Supplementary Material

August Jörding and Szabolcs Blazsek*

Systematic risk in publicly listed private equity: An empirical study using score-driven beta models

Abstract: We investigate systematic risk dynamics using the LPX Group's 10 indices, which represent publicly listed private equity (PLPE) market indices as well as different PLPE investment styles and PLPE performance at alternative geographic regions. PLPE is a hybrid asset class that combines private capital exposure with public market liquidity. Using daily data from 2002 to 2025 and a recent score-driven modeling framework, i.e., the autoregressive conditional beta (ACB) model for the t-distribution (t-ACB), we demonstrate that the betas and volatility are highly time-varying and LPX-specific, particularly during periods of macroeconomic or financial crisis. Contrary to classical theory, LPX betas do not converge in crises, revealing persistent heterogeneity across PLPE firms. These findings challenge the assumptions underlying static asset pricing models and conventional methods for estimating the cost of equity. By capturing how systematic risk evolves across indices and regimes, this paper extends the literature by offering a more accurate framework for valuing hybrid assets and managing risk in an increasingly institutionalized segment of the private equity market.

Keywords: publicly listed private equity (PLPE); systematic risk; asset pricing models; time-varying betas; dynamic conditional score (DCS) models; generalized autoregressive score (GAS) models

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Supplementary Material A

Table A1: Variable definitions, sources, frequencies, transformations, and uses of variables.

Variable	Type	Source	Frequency	Transformation applied	Used in
LPX indices	Market data	LPX Group	Daily	log-returns	Dependent variables
RF (risk-free rate)	Fama–French	Kenneth R. French Data Library	Daily	transform to log-return	Excess return computation
Mkt − RF (market minus risk-free rate)	Fama–French	Kenneth R. French Data Library	Daily	transform to log-return	Equation 1
SMB (small minus big)	Fama–French	Kenneth R. French Data Library	Daily	transform to log-return	Equation 1
HML (high minus low)	Fama–French	Kenneth R. French Data Library	Daily	transform to log-return	Equation 1
VIX	Macro-financial	Federal Reserve Economic Data	Daily	transform to log-return	Equation 2
Investment-grade spread	Macro-financial	Bloomberg	Daily	transform to log-return	Equation 2
High-yield spread	Macro-financial	Federal Reserve Economic Data	Daily	transform to log-return	Equation 2

