Effect of Finite Larmor Radius on the Rayleigh-Taylor Instability of Two Component Magnetized Rotating Plasma

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The Rayleigh-Taylor (R-T) instability of two superposed plasmas, consisting of interacting ions and neutrals, in a horizontal magnetic field is investigated. The usual magnetohydrodynamic equations, including the permeability of the medium, are modified for finite Larmor radius (FLR) corrections. From the relevant linearized perturbation equations, using normal mode analysis, the dispersion relation for the two superposed fluids of different densities is derived. This relation shows that the growth rate unstability is reduced due to FLR corrections, rotation and the presence of neutrals. The horizontal magnetic field plays no role in the R-T instability. The R-T instability is discussed for various simplified configurations. It remains unaffected by the permeability of the porous medium, presence of neutral particles and rotation. The effect of different factors on the growth rate of R-T instability is investigated using numerical analysis. Corresponding graphs are plotted for showing the effect of these factors on the growth of the R-T instability.

1. Introduction

The Rayleigh Taylor (R-T) instability of a plane interface separating two fluids when one is accelerated towards the other or when one is superposed over the other has been studied by several authors, and Chandrasekhar [1] has given a good account of these investigations. The solution of problems related to the development of the R-T instability is important in many areas in Physics. In the past two decades, the ablative R-T instability has been a subject of major interest in the context of inertial fusion [2]. Allen and Hughes [3] have studied the R-T instability in astrophysical fluids.

The classical R-T instability problem has also been reinvestigated in connection with magnetic fusion problems. A generalized theory of hydromagnetic stability of the interface between two infinitely conducting superposed fluids is given by Shivamoggi [4]. The classical problem of the small amplitude motion of two superposed viscous fluids, considered as an initial value problem in the stable and the unstable (R-T instability) case, was investigated by Prosperetti [5, 6]. Menikoff et al. [7] have analyzed the character of the growth rates of the normal modes for R-T instability of superposed incompressible viscous fluids in terms of dimensionless parameters and derived a simple R-T dispersion relation. In inertial confinement fusion, the R-T instability is crucial in order to achieve symmetrical implosions, and many authors have studied the effect of various factors on the growth rate of the R-T instability. Mikaelian [8] has studied the growth rates of the R-T instability in three layered fluids and extended the analysis of N-layered fluids.

The role of the finite Larmor radius (FLR) is also important, and the discussion of the plasma stability problem in the presence of finite ion Larmor radius corrections was initiated by Rosenbluth, Krall, and Rostoker [9] in connection with a plasma under gravity and also for a slowly rotating plasma. Roberts and Taylor [10] have also discussed the instability problem using single fluid hydrodynamic equations modified to include FLR effects.

In addition to this, Chandrasekhar [1] has discussed the R-T instability of two superposed magnetized incompressible fluids and also discussed the effect of rotation on the R-T instability. He has considered the horizontal and vertical direction of the magnetic field to the separating face of two fluids. Also the axis of rotation is assumed vertical for both fluids. Singh and Hans [11] have discussed the effect of magnetic viscosity of the R-T and K-H instability of superposed fluids with horizontal magnetic field. Kalra [12] has investigated the effect of finite ion Larmor radius corrections on the R-T stability of two superposed fluids with vertical magnetic fields. Hans [13] has carried out an investigation on the R-T instability of two superposed fluids with vertical magnetic field and discussed the effect of FLR and collisions of neutrals on the growth rate of the instability. He has shown

