On the Experimental Proofs of Relativistic Length Contraction and Time Dilation

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Recent advances in the theory of electromagnetic retardation have made it possible to derive the basic equations of the special relativity theory and to duplicate the most important practical results of this theory without using the concepts of relativistic length contraction and time dilation. Thus the reality of these concepts appears to be questionable. It is imperative therefore to reexamine the experimental evidence supporting these concepts. The calculations presented in this paper show that some of the experiments allegedly proving the reality of length contraction and time dilation can be unambiguously interpreted as manifestations of velocity-dependent dynamical interactions taking place within the systems involved in the experiments rather than as manifestations of length contraction or time dilation.

1. Introduction

In 1887, Michelson and Morley [1] carried out an experiment attempting to detect the "world ether", which was thought to be the invisible medium occupying the entire universe and transmitting electromagnetic effects and radiation. In spite of the great sensitivity of their apparatus, no ether was detected. In an attempt to explain the negative result of the experiment without abandoning the idea of the ether, Fitzgerald in 1889 [2] and Lorentz in 1892 [3] proposed the hypothesis that, because of an interaction with the ether, all bodies are contracted in the direction of their motion relative to the ether by a factor $(1-v^2/c^2)^{1/2}$. This hypothesis provided a plausible explanation of the Lorentz transformation equation

$$x' = \gamma(x - v\,t), \tag{1}$$

and the effect (albeit hypothetical) became known as "Lorentz contraction". In (1), x represents an x coordinate measured at a time t in a stationary reference frame, or "laboratory", while x' represents the corresponding x coordinate measured in an inertial reference frame moving with velocity v with respect to the laboratory along their common x axis; $\gamma=(1-v^2/c^2)^{-1/2}$, where c is the velocity of light.

Albert Einstein [4] in his famous 1905 article discarded the idea of world ether as "superfluous" and presented a derivation of the Lorentz transformation equations of coordinates and time on the basis of his postulates of relativity and of independence of the velocity of light of the velocity of the emitter. However, while rejecting the reality of ether, he accepted length contraction of moving bodies as an observable effect, and stated that, in accordance with (1), all moving objects "viewed" from a stationary system appear shortened in the ratio 1 to $1/\gamma$. He also suggested the following method for measuring the length of a moving object (rod): observers in the stationary system ascertain at what points of the stationary system the two ends of the moving rod are located at the same time t; the distance between these two points is the "length of the moving rod".

Einstein's measuring procedure was, of course, merely a "Gedankenexperiment," that is, an imaginary procedure, a verbalization of (1), that cannot be actually implemented [1]. The procedure has never been used, so that Einstein's "length of a moving body" has never been compared with the length of a body at rest. It is not surprising therefore that, ever since its inception, the idea of relativistic length contraction has been a subject of con-

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[1] To detect a contraction of a body 100 meters long moving with a velocity of 30 kilometers per second, for example, one would have to "ascertain" with an accuracy of 5×10^{-7} meter at what points the ends of this *moving* body are located at the same time t measured with an accuracy of 5×10^{-8} seconds, which is clearly an impossible task; since neither the trajectory nor the length of a moving body is known beforehand, the procedure requires that observers with clocks should be placed in each and every point of space, which is clearly impossible; to measure the lengths of moving microscopic particles (electrons, for example) the observers and the clocks would have to be of subatomic dimensions; etc, etc.

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siderable controversy and reevaluation [5]. However, there appears to be some experimental evidence supporting (albeit indirectly) the reality of relativistic length contraction.

It should be noted that although Einstein's hypothetical relativistic length contraction is a strictly kinematic effect and has nothing to do with the world ether, it continues to be known as the "Lorentz contraction" rather than as the "relativistic length contraction" as it should be properly called [6, 7].

