

# Laser-Phase-Noise-Induced Stochastic-Resonance Fluorescence\*

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Stochastic fluctuations of coherent and incoherent components of the resonance fluorescence intensity induced by Wiener-Levy laser phase noise are investigated. Statistical properties of the atomic dipole moment and stochastic Mollow spectra of atomic dipole fluctuations for different values of the laser linewidth and Rabi frequency are calculated. It is shown that these spectra exhibit a triplet structure which is purely classical and entirely laser-noise dependent.

## Summary

Resonance fluorescence of two-level atoms (TLA) induced by laser-phase noise has been investigated for many years. So far most of the theoretical work has concentrated on calculating the total resonance-fluorescence intensity  $I$  which is determined by the atomic population  $p(t) = w(t) + \frac{1}{2}$  of the radiating TLA. Recently the first experiment has been reported in which the noise-induced mean and variance of the total fluorescence intensity have been measured. It is the purpose of this paper to examine the problem if laser-phase noise-induced fluctuations of the coherent  $I_{\text{coh}}(t)$  and incoherent  $I_{\text{inc}}(t)$  fluctuations in resonance fluorescence.

The stochastic Bloch equations (with quantum averages already performed) have the following form:

$$\dot{d} = \left(-i\Delta - \frac{A}{2}\right)d - i\frac{\Omega}{2}e^{-i\phi(t)}w, \quad (1a)$$

$$\dot{d}^* = \left(i\Delta - \frac{A}{2}\right)d^* + i\frac{\Omega}{2}e^{i\phi(t)}w, \quad (1b)$$

$$\dot{w} = -A(1+w) + i\Omega(d^*e^{-i\phi(t)} - d e^{i\phi(t)}), \quad (1c)$$

where  $u = d + d^*$ ,  $v = i(d - d^*)$ , and  $w$  are three components of the Bloch vector driven by a stochastic field constant Rabi frequency  $\Omega$  and fluctuating phase  $\phi(t)$ . The radiative damping and detuning are denoted by

the Einstein  $A$  coefficient and  $\Delta$ , respectively. Phase fluctuations of the laser light are assumed to be described by a Wiener-Levy stochastic process leading to frequency fluctuations characterized by the zero mean and the following autocorrelation function:

$$\langle \dot{\phi}(t) \dot{\phi}(t') \rangle = 2\Gamma \delta(t - t'). \quad (2)$$

It is well known that the total resonance-fluorescence intensity consists of two contributions:

$$I = I_{\text{coh}} + I_{\text{inc}}. \quad (3)$$

The first one, originating from the motion of the dipole  $d(t)$ , driven by the laser field, is sometimes called the coherent or the Rayleigh scattered intensity,  $I_{\text{coh}} = d^*(t)d(t)$ . The second one, called the incoherent intensity, is due to the quantum fluctuations of the dipole moment produced by the vacuum field,  $I_{\text{inc}} = p(t) - d^*(t)d(t)$ .

We shall be concerned mainly with the statistical properties of resonance fluorescence generated by the atomic dipole fluctuations:  $\langle I_{\text{coh}} \rangle = \langle d^*(t)d(t) \rangle$ , and  $\langle I_{\text{inc}} \rangle = \langle p(t) \rangle - \langle d^*(t)d(t) \rangle$ .

In Fig. 1 we have plotted the total and the coherent-fluorescence intensity as a function of the detuning for different laser linewidth  $\Gamma$  and Rabi frequency  $\Omega$ . We see that for  $\Omega = 0.5 A$  and  $\Gamma = 1 A$ , the coherent fluorescence is practically identical to the total fluorescence. For higher values of the Rabi fluorescence, i.e., for  $\Omega = 2$  and  $3.5 A$  the differences between these two fluorescence components are more pronounced. We note that for  $\Omega > 1$  the coherent intensity has a power-broadened dip at exact resonance, which could be well-understood in the dressed state picture.

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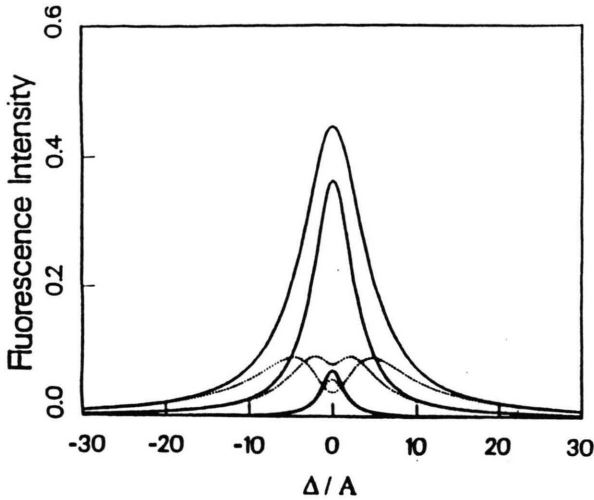


Fig. 1.  $\langle I \rangle$  and  $\langle I_{\text{coh}} \rangle$  as a function of laser detuning  $\Delta$  with  $\Gamma = 1$  A. The different solid lines from the bottom to the top correspond to  $\langle I \rangle$  with  $\Omega = 0.5, 2.0$ , and  $3.5$  A. The smallest Lorentzian dotted line curve corresponds to  $\Omega = 0.5$  A. The dotted line curves with a shallow dip and with the most pronounced dip corresponds to  $\Omega = 2.0$  and  $3.5$  A, respectively.