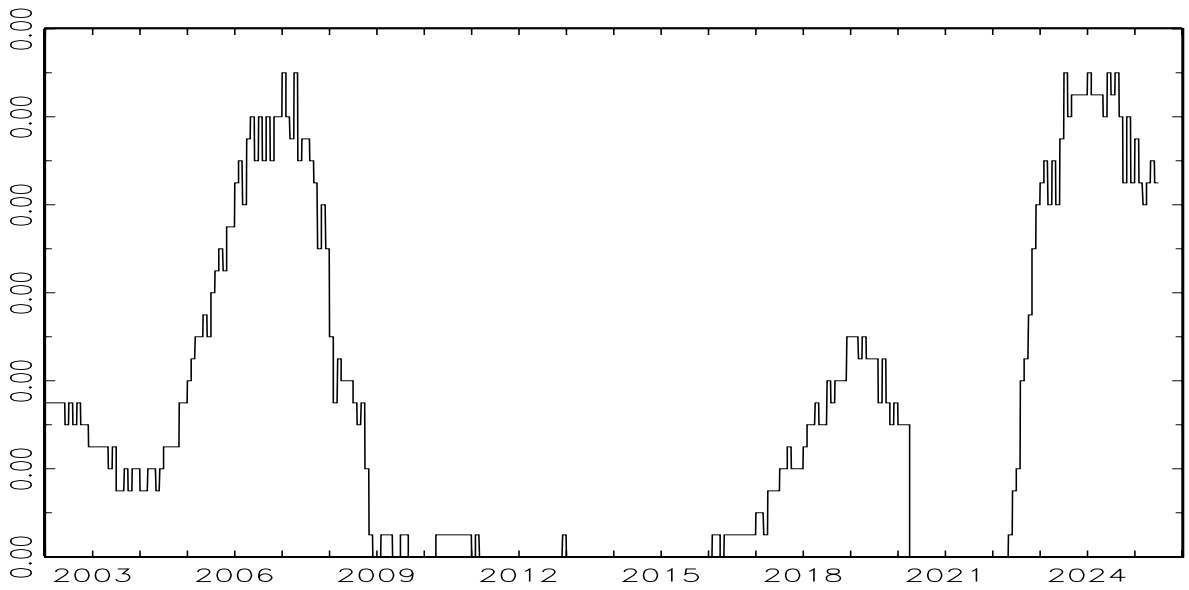

Figure A1: Risk-free rate based on 3-month US Treasury bill yield from January 2, 2002, to June 30, 2025.

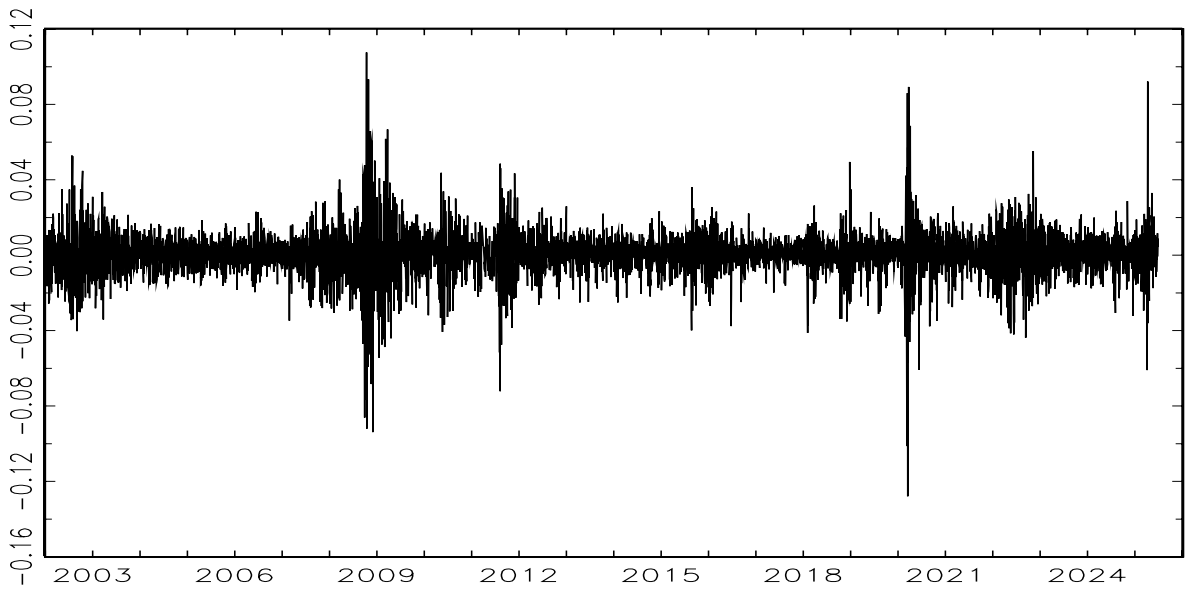

Figure A2: Daily market risk premium from the Fama–French three-factor model from January 2, 2002, to June 30, 2025.