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that the growth rate is suppressed if the collisional frequency of neutrals increases. Sharma [14, 15] has discussed the R-T instability of superposed fluids with a horizontal magnetic field and vertical rotation along with other parameters. Bhatia [16] has discussed the R-T instability of two superposed viscous conducting fluids in the presence of a horizontal magnetic field. Bhatia and Chhonkar [17] have investigated the R-T instability of two viscous superposed conducting fluids with horizontal magnetic field and rotation. They have considered rotation along the horizontal direction and magnetic field along the horizontal direction and concluded that FLR has a stabilizing influence on the superposed system. Vaghela and Chhajlani [18, 19] have analyzed the R-T problem of partially ionized superposed conducting fluids with FLR, rotation and surface tension. They have considered horizontal magnetic field and also vertical rotation. In addition to this, recently Gupta and Bhatia [20] have discussed the instability of partially ionized superposed fluids with horizontal magnetic fields. Also Sharma and Kumar [21] have discussed the R-T instability of viscous fluids in a porous medium. In the cited papers the direction of the magnetic field and that of rotation is either vertical or horizontal, and the condition of instability is different in these cases. Also the simultaneous effect of FLR, neutral collisions, rotation, and permeability of porous media has not yet been considered. This has motivated the present analysis with horizontal direction of rotation. We also know that FLR effects play the same role as the viscosity of fluids, the former being sometimes called magnetic viscosity. So, for simplicity of the analysis we assume that the contribution of the fluid-viscosity is negligible as compared to the magnetic viscosity and investigate the effect of FLR on the R-T instability. In Kelvin-Helmholtz (K-H) instability of superposed fluids, Shanghvi and Chhajlani [22] found that the FLR plays the same role as the horizontal magnetic field. Our object is to see the role of the FLR in the R-T instability and compare it with the K-H problem.

In the light of the above discussion, in the present paper we discuss the R-T instability of two superposed incompressible magnetized plasmas flowing over each other. The external magnetic field is assumed to be horizontal, and the plasma is assumed to be partially ionized. The magnetohydrodynamic equations are modified to include the effect of neutral particles. The effect of rotation is included and a porous medium and its permeability are also considered. The magnetohydrodynamic equations are modified to include the effect of finite Larmor corrections.

2. Linearized Perturbation Equations

Consider an incompressible, inviscid, infinitely conducting hydromagnetic fluid containing neutral particles. The fluid, having a horizontal surface, is assumed to be infinitely extended in the x and y directions. The magnetic field $\boldsymbol{H}(H, 0, 0)$, and $\boldsymbol{\Omega}=(\Omega, 0, 0)$, are in the x-direction (see Figure 1). The fluid is subject to gravity $\boldsymbol{g}(0, 0, -g)$. It is assumed that the density of the fluid is larger than that of the neutral particles. The effects of pressure gradient and gravity on the neutrals are negligible. The collisional force of neutrals with the ions is of the order of the pressure gradient of the ionized component.

The linearized perturbation equations under the above assumptions are

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla \delta p - \nabla \cdot \boldsymbol{\Pi} + \boldsymbol{g}\, \delta \rho + \frac{\mu_e}{4\pi} \cdot ((\nabla \times \boldsymbol{h}) \times \boldsymbol{H} + (\nabla \times \boldsymbol{H}) \times \boldsymbol{h}) + 2\rho(\boldsymbol{u} \times \boldsymbol{\Omega}) + \rho_d\, \nu_c\, (\boldsymbol{u}_d - \boldsymbol{u}) - \frac{\mu}{k_1} \boldsymbol{u}, \quad (1)$$

$$\frac{\partial \boldsymbol{u}_d}{\partial t} = -\nu_c\, (\boldsymbol{u}_d - \boldsymbol{u}), \quad (2)$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{H}), \quad (3)$$

$$\frac{\partial \delta \rho}{\partial t} = -(\boldsymbol{u} \cdot \nabla)\, \rho, \quad (4)$$

$$\nabla \cdot \boldsymbol{h} = 0, \quad \nabla \cdot \boldsymbol{u} = 0, \quad (5)$$

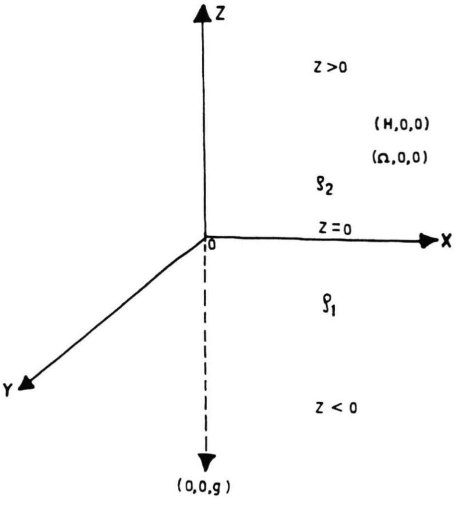

Fig. 1. The equilibrium configuration.

where $\boldsymbol{u}$ (u, v, w), $\boldsymbol{u}_d$ are the perturbation velocities of the ionized and neutral components respectively $\boldsymbol{h}$ (h_x, h_y, h_z), $\delta\rho$ and δp are the perturbations, in the magnetic field, the density and the pressure, respectively. k_1 and μ_e are the permeability of the porous medium and the magnetic permeability of the ionized component, respectively.

