Wave Function Collapse as a Real Physical Phenomenon Caused by Vacuum Fluctuations Near the Planck Scale

"My original decision to devote myself to science was a direct result of the discovery which has never ceased to fill me with enthusiasm since my early youth – the comprehension of the far from obvious fact that the laws of human reasoning coincide with the laws governing the sequences of the impressions we receive from the world around us; that, therefore, pure reasoning can enable man to gain an insight into the mechanism of the latter. In this connection, it is of paramount importance that the outside world is something independent from man, something absolute, and the quest for the laws which apply to this absolute appeared to me as the most sublime scientific pursuit in life."
Max Planck (from his scientific autobiography)

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Z. Naturforsch. **46 a,** 746–758 (1991); received January 27, 1991

It is hypothesized that the collapse of the wave function is a real physical phenomenon caused by vacuum fluctuations near the Planck scale. The hypothesis is suggested by a recently proposed model (Planck aether model) according to which the fundamental kinematic symmetry is the Galilei-group with the Lorentz invariance as a derived dynamic symmetry. The proposed model has the goal to derive all fields and their interactions from an exactly nonrelativistic operator field equation, resembling Heisenberg's relativistic spinor field equation. In this model the groundstate of the vacuum is a superfluid consisting of an equal number of positive and negative Planck masses interacting via delta function potentials and making the cosmological constant equal to zero. Gauge bosons come from transverse waves propagating in a lattice of quantized vortices, and spinors are explained in this model as exciton-like quasiparticles held together by gauge bosons. Because vector gauge bosons move in the model with the velocity of light, objects held together by the force fields of these bosons obey Lorentz invariance as a dynamic symmetry. With the longitudinal wave modes moving with a superluminal phase velocity at energies near the Planck scale, it is conjectured that the quantum mechanical wave function is real and that its collapse results from the entrapment of the wave function by these longitudinal superluminal wave modes. Because these modes occur near the Planck scale their very large zero point fluctuations might therefore trigger the collapse even through dense matter. But because the fluctuations are finite, and because the wave modes have a finite albeit very large phase velocity, the quantum mechanical correlations would be broken above a certain finite length. In the limit of a vanishing Planck length, and hence vanishing gravitational constant G, the phase velocity would become infinite, and the same would be true for the length above which the correlations are broken. One therefore may say that in the limit $G=0$ the collapse is infinitely fast and that in this limit the correlations are not broken even over arbitrarily large distances.

1. Introduction

According to the orthodox or Copenhagen interpretation of quantum mechanics, the ψ-function does not represent a real object by itself, but rather our knowledge about such an object. It is then argued, that in the way in which our knowledge can change abruptly, the collapse of the wave function is in reality a mathematical process, whereby new knowledge gained through a measurement can instantaneously change the ψ-function. As von Neumann has pointed out, this interpretation leads to fundamental conceptual difficulties for which no satisfactory solution has been found. Because the measuring device must as well be described by a ψ-function, this leads to the absurd conclusion that the collapse of the wave function is ultimately a mental process. Von Neumann proved that the mathematical apparatus of quantum mechanics represents a complete set of axioms which cannot be derived from an underlying reality governed by hidden variables, but this proof is based on the assumption that quantum mechanics is a nonrelativistic theory, with time always playing the role of a parameter [1]. It is in fact difficult to see how the Copenhagen interpretation, demanding the inclusion of the entire measuring apparatus, can be fully consistent with the theory of relativity, with the apparatus permitted to be arbitrarily large. The outcome of the two-photon correlation experiments [2, 3] illustrates this problem. Because a Lorentz transformation to another reference system can there change the se-

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quence of cause and effect, the fundamental principle of causality would be broken. Even though the predictions of quantum mechanics are statistical, these experiments demonstrate unambiguously that the correlations themselves are strictly deterministic. The outcome of the two photon correlation experiments is an example of the famous Einstein-Rosen-Podolsky paradox of quantum mechanics, and it was first noticed by Schrödinger that the correlations imply a weird action at a distance. Newton once stated "that one body my act upon another at a distance through a vacuum without the mediation of anything else... is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty for thinking, can ever fall into." The nonlocal action at a distance feature of quantum mechanics was later brought into a precise mathematical theorem by Bell [4], who showed that any experiment violating a certain inequality would prove the existence of nonlocal, but at least faster than light, interactions. It is important to remark that Bell's theorem is valid regardless whether quantum mechanics is true or not, and that a violation of Bell's inequality by some experiment would confirm the existence of nonlocal (or superluminal) connections. Applying Bell's theorem to the two-photon correlation experiments, the physical reality of the nonlocal (or superluminal) connections in nature is therefore established.

The problem with this weird quantum mechanical action at a distance is greatly amplified if one realizes that the correlations shall act over arbitrarily large astronomical distances. In the two-photon correlation experiments, however, this quantum mechanical action at a distance was confirmed experimentally only over several meters. An extrapolation from there to arbitrarily large distances is such a fantastic proposition, that it can not and should not be accepted easily. It rather appears much more plausible that the quantum mechanical correlations are broken above a certain length, and that this length can be determined experimentally. If such a length exists it would indicate that the collapse of the wave function is a real physical process, taking place at a superluminal (but not infinite) speed. The occurrence of a superluminal speed would not have to violate the results of the theory of relativity, as long as the stochastic nature of quantum mechanics excludes by this process the transmission of information. The conjectured existence of a superluminal speed, however, would suggest the existence of a substratum, or aether, permeating all of space, and which would be responsible for setting up the mechanism of the quantum mechanical correlations. In fact, the physical reality of such a substratum results from quantum mechanics itself. Because of Heisenberg's uncertainty relations, each field leads to zero point vacuum fluctuations, playing the role of an aether. Relativity requires that this zero point energy is infinite, having a divergent ω^3-frequency spectrum. Cutting off this spectrum at some high frequency (like the zero point energy spectrum in a Debye solid) would destroy Lorentz invariance by creating a preferred reference system in which the zero point energy spectrum is isotropic, but if the cut-off frequency is very high, all the predictions of relativity would still be valid at low energies, only to be violated in approaching the energy of the high-frequency cut-off. Einstein's interpretation of relativity, as a manifestation of a space-time symmetry, though, would have to be given up in favor of the older Lorentz-Poincaré interpretation, where it was seen as a dynamic symmetry, caused by true physical deformations of bodies in absolute motion through a three-dimensional space, with an absolute time. In this alternative, older interpretation, the Galilei group is the fundamental kinematic symmetry, with the Lorentz group as a derived dynamic symmetry.