The idea that some physical phenomena occur at a slower rate when the system in which the phenomena take place is moving with respect to the observer dates back to 1897, when Joseph Larmor, using transformations for length and time analogous to Lorentz transformations, concluded that the periods of orbiting electrons are shorter by the factor γ in the rest system than in the moving system [8]. Einstein, in his 1905 article, interpreted (actually verbalized) the Lorentz transformation equation of time

$$t' = \gamma(t - v\,x/c^2) \tag{2}$$

as indicating that the rate of a moving clock, "when viewed from the stationary system," is slower by the factor γ than the rate of the same clock at rest in the stationary system [4]. Later he generalized this statement by declaring that "a living organism after any lengthy flight could be returned to its original spot in a scarcely altered condition, while corresponding organism which had remained in the original position had already long since given way to new generations" and "every happening in a physical system slows down when this system is set in translational motion [9, 10]. Thus, according to Einstein, not only clocks run slow, but time itself is "dilated" in systems that move with respect to the system considered to be stationary (laboratory).

The idea of the slowing down of moving clocks as a strictly kinematic effect was unacceptable to many of Einstein's contemporaries [11], and the idea of time dilation remains to this day one of the most controversial aspects of Einstein's special relativity theory. However, experiments on the radioactive decay of fast mesons show that their decay occurs indeed at a rate slower by the factor γ (within experimental errors) than for resting or slowly-moving mesons [12]. And the observations of the rate of moving atomic clocks also appear to support the reality of relativistic time dilation [13].

Until very recently it was believed that it was impossible to reject relativistic length contraction and time dilation without rejecting the relativity theory itself. The dilemma has been now resolved with the help of the theory of electromagnetic retardation[2]. Recent advances in the theory of electromagnetic retardation have made it possible to derive all the equations of relativistic electrodynamics and relativistic mechanics as a direct consequence of Maxwellian electrodynamics and electromagnetic force relations without ever invoking the concepts of relativistic length contraction and time dilation [14, 15]. Thus the reality of these concepts appears doubtful. The question now arises therefore: How reliable are the experiments seemingly proving the reality of length contraction and time dilation?

In the next two sections of this paper we shall see a series of calculations, based on Maxwellian electrodynamics as well as on the theory of relativity, showing that all the experiments that have been presented and interpreted as proofs of relativistic length contraction and time dilation can be simply and rigorously interpreted as manifestations of dynamical interactions taking place within the systems involved in the experiments rather than as manifestations of any relativistic effects.

2. Alternative Interpretation of the Experimental Proofs of Relativistic Length Contraction

Although Einstein's prescription for the measurement of the length of moving bodies is only a "Gedankenexperiment" that is impossible to implement, many authors insist that the relativistic length contraction has been proved by experiments with charged particles moving at relativistic speeds. Here is an example of such a proof.

The Stanford accelerator is approximately 3 km long, and a beam of charged particles accelerated by it would spread out by mutual electrostatic repulsion of the particles and would disappear long before reaching its target. However, in reality the beam spreads very little. According to some authors [16], the beam does not spread because, in the frame of reference comoving with the beam, the accelerator is relativistically contracted. For instance, in the case of the beam of 10-Gev electrons, whose $\gamma \approx 2\times10^4$, the 3 km are contracted to just about

[2] The theory of electromagnetic retardation is an extension of Maxwellian electrodynamics to rapidly varying and rapidly moving charge and current distributions. The theory explicitly takes into account that electromagnetic effects propagate with finite speed, so that there always is a time delay before a change in electromagnetic conditions initiated at a point of space can produce an effect at any other point of space. This time delay is called electromagnetic retardation. Hence the name "theory of electromagnetic retardation."

15 cm, so that the beam "does not have enough time to spread.[3]

The problem with this proof is that it completely ignores the actual forces responsible for the spreading of the beam, erroneously attributing the spreading to an electrostatic repulsion between the particles and even ignoring the well-known rules for the relativistic transformation of forces.

In reality, the particles in the beam are subjected not to one but to two forces: an electric *repulsion* away from the axis of the beam and a magnetic attraction toward the axis of the beam. This interplay of the electric and magnetic forces had been discussed and correctly explained in terms of classical electrodynamics long before any particle accelerators were built. A very lucid explanation of these forces had been given, for example, by R. W. Pohl in his famous textbook "Elektrizitätslehre" [17]. The essence of this explanation is as follows [18].

