha\\
&-r^\intercal\partial^{-2}R_{L(\alpha)\beta\alpha}+r^\intercal\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal\alpha}+r^\intercal\partial^{-2}R_{qC_2^*r\alpha}+r^\intercal\partial^{-1}R_{r^\intercal C_2^*q^\intercal\alpha}\\
&-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\alpha\beta\alpha}+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)R_{E_{22}\alpha}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\alpha\beta\alpha}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\alpha}+L(r^\intercal)\partial^{-1}E_{22}R_\alpha\\
&-E_{22}\partial^{-1}qr\partial^{-1}r^\intercal R_\alpha-L(r^\intercal)\partial^{-2}R_{\alpha\beta\alpha}+L(r^\intercal)\partial^{-2}R_{\beta^\intercal\alpha^\intercal\alpha},\\
C_{14} &= \frac{3}{2}E_{22}R_{q^\intercal\alpha}+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}R_{q^\intercal\alpha}+\frac{1}{2}E_{22}\partial^{-1}R_{q^\intercal\alpha}\partial\\
&-\frac{1}{2}E_{22}\partial^{-1}R_{q_x^\intercal\alpha}-E_{22}\partial^{-1}qr\partial^{-1}R_{q^\intercal\alpha}-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{q^\intercal\alpha}\\
&+E_{22}\partial^{-2}R_{q^\intercal qr\alpha}+E_{22}\partial^{-2}R_{q^\intercal r^\intercal q^\intercal\alpha}-q\partial^{-2}rR_{(\int q^\intercal\alpha)\beta\alpha}\\
&+q\partial^{-2}rR_{(\int q^\intercal\beta^\intercal)\alpha^\intercal\alpha}+q\partial^{-2}rR_{C_2^*q^\intercal\alpha}-r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\alpha)\beta\alpha}\\
&r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\beta^\intercal)\alpha^\intercal\alpha}+r^\intercal\partial^{-2}q^\intercal R_{C_2^*q^\intercal\alpha}\\
C_{21} &= E_{22}\partial^2+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial+2qrE_{22}+2r^\intercal q^\intercal E_{22}-2\alpha_x\beta-\alpha\beta_x+2\beta_x^\intercal\alpha^\intercal+\beta^\intercal\alpha_x^\intercal\\
&+q\partial^{-1}(C_2^*L^*(r))+L(q)\partial^{-1}(C_2^*r)+r^\intercal\partial^{-1}(C_2^*L^*(q^\intercal))+L(r^\intercal)\partial^{-1}(C_2^*q^\intercal)\\
&-A\partial^{-1}\beta-B\partial^{-1}\alpha^\intercal-q\partial^{-1}R_{\beta r\alpha}-q\partial^{-2}r\alpha R_{\beta\alpha}-q\partial^{-2}rq\partial^{-1}rR_{\beta\alpha}\\
&-q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\beta\alpha}+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta\alpha}+q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\beta\alpha}-r^\intercal\partial^{-1}R_{\beta q^\intercal\alpha}\\
&-r^\intercal\partial^{-2}q^\intercal\partial R_{\beta\alpha}-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\beta\alpha}-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\beta\alpha}-L(r^\intercal)\partial^{-2}q^\intercal R_{\beta\alpha}\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\beta\alpha}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\beta\alpha}-L(q)\partial^{-2}rR_{\beta\alpha},
\end{aligned}
$$

$$
\begin{aligned}
C_{22} &= -q\partial^{-1}\alpha R_{r\alpha} - q\partial^{-1}(\int rL(\alpha))R_\alpha + q\partial^{-1}(\int rq)(\int r\alpha)R_\alpha \\
&\quad +q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\alpha)R_\alpha - r^\intercal\partial^{-1}\alpha R_{q^\intercal\alpha} - r^\intercal\partial^{-1}(\int q^\intercal L(\alpha))R_\alpha \\
&\quad +r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\alpha)R_\alpha + r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\alpha)R_\alpha \\
&\quad -L(q)\partial^{-1}(\int r\alpha)R_\alpha - L(r^\intercal)\partial^{-1}(\int q^\intercal\alpha)R_\alpha - A\partial^{-1}R_\alpha, \\