2. The Zero Point Vacuum Energy and the Planck Aether Hypothesis

To explain such low energy effects, like spontaneous emission, Lamb shift etc., one could assume a cut-off of the zero point vacuum energy not much higher than the rest mass energy of the electron. But, since results of high energy physics show that relativity is valid up to much higher energies, probably as high as 10^{16} GeV, a cut-off energy at least that high is needed. Most likely the cut-off energy is as high as the Planck energy of $\sim 10^{19}$ GeV, because at this energy space-time curls up providing a natural cut-off. But even a cut-off at the rest mass energy of the proton raises a serious problem, because it would give the vacuum a mass density of 10^{14} g/cm^3, leading to large gravitational fields, which obviously are not observed. If the cut-off is at the Planck energy, the vacuum mass density would be $\sim 10^{95}$ g/cm^3, large enough to put the mass of the known universe into a cube having the side length of one fermi. There must, therefore, be a mechanism by which the huge vacuum energy density is compensated. In fact, the vacuum energy must be close to absolute zero, to be in harmony with the extremely small (expressed in Planck units) cosmological constant, obtained from astronomical data. A possible solution to this problem is suggested by supersymmetric field theories, in which the positive zero point energy of the "Bose-part" is compensated by the negative zero point energy of the "Fermi-part". However, the supersymmetric particles these theories predict have never been found, and the hope that they might be observed above a certain very high energy does really not help, because it would leave uncompensated the vacuum energy up to this very high energy, again leading to very large vacuum mass densities.* However, the theoretical possibility to compensate the positive zero point energy through a negative zero point energy in supersymmetric theories gives a clue to a possible solution to this problem. As Schrödinger [5] had shown, the spin phenomenon can be understood by the negative energy (and hence negative mass) states of the Dirac equation. According to Schrödinger, the admixture of these negative mass states results in a large mass dipole superimposed on a mass monopole. The motion of such a pole-dipole particle leads to what Schrödinger has called "Zitterbewegung", possessing an orbital angular momentum as it is required to account for the particle spin. Schrödinger's analysis shows that the idea of negative masses not only has to be taken seriously, but that there is even compelling evidence for the existence of negative masses in nature.

To account for the experimentally verified vacuum fluctuations, but also for the vanishing of the cosmological constant, I have proposed as the probably most of simple model, the Planck aether hypothesis ** [6, 7]. It assumes that space is densely filled with an equal number of positive and negative Planck masses, each having the size of a Planck length, and interacting with each other by contact-type potentials. The underlying fundamental symmetry of such a model is the Galilei group, with the sea of Planck masses defining a preferred reference system. Because the Hamilton operator in nonrelativistic quantum mechanics commutes with the particle number operator, the total number of all Planck masses in conserved.* It was shown that such a model can reproduce not only Maxwell's theory of electromagnetism and Einstein's nonlinear gravitational field equation, but even Dirac spinors, including an expression for the mass of a typical Dirac spinor in terms of the Planck mass.

To discuss the Planck aether model in the context of quantum mechanics, we first note that with the exception of the fundamental Planck masses (Planckions) all particles are eliminated from it and replaced by wave packets of the aether resp. fundamental field. The proposed model therefore has great similarity with the situation existing in a solid, with the bosons resembling the phonons, and the fermions as a kind of excitons.

In the classical limit, the Planck aether can be entirely described by waves with the uncertainty relations of classical wave mechanics (k wave number, ω circular frequency)

$$\begin{aligned} &\Delta k \, \Delta q \geqq 1, \\ &\Delta \omega \, \Delta t \geqq 1. \end{aligned} \tag{2.1}$$

Quantization in the lowest approximation can then be simply understood by the establishment of standing wave patterns, as it was originally suggested by Schrödinger. Because all particles and physical objects are wave packets of the Planck aether, the interference effects so typical for quantum mechanics are explained as in classical wave theory. The quantum mechanical relationships, $E = \hbar\omega$ for energy, and $p = \hbar k$ for momentum, however, imply more. If combined with the classical uncertainty relations (2.1) they lead to the quantum mechanical uncertainty relations

$$\begin{aligned} &\Delta p \, \Delta q \geqq \hbar, \\ &\Delta E \, \Delta t \geqq \hbar \end{aligned} \tag{2.2}$$

* One therefore may say that breaking the supersymmetry generates a cosmological constant.

** Because the word "aether" was historically used as some kind of unspecified medium permeating all of space, it is important to stress that the Planck aether has little to do with these old ideas. It rather should be understood as the fundamental field from which all fields and particles are to be derived, in the same spirit as in Heisenberg's nonlinear relativistic spinor model of a unified field theory. Unlike a relativistic field theory, a nonrelativistic field theory has a finite zero point energy. It leads to a preferred reference system, which is the reason why we may speak of an "aether". In a relativistic field theory, with a Lorentz invariant but divergent ω^3-frequency spectrum of the zero point energy, the corresponding aether would have an infinite mass density. Accordingly, no preferred reference system could be assigned to do it.

* The important point that such a model must with necessity be nonrelativistic seems to have escaped Sakharov [8], who had proposed a very similar model of a vacuum filled with Planck masses (called by him maximons), and particles, called by him "ghost particles", to compensate for the huge vacuum energy density of the Planck masses. In our model the negative Planck masses are identical to Sakharov's "ghost particles."

resulting in a zero point energy. It has often been stated that this zero point energy can be interpreted by saying that through the quantum fluctuations energy can for a short time be borrowed out of nothing, a statement which sounds very strange. In the context of our model it can be understood by the coexistence of the positive with the negative Planck masses, because if some interaction is set up between positive and negative masses, a mass dipole consisting of a positive and a negative mass can, without the expenditure of energy, be accelerated to an arbitrarily large velocity, with the positive and negative kinetic energy compensating each other. This means the quantum fluctuations of the positive mass aether are always equal and opposite to the fluctuations of the negative mass aether, thereby cancelling each other out. This hypothesis would also explain why the zero point energy has no observable gravitational field.

