Magnetic Field Effects on Surface Ion Plasma Wave in Semi-bounded Magnetized Dusty Plasmas

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The effects of magnetic field strength on low-frequency surface ion plasma wave are investigated in semi-bounded magnetized dusty plasmas. The dispersion relation of the surface ion plasma wave is obtained by the plasma dielectric function with the specular reflection boundary condition. The results show that a transition from linear increase into the final level, i. e., the occurrence of a wave resonance. At a given frequency the phase velocity stays almost constant as long as the frequency remains below the respective resonance frequency.

Key words: Surface Waves; Magnetized Dusty Plasmas.

The investigation of surface waves [1 – 6] in plasmas has been of a great interest because their spectral frequency spectra provide useful information on plasma parameters for spatially bounded plasmas. In recent years there has been considerable interest in the dynamics of gases and plasmas containing dust particles or highly charged aerosols, including collective effects and strong electrostatic interaction between charged grains. Various physical processes in dusty plasmas have been investigated in order to obtain information on plasma parameters in dusty plasmas [7 – 10]. However, to the best of our knowledge the dispersion properties of the surface ion plasma wave in semi-bounded magnetized dusty plasmas have not yet been investigated. Here we consider the propagation of a surface ion plasma wave along the direction of the magnetic field in dusty plasmas. The calculation of the dispersion of surface waves in dusty plasmas can be a useful tool for investigating the structure and physical properties of such plasmas. In this paper we investigate the dispersion properties of low-frequency surface ion plasma waves in semi-bounded magnetized dusty plasmas.

The dispersion relation [1, 3] of surface waves propagating in the z-direction in semi-bounded isotropic plasmas with the plasma-vacuum interface at $x = 0$ is given by

$$\sqrt{\frac{k_z^2 c^2}{\omega^2} - 1} + \frac{\omega}{\pi c}\int_{-\infty}^{\infty} \frac{\mathrm{d}k_\perp}{k^2}\left[\frac{k_z^2 c^2}{\omega^2 \varepsilon_\mathrm{l}(\omega,k)} - \frac{k_\perp c^2}{k^2 c^2 - \omega^2 \varepsilon_\mathrm{t}(\omega,k)}\right] = 0, \quad (1)$$

where $\varepsilon_\mathrm{l}(\omega,k)$ and $\varepsilon_\mathrm{t}(\omega,k)$ are the longitudinal and transverse components of the plasma dielectric function, ω is the frequency, c is the speed of the light, $k^2 = k_\perp^2 + k_z^2$, $k_\perp (= k_x)$ and k_z are, respectively, the perpendicular and parallel components of the wave vector $\boldsymbol{k}$. The physical properties of an electrostatic wave in a dusty plasma can be obtained by the plasma dielectric function [9, 10], which is expressed as

$$\varepsilon_\mathrm{l}(\omega,k) = 1 + \chi_\mathrm{e} + \chi_\mathrm{i} + \chi_\mathrm{d}, \quad (2)$$

where $\chi_s (s = \mathrm{e,i,d})$ are the dielectric susceptibilities for electrons (e), ions (i), and dust grains (d):

$$\chi_s = \frac{\omega_\mathrm{ps}^2}{k^2 v_\mathrm{Ts}^2}\left[1 - \omega \sum_{n=-\infty}^{\infty} I_n\left(\frac{k_\perp^2 v_\mathrm{Ts}^2}{\omega_\mathrm{cs}^2}\right)\exp\left(-\frac{k_\perp^2 v_\mathrm{Ts}^2}{\omega_\mathrm{cs}^2}\right) \cdot \int_{-\infty}^{\infty}\frac{\mathrm{d}v_z f_s(v_z)}{\omega - k_z v_z - n\omega_\mathrm{cs}}\right]. \quad (3)$$

Here, ω_ps is the plasma frequency of the species s, v_Ts the thermal velocity, I_n the modified Bessel function of order n, ω_cs the cyclotron frequency, and $f_s(v_z)$ $[= (2\pi v_\mathrm{Ts}^2)^{-1/2}\exp(-v_z^2/2v_\mathrm{Ts}^2)]$ is the distribution function. For low-frequency modes, ω_cd, $kv_\mathrm{Ti}, kv_\mathrm{Td} \ll \omega \ll k_z v_\mathrm{Te}$, $\omega_\mathrm{ce} k_z/k_\perp$, $kv_\mathrm{Ts} \ll \omega_\mathrm{cs}$, and $Z_\mathrm{d} \ll m_\mathrm{d}/m_\mathrm{i}$, where Z_d is the charge number of the dust grain and m_s the mass of the species s. The plasma dielectric function [9] is found to be