Consider a long cylindrical beam of charged particles moving with velocity v along its length. Let the radius of the beam be a, and let the charge density of the beam be ρ (line charge density λ). Let there be a particle of charge q on the surface of the beam, and let the polarity of this particle be the same as that of the beam. The electric field at the location of q is (see e.g. [18], pp. 89–90)

$$E = \frac{\lambda}{2\pi\varepsilon_0 a} = \frac{\rho\pi a^2}{2\pi\varepsilon_0 a} = \frac{\rho a}{2\varepsilon_0} \tag{3}$$

and is normal to the beam. The force associated with this field *repels* the charge q from the beam. The magnetic field produced by the beam at the location of q is (see e.g. [18], pp. 331–332]

$$\boldsymbol{B} = \frac{v\rho a}{2\varepsilon_0 c^2} \tag{4}$$

and is in a circular direction relative to the beam. According to the Lorentz force law (see e.g. [18], p. 417)

$$\boldsymbol{F} = q(\boldsymbol{E} + v \times \boldsymbol{B}), \tag{5}$$

the total force experienced by q is then (observe that q and the beam move with the same velocity and that the magnetic force on q is directed *into* the beam)

$$F = \frac{q\rho a}{2\varepsilon_0}(1 - \varepsilon_0\mu_0 v^2) = \frac{q\rho a}{2\varepsilon_0}(1 - v^2/c^2), \tag{6}$$

where we have used $\varepsilon_0\mu_0 = 1/c^2$.

[3] It may be noted that the same argument is frequently used to "prove" the relativistic time dilation; see Section 3.

Although the force given by (6) repels q from the beam, the repulsion is smaller than the ordinary electrostatic repulsion, because the magnetic force *attracts* q toward the beam. In fact, if the beam could move with the velocity of light, the charges on the surface of the beam and inside the beam would experience zero radial force, and the beam would not spread at all.

Thus the beam in the Stanford accelerator remains intact because the electric repulsion forces acting on its particles are to a large degree counterbalanced by the magnetic attraction forces, rather than because the accelerator is "contracted to 15 cm.".

To complete our explanation of the beam phenomenon, we shall now derive (6) by using the theory of relativity itself.

In the reference frame Σ' comoving with the beam, the force on the charge q is purely electrostatic, normal to the beam, and is, by (3),

$$F' = \frac{q\lambda'}{2\pi\varepsilon_0 a} = \frac{q\rho'\pi a^2}{2\pi\varepsilon_0 a} = \frac{q\rho' a}{2\varepsilon_0}. \tag{7}$$

The relativistic transformation equation for the charge density is (see e.g. [14], p. 150)

$$\rho = \gamma[\rho' + (v/c^2) J'_x], \tag{8}$$

and the relativistic transformation equation for the perpendicular component of the force is (see e.g. [14], pp. 196–197)

$$F = \frac{1}{\gamma(1 + v u'_x/c^2)} F'. \tag{9}$$

Assuming that the beam moves along the x axis, taking into account that in Σ' the charges of the beam do not move in the direction of the beam, so that the current $J'_x = 0$, taking into account that in Σ' the x component of the velocity of the charge q is $u'_x = 0$, and substituting (8) and (9) into (7), we obtain for the force acting on q in the laboratory reference frame

$$F = \frac{q\rho a}{2\varepsilon_0\gamma^2} = \frac{q\rho a}{2\varepsilon_0}(1 - v^2/c^2), \tag{10}$$

which is exactly the same as (6) that we obtained by directly considering the forces acting on q in the accelerator.

Thus the experimental proof of relativistic length contraction discussed above ignores the basic laws of electromagnetism and even ignores the basic rules of the relativity theory itself.

3. Alternative Interpretation of the Experimental Proofs of Relativistic Time Dilation

As a physical entity, time is defined in terms of specific measurement procedures, which may be described simply as "observing the rate of clocks." But a clock is a physical apparatus or device and is subject to the laws of physics in accordance with which the clock is constructed. Therefore. if a clock slows down when it moves, its slower rate should be explainable on the basis of the specific laws responsible for the operation of the clock.