While the single particle Schrödinger equation can explain the interference effects, for example in the double slit experiment, in can not explain why a measurement localizes a particle within a small volume in space. The wave function may have spread out over a much larger volume prior to the measurement, than the volume the particle actually occupies in the moment of the measurement. This phenomenon, known as the collapse of the wave function, has always been a mystery, but in our model this mystery is not so great because a single particle would have to be seen as a collective state of a very large number of Planck masses. The description by a single particle wave function would there always have to be seen as an approximation. In the anti-Copenhagen view taken here, the collapse of the wave function implies the existence of correlations through superluminal signals. We know of a similar situation in classical fluid dynamics, where the pressure in an incompressible fluid transmits signals at an infinite speed, thereby setting up correlations between widely separated fluid elements. In the sense that the Planck aether acts like a fluid, it is plausible to suspect that it might establish the quantum mechanical correlations at a superluminal speed, and it might therefore provide a mechanism for the collapse of the wave function. Going from a one-body system, described by a one-body Schrödinger equation to a many-body system, described by a many-body Schrödinger equation, would mean that a many-body Schrödinger wave function in configuration space can as well be only an approximate description of the true situation, with the many-body system in reality a collective structure involving a much larger number of Planck masses. In our model, the only correct description would be by a many-body Schrödinger equation in the high-dimensional configuration space of all Planck masses. The paradoxes of quantum mechanics would then arise as a result of such an approximation. The quantum mechanical correlations would still be present but they would be reduced to the correlations of all the Planck masses. Because a system of an infinite number of Planck masses can in its classical limit be described by a nonlinear field equation, this could mean that the collapse of the wave function results from this nonlinearity. If our conjecture is true, it should leave quantum mechanics intact, and would only mean a change in the model, in the sense that a correct quantum mechanical treatment would always involve the quantum mechanical infinite-many-body problem of all Planck masses.

3. The Fundamental Field Equation for the Planck Aether

If we assume that all interactions between the Planck masses are delta-function contact type interactions, repulsive between Planck masses of equal sign and attractive between those of opposite sign, the Planck aether can be described by an equation for the field operators $\psi_\pm$ of its positive and negative mass components [6]:

$$\begin{aligned} i\hbar \frac{\partial \psi_\pm}{\partial t} &= \mp \frac{\hbar^2}{2m_p} \nabla^2 \psi_\pm \\ &\quad + 2\hbar c r_p^2 (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-)\psi_\pm , \end{aligned} \tag{3.1}$$

where the field operators have to obey the canonical commutation reactions

$$\begin{aligned} &[\psi_\pm(\boldsymbol{r})\psi_\pm^\dagger(\boldsymbol{r}')] = \delta(\boldsymbol{r}-\boldsymbol{r}'), \\ &[\psi_\pm(\boldsymbol{r})\psi_\pm(\boldsymbol{r}')] = [\psi_\pm^\dagger(\boldsymbol{r})\psi_\pm^\dagger(\boldsymbol{r}')] = 0. \end{aligned} \tag{3.2}$$

In (3.1) $m_p = \sqrt{\hbar c/G} \simeq 2.2 \times 10^{-5}$ g and $r_p = \sqrt{\hbar G/c^3} \simeq 1.6 \times 10^{-33}$ cm, are the Planck mass and length derived from the two fundamental Planck relations [9] $G m_p^2 = \hbar c$ and $m_p r_p c = \hbar$. Equation (3.1) resembles Heisenberg's nonlinear spinor wave equation, with the important difference that Heisenberg's equation is exactly relativistic, whereas (3.1) is exactly nonrelativistic. Equation (3.1), having the structure of a nonlinear Schrödinger equation, actually is the Heisenberg equation of motion for the field operators $\psi_\pm$.

Another way of looking at (3.1) is to identify the operators with their expectation values, which amounts to make a Hartree approximation. One can then view the Planck aether as composed of two superfluids, one with a positive and the other with a negative mass. In each superfluid vortices can form with the circulation quantization given by

$$\oint \boldsymbol{v}_{\pm} . \, \mathrm{d}\boldsymbol{r} = n\,h/m_{\mathrm{p}}, \quad n = \pm 1, \ \pm 2, \ \pm 3, \ \ldots \tag{3.3}$$

and with the radius of the vortex core given by the Planck length r_{p}. From (3.3) immediately follows the Helmholtz vortex theorem

$$\frac{\mathrm{d}}{\mathrm{d}t} \oint \boldsymbol{v}_{\pm} \cdot \mathrm{d}\boldsymbol{r} = 0. \tag{3.4}$$

It simply states that vortices embedded in the superfluid Planck aether can not be annihilated by dissipative processes.

Because the Planck aether consists of both positive and negative masses, it can without the expenditure of energy form pairs of vortex rings possessing opposite mass. Ultimately, it can form a lattice composed of such pairs of vortex rings, with the vortex core radius equal to r_{p} and the ring radius set equal to $r_0 \gg r_{\mathrm{p}}$. As Kelvin had already shown 100 years ago, a frictionless aether consisting of a lattice of vortex rings can transmit mechanical waves, which for small amplitudes have the same property as the waves derived from Maxwell's electromagnetic field equations. By an almost trivial generalization of Kelvin's derivation of electromagnetic waves, one can deduce a second transverse wave mode which has the same property as small amplitude gravitational waves derived from Einstein's gravitational field equations [7]. Because experiments in high energy physics suggest a unification of all interactions at an energy of $\simeq 10^{16}$ GeV (GUT scale), it is plausible that the ring radius (and therefore lattice constant) is equal to the GUT scale length of $r_0 \sim 10^{-30}$ cm. This scale would determine the smallest possible wavelength for a transverse wave (for example an electromagnetic wave) propagating through the vortex lattice.

Through the zero point fluctuations of the Planck masses bound in the vortex filaments, the Planck masses become the source of longitudinal waves, leading to a scalar force coupling the vortex rings with the coupling constant turning out to be equal to Newton's constant. The Planck aether model, therefore, not only can explain Maxwell- and Einstein-type waves as mechanical vortex waves, but can even provide a completely mechanistic explanation for what is charge.

Fermions can in this model be understood as excitons resulting from the resonance energy of the vortex rings by their elliptic deformation. The positive energy resonance of a positive mass vortex together with the negative energy resonance of a negative mass vortex can form what has been called a pole-dipole particle and which can reproduce a Dirac spinor. The mass of such a pole-dipole particle results from the positive gravitational energy of two masses with opposite sign. This hypothesis for the origin of Dirac spinors is supported by the surprisingly good agreement of the mass ratio, Planck mass to spinor mass, derived under this assumption.