C_{23} &= q\partial^{-1}\beta^\intercal R_{r\alpha} + q\partial^{-1}(\int rL(\beta^\intercal))R_\alpha - q\partial^{-1}(\int rq)(\int r\beta^\intercal)R_\alpha \\
&\quad -q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\beta^\intercal)R_\alpha + r^\intercal\partial^{-1}\beta^\intercal R_{q^\intercal\alpha} + r^\intercal\partial^{-1}(\int q^\intercal L(\beta^\intercal))R_\alpha \\
&\quad -r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\beta^\intercal)R_\alpha - r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\beta^\intercal)R_\alpha \\
&\quad +L(q)\partial^{-1}(\int r\beta^\intercal)R_\alpha + L(r^\intercal)\partial^{-1}(\int q^\intercal\beta^\intercal)R_\alpha - B\partial^{-1}R_\alpha \\
C_{24} &= q\partial^{-1}R_{\alpha^\intercal r\alpha} + q\partial^{-2}r\partial R_{\alpha^\intercal\alpha} + q\partial^{-2}rq\partial^{-1}rR_{\alpha^\intercal\alpha} + q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal\alpha} \\
&\quad -q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^\intercal\alpha} - q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal\alpha} + r^\intercal\partial^{-1}R_{\alpha^\intercal q^\intercal\alpha} \\
&\quad +r^\intercal\partial^{-2}q^\intercal\partial R_{\alpha^\intercal\alpha} + r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\alpha^\intercal\alpha} + r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal\alpha} \\
&\quad -r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\alpha^\intercal\alpha} - r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal\alpha} \\
&\quad +L(q)\partial^{-2}rR_{\alpha^\intercal\alpha} + L(r^\intercal)\partial^{-2}q^\intercal R_{\alpha^\intercal\alpha},
\end{aligned}
$$

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\begin{aligned}
D_{11} &= (\beta^\intercal\alpha^\intercal - \alpha\beta)\partial^{-1}R_{r\beta} + E_{22}R_{r\beta} + \frac{1}{2}E_{22}\partial^{-1}R_{r\beta}\partial - \frac{1}{2}E_{22}\partial^{-1}R_{r_x\beta} \\
&\quad +\frac{1}{2}E_{22}R_{r\beta} - E_{22}\partial^{-1}qr\partial^{-1}R_{r\beta} - E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{r\beta} + E_{22}\partial^{-2}R_{rqr\beta} \\
&\quad E_{22}\partial^{-2}R_{rr^\intercal q^\intercal\beta} - q\partial^{-2}rR_{\int r\alpha)\beta\beta} + q\partial^{-2}rR_{(\int r\beta^\intercal)\alpha^\intercal\beta} + q\partial^{-2}rR_{C_2^*r\beta} \\
&\quad -r^\intercal\partial^{-2}q^\intercal R_{(\int r\alpha)\beta\beta} + r^\intercal\partial^{-2}q^\intercal R_{C_2^*r\beta} + r^\intercal\partial^{-2}q^\intercal R_{\int r\beta^\intercal\alpha^\intercal\beta}, \\
D_{12} &= E_{22}qR_\beta + (\beta^\intercal\alpha^\intercal - \alpha\beta)\partial^{-1}qR_\beta + \frac{1}{2}E_{22}\partial^{-1}q_xR_\beta - \frac{1}{2}E_{22}\partial^{-1}qR_\beta\partial \\
&\quad +\frac{1}{2}E_{22}qR_\beta - E_{22}\partial^{-1}qr\partial^{-1}qR_\beta - E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}qR_\beta \\
&\quad +E_{22}\partial^{-2}qR_{qr\beta} + E_{22}\partial^{-2}qR_{r^\intercal q^\intercal\beta} + q\partial^{-1}E_{22}\partial R_\beta + q\partial^{-1}\beta^\intercal\alpha^\intercal R_\beta \\
&\quad -q\partial^{-1}\alpha\beta R_\beta - q\partial^{-2}R_{L(\alpha)\beta\beta} + q\partial^{-2}R_{qC_2^*r\beta} + q\partial^{-2}R_{r^\intercal C_2^*q^\intercal\beta}
\end{aligned}
$$

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\begin{aligned}