The pressure tensor $\mathbb{P}$, taking in account the effect of the finite ion-gyration radius for a given magnetic field along the x-direction with components is given as

$$\begin{aligned}
\Pi_{xx} &= 0,\\
\Pi_{yy} &= -\rho\, \nu_0 \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right),\\
\Pi_{zz} &= \rho\, \nu_0 \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right),\\
\Pi_{xy} &= \Pi_{yx} = -2\rho\, \nu_0 \left(\frac{\partial u}{\partial z}\right),\\
\Pi_{xz} &= \Pi_{zx} = 2\rho\, \nu_0 \left(\frac{\partial u}{\partial y}\right),\\
\Pi_{zy} &= \Pi_{yz} = \rho\, \nu_0 \left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right).
\end{aligned} \tag{6}$$

We seek solutions into normal modes whose dependence on y, z, and t is given by

$$f(z) \exp(iky + nt), \tag{7}$$

where k is the wave number in the y-direction and n is the growth rate of the disturbance. We have neglected the contribution of viscosity in the momentum transfer equation, assuming it to be small as compared to finite Larmor radius corrections, which in fact are of similar nature with anisotropy in character. This is also more realistic in discussions of magnetized plasma fluids. We also find that Gupta and Bhatia [20] have not considered the effect of neutral particles on the R-T instability, assuming $n' = n[1 + \alpha\, \nu_c/(n + \nu_c)]$. We find that this is not true in case of dust in the plasma, as the contribution of uncharged dust is important in interstellar space.

3. Dispersion Relation

With the help of (7), and writing D for d/dz, (1–5) are conveniently written explicitly in terms of the components v and w of the velocity (in the y- and z-directions, respectively) as

$$\begin{aligned}
\rho\, v [n n' + \nu / k_1] = {}& -ik\,\delta p + \frac{\mu_e}{4\pi}\left[H\left(ik\frac{w}{n}\right)(DH)\right]\\
&+ 2\rho\, w\, \Omega + \nu_0\, [\rho (D^2 - k^2)\, w\\
&+ (D\rho)(Dw) - ik\,(D\rho)\, v],
\end{aligned} \tag{8}$$

$$\begin{aligned}
\rho\, w [n n' + \nu / k_1] = {}& -D\delta p + \frac{\mu_e}{4\pi}\left[\frac{H}{n} D(w\, DH)\right]\\
&- 2\rho\, v\, \Omega - \nu_0\, [\rho (D^2 - k^2)\, v\\
&+ (D\rho)(Dv) + ik\,(D\rho)\, w] - g\,\delta\rho,
\end{aligned} \tag{9}$$

where

$$n' = \left[1 + \frac{\alpha\, \nu_c}{(n + \nu_c)}\right] \quad \text{and} \quad \alpha = \rho_d / \rho.$$

From (3) and (7) it follows that

$$h_x = -\frac{w}{n}(DH), \quad h_y = h_z = 0. \tag{10}$$

From (5) and (7)

$$v = i\frac{Dw}{k}. \tag{11}$$

From (4) we get

$$\delta\rho = -\frac{w}{n}(D\rho). \tag{12}$$

With the help of the above equations, (8) and (9) may be written in the form

$$\begin{aligned}
\rho \frac{i}{k}(Dw)[n n' + \nu / k_1] = {}& -ik\,\delta p + \frac{\mu_e}{4\pi}\left(ikH\frac{w}{n}\right)DH\\
&+ 2\rho\, w\, \Omega + \nu_0\, [\rho (D^2 - k^2)\, w\\
&+ 2(D\rho)(Dw)],
\end{aligned} \tag{13}$$

$$\begin{aligned}
\rho\, w [n n' + \nu / k_1] = {}& -D\delta p + \frac{\mu_e}{4\pi}\left[\frac{H}{n} D(w\, DH)\right] - \frac{2\rho\,\Omega\, i}{k} Dw\\
&- \nu_0\left[\rho (D^2 - k^2)\frac{i}{k}(Dw) + \frac{i}{k}(D\rho)(D^2 w)\right.\\
&\left. + ik\,(D\rho)\, w\right] - \frac{g}{n} w (D\rho) .
\end{aligned} \tag{14}$$

In writing these equations we have made use of the fact that ρ depends only on z.