In the Hartree approximation, whereby the field operators in (3.1) are replaced by their expectation values

$$\begin{aligned} \varphi_{\pm} &= \langle \psi_{\pm} \rangle, \\ \varphi_{\pm}^{*} &= \langle \psi_{\pm}^{\dagger} \rangle, \\ \varphi_{\pm}^{*}\,\varphi_{\pm}^{2} &= \langle \psi_{\pm}^{\dagger}\,\psi_{\pm}\,\psi_{\pm} \rangle \end{aligned} \tag{3.5}$$

the following nonlinear wave equation for $\varphi_{\pm}$ is obtained:

$$\begin{aligned} i\hbar \frac{\partial \varphi_{\pm}}{\partial t} &= \mp \frac{\hbar^2}{2 m_{\mathrm{p}}} \nabla^2 \varphi_{\pm} \\ &\quad + 2\hbar r_{\mathrm{p}}^2 \left[\varphi_{+}^{*}\,\varphi_{+} - \varphi_{-}^{*}\,\varphi_{-}\right] \varphi_{\pm}. \end{aligned} \tag{3.6}$$

Since the Planck aether resembles a fluid, it is convenient to adopt the hydrodynamic formulation [10]. One there puts

$$\begin{aligned} \varphi_{\pm} &= A_{\pm}\, e^{iS_{\pm}}, \\ n_{\pm} &= A_{\pm}^2, \\ \boldsymbol{v}_{\pm} &= \pm \frac{\hbar}{m_{\mathrm{p}}} \operatorname{grad} S_{\pm} \end{aligned} \tag{3.7}$$

and obtains

$$\begin{aligned} &\frac{\partial n_{\pm}}{\partial t} + \operatorname{div}(n_{\pm}\, \boldsymbol{v}_{\pm}) = 0, \\ &\frac{\partial \boldsymbol{v}_{\pm}}{\partial t} + (\boldsymbol{v}_{\pm} \cdot \nabla)\, \boldsymbol{v}_{\pm} = \mp \frac{1}{m_{\mathrm{p}}} \operatorname{grad}(V + Q_{\pm}), \end{aligned} \tag{3.8}$$

$$\begin{aligned} V &= 2\hbar c\, r_{\mathrm{p}}^2 [n_{+} - n_{-}], \\ Q_{\pm} &= \mp \frac{\hbar^2}{2 m_{\mathrm{p}}} \frac{\nabla^2 \sqrt{n_{\pm}}}{\sqrt{n_{\pm}}} \end{aligned} \tag{3.9}$$

with V the ordinary and $Q_{\pm}$ the quantum potential.

4. Longitudinal Waves and Einstein's Ghost Field

We are first considering small amplitude longitudinal waves with a wavelength $r_p \ll \lambda \ll r_0$. Because $|Q_\pm|/V| \sim (r_p/r_0)^2$ we can neglect $Q_\pm$ against V. In the undisturbed state of the Planck aether there are $n_\pm = 1/2\, r_p^3\, (n' \ll n_+ \equiv n)$, Planck masses per unit volume. Keeping n_- fixed and superimposeing a small disturbance n' on $n_+\, (n' \ll n_+ \equiv n)$ then leads to a velocity disturbance $\boldsymbol{v}'_+ \equiv \boldsymbol{v}$, with both disturbances described by the linearized Euler and continuity equations (3.8)

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{2\hbar c r_p^2}{m_p} \nabla n', \tag{4.1}$$

$$\frac{\partial n'}{\partial t} + n \operatorname{div} \boldsymbol{v} = 0. \tag{4.2}$$

Putting $\theta \equiv \operatorname{div} \boldsymbol{v}$ one obtains the wave equation

$$-(1/c^2)\,\partial^2\theta/\partial t^2 + \nabla^2\theta = 0 \tag{4.3}$$

which shows that these compression waves move with the velocity of light.

To show that the zero point fluctuations of the Planck masses bound in the vortex filaments lead to a Newtonian attractive inverse square law force, we compute the average kinetic energy ΔE of the Planck masses confined in the vortex filaments:

$$\Delta E = \mp \overline{(1/2)\, m_p v^2} = \mp (1/4)\, \hbar c/r_p. \tag{4.4}$$

Because the vortex core is hollow, this energy is negative for the positive mass component of the Planck aether and positive for its negative mass component. With a volume $\pi r_p^2 . r_p = \pi r_p^3$ occupied by each Planck mass within the vortex filament, the value for the kinetic energy density ε_k is

$$\varepsilon_k = \mp (1/4\pi)\, \hbar c/r_p^4. \tag{4.5}$$

This energy density is the same as that of a Newtonian gravitational field produced by the masses in the vortex core

$$g = \mp 2\sqrt{G}\,(m_p/r_p^3)\, r \tag{4.6}$$

with an average energy density

$$\varepsilon_g = \mp \overline{g^2/8\pi} = \mp (1/4\pi)\, \hbar c/r_p^4. \tag{4.7}$$

The gravitational charges of the Planck masses distributed along the vortex rings lead to a modification of the longitudinal waves with a wavelength of the order r_0. The number of charges per unit volume attached to the vortex rings of radius r_0 in a vortex lattice with a lattice constant $\sim r_0$ is $\sim (r_p/r_0)^2\, n$, where $n = 1/2\, r_p^3$. The Planck masses, therefore, can give rise to a field determined by the Poisson-type equation

$$\operatorname{div} \boldsymbol{g} = 4\pi \sqrt{2\hbar c}\, (r_p/r_0)^2\, n'. \tag{4.8}$$

The force is here repulsive, because an increased aether density displaces and thereby dilutes the density of the vortex filaments embedded in it. With (4.8) the Euler equation (4.1) is modified as follows:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{2\hbar c r_p^2}{m_p} \nabla n' + \frac{\sqrt{2\hbar c}}{m_p}\, \boldsymbol{g} \tag{4.9}$$

with the continuity equation (4.2) unchanged. With $\theta \equiv \operatorname{div} \boldsymbol{v}$ we then obtain the modified wave equation

$$-(1/c^2)\,\partial^2\theta/\partial t^2 + \nabla^2\theta - (\omega_0^2/c^2)\,\theta = 0, \tag{4.10}$$

where

$$\omega_0^2 = 4\pi\,(c/R)^2 \tag{4.11}$$

The wave equation (4.10) has the dispersion relation

$$\omega^2 = c^2 k^2 + \omega_0^2 \tag{4.12}$$

with a phase velocity

$$\boldsymbol{v}_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{1-(\omega_0/\omega)^2}} \tag{4.13}$$

and a group velocity

$$v_{gr} = \frac{d\omega}{dk} = c\sqrt{1-(\omega_0/\omega)^2} \tag{4.14}$$

such that $v_{ph} v_{gr} = c^2$. It follows from (4.13) that the longitudinal waves have an upper cut-off at the wavelength $\lambda \sim r_0$. The longitudinal waves are, therefore, restricted to wavelengths $r_p < \lambda < r_0$. If quantized, these waves lead to very energetic zero spin bosons in the energy range $10^{16}\,\text{GeV} \lesssim E \lesssim 10^{19}\,\text{GeV}$.