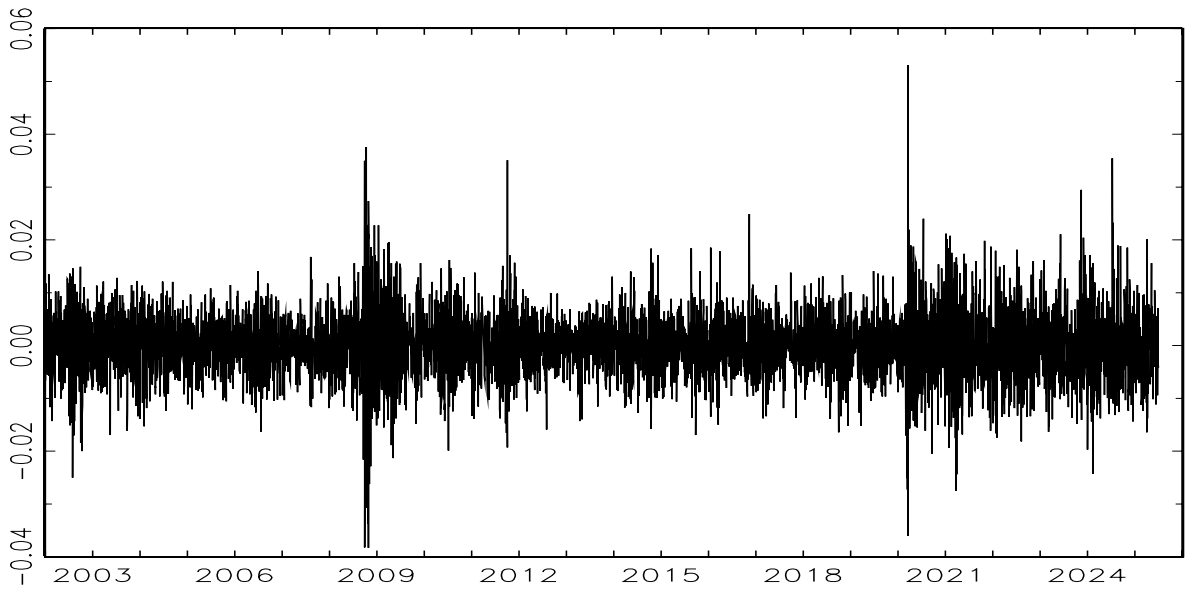

Figure A3: Daily small minus big (SMB) factor from the Fama–French three-factor model from January 2, 2002, to June 30, 2025.

