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Numerical Solution for the Effect of Suction or Injection on Flow of Nanofluids Past a Stretching Sheet

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Abstract: The flow of nanofluids past a stretching sheet has attracted much attention owing to its wide applications in industry and engineering. Numerical solution has been discussed in this article for studying the effect of suction (or injection) on flow of nanofluids past a stretching sheet. The numerical results carried out using Chebyshev collocation method (ChCM). Useful results for temperature profile, concentration profile, reduced Nusselt number, and reduced Sherwood number are discussed in tabular and graphical forms. It was also demonstrated that both temperature and concentration profiles decrease by an increase from injection to suction. Moreover, the numerical results show that the temperature profiles decrease at high values of Prandtl number Pr. Finally, the present results showed that the reduced Nusselt number is a decreasing function, whereas the reduced Sherwood number is an increasing function at fixed values of Prandtl number Pr, Lewis number Le and suction (or injection) parameter *s* for variation of Brownian motion parameter Nb, and thermophoresis parameter Nt.

Keywords: Boundary Layer; Nanofluids; Numerical Solution; Stretching Sheet; Suction or Injection.

1 Introduction

Nanotechnology has been widely used in industry as materials with sizes of nanometers possess unique physical and chemical properties. Nanotechnology is considered by many to be one of the significant forces that drive the next major industrial revolution of this century. Nano-scale particle-added fluids are called as nanofluid. It represents the most relevant technological cutting edge currently being explored. It aims at manipulating the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public endeavour including biological sciences, physical sciences, electronics cooling, transportation, the environment, and national security. Choi [1] is the first author to use the term nanofluid that refers to the fluid with suspended nanoparticles. In [2], the author proved that the addition of small amount <1 % by volume of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. Each of the authors [3–6] reported that with low nanoparticles concentrations (1–5 Vol%), the thermal conductivity of the suspensions can increase more than 20 %. In recent years, some interest has been given to the study of the boundary layer flow of a nanofluid and some useful results have been introduced by the authors Kakac and Pramuanjaroenkij [7], Abu-Nada [8], Oztop and Abu-Nada [9], Nield and Kuznetsovand [10], and Kuznetsov and Nield [11]. The aim of this article is to modify a similarity solution of the work of Khan and Pop [12] to become as the same as in the work of Kuznetsov and Nield [11] for studying the effect of suction or injection on flow of nanofluids past a stretching sheet. Where the numerical results are deduced at some values of the investigating physical parameters. They are plotted using Chebyshev collocation method (ChCM).

2 Analysis

A similarity transform according to the work of Kuznetsov and Nield [11] is applied in the model of Khan and Pop [12] to convert the basic steady conservation of mass, momentum, thermal energy, and nanoparticales equations for nanofluids into the followig nonlinear ordinary differential equations:

$$f''' + \left(\frac{1}{4Pr}\right)[3f\,f'' - 2(f')^2] = 0, \tag{1}$$

$$\theta'' + \frac{3}{4} f\,\theta' + Nb\,\phi'\theta' + Nt\,(\theta')^2 = 0, \tag{2}$$

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$$\phi'' + \frac{3}{4} Le\, f\, \phi' + \frac{Nt}{Nb} \theta'' = 0, \tag{3}$$

with the boundary conditions:

$$f(0)=s, \quad f'(0)=1, \quad \theta(0)=1, \quad \phi(0)=1, \tag{4}$$

$$f'(\infty)=0, \quad \theta(\infty)=0, \quad \phi(\infty)=0, \tag{5}$$

where primes denote to differentiation with respect to η and Pr, Nb, Nt, Le, and s are Prandtl number, Brownian motion parameter, thermophoresis parameter, Lewis number, and suction (or injection), respectively. Moreover, f, θ, and ϕ are the dimensionless of the stream function, temperature, and nanoparticle volume fraction, respectively. Further, as in Kuznetsov and Nield [11], the quantities of practical interest in this study are the reduced Nusselt number Nur and the reduced Sherwood number Shr which are defined, respectively, by

