

## A New Type of Chaotic Attractor

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An experimentally discovered inverted spiral-type chaotic attractor is reproduced by a model equation.

Does there exist a simple equation for the attractor recently found both in an electronic and a hydrodynamical system [1]? After a first, unpublished attempt by Sven Sahle to mirror a classical spiral-type attractor using a tube put into the middle, which yielded "messy" equations, a fairly simple ordinary differential equation (ODE) was found and will be presented in the following.

An experimental result is reproduced in Figure 1. It

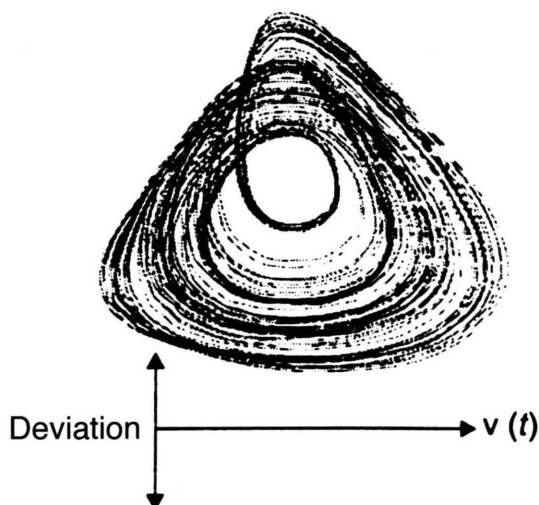


Fig. 1. Experimental attractor obtained with a hydrodynamical system, cf. [1]

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shows a typical inverted spiral-type attractor obtained in the hydrodynamical system described in [1].

The 3-variable ODE:

$$\begin{aligned}\dot{x} &= -y + z, \\ \dot{y} &= x - ay, \\ \dot{z} &= b - (x^2 + y^2)z + cz.\end{aligned}\quad (1)$$

Figures 1 and 2 show an analogous type of chaotic attractor. The main point in common is that the spiral is traversed "from the outside to the inside", in contrast to the attractor described in 1976 by Rössler [2].

Two questions are currently open:

1) Can the equations be simplified (so that eventually perhaps only a single quadratic term is left)?

2) Is a Shil'nikov flow, with a homoclinic trajectory emerging from a central saddle focus [3], as it has been observed experimentally by N. Kleiner (work in preparation), reproducible by a similarly simple equation?

To conclude, we have presented a simple 3-variable ODE showing a prototype "inverted spiral attractor".

We thank Sven Sahle and Otto E. Rössler for discussions.

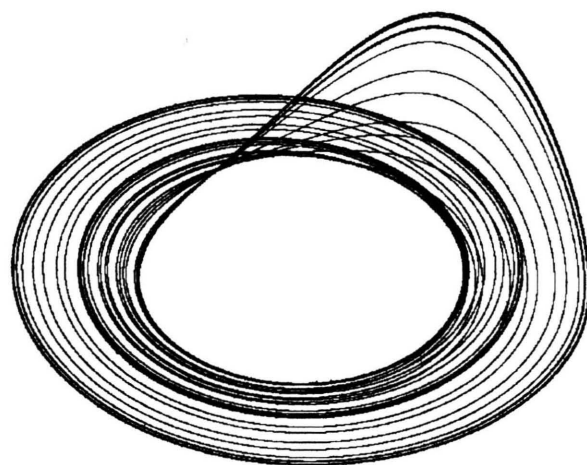


Fig. 2. Numerical simulation of (1), using  $a = 0.1$ ,  $b = 0.1$ , and  $c = 4.2$ .

- [1] N. Kleiner, Das deterministische Chaos im Experiment, in: Komplexität und Selbstorganisation, ed. by H. Krapp and T. Wägenbaur (Fink, München 1997), p. 259.  
[2] O. E. Rössler, An equation for continuous chaos, Phys. Lett. A **57**, 397 (1976).

- [3] L. P. Sil'nikov, A contribution to the problem of the structure of an extended neighborhood of a structurally stable equilibrium of saddle-focus type, Math. USSR Sbornik **10**, p. 91 (1970).