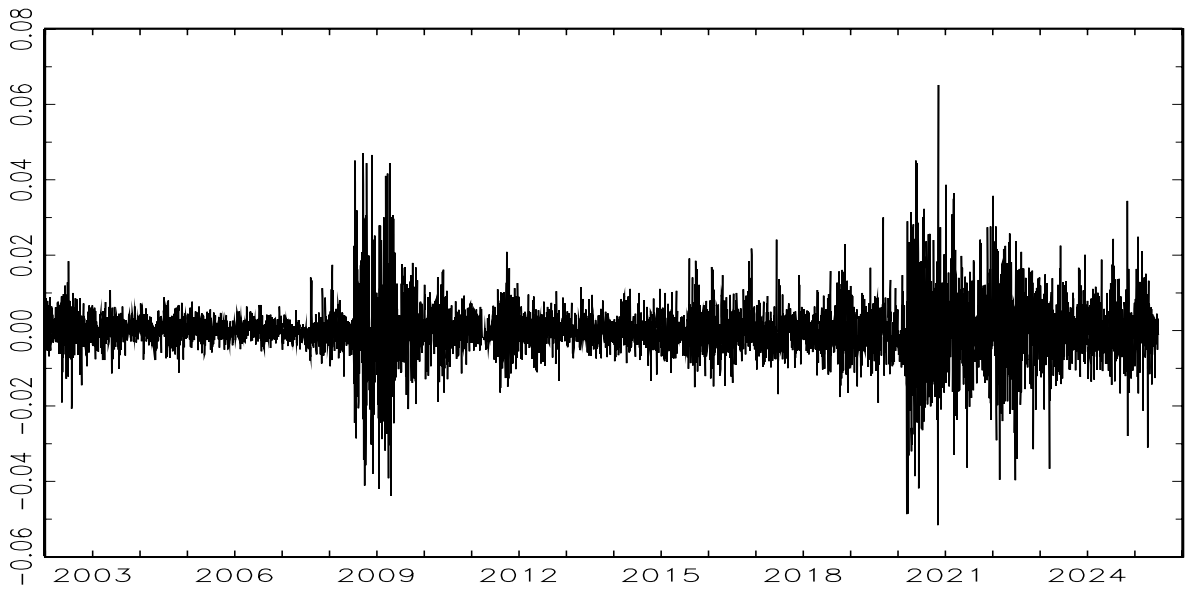

Figure A4: Daily high minus low (HML) factor from the Fama–French three-factor model from January 2, 2002, to June 30, 2025.

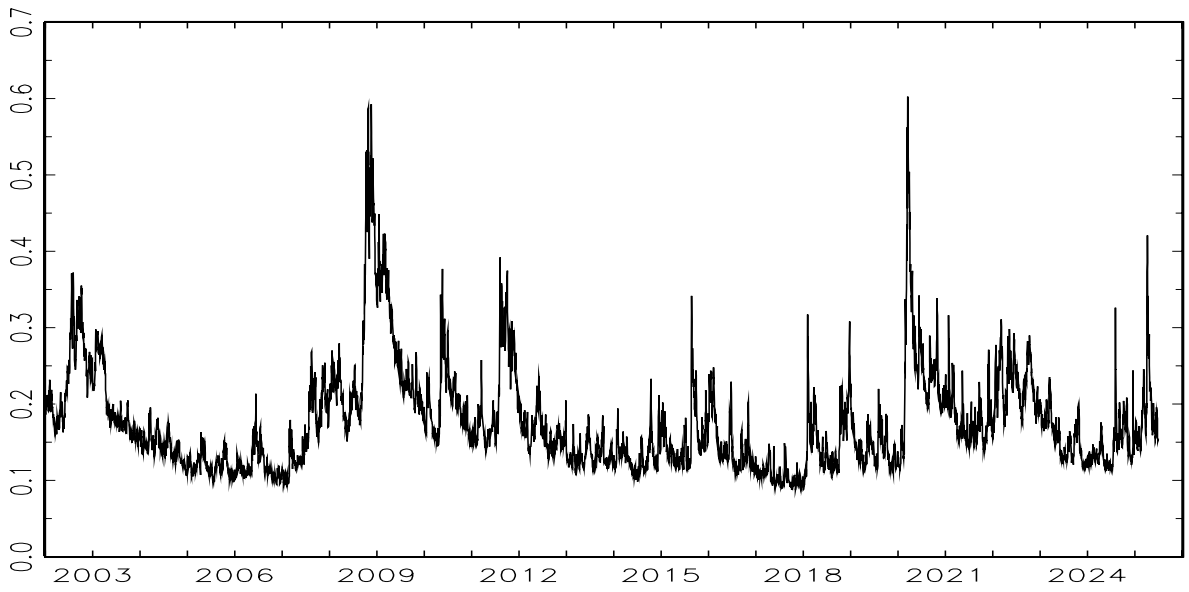

Figure A5: VIX index from January 2, 2002, to June 30, 2025. *Notes:* VIX is the market's 30-day forward-looking volatility, calculated using S&P 500 option prices.