Next we look for small amplitude solutions involving the simultaneous, and in magnitude equal, disturbance of the positive and negative Planck masses. In the limit where these disturbances are in phase, the combined disturbance carries no energy. Putting

$$\begin{aligned} n_+ &= n + n' \\ n_- &= n - n' \end{aligned} \tag{4.15}$$

with $n' \ll n$, the linearized equations of motion and continuity are now

$$\frac{\partial \boldsymbol{v}}{\partial t} = \mp \frac{4\hbar c r_p^2}{m_p} \nabla n' \tag{4.16}$$

$$\pm \frac{\partial n'}{\partial t} + n \operatorname{div} \boldsymbol{v}_\pm = 0, \tag{4.17}$$

from which one obtains the wave equation

$$-\frac{1}{c_0^2}\frac{\partial^2 \boldsymbol{v}_\pm}{\partial t^2} + \nabla^2 \boldsymbol{v}_\pm = 0, \tag{4.18}$$

where

$$c_0^2 = \frac{4\hbar c r^2 n}{m_p} = 2c^2, \tag{4.19}$$

hence

$$c_0 = \sqrt{2}\, c. \tag{4.20}$$

The modification of the wave equation (4.18) for wavelengths $\lambda \lesssim r_0$ is done in the same way as for the longitudinal disturbances of the positive Planck masses only. Instead of (4.8) we now have

$$\begin{aligned}\operatorname{div} \boldsymbol{g}_\pm &\simeq 4\pi\sqrt{2\hbar c}\,(n_\pm - n_\mp)(r_p/r_0)^2 \\ &= \pm 8\pi\sqrt{2\hbar c}\,(r_p/r_0)^2\, n',\end{aligned} \tag{4.21}$$

where $\boldsymbol{g}_+$ is the field acting on the positive and $\boldsymbol{g}_-$ the one acting on the negative mass waves. The equations of motion which we then obtain are

$$\frac{\partial \boldsymbol{v}_\pm}{\partial t} = \mp \frac{4\hbar c r_p^2}{m_p}\nabla n' + \frac{\sqrt{2\hbar c}}{m_p}\boldsymbol{g}_\pm . \tag{4.22}$$

Putting

$$\operatorname{div} \boldsymbol{v}_\pm = \theta_\pm, \tag{4.23}$$

one obtains from (4.17), (4.21), and (4.22) a wave equation for $\theta_\pm$:

$$-\frac{1}{c_0^2}\frac{\partial^2 \theta_\pm}{\partial t^2} + \nabla^2 \theta_\pm - \frac{\omega_0^2}{c_0^2}\theta_\pm = 0, \tag{4.24}$$

where

$$\omega_0^2 = 4\pi (c_0/r_0)^2. \tag{4.25}$$

Equation (4.24) has the dispersion relation

$$\omega^2 = c_0^2 k^2 + \omega_0^2 \tag{4.26}$$

with the phase velocity

$$v_{ph} = \frac{\omega}{k} = \frac{c_0}{\sqrt{1-(\omega_0/\omega)^2}} \tag{4.27}$$

and the group velocity

$$v_{gr} = \frac{d\omega}{dk} = c_0\sqrt{1-(\omega_0/\omega)^2} \tag{4.28}$$

such that $v_{ph}\, v_{gr} = c_0^2$.

We therefore see that these longitudinal waves also have a superluminal phase velocity going to infinity at a wave length which by order of magnitude is equal to the vortex ring radius r_0, and have cut-off for wavelengths larger that r_0, but are otherwise always faster than c by the factor $\sqrt{2}$. These waves, carrying no energy (and hence no information), remind us of Einstein's conjecture that for the collapse of the wave function a "ghost field" (Gespensterfeld) seems to be responsible. In Einstein's view it is the Schrödinger wave function which plays the role of this ghost field and which somehow guides a particle on its trajectory. A model along this line of thought (but which Einstein rejected as "too cheap") was proposed by DeBroglie and later by Bohm. In our view the Schrödinger wave function is real but only in the sense of an approximation for a system involving an infinite number of Planck masses. In the Copenhagen interpretation the collapse process is ultimately mental, something which is difficult to believe to be true, but DeBroglie's guiding wave field model is not very satisfactory either because it is difficult to see how this model can work for a quantum mechanical system of more that one particle, since such a system is described by a Schrödinger wave function in a highly abstract configuration space, not ordinary space.

We take here the view that the Schrödinger wave is physically real, and that a particle localized in space is literally "produced" by a real collapse of the Schrödinger wave function into a small volume element. This collapse, as the experiments show, must occur at superluminal speed and can even take place through dense matter*. Such a process would have to be highly nonlinear, which is certainly true if for the mechanism of the collapse process all Planck masses must be taken into account, described in our model by a nonlinear operator field equation. As we have shown, this field equation leads to longitudinal wave modes of very short wavelengths carrying no energy. It is therefore our conjecture that the wave function collapse might be triggered through chaotic entrapment of the Schrödinger wave function in these longitudinal non-

* That this is the only remaining, physically meaningful interpretation of quantum mechanics was emphasized by Renninger many years ago [11].

energy-carrying waves, which for a wavelength of $\sim 10^{-30}$ cm have a diverging superluminal phase velocity**.

5. Model for the Collapse of the Wave Function by Chaotic Entrapment

As Ehrenfest has shown, the motion of a wave packet under the influence of an outside force resembles that of the motion of a particle in classical physics [13]. In the Planck aether hypothesis, in which special relativity is a derived dynamic symmetry for objects held together by vector gauge bosons described by Maxwell-type field equations, only the wave packet as a whole would for this reason be subject to the relativistic law of motion. In the dynamic interpretation of special relativity, the inner motion of the wave packet, representing the particle, does therefore not have to obey the laws of relativistic mechanics. As long as the center of mass of a wave packet follows the relativistic laws of motion, there is no reason why its parts can not assume superluminal velocities. In fact, trying to analyze the double slit experiment in a way consistent with Newtonian realism shows that the wave packet, while having spread out over a large volume, suddenly collapses with superluminal speed into a small volume element producing a spot on the screen. In the proposed model, the probability for the wave packet to condense into a small volume is still given by the Schrödinger equation, but the ψ-function would now present a real physical entity, not just a probability amplitude. The connection with the probability density $\psi^* \psi$ would simply mean that the probability for the collapse of the wave function to a particular point is proportional to the value of $\psi^* \psi$ at this point. It is, of course, very plausible that the collapse, during which the density becomes very large, is enhanced at those locations where the density $\psi^* \psi$ prior to the collapse is large*.

The inner degrees of freedom of the wave packet describing the collapse would represent a kind of hidden variable, but since the collapse of the wave function is now considered real, taking place with superluminal speed in a finite time; von Neumann's nonexistence theorem for hidden variables does not apply. In standard one- or many-body quantum mechanics the collapse is infinitely fast. Therefore, if the superluminal speed with which the collapse occurs is very large, many body quantum mechanics would remain an extremely good approximation.

