Reflexion zeigt wie Kristalle von teilweise oxidiertem Magnus-Salz bzw. K_2PtCl_4. Die erstere der genannten Verbindungen wurde bisher fälschlicherweise als Schwefelsäure-Addukt, anstatt als Oxidationsprodukt, aufgefaßt [24]. Die beiden letzteren isolierten wir erstmals im Zusammenhang mit den hier diskutierten Untersuchungen [19, 25]. Des weiteren zählen die von Malatesta erstmals dargestellten Halogenocarbonyliridate mit „gemischter" Wertigkeit [26] ($K[Ir_2(CO)_4Cl_4]$) sowie ein aus dieser Verbindung von uns synthetisiertes Dimethylglyoxim-Derivat der Stöchiometrie $K[Ir_2(CO)_2(DMGH)_2Cl_2]$ [27], zu dieser besonderen Klasse von Festkörpern, wovon man sich leicht beim Beobachten der metallischen Reflexion durch einen Analysator überzeugen kann. Die Reihe ließe sich z. B. mit dem von WERNER erstmals erwähnten teilweise oxidierten $K[PtpyCl_3]$ [28] (py = pyridin) und „Platinblau" [24, 29, 30] fortsetzen, wobei das „optische" Kriterium im letzteren Fall durch die besondere Breite des ^{195}Pt-NMR-Signals gestützt wird [31].

Dieses optische Erkennungsmerkmal ist insofern eindeutig, als die verschiedenen, ebenfalls eingehend untersuchten „kolumnarstrukturierten" Komplexe der d^8-konfigurierten Ionen Ni(II), Pd(II), Pt(II), Magnussches Salz und Analoge [4], Bis(dioximato)metall(II)-Verbindungen [2], $Ir(CO)_2(acac)$, $Rh(CO)_2(acac)$, $Ir(CO)_3Cl$ zwar stark anisotrope optische Absorption zeigen, jedoch keine für einen eindimensional metallischen Zustand charakteristische in Kettenrichtung linear polarisierte Reflexion. Die untaugliche Leitfähigkeitsmethode sollte deshalb durch die wesentlich weniger aufwendige optische Methode ersetzt werden.

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On the Motion of a Charged Particle in an Axially Symmetric Magnetic Field

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The object of this note is to study the motion of a charged particle entering the magnetic field due to a steady current inside a plasma column. Even if the particles of the plasmas stream mutually interact. their distribution across the stream can be such that the interaction forces are balanced by the transverse pressure gradient in the plasma [1]. It is shown here that the test charge entering the plasma stream remains bounded to the stream between two coaxial cylindrical surfaces and further that under suitable conditions it may remain trapped inside a cylindrical box. FISSER and KIPPENHAHN [2] have discussed the general problem of the motion of a charged particle in an axially symmetric magnetic field in configuration space. The following is a particular case of that problem. HERTWECK [3] has investigated the motion of a test charge in the magnetic field due to a line current. His results are applicable here if the test charge remains outside the plasma stream.

The Equations of Motion

If $J\hat{z}$ is the constant plasma current density in a cylindrical coordinate system (ϱ, φ, z), then the magnetic field is

$$\boldsymbol{B} \equiv \left(0, \frac{2\pi J}{c}\varrho, 0\right).$$

The first integrals of the equations of motion are

$$\begin{aligned} &m\,\varrho^2\,\dot{\varphi} = h = m\,\varrho_0^2\,\dot{\varphi}_0\,, \\ &m\,\dot{z} - \frac{\pi e J}{c^2}\,\varrho^2 = k = m\,v_{||} - \frac{\pi e J}{c^2}\,\varrho_0^2, \qquad (1)\\ &\dot{\varrho} + \varrho^2\,\dot{\varphi}^2 + \dot{z}^2 = v^2 \end{aligned}$$

where h, k and $\boldsymbol{v}\cdot\boldsymbol{v} = v^2$ are constants, $v_{||} = \boldsymbol{v}\cdot\hat{z}$ and $\varrho_0 = \varrho(t=0)$, $\dot{\varphi}_0 = \dot{\varphi}(t=0)$. For relativistic particles m is to be replaced by $m[1-(v^2/c^2)]^{-1/2}$. We have taken $\dot{\varrho} = 0$ at $\varrho = \varrho_0$, $t = 0$, and written $\varrho_0\,\dot{\varphi}_0 = v_\perp$. Let us introduce the following substitutions,