We shall now derive an equation for w by eliminating δp from (14) with the help of (13):

$$[n n' + \nu / k_1] D(\rho Dw) + 2 i k \Omega (D\rho) w + 2 i k \nu_0 [D((D\rho)(Dw)) - k^2 (D\rho) w] - k^2 [n n' + \nu / k_1] \rho w + \frac{g k^2}{n} w(D\rho) = 0 . \quad (15)$$

This is the required equation for w.

It should be remarked here that the density of the neutral particles in the two regions $z<0$ and $z>0$ is assumed to be the same.

4. The Case of Two Superposed Fluids Separated Horizontal Boundary

We shall now investigate the solutions of (15) in the case of two superposed fluids, in each of which the density and the speed of sound are constant and independent of z. We shall suppose that the common boundary which separates the two fluids is located in the plane $z=0$, and we shall suppose that the fluids are of infinite extent above and below this interface. The assumption that the density and the speed of sound are constant is rather artificial (it requires unusual equations of state in the two fluids), but it may be expected to illustrate the general features of the physical situation in which incompressibility plays a role.

We now consider two superposed partially ionized plasmas of uniform densities ρ_1 and ρ_2, separated by a horizontal boundary at $z=0$. Then in each region of constant ρ, (15) becomes

$$(D^2 - k^2) w = 0 . \quad (16)$$

Since w must vanish both when $z \to -\infty$ (in the lower plasma) and $z \to \infty$ (in the upper plasma), we can write the solutions, appropriate to the two regions, as

$$w_1 = A_1 e^{kz}, \quad (z<0), \quad (17)$$

$$w_2 = A_2 e^{-kz}, \quad (z>0), \quad (18)$$

The solutions of (15) must satisfy a certain boundary condition on the horizontal planes within which the fluid is confined. Clearly, the appropriate condition is

$$w = 0, \quad \text{(on a bounding surface)}. \quad (19)$$

Additional conditions must be satisfied on any horizontal plane ($z = z_0$, say) on which there are discontinuities in the density and the speed of sound. Evidently, one condition is that

$$Dw = 0, \quad \text{(continuous on the interface)}. \quad (20)$$

A second condition is obtained by integrating (15) across the boundary $z = z_0$ between $(z_0 - \varepsilon)$ and $(z_0 + \varepsilon)$, then assuming $(\varepsilon \to 0)$. In view of the continuity of w across $z = z_0$ and the boundedness of ρ, this limiting process leads to

$$[n n' + \nu / k_1] \Delta_0 (\rho Dw) + 2 i k \Omega \Delta_0 (\rho) w_0 - 2 i k \nu_0 [k^2 \Delta_0 (\rho) w_0] + \frac{g k^2}{n} \Delta_0 (\rho) w_0 = 0 , \quad (21)$$

where (Δf) denotes the jump in the quantity f across the interface and $(w)_0$ is the common value of w on the interface.

If we apply the conditions (19) and (20) to the solutions (17) and (18), and using them in (21), we get

$$[n n' + \nu / k_1][\rho_2 Dw_2 - \rho_1 Dw_1] + 2 i k \Omega [(\rho_2 - \rho_1) w_0] - 2 \nu_0 i k^3 [(\rho_2 - \rho_1) w_0] + \frac{g k^2}{n} [(\rho_2 - \rho_1) w_0] = 0 . \quad (22)$$

5. Discussion

Now substituting the value of n', (22) becomes

$$n^3 + n^2 [\nu_c (1 + \alpha) + 2 i \Omega (\beta_1 - \beta_2) - 2 i k^2 \nu_0 (\beta_1 - \beta_2) + \nu / k_1] + n \left[2 i \Omega \nu_c (\beta_1 - \beta_2) - 2 i k^2 \nu_0 \nu_c (\beta_1 - \beta_2) + \frac{\nu \nu_c}{k_1} + g k (\beta_1 - \beta_2) \right] + g k \nu_c (\beta_1 - \beta_2) = 0 , \quad (23)$$

where

$$\beta_1 = \frac{\rho_1}{(\rho_1 + \rho_2)}, \quad \beta_2 = \frac{\rho_2}{(\rho_1 + \rho_2)},$$

Equation (23) is the general dispersion relation including the effect of permeability, neutral particles, FLR and rotation for the R-T instability of two superposed magnetized fluids. In the absence of rotation this reduces to the dispersion relation obtained by Chhajlani and Vaghela [19] without viscosity and surface tension. In the absence of the neutral particles this reduces to the dispersion relation obtained by Bhatia and Chhonkar [17] without the viscosity effect.