It is also very interesting to study the power spectra of dipole-moment fluctuations responsible for the coherent fluorescence intensity. The quantum correlation of atomic dipole moments leads to the well-known Mollow spectrum (MS). Now we are dealing with the classical correlation of atomic dipole moments driven by stochastic fluctuations of the driving field. We shall call the resulting power spectrum a Stochastic Mollow Spectrum (SMS).

In Fig. 2 we have plotted the SMS for weak-field excitation  $\Omega = 0.2$  A, detuned by  $\Delta = 10$  A and with a laser linewidth  $\Gamma = 0.1$  A. We see that in addition to the Rayleigh component centered around  $\omega = \omega_L$  we have a purely inelastic component centered around  $\omega = \omega_0$ .

A remarkable feature of the SMS emerges when the Rabi oscillations are considerably increased. In Fig. 3 we have plotted the dipole-moment spectrum for  $\Omega = 60$  A, at exact resonance and for different values of laser linewidth. The striking feature of these curves is the presence of a triplet. Note that the appearance of these three peaks is similar to the structure of the famous MS. The fundamental difference between the stochastic spectrum and the quantum spectrum is in

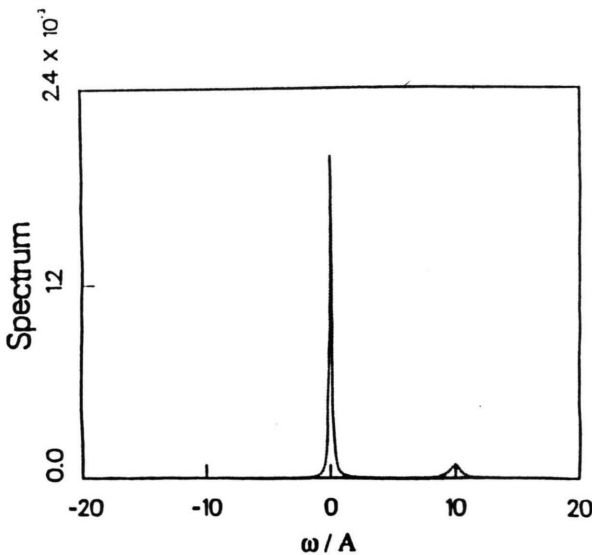


Fig. 2. The SMS for weak field  $\Omega = 0.2$  A,  $\Delta = 10$  A, and  $\Gamma = 0.1$  A.

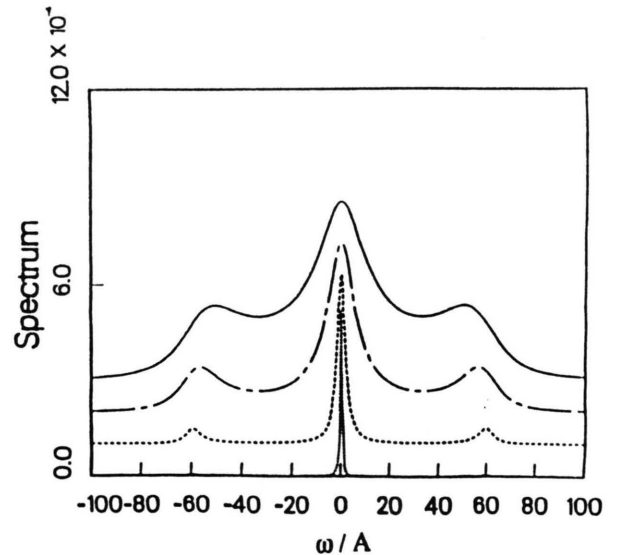


Fig. 3. The SMS for  $\Omega = 60$  A,  $\Delta = 0$ , and different values of laser linewidth. The dotted line corresponds to  $\Gamma = 0.5$  A. The dashed line, dash-dotted line, and solid line correspond to  $\Gamma = 2.0, 6.0$ , and  $10$  A, respectively, and the figures have been shifted by  $0.0001, 0.0002$ , and  $0.0003$  and magnified by  $3.0, 6.0$ , and  $7.5$ , respectively.

the source of fluctuations. The MS results from vacuum fluctuations, while the SMS results from phase-laser fluctuations. We see that when  $\Gamma$  is small, there is basically one central Rayleigh peak, but when  $\Gamma$  increases, two sideband peaks appear, and their heights grow with  $\Gamma$ . This indicates that the triplet structure is purely noise dependent. Correlations of

the atomic dipole result in this case from purely classical fluctuations induced by the laser-phase noise.

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