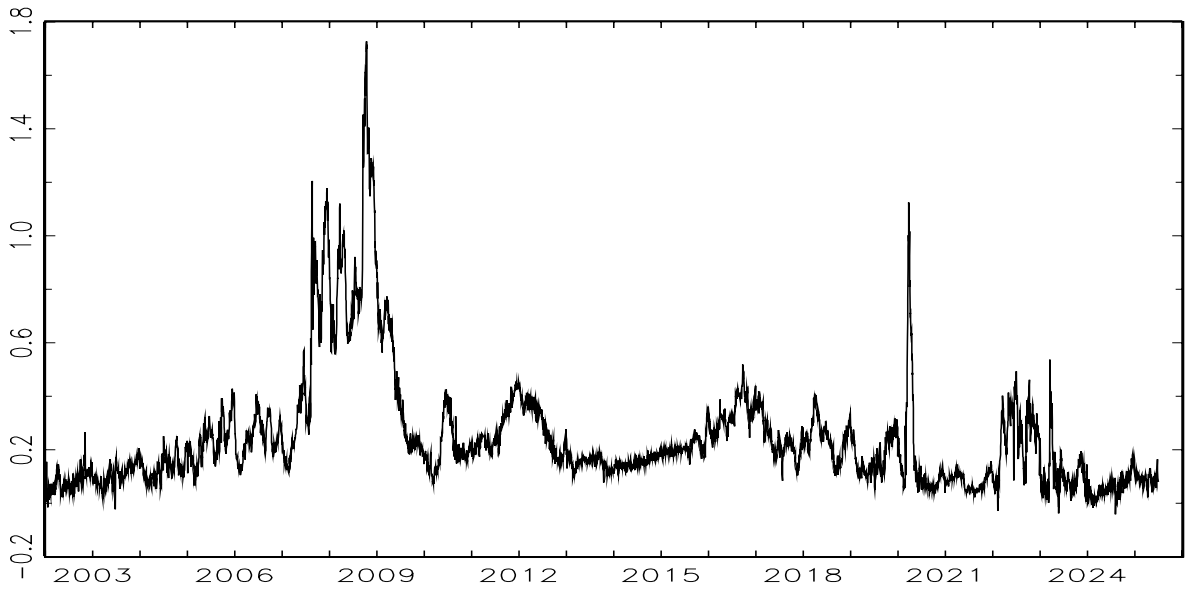

Figure A6: Investment-grade credit spread, BICLUSSP Index (BP) from January 2, 2002, to June 30, 2025. *Notes:* This spread serves as a proxy for credit market risk conditions.