&+q\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal\beta}-q\partial^{-2}rq\partial^{-1}R_{\alpha\beta\beta}+q\partial^{-2}rq\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}\\
&+q\partial^{-1}(\int rq)R_{E_{22}\beta}+q\partial^{-1}(\int rq)\partial^{-1}R_{\alpha\beta\beta}-q\partial^{-1}(\int rq)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}\\
&-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\alpha\beta\beta}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}+r^\intercal\partial^{-1}(\int q^\intercal q)R_{E_{22}\beta}\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\alpha\beta\beta}-r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}+L(q)\partial^{-1}E_{22}R_\beta\\
&-L(q)\partial^{-2}R_{\alpha\beta\beta}+L(q)\partial^{-2}R_{\beta^\intercal\alpha^\intercal\beta},\\
D_{13} &= E_{22}r^\intercal R_\beta+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}r^\intercal R_\beta+\frac{1}{2}E_{22}\partial^{-1}r_x^\intercal R_\beta-L(r^\intercal)\partial^{-2}R_{\alpha\beta\beta}\\
&-\frac{1}{2}E_{22}\partial^{-1}r^\intercal R_\beta\partial+\frac{1}{2}E_{22}r^\intercal R_\beta-E_{22}\partial^{-1}qr\partial^{-1}r^\intercal R_\beta+L(r^\intercal)\partial^{-2}R_{\beta^\intercal\alpha^\intercal\beta}\\
&-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}r^\intercal R_\beta+E_{22}\partial^{-2}r^\intercal R_{qr\beta}+E_{22}\partial^{-2}r^\intercal R_{r^\intercal q^\intercal\beta}-q\partial^{-2}rr^\intercal\partial^{-1}R_{\alpha\beta\beta}\\
&+q\partial^{-2}rr^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}+q\partial^{-1}(\int rr^\intercal)R_{E_{22}\beta}+q\partial^{-1}(\int rr^\intercal)R_{\alpha\beta\beta}\\
&-q\partial^{-1}(\int rr^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}+r^\intercal\partial^{-1}E_{22}\partial R_\beta+r^\intercal\partial^{-1}\beta^\intercal\alpha^\intercal R_\beta-r^\intercal\partial^{-1}\alpha\beta R_\beta\\
&-r^\intercal\partial^{-2}R_{L(\alpha)\beta\beta}+r^\intercal\partial^{-2}R_{L(\beta^\intercal)\alpha^\intercal\beta}+r^\intercal\partial^{-2}R_{qC_2^*r\beta}+r^\intercal\partial^{-1}R_{r^\intercal C_2^*q^\intercal\beta}\\
&-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\alpha\beta\beta}+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)R_{E_{22}\beta}\\
&+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\alpha\beta\beta}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}R_{\beta^\intercal\alpha^\intercal\beta}+L(r^\intercal)\partial^{-1}E_{22}R_\beta,\\
D_{14} &= \frac{3}{2}E_{22}R_{q^\intercal\beta}+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial^{-1}R_{q^\intercal\beta}+\frac{1}{2}E_{22}\partial^{-1}R_{q^\intercal\beta}\partial\\
&-\frac{1}{2}E_{22}\partial^{-1}R_{q_x^\intercal\beta}-E_{22}\partial^{-1}qr\partial^{-1}R_{q^\intercal\beta}-E_{22}\partial^{-1}r^\intercal q^\intercal\partial^{-1}R_{q^\intercal\beta}\\
&+E_{22}\partial^{-2}R_{q^\intercal qr\beta}+E_{22}\partial^{-2}R_{q^\intercal r^\intercal q^\intercal\beta}-q\partial^{-2}rR_{(\int q^\intercal\alpha)\beta\beta}\\
&+q\partial^{-2}rR_{(\int q^\intercal\beta^\intercal)\alpha^\intercal\beta}+q\partial^{-2}rR_{C_2^*q^\intercal\beta}-r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\alpha)\beta\beta}\\
&r^\intercal\partial^{-2}q^\intercal R_{(\int q^\intercal\beta^\intercal)\alpha^\intercal\beta}+r^\intercal\partial^{-2}q^\intercal R_{C_2^*q^\intercal\beta}\\
D_{21} &= -q\partial^{-1}R_{\beta r\beta}-q\partial^{-2}r\alpha R_{\beta\beta}-q\partial^{-2}rq\partial^{-1}rR_{\beta\beta}-q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\beta\beta}\\