$$\mathrm{Nur} = -\theta'(0), \quad \mathrm{Shr} = -\phi'(0). \tag{6}$$

2.1 Numerical

As described in the literature review, Canuto et al. [13] and Peyret [14], Chebyshev collocation method (ChCM) can be considered as a suitable choice for many practical problems. Therefore, (1–3) with the boundary conditions (4) and (5) have been solved numerically by applying ChCM ([15] and [16]). ChCM is applicable for a wide area of nonlinear differential equations. Accordingly, ChCM will be applied for the presented model. The derivatives of the function $f(x)$ at the Gauss–Lobatto points, $x_k = \cos\left(\frac{k\pi}{L}\right)$, which are the linear combination of the values of the function $f(x)$ [17]

$$f^{(n)} = D^{(n)} f,$$

where,

$$\underline{f} = [f(x_0),\ f(x_1),\ \ldots,\ f(x_L)]^T,$$

and

$$\underline{f}^{(n)} = [f^{(n)}(x_0),\ f^{(n)}(x_1),\ \ldots,\ f^{(n)}(x_L)]^T.$$

Where,

$$D^{(n)} = [d_{k,j}^{(n)}],$$

or

$$f^{(n)}(x_k) = \sum_{j=0}^{L} d_{k,j}^{(n)} f(x_j),$$

where,

$$d_{k,j}^{(n)} = \frac{2\gamma_j^*}{L} \sum_{l=n}^{L} \sum_{\substack{m=0 \\ (m+l-n)even}}^{l-n} \gamma_l^* a_{m,l}^n (-1)^{\left[\frac{lj}{L}\right] + \left[\frac{mk}{L}\right]} x_{lj - L\left[\frac{lj}{L}\right]} x_{mk - L\left[\frac{mk}{L}\right]},$$

where,

$$a_{m,l}^n = \frac{2^n l}{(n-1)!\, c_m} \frac{(s-m+n-1)!\,(s+n-1)!}{(s)!\,(s-m)!},$$

such that $2s = l+m-n$ and $c_0 = 2$, $c_i = 1$, $i \geq 1$, where $k, j = 0, 1, 2, \ldots, L$ and $\gamma_0^* = \gamma_l^* = \frac{1}{2}$, $\gamma_j^* = 1$ for $j = 1, 2, 3, \ldots, L-1$. The round off errors incurred during computing differentiation matrices $D^{(n)}$ are investigated in [17].

2.2 Description of the Chebyshev Collocation Method

The grid points (x_i, x_j) in this situation are given as $x_i = \cos\left(\frac{i\pi}{L_1}\right)$, $x_j = \cos\left(\frac{j\pi}{L_2}\right)$ for $i = 1, \ldots, L_1 - 1$ and $j = 1, \ldots, L_2 - 1$. The domain in the x-direction is $[0, x_{\max}]$ where $x_{\max}$ is the length of the dimensionless axial coordinate and the domain in the η-direction is $[0, \eta_{\max}]$ where $\eta_{\max}$ corresponds to η_∞. The domain $[0, x_{\max}] \times [0, \eta_{\max}]$ is mapped into the computational domain $[0, x_{\max}] \times [-1, 1]$. Thus, by applying the Chebyshev collocation approximation to (1–3), the following Chebyshev collocation equations can be obtained:

$$\left(\frac{2}{\eta_{\max}}\right)^3 \left(\sum_{l=0}^{L^*} d_{j,l}^{(3)} f_l\right) + \left(\frac{3}{4Pr}\right) f_j \left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(2)} f_l\right) - \left(\frac{2}{4Pr}\right) \left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(1)} f_l\right)^2 = 0, \tag{7}$$

$$\left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(2)} \theta_l\right) + \left(\frac{3}{4}\right) f_j \left(\frac{2}{\eta_{\max}}\right) \left(\sum_{l=0}^{L^*} d_{j,l}^{(1)} \theta_l\right) + Nb \left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(1)} \phi_l\right) \left(\sum_{l=0}^{L^*} d_{j,l}^{(1)} \theta_l\right) + Nt \left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(1)} \theta_l\right)^2 = 0, \tag{8}$$

$$\left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(2)} \phi_l\right) + \left(\frac{3}{4}\right) \mathrm{Le}\, f_j \left(\frac{2}{\eta_{\max}}\right) \left(\sum_{l=0}^{L^*} d_{j,l}^{(1)} \phi_l\right) + \frac{Nt}{Nb} \left(\frac{2}{\eta_{\max}}\right)^2 \left(\sum_{l=0}^{L^*} d_{j,l}^{(2)} \theta_l\right) = 0. \tag{9}$$