Now, from (23) we find that this equation is a cubic equation in "n". From the constant term of this equation, i.e. if $(\beta_1-\beta_2)<0$ or $\rho_1<\rho_2$, there will be one root giving instability and the configuration will be unstable. It is obvious from our configuration that if the density of the upper fluid is higher than that of the lower fluid, the system will be unstable.

Now if $\rho_1>\rho_2$, the system will be stable. But the dispersion relation is a cubic equation, and for the necessary condition of the stability each coefficient should be positive, and it will be positive when $\rho_1>\rho_2$ and $[\Omega(\beta_1-\beta_2)-k^2\nu_0(\beta_1-\beta_2)]>0$.

To see the effect of the various factors on the condition of instability, let us reduce (23), eliminating the factors one after the other. For the FLR correction $\nu_0=0$, (23) becomes

$$n^3+n^2[\nu_c(1+\alpha)+2i\Omega(\beta_1-\beta_2)+\nu/k_1] + n\left[2i\Omega\nu_c(\beta_1-\beta_2)+\frac{\nu\nu_c}{k_1}+gk(\beta_1-\beta_2)\right] + gk\nu_c(\beta_1-\beta_2)=0. \quad (24)$$

For $\nu_c=0$, (24) reduces to

$$n^2+n[2i\Omega(\beta_1-\beta_2)+\nu/k_1]+gk(\beta_1-\beta_2)=0. \quad (25)$$

If in (25) $\nu=0$, we get

$$n^2+n[2i\Omega(\beta_1-\beta_2)]+gk(\beta_1-\beta_2)=0. \quad (26)$$

When $\Omega=0$, then (26) becomes

$$n^2+gk(\beta_1-\beta_2)=0. \quad (27)$$

The condition of instability for (24), (25), (26) and (27) are same, for $\rho_2>\rho_1$ the system becomes unstable. From these we find that the permeability, collisional frequency of neutral particles and rotation in the horizontal direction do not affect the condition of instability, they do have the damping effect there by reducing the growth rate of instability, and thus they tend to stabilize the system.

If $\nu=\nu_c=0$ and $\Omega=0$, (23) reduces to

$$n^2-n[2ik^2\nu_0(\beta_1-\beta_2)]+gk(\beta_1-\beta_2)=0. \quad (28)$$

This dispersion relation has already been discussed by Singh and Hans [11]. The solution of (28) is given by

$$n=i\nu_0k^2(\beta_1-\beta_2) \mp [gk(\beta_1-\beta_2)+\nu_0^2k^4(\beta_1-\beta_2)^2]^{1/2}. \quad (29)$$

Equation (29) gives the result obtained by Kalra [12] in his eq. (10), for static superposed fluids under gravity. We find that the effect of the Larmor radius is to reduce the growth rate of the system otherwise monotonically unstable; the perturbation makes them over stable.

To show the effect of rotation in the horizontal directions with FLR, when $\nu=\nu_c=0$ we have

$$n^2+n[2i\Omega(\beta_1-\beta_2)-2ik^2\nu_0(\beta_1-\beta_2)] +gk(\beta_1-\beta_2)=0. \quad (30)$$

The solution of (30) is

$$n=-i(\beta_1-\beta_2)(\Omega-k^2\nu_0) \mp[gk(\beta_1-\beta_2)+(\beta_1-\beta_2)^2(\Omega-k^2\nu_0)^2]^{1/2}. \quad (31)$$

This is the modified result which includes the effect of rotation along with FLR.

Now we wish to look into the effect of various factors on the growth rate of instability. For this discussion our dispersion relation (23) is put in a non-dimensional form by the substitutions

$$\bar{n}=\frac{n}{(gk)^{1/2}},\quad \bar{k}=\frac{k}{(gk)^{1/2}},\quad \bar{\nu}_c=\frac{\nu_c}{(gk)^{1/2}},$$
$$\bar{\Omega}=\frac{\Omega}{(gk)^{1/2}},\quad \bar{\nu}_0=\nu_0(gk)^{1/2},\quad \bar{\nu}_p=\frac{\nu_p}{(gk)^{1/2}},$$

where $\nu_p=\dfrac{\nu}{k_1}$.