$$\varepsilon_{\mathrm{l}}(\omega,k_\perp,k_z) = 1+\frac{1}{k^2\lambda_{\mathrm{D}}^2}-\frac{\omega_{\mathrm{pi}}^2k_\perp^2}{(\omega^2-\omega_{\mathrm{ci}}^2)k^2}-\frac{\omega_{\mathrm{pi}}^2k_z^2}{\omega^2k^2}-\frac{\omega_{\mathrm{pd}}^2}{\omega^2}, \tag{4}$$

where λ_{D} is the electrons Debye length. Then, in the quasi-static limit $(\omega^2\varepsilon/c^2 \ll k^2)$, the dispersion equation for low-frequency surface ion plasma wave in semi-bounded magnetized plasmas is expressed as

$$\frac{k_z}{\pi}\int_{-\infty}^{\infty}\mathrm{d}k_\perp\left[\left(1-\frac{\omega_{\mathrm{pd}}^2}{\omega^2}\right)(k_\perp^2+k_z^2)+\frac{1}{\lambda_{\mathrm{D}}^2}-\frac{\omega_{\mathrm{pi}}^2k_\perp^2}{(\omega^2-\omega_{\mathrm{ci}}^2)}-\frac{\omega_{\mathrm{pi}}^2k_z^2}{\omega^2}\right]^{-1}=-1. \tag{5}$$

After some manipulations, using the contour integration in the $k_\perp$-plane, the dispersion relation is obtained as

$$\frac{\omega^2(\omega^2-\omega_{\mathrm{ci}}^2)k_z^2}{\omega^2(\omega^2-\omega_{\mathrm{ci}}^2)-\omega_{\mathrm{pd}}^2(\omega^2-\omega_{\mathrm{ci}}^2)-\omega^2\omega_{\mathrm{pi}}^2} = \frac{1}{\lambda_{\mathrm{D}}^2}+\frac{[\omega^2-(\omega_{\mathrm{ci}}^2+\omega_{\mathrm{pd}}^2)]k_z^2}{\omega^2}. \tag{6}$$

When $\omega \ll \omega_{\mathrm{ci}}$, the dispersion relation for the surface ion plasma wave leads to

$$\begin{aligned}&\omega^4[\lambda_{\mathrm{D}}^{-2}(\omega_{\mathrm{ci}}^2+\omega_{\mathrm{pi}}^2)+k_z^2\omega_{\mathrm{pi}}^2]\\&-\omega^2[(\lambda_{\mathrm{D}}^{-2}+k_z^2)\omega_{\mathrm{pd}}^2\omega_{\mathrm{ci}}^2\\&+k_z^2(\omega_{\mathrm{pi}}^2+\omega_{\mathrm{pd}}^2)(\omega_{\mathrm{ci}}^2+\omega_{\mathrm{pi}}^2)]\\&+k_z^2(\omega_{\mathrm{pi}}^2+\omega_{\mathrm{pd}}^2)\omega_{\mathrm{pd}}^2\omega_{\mathrm{ci}}^2=0.\end{aligned} \tag{7}$$

Thus, the low-frequency mode solution of the dispersion relation for the surface ion plasma wave is found to be

$$\begin{aligned}\frac{\omega}{\omega_{\mathrm{pi}}}=&\left[2\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2+2(k_z\lambda_{\mathrm{D}})^2+2\right]^{-1/2}\\&\cdot\Bigg\{\left[[1+(k_z\lambda_{\mathrm{D}})^2]\left(\frac{\omega_{\mathrm{pd}}}{\omega_{\mathrm{pi}}}\right)^2\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2+(k_z\lambda_{\mathrm{D}})^2\left[1+\left(\frac{\omega_{\mathrm{pd}}}{\omega_{\mathrm{pi}}}\right)^2\right]\left[1+\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2\right]\right]\\&-\Bigg[\left[[1+(k_z\lambda_{\mathrm{D}})^2]\left(\frac{\omega_{\mathrm{pd}}}{\omega_{\mathrm{pi}}}\right)^2\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2+(k_z\lambda_{\mathrm{D}})^2\left[1+\left(\frac{\omega_{\mathrm{pd}}}{\omega_{\mathrm{pi}}}\right)^2\right]\left[1+\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2\right]\right]^2\\&-4\left[\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2+(k_z\lambda_{\mathrm{D}})^2+1\right](k_z\lambda_{\mathrm{D}})^2\left[1+\left(\frac{\omega_{\mathrm{pd}}}{\omega_{\mathrm{pi}}}\right)^2\right]\left(\frac{\omega_{\mathrm{pd}}}{\omega_{\mathrm{pi}}}\right)^2\left(\frac{\omega_{\mathrm{ci}}}{\omega_{\mathrm{pi}}}\right)^2\Bigg]^{1/2}\Bigg\}^{1/2}.\end{aligned} \tag{8}$$