This system of equations for the unknowns f_j, θ_j, and ϕ_j where $j=1(1)L^*$ (take $L^*=32$) with the boundary conditions (4–5) is solved by Newton–Raphson iteration technique [14].

3 Results and Discussion

Equations (1–3) with the boundary conditions (4) and (5) have been solved numerically, using ChCM. In Table 1, the numerical results are computed for the reduced Nusselt number $\mathrm{Nur}=-\theta'(0)$ and the reduced Sherwood number $\mathrm{Shr}=-\phi'(0)$ at Nt = 0.1, 0.3, 0.5 for various values of Nb, when Pr = Le = 10 and $s=1$. It is noted that the reduced Nusselt number $\mathrm{Nur}=-\theta'(0)$ is a decreasing function while the reduced Sherwood number $\mathrm{Shr}=-\phi'(0)$ is an increasing function. Figure 1 shows plots of variation of dimensionless similarity functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ for the case Pr = Le = 1, Nb = Nt = 0.1, $d=1$, and $s=0$ in this study and previous published work of Khan and Pop [12]. Figures 2–4 show the effects of different physical parameters on both the temperature and concentration distributions. It is noticed that both the temperature and the concentration profiles start from unity near the wall and reach to vanish as the distance increases from the solid boundaries. Figure 2a and b illustrates the present numerical results for the effect of $s=-1, 0, 1$ on $\theta(\eta)$ and $\phi(\eta)$ in the case of Nt = Nb = 0.1, Pr = Le = 1. It is shown that these profiles decrease with the increase in s. However, η_∞ increase with the increase in s and Pr = Le = 1. The numerical results of the profiles of $\theta(\eta)$ and $\phi(\eta)$ are shown in Figure 3a and b for (a) Pr = 0.07, 1, 10, 10^5 at Nt = Nb = 0.1, Le = 10, and $s=1$ and (b) Le = 1, 2, 10, 20 at Nt = Nb = 0.1, Pr = 10, and $s=1$. It is clear that the temperature function decreases at high values of the Prandtl number Pr as shown in Figure 3a. Besides the typical matching of temperature profiles at values ($10 \prec \mathrm{Pr} \le 10^5$). It should be noted that in Figure 3b the concentration function decreases with the increase in the Lewis number Le. Figure 4a and b depicts

Table 1: Variation of numerical results for the reduced Nusselt number $\mathrm{Nur}=-\theta'(0)$ and the reduced Sherwood number $\mathrm{Shr}=-\phi'(0)$ at Nt = 0.1, 0.3, 0.5 for various values of Nb, when Pr = Le = 10 and $s=1$.

Nt = 0.1		Nt = 0.3		Nt= 0.5	
Nb	**Nur**	**Nb**	**Nur**	**Nb**	**Nur**
0.1	1.03564	0.1	0.947237	0.1	0.868555
0.3	0.866975	0.3	0.792816	0.3	0.726846
0.5	0.724577	0.5	0.662493	0.5	0.607285
Nt = 0.1		**Nt = 0.3**		**Nt = 0.5**	
Nb	**Shr**	**Nb**	**Shr**	**Nb**	**Shr**
0.1	7.6557	0.1	6.57938	0.1	5.76259
0.3	8.14513	0.3	7.88036	0.3	7.686
0.5	8.23787	0.5	8.12638	0.5	8.04899