We now propose a mechanism for the collapse. We first note that the particle, which would here always be seen as a wave packet (very much as it was believed to be true by Schrödinger) is continuously exposed to the violent quantum fluctuations of the Planck aether, all the way up to the Planck energy, but in particular to fluctuations of the longitudinal waves near the wavelength $\lambda_0 \simeq r_0$ where the phase velocity of these waves diverges. Because these fluctuating waves are longitudinal, they can entrap and accelerate to superluminal velocities the wave packet representing a particle, with the center of mass of the packet remaining subluminal. The region onto which the wave packet collapses must be occupied by a fermion absorbing the packet. The wavelengths of the fluctuating longitudinal waves are extremely short, and their pattern therefore changes very rapidly, generating within a short period of time a chaotic wave pattern of almost any possible form. Therefore, if from the many violently fluctuating configurations, one representing a longitudinal wave field having superluminal phase velocity and converging onto a fermion, is suddenly offered to the wave packet, the different internal parts of the packet are entrapped in this convergent longitudinal wave field and are accelerated with superluminal speed onto the absorbing fermion. Because the wavelengths of the fluctuating longitudinal waves represent quantum fluctuations at

** The heuristic power of the (in the space and time average) massless Planck aether has been previously demonstrated to explain the Aharonov-Bohm effect, another "mystery" of quantum mechanics. It was shown that our model permits a simple local explanation of the Aharonov-Bohm effect through the formation of a massless potential vortex in the aether shifting the phase [12].

* We remark that the wave equation for ψ^* can be formally obtained by changing in the Schrödinger equation the sign for the mass. That the density function turns out to be $\psi^* \psi$, may in a strange way explain the fact that the Schrödinger equation describes a particle, resp. wave packet, which is a bound state made up from the positive and negative masses of the Planck aether. As it would be expected, no such density function exists for zero rest mass bosons which in our model are interpreted as vortex waves of the positive mass Planck aether.

$\sim 10^{16}$ GeV, the collapse of the wave function can take place without appreciable disturbance even through very dense matter*.

In order to see if this idea makes any sense, we are now trying to explore it in some quantitative detail. For simplicity, we first apply it to the collapse of a photon (or another zero rest mass particle) wave function. A particle with zero rest mass is as usual represented by a wave packet of wave frequency ω with a spread in time of the order $\Delta t \sim 1/\omega$. Near the frequency ω_0 (with $\hbar\omega_0 \simeq 10^{16}$ GeV), the wave packet, of frequency ω, is exposed during the time interval Δt to a very large number of chaotic fluctuations of the Planck aether given by the ratio of the spherical surfaces in frequency space:

$$Z = (\omega_0/\omega)^2. \tag{5.1}$$

To get an idea about the magnitude of this number, we apply it to a photon of ~ 1 eV. With $\hbar\omega_0 \simeq 10^{16}$ GeV it follows that $Z \simeq 10^{50}$. If one of these longitudinal wave fluctuations has the form of a convergent spherical wave expressed by

$$\theta = \theta_0 \frac{\exp\{i(kr+\omega t)\}}{r}, \quad \frac{\omega}{k} = v_{\text{ph}}, \tag{5.2}$$

it would entrap the wave packet collapsing it with superluminal phase velocity towards $r=0$. For the collapse actually to take place, the total energy of the wave packet $E=\hbar\omega$ must match the energy made available by the field of the convergent longitudinal wave. Since the energy of these aether waves is of the order $\hbar\omega_0 \sim 10^{16}$ GeV, the matching would not be possible, would these waves not be a superposition of a positive and a negative energy wave. Since the phase velocity of these waves diverges at $\omega \sim \omega_0$, and since the collapse requires a very large phase velocity, we may assume that the positive energy wave has the energy $\hbar(\omega_0+\omega)$ and the negative energy wave the energy $-\hbar\omega_0$, such that both combined give the energy $\hbar\omega$. The phase velocity of the negative energy wave at $\omega=\omega_0$ is infinite, but the phase velocity of the positive energy wave with a frequency $\omega_0+\omega$, with $\omega \ll \omega_0$, is according to (4.27) given by

$$v_{\text{ph}} = c\sqrt{\omega_0/\omega}. \tag{5.3}$$

We now assume that after the wave packet has been offered a wave field having the structure (5.2), it forms together with this wave field a compound state. The lifetime of this compound state is $\tau \sim 1/\omega$, which is about equal the spread in time of the wave packet. It then follows that the largest length L, over which the wave packet can spread prior to its collapse is

$$L \simeq v_{\text{ph}}\,\tau = (c/\omega)\sqrt{\omega_0/\omega}. \tag{5.4}$$

This is the length above which the correlation for photons (for example in Aspect's two photon correlation experiment) would be broken. For $\hbar\omega \sim 1$ eV photons, this length would be of the order ~ 100 km.

We now show that for this correlation length L, the probability that the wave packet is offered a longitudinal wave field causing its collapse, is of order unity. This surprising result emerges due to the immense magnitude Z of the chaotic wave pattern. The probability for a wave pattern to emerge out to these chaotic fluctuations, having the structure of a convergent wave field of wave number k within a volume L^3, and directed within an angle $\lambda/L \sim 1/Lk$ towards a center of convergence, is

$$P = (Lk)^{-3}(Lk)^{-1} = (Lk)^{-4} = (\omega/\omega_0)^2, \tag{5.5}$$

such that

$$ZP = 1. \tag{5.6}$$

To generalize the expression of the correlation length obtained for zero rest mass particles to those of finite rest mass, we insert into the expression (5.3) for v_{ph} the DeBroglie relationship:

$$\omega/k = c^2/v, \tag{5.7}$$

where $v<c$ is the particle velocity, with the result that

$$v_{\text{ph}} = \sqrt{\omega_0 v/k}. \tag{5.8}$$

For τ we have to put

$$\tau = \hbar/(m-m_0)c^2, \tag{5.9}$$

hence

$$L = \sqrt{\omega_0 v/k}\,\hbar/(m-m_0)c^2 \tag{5.10}$$

or with $mv/\hbar = k$

$$L = \sqrt{\hbar\omega_0/mc^2}\,\hbar/(m-m_0)c. \tag{5.11}$$

* The proposed collapse process does not have to contradict the law of entropy because the Schrödinger equation is symmetric under time reversal. More precisely this is only true if ψ and ψ^* are interchanged at the same time, but since in a measurement one is always interested only in the product of $\psi^*\psi$, the time reversal symmetry is perfect and the spreading out of the wave function is therefore a reversible process.