In order to investigate the magnetic field effects on the dispersion relation, we choose three cases of the parameter $\omega_{\mathrm{pd}}/\omega_{\mathrm{pi}} = 10^{-5}$, 5×10^{-5}, and 10^{-4}. Figure 1 shows the dispersion relation as a function of the scaled wave number $(k_z\lambda_{\mathrm{D}})$ for various values of the ion cyclotron frequency. As we see in this ω/k diagram, a transition from linear increase to a final level is found, i.e., the occurrence of a wave resonance. The value of saturation is increasing with the magnetic field strength. At a given frequency the phase velocity stays almost constant as long as the frequency remains below the respective resonance frequency. Figure 2 represents the three-dimensional plot of the dispersion relation as a function of the scaled ion cyclotron frequency $(\omega_{\mathrm{ci}}/\omega_{\mathrm{pi}})$ and the scaled wave number $(k_z\lambda_{\mathrm{D}})$. With increasing the magnetic field strength, i. e., increasing the cyclotron frequency, the rotation energy becomes strongly coupled to the plasma oscillation. Thus, the transition between the linear increase and the final level exhibits a coupling between the magnetic field and charges that are placed onto the plasma-vacuum interface in a semi-bounded dusty plasma. As it is seen in Fig. 1, the transition position of the dispersion curve is shifted to larger wave numbers with increasing the magnetic field strength due to the strong coupling with the plasma oscillation. Thus, it can be understood that the size of the Larmor radius is closely related to the size of the wavelength near the transi-