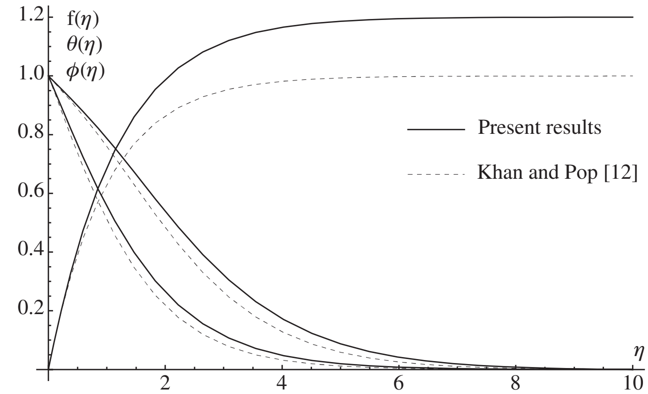

Figure 1: Plots of dimensionless similarity functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ for the case Pr = Le = 1, Nb = Nt = 0.1, and $s=0$ in the present study and previous published work of Khan and Pop [12].

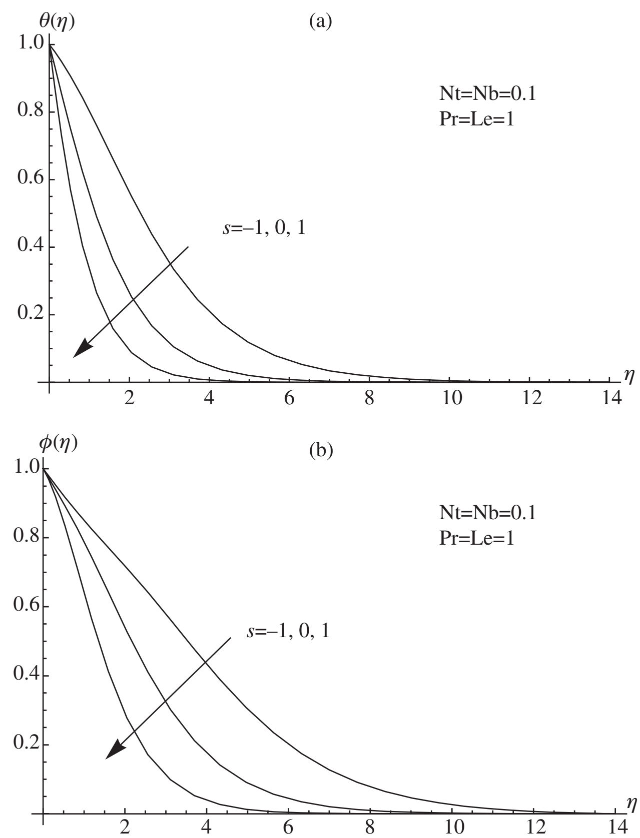

Figure 2: Effect of s on (a) $\theta(\eta)$ and (b) $\phi(\eta)$.

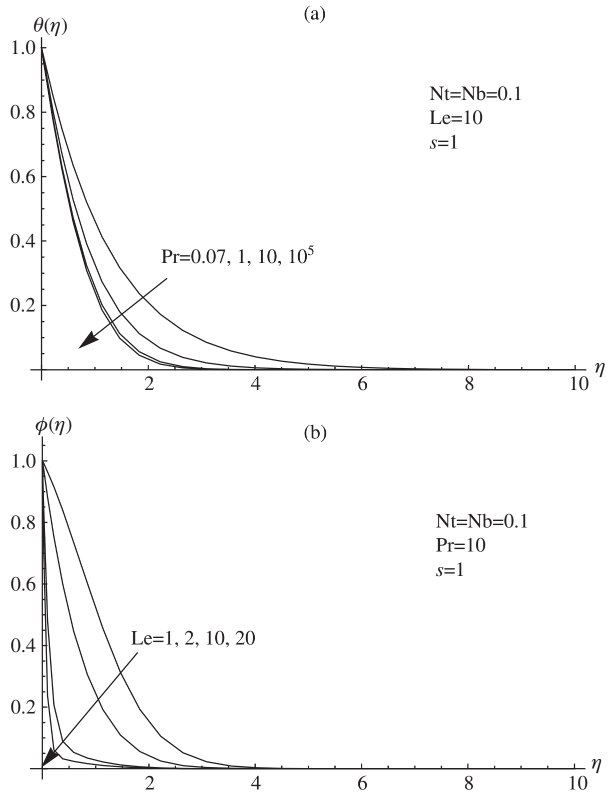

Figure 3: Effect of Pr on $\theta(\eta)$ in (a) and Le on $\phi(\eta)$ in (b).