In the preceding section we saw that certain electromagnetic interactions between rapidly moving charges are easily overlooked. It is conceivable therefore that moving clocks may run slower than stationary clocks as a result of some heretofore ignored interactions affecting moving clocks, rather than as a result of their motion as such. And if the slow rate of moving clocks can be indeed explained as a dynamic cause-and-effect phenomenon rather than as the kinematic effect enunciated by Einstein, then the slow rate of moving clocks cannot be interpreted as a proof of time dilation. Let us illustrate this point by means of the following example.

Consider a ring of radius a carrying a uniformly distributed charge q_1. Let the axis of the ring be the x axis, and let the center of the ring be the origin of rectangular coordinates (Figure 1). The electric field on the axis of the ring is (see e.g. [18], pp. 90–100)

$$\boldsymbol{E} = \frac{q_1\, x}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}\, \boldsymbol{i}\,, \tag{11}$$

where $\boldsymbol{i}$ is a unit vector along the x axis. A point charge q_2, whose polarity is opposite to that of q_1 and whose mass is m_0, is placed on the x axis near the center of the ring at a distance x from the center and is constrained to move only along the axis. By (11), if q_2 is sufficiently close to the center, so that $x \ll a$, which we assume to be the case, the force on q_2, $\boldsymbol{F} = q_2 \boldsymbol{E}$, is essentially

$$\boldsymbol{F} = -\frac{q_1 q_2\, x}{4\pi\varepsilon_0 a^3}\, \boldsymbol{i}\,. \tag{12}$$

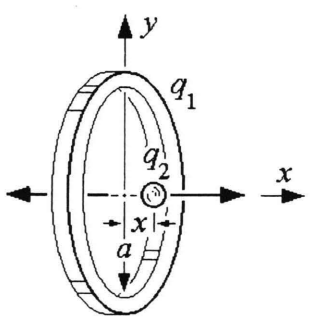

Fig. 1. A point charge q_2 is placed on the axis of an oppositely charged ring carrying a charge q_1. The point charge oscillates along the x axis about the center of the ring. This system constitutes a primitive clock.

Let the ring be fixed in the laboratory reference frame Σ. Since the force given by (12) is a linear restoring force, the ring and the charge constitute a simple harmonic oscillator, and the period of the oscillations of q_2 is

$$T = 2\pi\left(\frac{m_0}{F/x}\right)^{1/2} = 4\pi^{3/2} a^{3/2}\left(\frac{m_0\,\varepsilon_0}{q_1\, q_2}\right)^{1/2}. \tag{13}$$

Clearly, the ring and the charge may be considered to constitute a clock and can be used for measuring time in terms of the period of oscillations T.

Let us now assume that the same ring and the charge q_2 are located in a reference frame Σ' moving along the x axis with velocity $\boldsymbol{v} = v\,\boldsymbol{i}$ relative to the laboratory. By symmetry, the electric field on the axis of the ring is the same as the x component of the electric field of a moving point charge q_1 whose perpendicular distance from the axis is a. The electric field of a moving point charge q is given by the Heaviside's formula [19]

$$\boldsymbol{E} = \frac{q\,(1 - v^2/c^2)}{4\pi\varepsilon_0\, r^3 [1 - (v^2/c^2)\sin^2\theta)]^{3/2}}\, \boldsymbol{r}\,, \tag{14}$$

where v is the velocity of the charge, c is the velocity of light, $\boldsymbol{r}$ is the vector from the present position of the charge to the point of observation, and θ is the angle between $\boldsymbol{r}$ and $\boldsymbol{v}$. Since $r^2 = (a^2 + x^2)$ and since $\sin^2\theta = a^2/(a^2 + x^2)$, we have for the field on the axis of the moving ring

$$\boldsymbol{E}_m = \frac{q_1\,(1 - v^2/c^2)\, x}{4\pi\,\varepsilon_0\,(a^2 + x^2)^{3/2}\,[1 - v^2 a^2/c^2(a^2 + x^2)]^{3/2}}\, \boldsymbol{i}\,. \tag{15}$$

The subscript m is used to indicate that the field under consideration is that of the moving ring-charge.