Equation (23) takes the following form after substitutions:

$$\bar{n}^3+\bar{n}^2[\bar{\nu}_c(1+\alpha)+2i\bar{\Omega}(\beta_1-\beta_2) -2i\bar{k}^2\bar{\nu}_0(\beta_1-\beta_2)+\bar{\nu}_p] +\bar{n}[2i\bar{\Omega}\bar{\nu}_c(\beta_1-\beta_2) -2i\bar{k}^2\bar{\nu}_0\bar{\nu}_c(\beta_1-\beta_2) +\bar{\nu}_p\bar{\nu}_c+(\beta_1-\beta_2)]+\bar{\nu}_c(\beta_1-\beta_2)=0. \quad (32)$$

Equation (32) is a cubic equation in $\bar{n}$ with complex coefficients. We solve this equation for $(\beta_2-\beta_1)>0$ for various values of $\bar{\Omega}$, $\bar{\nu}_c$ and $\bar{\nu}_0$. For $\alpha=0.1$, $\beta_1=0.2$, $\beta_2=0.8$ (potentially unstable configuration), these calculations are presented in Table 1, where we have given the growth rate (real positive part of $\bar{n}$) against the wave number $\bar{k}$, (from $\bar{k}=0$ to $\bar{k}=1$) for $\bar{\Omega}$ (angular velocity)=5, 10, 15, $\bar{\nu}_0$ (FLR)=1, 2, 3 and $\bar{\nu}_c$ (collision frequency)=0.2, 0.4, 0.6. It can be clearly seen from Table 1 that, as $\bar{\nu}_0$ and $\bar{\Omega}$ are increasing, $\bar{n}$ decreases showing thereby the stabilizing character of the effect of FLR as well as rotation. Also we see from Table 1 that the growth rate increases

Table 1.

$\bar{k}$	Values of growth rate (multiplied by 100)						
	$\bar{\nu}_c=0.2$ $\bar{\nu}_0=1.0$ $\bar{\Omega}=5.0$	$\bar{\nu}_c=0.4$ $\bar{\nu}_0=1.0$ $\bar{\Omega}=5.0$	$\bar{\nu}_c=0.6$ $\bar{\nu}_0=1.0$ $\bar{\Omega}=5.0$	$\bar{\nu}_c=0.2$ $\bar{\nu}_0=2.0$ $\bar{\Omega}=5.0$	$\bar{\nu}_c=0.2$ $\bar{\nu}_0=3.0$ $\bar{\Omega}=5.0$	$\bar{\nu}_c=0.2$ $\bar{\nu}_0=1.0$ $\bar{\Omega}=10$	$\bar{\nu}_c=0.2$ $\bar{\nu}_0=1.0$ $\bar{\Omega}=15$
0.0	24.691	40.475	54.132	24.691	24.691	21.730	20.878
0.2	24.414	40.011	53.516	24.141	23.872	21.584	20.777
0.4	23.607	38.652	51.701	22.586	21.621	21.151	20.477
0.6	22.339	36.495	48.791	20.273	18.452	20.455	19.989
0.8	20.710	33.695	44.967	17.533	14.965	19.529	19.329
1.0	18.838	30.445	40.479	14.679	11.630	18.415	18.518

Table 2.

$\bar{k}$	Values of growth rate (multiplied by 100		
	$\beta_1=0.2$, $\beta_2=0.8$, $\bar{\nu}_p=1$, $\bar{\nu}_c=0.2$, $\bar{\nu}_0=1$, $\bar{\Omega}=5$		
	$\alpha=0.2$	$\alpha=0.4$	$\alpha=0.6$
0.0	24.683	24.666	24.647
0.2	24.407	24.391	24.374
0.4	23.602	23.591	23.579
0.6	22.337	22.332	22.326
0.8	20.711	20.713	20.714
1.0	18.842	18.850	18.858