$$\begin{aligned} j &= \pi(e\,J/m\,c^2), & u &= z/\varrho_0\,, \\ \tau &= j\,\varrho_0\,t\,, & \alpha &= v_\perp/j\,\varrho_0^2, \qquad (2)\\ x &= (\varrho/\varrho_0)^2, & \beta &= v_{||}/j\,\varrho_0^2. \end{aligned}$$

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Hence for Eqs. (1) we have,

$$\mathrm{d}\varphi/\mathrm{d}\tau = \alpha/x\,, \qquad \mathrm{d}u/\mathrm{d}\tau = x+\beta-1\,,$$
$$\tfrac{1}{4}\,(\mathrm{d}x/\mathrm{d}\tau)^2 = (1-x)\,(x-p_+)\,(x-p_-)\,, \tag{3}$$

where $p_\pm = \tfrac{1}{2}\{(1-2\beta) \pm \sqrt{(1-2\beta)^2+4\alpha^2}\}\,.$ (4)

It is clear that $v_{\|}$ is the maximum velocity in the z-direction and that $\dot{z}$ may change sign between ϱ_0 and $\varrho_{\min}$. Also φ oscillates, though $\dot{\varphi}$ increases or decreases according as $h \gtrless 0$.

Special Cases

(i) Motion in the meridional plane. For $\alpha=0$, the trajectory of the particle will touch the z-axis if $2\beta \geqq 1$, $p_+=0$ and will not touch the z-axis if $2\beta<1$ (Fig. 1).

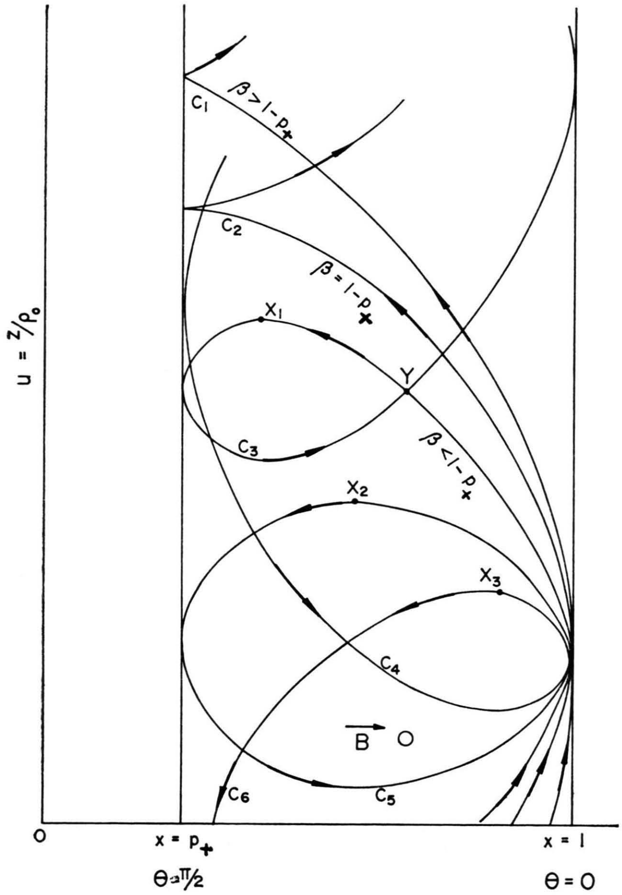

Fig. 1.

(ii) Helical motion. Since $p_+<1$; $2\beta>\alpha^2$. If $\tan\gamma = v_\perp/v_{\|}$ gives the initial direction in which the particle starts out at $\varrho=\varrho_0$, $t=0$ then for $\tan\gamma=2/\alpha$; $\varrho=\varrho_0$, $\dot{z}=v_{\|}$, $\varrho_0\,\dot{\varphi}=v_\perp=$ constant. Thus the trajectory is a helix with a pitch $\varrho_0\,\alpha$ and period $2\pi/j\,\varrho_0\,\alpha$. The particle is restricted to move on a surface over which the magnetic field is constant, the centrifugal force being balanced by the Lorentz force. We also note that $j\,\varrho_0/\pi$ is the cyclotron frequency in the magnetic field $(2\,m\,c/e)\,j\,\varrho_0$ on the cylindrical surface $\varrho=\varrho_0$.