Assuming, as before, that $x \ll a$, we then have for the force on q_2

$$\boldsymbol{F}_m = -\frac{q_1 q_2\, x}{4\pi\,\varepsilon_0\, a^3\,(1 - v^2/c^2)^{1/2}}\, \boldsymbol{i}\,. \tag{16}$$

Let us also assume that the velocity v of the moving reference frame is much larger than the maximum velocity of q_2 relative to the ring. In this case the velocity of q_2 relative to the laboratory is essentially v. The *longitudinal mass* [20] of q_2 is then

$$m_{\parallel} = \frac{m_0}{(1 - v^2/c^2)^{3/2}}\,. \tag{17}$$

The period of the oscillations of q_2 is therefore

$$T_m = 2\pi\left(\frac{m_{\|}}{F_m/x}\right)^{1/2} = 2\pi\left[\frac{m_0\, 4\pi\varepsilon_0\, a^3\,(1-v^2/c^2)^{1/2}}{(1-v^2/c^2)^{3/2}\, q_1\, q_2}\right]^{1/2} = 4\pi^{3/2}\, a^{3/2}\left[\frac{m_0\,\varepsilon_0}{(1-v^2/c^2)\, q_1\, q_2}\right]^{1/2}, \tag{18}$$

so that

$$T_m = \frac{1}{(1-v^2/c^2)^{1/2}}\, T. \tag{19}$$

Thus the period of the oscillations of q_2 located in the moving reference frame, as observed from the laboratory (stationary) reference frame, is by the factor $(1-v^2/c^2)^{-1/2}$ *longer* than the period of the oscillations of q_2 in the laboratory. Hence our clock consisting of the charged ring and the point charge runs *slower* when the clock is moving, and the rate of the moving clock is $(1-v^2/c^2)^{-1/2}=\gamma$ times the rate of the same stationary clock. However, the slow rate of our moving clock is a result of dynamic interactions (a cause-and-effect phenomenon) rather than a kinematic effect of time dilation enunciated by Einstein. Of course, the force acting on the point charge in our clock may be very different from the forces responsible for the decay of mesons or for the operation of the atomic clocks, but whatever the nature of these latter forces may be, they cannot be ignored.

It is clear now that just as the proof of length contraction discussed in Sect. 2, the experimental proofs of time dilation have a monumental flaw: they completely ignore dynamic interactions taking place in the systems under consideration. In this connection it is important to note that some authors interpret the absence of a strong spread of the beam of rapidly moving charged particles (see Sect. 2) as a proof of time dilation *as well* as a proof of length contraction.

4. Conclusion

The experiments that are interpreted as proofs of the reality of relativistic length contraction and time dilation have a simple alternative interpretation in terms of velocity-dependent forces present in the systems under consideration. Of course, we do not know what forces are responsible for the decay of elementary particles (mesons) and we know little about the force responsible for the functioning of atomic clocks. But there can be no doubt that the meson decay and the atomic clocks are controlled by some kind of forces and that these forces cannot be ignored. In the light of the calculations presented in Sect. 2 and 3 and of similar calculations presented in [21] it is much more natural and prudent to interpret the experiments allegedly proving the reality of relativistic length contraction and time dilation as manifestations of velocity-dependent interactions in the systems under consideration rather than as proofs of length contraction and time dilation. More experiments are definitely needed in order to elucidate the essence of these interactions and in order to establish their exact quantitative characteristics. One thing is obvious, however: it is fallacious to interpret any experiment as a proof of length contraction or time dilation as long as the experiment has a simple and clear alternative interpretation.

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[6] In this connection it is important to note that in recent years an important alternative to Einstein's special relativity theory has been developed. It is the "Galilei Covariant Electrodynamics" created by H. E. Wilhelm. See H. E. Wilhelm, Z. Naturforsch. **45a,** 736 (1990); H. E. Wilhelm, Apeiron, Nr. **18,** 9 (1994). Wilhelm's theory is based on accepting the world ether as the carrier of electromagnetic effects. According to this theory, length contraction and time dilation are dynamical rather than kinematic effects. Wilhelm's theory essentially completes the work initiated by Fitzgerald, Larmor, and Lorentz at the end of the last century, which has been interrupted since 1905.

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