For $v \ll c$ this is

$$L = 2\sqrt{m\omega_0/\hbar}\, k^{-2}, \quad (5.12)$$

and one there has

$$P = (Lk)^{-4} = (E/E_0)^2, \quad (5.13)$$

where $E = mv^2/2$ and $E_0 = \hbar\omega_0$. Replacing in (5.1) ω by $\tau^{-1} = E/\hbar$ one finds that

$$Z = (E_0/E)^2 \quad (5.14)$$

and hence $PZ = 1$.

Putting in (5.11) $\hbar\omega_0 = (r_0/r_p)\hbar\omega_p$, where $\omega_p = c/r_p$, we can express (5.11) in the following way:

$$L = \left(\frac{r_0}{r_p}\right)^{1/2}\left(\frac{\hbar}{m c r_p}\right)^{1/2}\frac{\hbar}{(m-m_0)c} \quad (5.15)$$

or with $r_p = \sqrt{\hbar G/c^3}$

$$L = \left(\frac{c^3}{\hbar G}\right)^{1/4}\left(\frac{r_0}{r_p}\right)^{1/2}\left(\frac{\hbar}{mc}\right)^{1/2}\frac{\hbar}{(m-m_0)c}. \quad (5.16)$$

In the form (5.16) one sees that $L \to \infty$ *for* $G \to 0$, or $m_p \to \infty$ *.

The calculation of the correlation length was done for a single particle described by a one-body Schrödinger equation. For a many-body problem described by a many-body Schrödinger equation, the collapse would have to be computed in configuration space. Since a many-body wave function can in its lowest order be approximated by a product of one-body wave functions, the value of the correlation length should be the same. Applied to the two photon correlation experiment with $mc^2 \simeq 5$ eV one finds that $L \simeq 30$ km, but for nonrelativistic electrons with $E \simeq 100$ eV one finds that $L \simeq 4$ m. This second example shows that for a double slit experiment with 100 eV electrons the correlation would be broken above a separation of slits by a few meters. For a smaller separation of the slits the breaking of the correlations may be incomplete, but perhaps detectable**.

To confirm the breaking of the correlations would be easier with gamma rays. If we consider a two photon correlation experiment with two gamma quants emitted in an electron-positron annihilation experiment, on there would have only $L \sim 1$ mm. Such experiments are, however, difficult to carry out.

Inserting $v = \hbar k/m$ into (5.8) we obtain for the superluminal phase velocity causing the collapse

$$v_{ph}/c = \sqrt{\hbar\omega_0/mc^2}, \quad (5.17)$$

for which we can also write

$$v_{ph}/c = (r_p/r_0)^{1/2}(m_p/m)^{1/2}. \quad (5.18)$$

With $m_p = \sqrt{\hbar c/G}$ we can write

$$v_{ph}/c = (r_p/r_0)^{1/2}(\hbar c/G m^2)^{1/4}, \quad (5.19)$$

which shows that in the limit $G \to 0$ one has $v_{ph} \to \infty$. We remark that because of the uncontrollable nature of the quantum fluctuations, these superluminal velocities cannot be used to transmit information, and because the center of mass of the wave packets must move in accordance with the relativistic equation of motion, no superluminal energy transport is possible either.

The paradox of delayed choice experiment is easily explained with our model, because at all times prior to its absorption, the wave packet is spread out over space, and therefore can never be thought of as a particle concentrated into a small volume. This explanation, of course, requires that the wave function can collapse at superluminal velocities and through solid walls. Since the wave function collapse is always a real physical phenomenon, taking place through real physical interactions, negative result experiments simply mean that the collapse must have already taken place somewhere else, and not by recording the negative result. Schrödinger's cat is dead or alive, depending on whether the collapse of the wave function has taken place in the radioactive nucleus, not in the mind of the observer after opening the box. Within macroscopic objects (like Schrödinger's cat), which can be seen as consisting of a very large number of interacting atomic systems, there is a steady collapse of the many atom wave functions at the atomic level. A rock sitting for billions of years undisturbed on the far side of the moon did not suffer a wave function collapse in the moment a conscious observer had looked at a photographic plate recording the existence of the rock for the first time. The rock must rather be described by a wave function, which during the billions of years collapses all the time in an extremely rapid sequence, sustaining the feature of the rock. For macroscopic bodies consisting of many atomic objects, the collapse

* We remark that R. Penrose [14] has also conjectured that the wave function collapse has to do with a finite Planck mass, but as it appears on very different grounds.

** If $\hbar\omega_0 \sim 10^{15}$ GeV, as some GUT theories suggest, rather than $\sim 10^{16}$ GeV, which would make r_0/r_p about 10 times larger, this would make the maximum correlation length about 3 times larger.

results from the interaction between the many atomic systems. They act as an ever present perturbing influence causing the collapse. In the two photon correlation experiment great care is taken to eliminate any perturbing influence, with the result that the collapse is artificially delayed. The same is true for the carefully delayed choice experiment. In our interpretation of the quantum mechanical measuring process the collapse of the wave function is caused by an instability triggered by the presence of physical objects which can perturb or absorb a wave packet. These objects can be the many atoms in a macroscopic body, through which the individual wave functions of all the atoms are steadily forced to collapse, but they can be also a measuring apparatus observing a single atomic object. If the collapse is delayed to such a degree that the wave packet is spread out over a distance larger than the correlation length L, a collapse can still occur provided not just one, but rather many wave packets have spread out over a distance larger than L, with the total energy from all wave packets within the volume L^3 at least as large as the energy would be for one wave spread out over the distance L. If this condition is met, the proposed mechanism can still collapse the energy within the volume L. An example is provided in the absorption of light from a distant source, like a distant star. A photon absorbed can there hardly be a particle emitted from the source, because the spread of its wave packet over the large distance travelled would become very large. What happens instead is that out of the wave field of frequency ν emitted and spread out over space, a piece would be absorbed with the energy $h\nu$, which would give the erroneus conclusion that this energy was the energy of a photon of energy $h\nu$ having travelled over several light years.

6. Discussion

The existence of superluminal velocities is in a sense not only verified by the outcome of the two-photon correlation experiments but also by an interpretation of the double slit experiment, if these experiments are analyzed by pure realism. The thusly observed superluminal velocities do not lead to logical contradictions with the theory of relativity, provided the dynamic Lorentz-Poincaré interpretation is adopted. In the dynamic Lorentz-Poincaré interpretation of relativity (unlike Einstein's kinematic interpretation) there is an absolute time measured in a system at rest with the aether, neatly fitting the fact that in quantum mechanics time is a universal parameter. Travel back in time with violation of causality, which in the Einsteinian view would be inevitable assuming superluminal velocities, is in the Lorentz-Poincaré view seen as an illusion.