The Functions $\varrho(t)$, $\varphi(t)$, and $z(t)$

(i) The solution of Eq. (3 iii) is

$$x = (\varrho/\varrho_0)^2 = 1-(1-p_+)\,\mathrm{sn}^2\,\omega\,\tau\,. \tag{5}$$

This oscillation has a period $(2/\omega)\,F(\tfrac{1}{2}\pi,k)$, where $k^2=(1-p_+)_1(1-p_-)<1$, $\omega^2=1-p_-$ and $F(\tfrac{1}{2}\pi,k)$ is the complete elliptic integral of the first kind.

(ii) With Eq. (5), Eq. (3 ii) has the integral

$$u-u_0 = \frac{1}{\varrho_0}\,(z-z_0) = \beta\,\tau-(1-p_+)\int \mathrm{sn}^2\,\omega\,\tau\,\mathrm{d}\tau_0\,. \tag{6}$$

This is the familiar solution in the problem of the spherical pendulum. Similarly Eq. (3 i) has the solution

$$\varphi-\varphi' = \alpha\int\{1-(1-p_+)\,\mathrm{sn}^2\,\omega\,\tau\}^{-1}\,\mathrm{d}\tau\,. \tag{7}$$

The Nature of the Orbits

(i) Projection on the $\varrho-z$ plane: From Eq. (3) and Eq. (5),

$$\frac{\mathrm{d}u}{\mathrm{d}\theta} = \left(\frac{\beta-\omega^2}{\omega}\right)(1-k^2\sin^2\theta)^{-1/2}+\omega\,(1-k^2\sin^2\theta)^{+1/2}. \tag{8}$$

At $x=1$, $\mathrm{d}u/\mathrm{d}\theta=\beta/\omega>0$, and at $x=p_+$, $\mathrm{d}u/\mathrm{d}\theta = (\beta-\omega^2 k^2)/\omega\,\sqrt{1-k^2}$. So if $\beta \geqq \omega^2 k^2 = 1-p_+$, then u increases steadily during the time the particle travels from the outer turning point to the inner turning point (the curves C_1 and C_2 in Figure 1). For $\beta<1-p_+$, $\mathrm{d}u/\mathrm{d}\theta$ has a turning point at $\theta=\theta_1$, and from Eq. (8), $\theta_1=\sin^{-1}\sqrt{\beta/\omega^2 k^2}$ (points X_1, X_2, X_3 in Fig. 1). Integration of Eq. (8) gives

$$u-u_0 = \left(\frac{\beta-\omega^2}{\omega}\right)F(\theta,k)+\omega\,E(\theta,k) \tag{9}$$

where $F(\theta,k)$ and $E(\theta,k)$ are the elliptic integrals of the first and second kind respectively.

Incidentally, the curves in Fig. 1 are similar to those given by HERTWECK[3], though the magnetic field there is as $1/\varrho$ while here it is as ϱ.

(ii) Projection on the transverse plane. This is given by

$$\varphi-\varphi_0 = \Pi(\theta,k,\omega^2 k^2)\,, \tag{10}$$

where $\Pi(\theta,k,\omega^2k^2)$ is the elliptic integral of the third kind. At the point $x=1$, the minimum angular velocity of the particle is $\varrho_0\,v_\perp$ and its maximum is at $x=p_+$. The angle covered as the particle goes from $x=1$ to $x=p_+$ is from Eq. (10),

$$\varphi_1 = \frac{\alpha}{\omega}\,\Pi(\tfrac{1}{2}\pi,k,\omega^2k^2)\,. \tag{11}$$

It can be seen that $\varphi_1<\tfrac{1}{2}\pi$ and so in turning through an angle 2π, the particle oscillates more than once