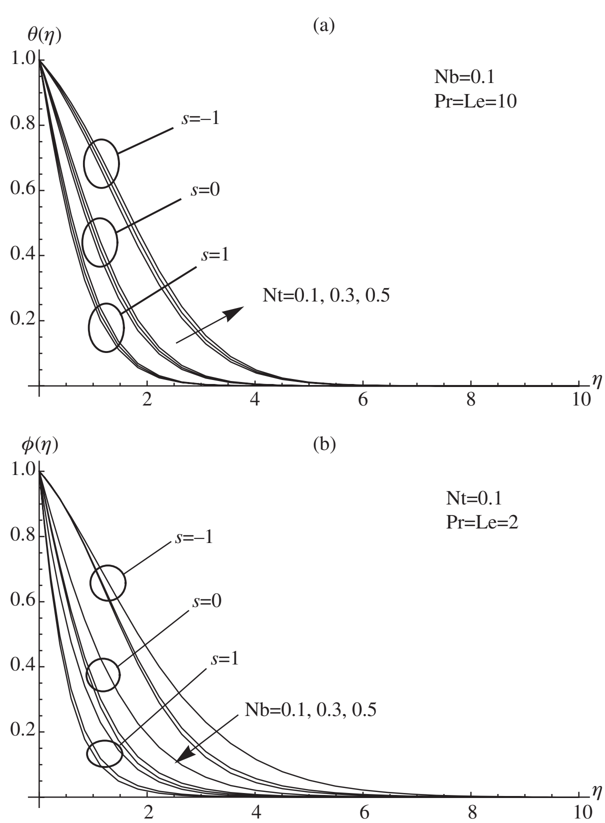

Figure 4: Effect of s and Nt in (a) on $\theta(\eta)$ and s and Nb in (b) on $\phi(\eta)$.

the numerical solutions for $\theta(\eta)$ and $\phi(\eta)$ for (a) at different values of s and Nt when Nb = 0.1 and Pr = Le = 10 and (b) at different values of s and Nb when Nt = 0.1 and Pr = Le = 2. It is known that an increase in Nt, the temperature profile $\theta(\eta)$ increases while the concentration profile $\phi(\eta)$ decreases with the increasing in Nb. It is noted that the thickness of the thermal boundary layer for the temperature profile $\theta(\eta)$ is less than thickness of the boundary layer for the concentration profile $\phi(\eta)$. But with the increase in the parameter s each of the temperature profiles $\theta(\eta)$ at Nt = 0.1, 0.3, 0.5 and the concentration profiles $\phi(\eta)$ at Nb = 0.1, 0.3, 0.5 decrease as shown in Figure 4.

4 Conclusion

Numerical solution have been analysed for studying the effect of suction or injection on flow of nanofluids past a stretching sheet. A system of nonlinear ordinary differential equations has been solved numerically using ChCM at some values of the physical parameters; Pr, Nb, Nt, Le and s. It has been concluded from the previous results that:

1. It was found that the present results show that the reduced Nusselt number Nur $= -\theta'(0)$ is a decreasing function while the reduced Sherwood number Shr $= -\phi'(0)$ is an increasing function for variation of Nt with Nb, when Pr = Le = 10 and $s = 1$.
2. The increase in the parameter s (from injection to suction) decelerates the fluid motion and decreases the temperature and the concentration along a stretching sheet.
3. Lewis number Le $\succ 1$ has strong effect on the concentration profile $\phi(\eta)$, where an increase in the Lewis number Le the concentration profile $\phi(\eta)$ decreases while the Prandtl number Pr has no effect on the temperature profile $\theta(\eta)$ when $(10 \prec \text{Pr} \le 10^5)$.

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