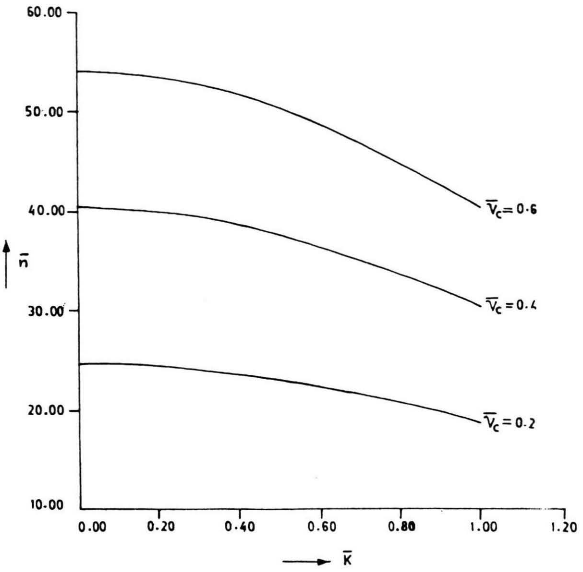

Fig. 2. The growth rate $\bar{n}$ of the unstable mode, plotted against the wave number $\bar{k}$ for the collision frequencies $\bar{\nu}_c=0.2, 0.4$ and 0.6 with $\bar{\Omega}=5$, $\bar{\nu}_0=1$, $\beta_2=0.8$, $\beta_1=0.2$ and $\alpha=0.1$.

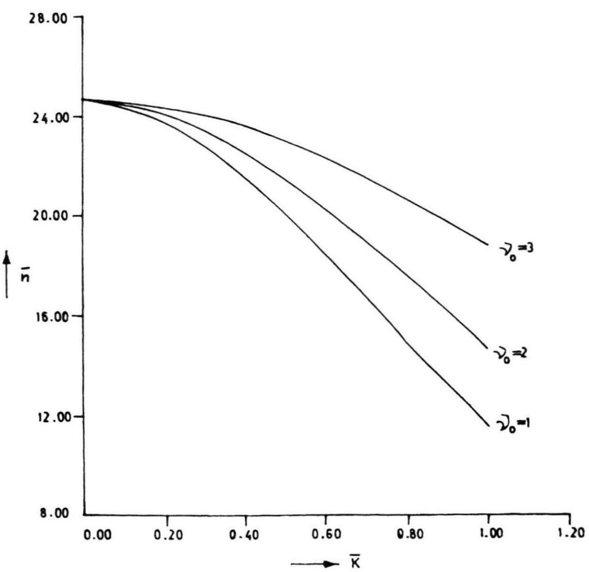

Fig. 3. The growth rate $\bar{n}$ of the unstable mode, plotted against the wave number $\bar{k}$ for the FLR $\bar{\nu}_0=1$, 2 and 3 with $\bar{\Omega}=5$, $\bar{\nu}_c=0.2$, $\beta_2=0.8$, $\beta_1=0.2$ and $\alpha=0.1$.

as $\bar{\nu}_c$ increases, for the same $\bar{k}$, thereby exhibiting the destabilizing character of the collision frequency in the presence of FLR and Coriolis forces. The results are shown in the Figs. 2, 3 and 4.

It is seen from Table 2 that for small wave numbers the growth rate decreases as α (the dust density) increases, while for large wave numbers the growth rate increases with α. We thus see that the dust density has a dual role, stabilizing for small wave numbers (large wave length) and destabilizing for large wave numbers (small wave length). Further it can be easily seen that the range of the wave numbers for which the dust density has a stabilizing influence increases with the dust density.

In the absence of FLR, the permeability and rotation equation (32) becomes

$$\bar{n}^3+\bar{n}^2\,[\bar{\nu}_c(1+\alpha)]+\bar{n}\,[(\beta_1-\beta_2)]+\bar{\nu}_c(\beta_1-\beta_2)=0\,. \quad (33)$$

This equation has been solved numerically for various values of the non-dimensional parameters. The results are presented in Figure 5. The stabilizing effect of the collision frequency is shown in this figure, where we plot the real positive root (leading to instability) for various values of α against the non-dimensionalized collision frequency $\bar{\nu}_c$, taking $\beta_2=0.8$ and $\beta_1=0.2$. We find that the growth rate is superposed with the increase of collision frequency and of dust density. When we add the Coriolis force, Fig. 5 is modified, and results are shown in Fig-