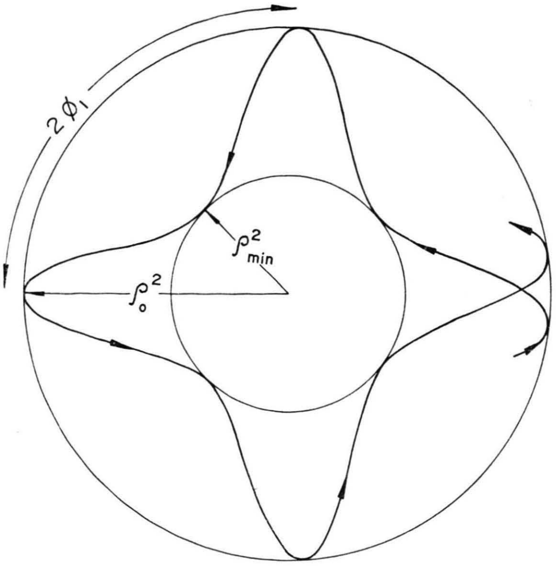

Fig. 2.

between the two turning points. Thus these projections are not closed curves. Figure 2 shows this typical curve.

Confinement of the Particle

The net u-displacement may be zero if the trajectory shows a loop in the $\varrho-z$ plane for $\beta < 1-p_+$ and

$$(\omega^2-\beta)\, F(\tfrac{1}{2}\pi, k) = \omega^2\, E(\tfrac{1}{2}\pi, k)\,. \quad (12)$$

This means that the particle moves between two coaxial cylinders and gets trapped between two transverse planes whose separation is

$$D = 2\,\{(\beta-\omega^2/\omega)\, F(\theta_1, k) + \omega\, E(\theta_1, k)\}\,. \quad (13)$$

The particle moves on a doughnut shaped surface whose section in the $\varrho-z$ plane is the curve C_5.

Curve C_6 shows that the particle may travel even against the plasma current. The flux invariance is obvious as the motion has z-periodicity.

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Nuclear Charge Radius Differences of $^{15}N - {}^{14}N$ and $^{18}O - {}^{16}O$

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The relative differences of the rms ground state nuclear charge radii of $^{15}N-{}^{14}N$ and $^{18}O-{}^{16}O$ have been measured by low energy elastic electron scattering to be $(1.3 \pm 0.7)\%$ and $(2.4 \pm 0.6)\%$, respectively. Both values are less than those following from an $A^{1/3}$ dependence.

The Darmstadt electron linear accelerator ($E \leqq 65$ MeV) has been used to measure the differences of the rms ground state nuclear charge radii R_m of $^{15}N-{}^{14}N$ and $^{18}O-{}^{16}O$ by elastic electron scattering at low momentum transfer ($q^2 < 0.25$ fm^{-2}). Measurements at low momentum transfer have the advantage that radii and radius differences can be determined in a nearly model independent way [1, 2]. The experimental arrangement was that described by GUDDEN et al. [3], except for the 20-channel detector system in the focal plane of the spectrometer [4], the quadrupole doublet behind the scattering chamber [5], and a Faraday cage [6] instead of the old beam dump. Data were taken at a fixed incident electron energy of 50 MeV and scattering angles varying between 81° and 141° as well as at a fixed scattering angle of 93° and energies ranging from 30 to 65 MeV, giving a total of 13 data points.

The measurements were made on gaseous targets [7]. Cylindrical aluminum vessels whose walls were machined down to a thickness of about 200 μm were used as target cells. The gases were filled in through copper pipes which were pinched off after measuring the pressure. The pressure was measured by a precision quartz pressure gauge with a relative accuracy of better than 0.05%. It was choosen within the range from 0.4 to 0.8 atmospheres.

Three target cells were used in the experiment. Two of them contained either isotope, a third one was empty to determine the background. Effects due to differences ($< 5\%$) in the wall thicknesses of the two filled target cells were eliminated by repeating the measurement for each data point with the isotopes interchanged. The gas pressure in both cells was adjusted to the same value in order to avoid systematic errors. Thus, the local decrease in target gas density stemming from the heat generated by the beam along its path across the gas as well as dead time losses cancelled to a high

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