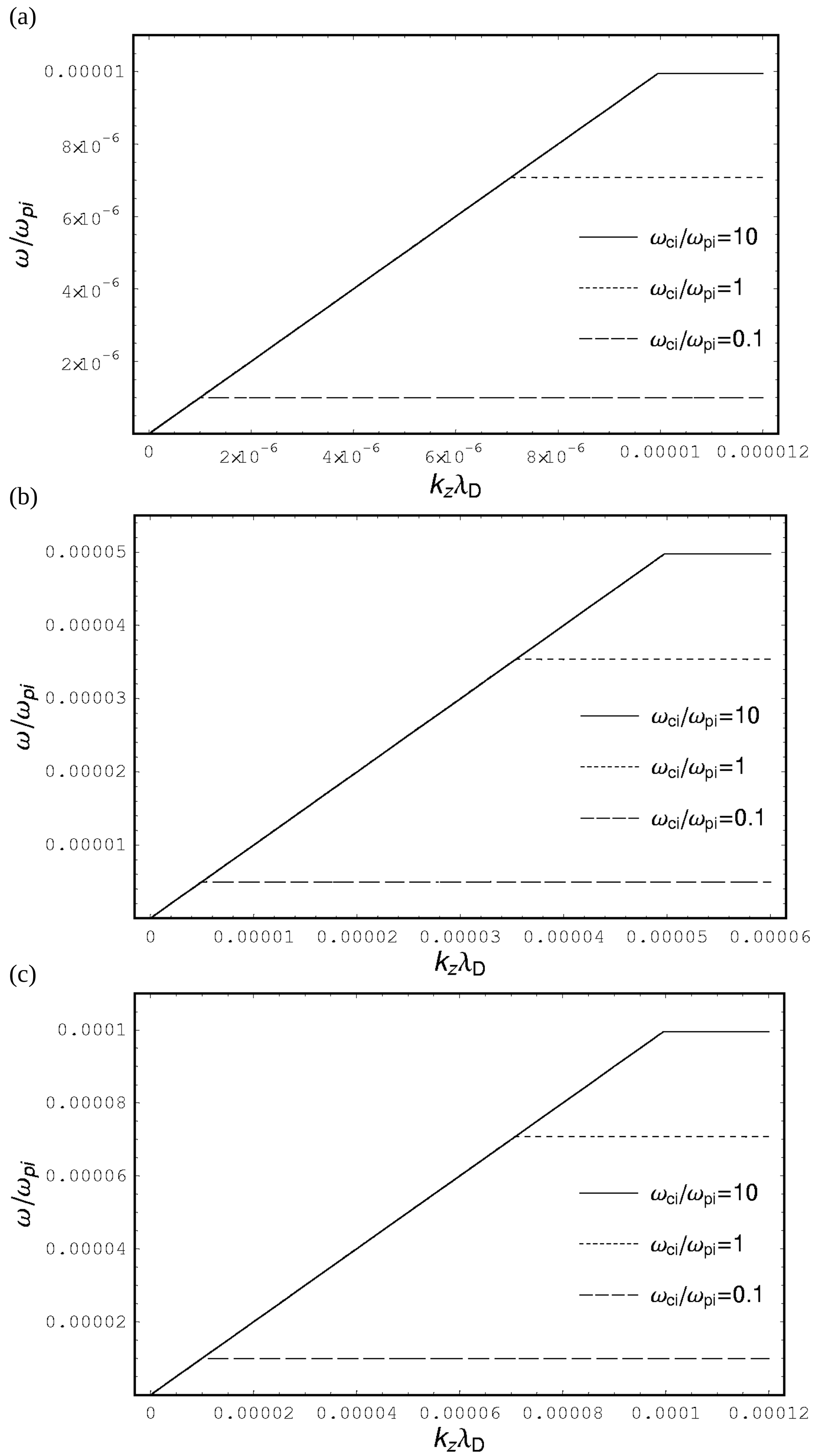

Fig. 1. The dispersion relation $(\omega/\omega_{\rm pi})$ as a function of the scaled wave number $(k_z\lambda_{\rm D})$ for various values of the ion cyclotron frequency. The solid line represents the case of $\omega_{\rm ci}/\omega_{\rm pi} = 10$. The dotted line represents the case of $\omega_{\rm ci}/\omega_{\rm pi} = 1$. The dashed line represents the case of $\omega_{\rm ci}/\omega_{\rm pi} = 0.1$. (a) $\omega_{\rm pd}/\omega_{\rm pi} = 10^{-5}$; (b) $\omega_{\rm pd}/\omega_{\rm pi} = 5 \times 10^{-5}$; (c) $\omega_{\rm pd}/\omega_{\rm pi} = 10^{-4}$.

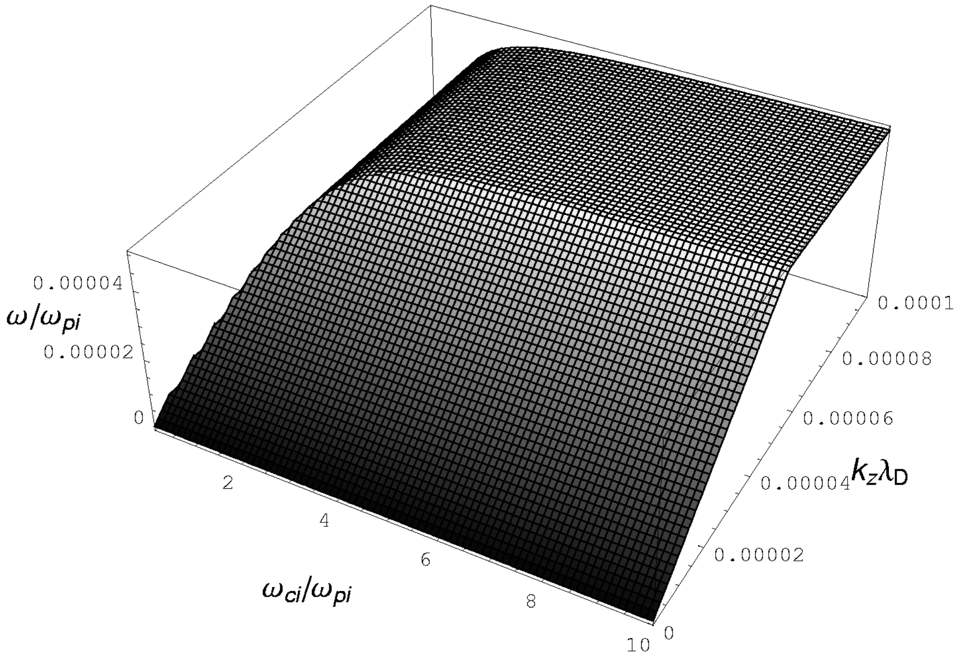

Fig. 2. The three-dimensional plot of the dispersion relation as a function of the scaled ion cyclotron frequency $(\omega_{ci}/\omega_{pi})$ and the scaled wave number $(k_z\lambda_D)$ for $\omega_{pd}/\omega_{pi} = 5\times10^{-5}$.

tion region. It is interesting to note that for small wave numbers the phase velocity of the wave is found to be independent of the variation of the ion cyclotron frequency, i.e., the strength of the magnetic field. It is also found that the phase velocity decreases with increasing wave number. In this work, the Landau damping effects on the plasma dielectric function has been neglected since the Landau damping term [11] is known to be quite small for small wave numbers. These results provide useful information on the magnetic field effects on low-frequency surface plasmas in magnetized dusty plasmas.

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