&+q\partial^{-1}(\int rq)\partial^{-1}rR_{\beta\beta}+q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\beta\beta}-r^\intercal\partial^{-1}R_{\beta q^\intercal\beta}\\
&-r^\intercal\partial^{-2}q^\intercal\partial R_{\beta\beta}-r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\beta\beta}-r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\beta\beta}-L(r^\intercal)\partial^{-2}q^\intercal R_{\beta\beta}\\
&+r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\beta\beta}+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\beta\beta}-L(q)\partial^{-2}rR_{\beta\beta},\\
D_{22} &= E_{22}\partial^2+(\beta^\intercal\alpha^\intercal-\alpha\beta)\partial+2qrE_{22}+2r^\intercal q^\intercal E_{22}-2\alpha_x\beta-\alpha\beta_x+2\beta_x^\intercal\alpha^\intercal+\beta^\intercal\alpha_x^\intercal
\end{aligned}
$$

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\begin{aligned}
&+q\partial^{-1}(C_2^*L^*(r))+L(q)\partial^{-1}(C_2^*r)+r^\intercal\partial^{-1}(C_2^*L^*(q^\intercal))+L(r^\intercal)\partial^{-1}(C_2^*q^\intercal)-A\partial^{-1}\beta\\
&-B\partial^{-1}\alpha^\intercal-q\partial^{-1}\alpha R_{r\beta}-q\partial^{-1}(\int rL(\alpha))R_\beta+q\partial^{-1}(\int rq)(\int r\alpha)R_\beta\\
&+q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\alpha)R_\beta-r^\intercal\partial^{-1}\alpha R_{q^\intercal\beta}-r^\intercal\partial^{-1}(\int q^\intercal L(\alpha))R_\beta\\
&+r^\intercal\partial^{-1}(\int q^\intercal\beta)(\int r\alpha)R_\beta+r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\alpha)R_\beta\\
&-L(q)\partial^{-1}(\int r\alpha)R_\beta-L(r^\intercal)\partial^{-1}(\int q^\intercal\alpha)R_\beta-A\partial^{-1}R_\beta,\\
D_{23} \;=\;& q\partial^{-1}\beta^\intercal R_{r\beta}+q\partial^{-1}(\int rL(\beta^\intercal))R_\beta-q\partial^{-1}(\int rq)(\int r\beta^\intercal)R_\beta\\
&-q\partial^{-1}(\int rr^\intercal)(\int q^\intercal\beta^\intercal)R_\beta+r^\intercal\partial^{-1}\beta^\intercal R_{q^\intercal\beta}+r^\intercal\partial^{-1}(\int q^\intercal L(\beta^\intercal))R_\beta\\
&-r^\intercal\partial^{-1}(\int q^\intercal q)(\int r\beta^\intercal)R_\beta-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)(\int q^\intercal\beta^\intercal)R_\beta\\
&+L(q)\partial^{-1}(\int r\beta^\intercal)R_\beta+L(r^\intercal)\partial^{-1}(\int q^\intercal\beta^\intercal)R_\beta-B\partial^{-1}R_\beta\\
D_{24} \;=\;& q\partial^{-1}R_{\alpha^\intercal r\beta}+q\partial^{-2}r\partial R_{\alpha^\intercal\beta}+q\partial^{-2}rq\partial^{-1}rR_{\alpha^\intercal\beta}+q\partial^{-2}rr^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal\beta}\\
&-q\partial^{-1}(\int rq)\partial^{-1}rR_{\alpha^\intercal\beta}-q\partial^{-1}(\int rr^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal\beta}+r^\intercal\partial^{-1}R_{\alpha^\intercal q^\intercal\beta}\\
&+r^\intercal\partial^{-2}q^\intercal\partial R_{\alpha^\intercal\beta}+r^\intercal\partial^{-2}q^\intercal q\partial^{-1}rR_{\alpha^\intercal\beta}+r^\intercal\partial^{-2}q^\intercal r^\intercal\partial^{-1}q^\intercal R_{\alpha^\intercal\beta}\\
&-r^\intercal\partial^{-1}(\int q^\intercal q)\partial^{-1}rR_{\alpha^\intercal\beta}-r^\intercal\partial^{-1}(\int q^\intercal r^\intercal)\partial^{-1}q^\intercal R_{\alpha^\intercal\beta}\\
&+L(q)\partial^{-2}rR_{\alpha^\intercal\beta}+L(r^\intercal)\partial^{-2}q^\intercal R_{\alpha^\intercal\beta},
\end{aligned}
$$