In the Planck aether hypothesis there are in reality no such objects as particles or waves. The reality is rather described by the completely symmetric wave functions $\Phi_\pm(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots \boldsymbol{r}_N)$ for all Planck masses, one for those of positive and the other one for those of negative sign, and obtained from the solution of

$$i\hbar \frac{\partial \Phi_\pm}{\partial t} = H_\pm \Phi_\pm, \tag{6.1}$$

where

$$H_\pm = \pm \int \Bigg[\frac{\hbar^2}{2m_{\mathrm{p}}} \nabla\psi_\pm^\dagger . \nabla\psi_\pm \pm 2\hbar c r_{\mathrm{p}}^2 (\psi_\pm^\dagger \psi_\pm - \psi_\mp^\dagger \psi_\mp) \psi_\pm^\dagger \psi_\pm \Bigg] \mathrm{d}\boldsymbol{r}. \tag{6.2}$$

The quantum mechanical correlations in a many-body problem would always have to be seen as a result of the correlations between Planck masses, and the exact description of the correlations could only be obtained from the solution of the many body Schrödinger equation for all Planck masses. The quantum mechanical correlations between the Planck masses would remain, but all other quantum mechanical correlations would be reduced to them alone. This reduction to the correlations of all Planck masses can at least make plausible the long range of the correlations, as they are observed in the Aspect experiment.* The zero point energy fluctuations become plausible through the coexistence of the positive and negative mass Planck aether keeping the total energy equal to zero in every instant.

In this regard the expression (5.18) for the superluminal collapse velocity gives us an interesting insight into the nature of the hypothetical hidden parameters because the proposed model for the collapse of the wave function suggests that these hidden parameters are longitudinal compression waves of the Planck aether near the cut-off frequency ω_0. The collapse velocity, which is set equal to these phase velocity of the compression waves, goes to infinity if $m_{\mathrm{p}} \to \infty$. The

* A very similar situation exists in superfluid Helium [15].

role of the hidden parameters would therefore greatly resemble the pressure in hydrodynamics, where in the limit of an incompressible fluid the speed of the compression waves goes to infinity. As it is in this limit possible to eliminate the pressure from the equations of motion with the help of the incompressibility condition, the hidden parameters can likewise be eliminated in the limit $m_p \to \infty$. The von Neumann non-existence theorem for the hidden parameters would then be valid only in this limit, very much as in the analogous situation in hydrodynamics where the "hidden parameter" pressure can only be eliminated in the limit of an incompressible fluid. Such an interpretation of the hidden parameters as some kind of a chaotically fluctuating pressure disturbance, would also make understandable the quantum correlations as a result of these fluctuations. In an incompressible fluid, these hydrodynamic correlations are transmitted through the repulsive contact interactions acting between the atoms of the fluid. For the quantum mechanical correlations the contact interactions between Planck masses would likewise be responsible.

The superluminal velocities needed to explain the quantum mechanical correlations can be illustrated by a mechanical model which simulates the Einstein-Podolsky-Rosen paradox. In this model the spins of two particles flying apart are thought to be attached to a massless rigid shaft connecting both particles. The spins of both particles can still fluctuate in accordance with the laws of quantum mechanics, but the shaft always keeps these fluctuations exactly in phase such that the total spin of both particles vanishes in each moment. The shaft, of course, implies a coupling of both spins through a torsional wave along the shaft, which for a rigid shaft propagates infinitely fast. In our interpretation of the collapse of the wave function both particles are represented by extended wave packets instead, with their phases entangled by the nonlinear coupling term in (3.1), resp. (3.6). But because the aether represented by these equations is not rigid, the nonlinear coupling of the wave packets leads to a finite velocity, not infinite velocity, coupling the phases of both packets, thereby sustaining their correlations.

Appendix

As we had remarked, the strange action at a distance of quantum mechanics is not so foreign to physics as it may appear, because it somehow resembles nonlocal interactions occuring in the theory of incompressible fluids. Of course, it is there only the result of the physically unrealistic assumption of an incompressible fluid. This nonlocal interaction can be readily demonstrated from the Euler and continuity equation for an incompressible fluid

$$\frac{\partial \boldsymbol{v}}{\partial t} + \nabla\left(\frac{v^2}{2}\right) - \boldsymbol{v} \times \operatorname{curl} \boldsymbol{v} = -\frac{1}{\varrho} \nabla p, \tag{A.1}$$

$$\operatorname{div} \boldsymbol{v} = 0. \tag{A.2}$$

From (A.1) and (A.2) we can eliminate the pressure, first by applying the div-operator on (A.1) and using (A.2). We thus find

$$\nabla^2\left(\frac{p}{\varrho} + \frac{v^2}{2}\right) = \operatorname{div}(\boldsymbol{v} \times \operatorname{curl} \boldsymbol{v}) \tag{A.3}$$

having the solution

$$\frac{p}{\varrho} + \frac{v^2}{2} = -\int \frac{\operatorname{div}(\boldsymbol{v} \times \operatorname{curl} \boldsymbol{v})}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}\boldsymbol{r}'. \tag{A.4}$$

With the help of (A.4) the pressure can now be eliminated from (A.1) resulting in an integral equation for $\boldsymbol{v}$

$$\frac{\partial \boldsymbol{v}}{\partial t} - \boldsymbol{v} \times \operatorname{curl} \boldsymbol{v} = \nabla \int \frac{\operatorname{div}(\boldsymbol{v} \times \operatorname{curl} \boldsymbol{v})}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}\boldsymbol{r}' \tag{A.5}$$

It contains a nonlocal instantaneous interaction term between $\boldsymbol{r}$ and $\boldsymbol{r}'$. In reality, though, there is no such thing as an incompressible fluid, and any interaction is transmitted with the velocity of sound. Still, since this velocity is often very large compared with the fluid velocity v, the assumption of an incompressible fluid can be very good. For an incompressible fluid, the pressure therefore plays the role of a kind of a hidden parameter, which can be completely eliminated from the equations of motion and continuity. This result is not surprising, if we realize that for an incompressible fluid the pressure can be seen as a Lagrange multiplier, multiplied with the constraint $\operatorname{div} \boldsymbol{v} = 0$, resulting in the Lagrangian

$$\mathscr{L} = \tfrac{1}{2} \varrho \boldsymbol{v}^2 + \lambda \operatorname{div} \boldsymbol{v}, \tag{A.6}$$

where with the definition

$$p = -\mathrm{d}\lambda / \mathrm{d}t \tag{A.7}$$

one obtains the Euler equation (A.1) from Hamilton's principle.

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