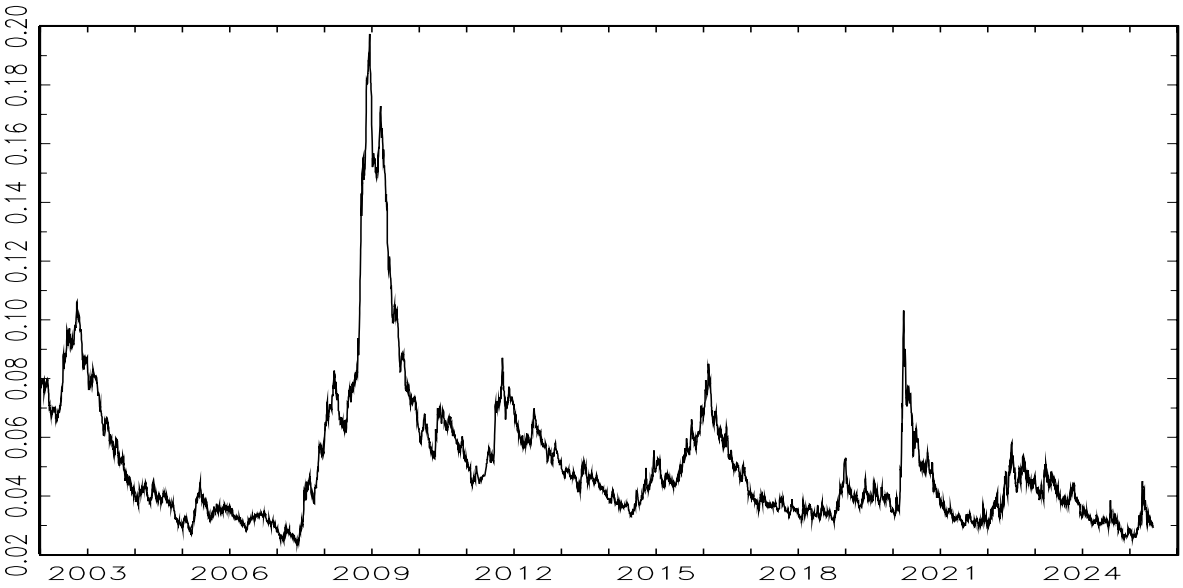

Figure A7: ICE BofA US high-yield index option-adjusted spread from January 2, 2002, to June 30, 2025. *Notes:* The option-adjusted spread is measured as the yield differential between high-yield corporate bonds and comparable US Treasury securities.

Supplementary Material B

For the t-ACB model, the log density of $y_t|\mathcal{F}_{t-1}$ is:

$$\ln f(y_t|\mathcal{F}_{t-1},\Theta) = \ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\ln(\pi\nu) - \lambda_t \tag{B.1}$$

$$-\frac{\nu+1}{2}\ln\left[1+\frac{(y_t-\alpha_t-\beta_{1,t}m_{1,t}-\ldots-\beta_{p,t}m_{p,t})^2}{\nu\exp(2\lambda_t)}\right]$$

where $\Gamma(\cdot)$ is the gamma function, $\mathcal{F}_{t-1} = \sigma(\tilde{\alpha}_1, \tilde{\beta}_{1,1}, \ldots, \tilde{\beta}_{p,1}, \tilde{\lambda}_1, y_1, \ldots, y_{t-1})$ is the information set, and $\Theta = [\alpha, (\beta_i : i = 1,\ldots,p), \lambda, (\Gamma_{i,j} : i = 1,\ldots,p+2; j = 1,\ldots,q), (\Phi_{i,j} : i,j = 1,\ldots,p+2), (\Psi_{i,j} : i,j = 1,\ldots,p+2), \nu]'$ is the $S \times 1$ parameter vector.

We define the score functions for the t-ACB model as follows: (i) Score function $u_{\alpha,t}$ is

$$\frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\alpha_t} = K(\lambda_t)\times u_{\alpha,t} = \frac{\nu+1}{\nu\exp(2\lambda_t)}\times u_{\alpha,t} \tag{B.2}$$

where scaling parameter $K(\lambda_t)$ is defined by the second equality, and

$$u_{\alpha,t} = \frac{\nu\exp(2\lambda_t)(y_t-\mu_t)}{\nu\exp(2\lambda_t)+(y_t-\mu_t)^2}\left[1+\frac{(y_t-\mu_t)^2}{\nu\exp(2\lambda_t)}\right]^{-1}(y_t-\mu_t) \tag{B.3}$$

$$= \left[1+\frac{v_t^2}{\nu\exp(2\lambda_t)}\right]^{-1} v_t = \frac{\nu\exp(\lambda_t)\epsilon_t}{\nu+\epsilon_t^2}$$

(ii) Score functions $u_{\beta,i,t}$ for $i = 1,\ldots,p$ are

$$\frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\beta_{i,t}} = K(\lambda_t)\times u_{\beta,i,t} = \frac{\nu+1}{\nu\exp(2\lambda_t)}\times u_{\beta,i,t} \tag{B.4}$$

for $i = 1,\ldots,p$, where

$$u_{\beta,i,t} = \frac{\nu\exp(2\lambda_t)(y_t-\mu_t)}{\nu\exp(2\lambda_t)+(y_t-\mu_t)^2}\left[1+\frac{(y_t-\mu_t)^2}{\nu\exp(2\lambda_t)}\right]^{-1}(y_t-\mu_t)\times m_{i,t} \tag{B.5}$$

$$= \left[1+\frac{v_t^2}{\nu\exp(2\lambda_t)}\right]^{-1} v_t\times m_{i,t} = \frac{\nu\exp(\lambda_t)\epsilon_t}{\nu+\epsilon_t^2}\times m_{i,t}$$