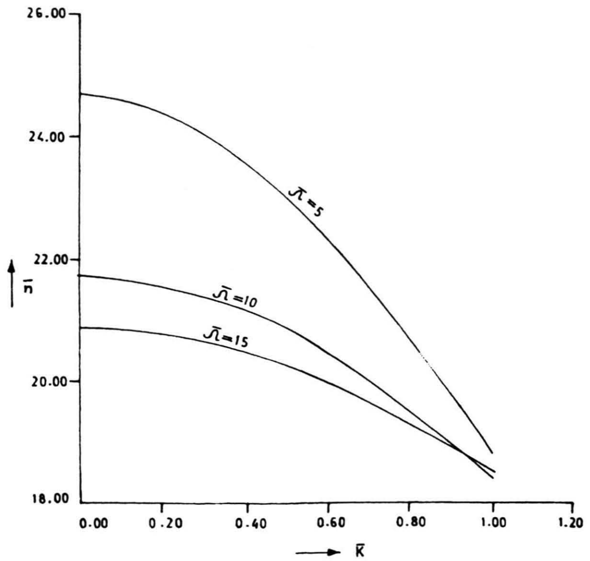

Fig. 4. The growth rate $\bar{n}$ of the unstable mode, plotted against the wave number $\bar{k}$ for the angular velocities $\bar{\Omega}=5$, 10 and 15 with $\bar{v}_c=0.2$, $\bar{v}_0=1$, $\beta_2=0.8$, $\beta_1=0.2$ and $\alpha=0.1$.

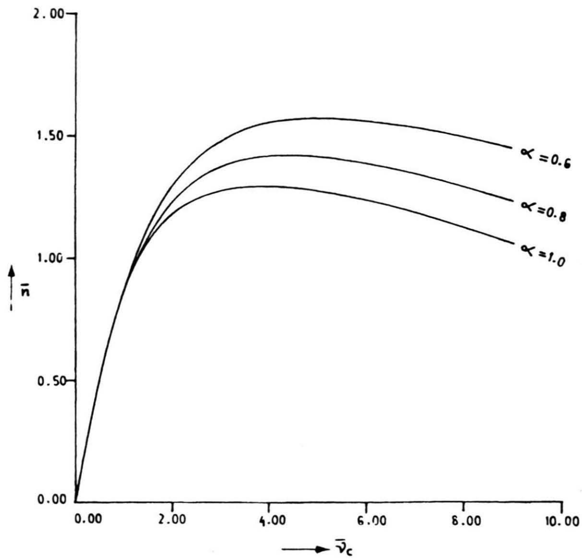

Fig. 6. The growth rate $\bar{n}$, plotted against the wave number $\bar{k}$ for $\alpha=0.6$, 0.8 and 1.0 with $\bar{\Omega}=5$, $\beta_2=0.8$ and $\beta_1=0.2$.

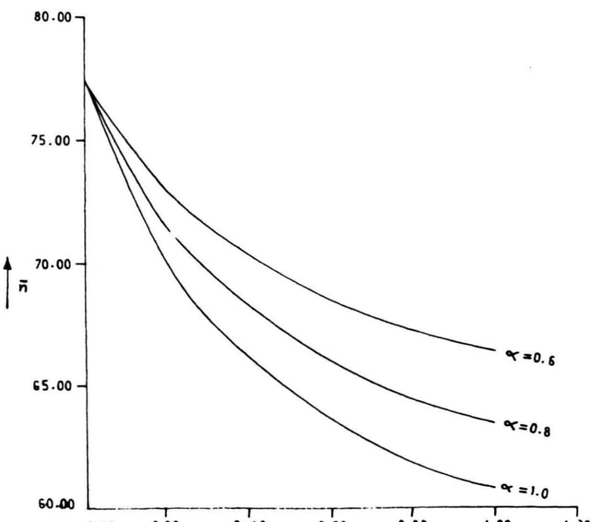

Fig. 5. The growth rate $\bar{n}$, plotted against the wave number $\bar{k}$ for $\alpha=0.6$, 0.8 and 1.0 with $\beta_2=0.8$ and $\beta_1=0.2$.

ure 6. In this case the growth rate is superposed with the increase of collision frequency and of dust density, showing thereby the stabilizing character of the effect of collision frequency.

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