for $i = 1, \ldots, p$. (iii) Score function $u_{\lambda,t}$ is the conditional score with respect to λ_t:

$$u_{\lambda,t} = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1}, \Theta)}{\partial \lambda_t} = \frac{(\nu+1)(y_t - \mu_t)^2}{\nu \exp(2\lambda_t) + (y_t - \mu_t)^2} - 1 \tag{B.6}$$

$$= \frac{(\nu+1)v_t^2}{\nu \exp(2\lambda_t) + v_t^2} - 1 = \frac{(\nu+1)\epsilon_t^2}{\nu + \epsilon_t^2} - 1$$

For the Gaussian-ACB model, the log density of $y_t|\mathcal{F}_{t-1}$ is

$$\ln f(y_t|\mathcal{F}_{t-1}, \Theta) = \frac{\ln(2\pi)}{2} - \lambda_t - \frac{(y_t - \alpha_t - \beta_{1,t}m_{1,t} - \ldots - \beta_{p,t}m_{p,t})^2}{2\exp(2\lambda_t)} \tag{B.7}$$

The score functions are given as follows: (i) Score function $u_{\alpha,t}$ is

$$u_{\alpha,t} = \exp(\lambda_t)\epsilon_t \tag{B.8}$$

(ii) Score functions $u_{\beta,i,t}$ for $i = 1, \ldots, p$ are:

$$u_{\beta,i,t} = \exp(\lambda_t)\epsilon_t \times m_{i,t} \tag{B.9}$$

for $i = 1, \ldots, p$. (iii) Score function $u_{\lambda,t}$ is the conditional score with respect to λ_t:

$$u_{\lambda,t} = \epsilon_t^2 - 1 \tag{B.10}$$

It can be shown that if $\nu \to \infty$, then u_t defined by $\epsilon_t \sim t(\nu)$ will converge in probability to u_t defined by $\epsilon_t \sim N(0,1)$. Hence, u_t of the Gaussian-ACB model is a special case of u_t for the t-ACB model.

Supplementary Material C

Table C1: Parameter estimates for the Gaussian-ACB model for LPX 50 (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.3491^{***}	0.0982
	β_2 (SMB)	0.0641	0.0509
	β_3 (HML)	0.0252	0.0579
	λ	-5.8691^{***}	0.0479
Macro sensitivity	$\Gamma_{2,1}$	-0.7363^{**}	0.2959
(Equation 2)	$\Gamma_{2,2}$	0.1038	0.0741
	$\Gamma_{2,3}$	3.2735^{***}	1.2362
	$\Gamma_{3,1}$	0.7304^{*}	0.3992
	$\Gamma_{3,2}$	-0.3353^{***}	0.1103
	$\Gamma_{3,3}$	−0.4708	1.2487
	$\Gamma_{4,1}$	1.9587^{***}	0.2249
	$\Gamma_{4,2}$	0.0547	0.1169
	$\Gamma_{4,3}$	-2.4960^{+}	1.5842
	$\Gamma_{5,1}$	5.8004^{***}	0.3057
	$\Gamma_{5,2}$	0.0539	0.0769
	$\Gamma_{5,3}$	-2.0361^{**}	0.9871
Persistence	$\Phi_{2,2}$	0.9995^{***}	0.0002
(Equation 3)	$\Phi_{3,3}$	0.9898^{***}	0.0019
	$\Phi_{4,4}$	0.9955^{***}	0.0021
	$\Phi_{5,5}$	0.9680^{***}	0.0064
Score dynamics	$\Psi_{2,2}$	70.5356^{***}	10.8876
(Equation 3)	$\Psi_{3,3}$	260.2168^{***}	39.8858
	$\Psi_{4,4}$	84.5073^{***}	22.2837
	$\Psi_{5,5}$	0.0197^{***}	0.0021

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

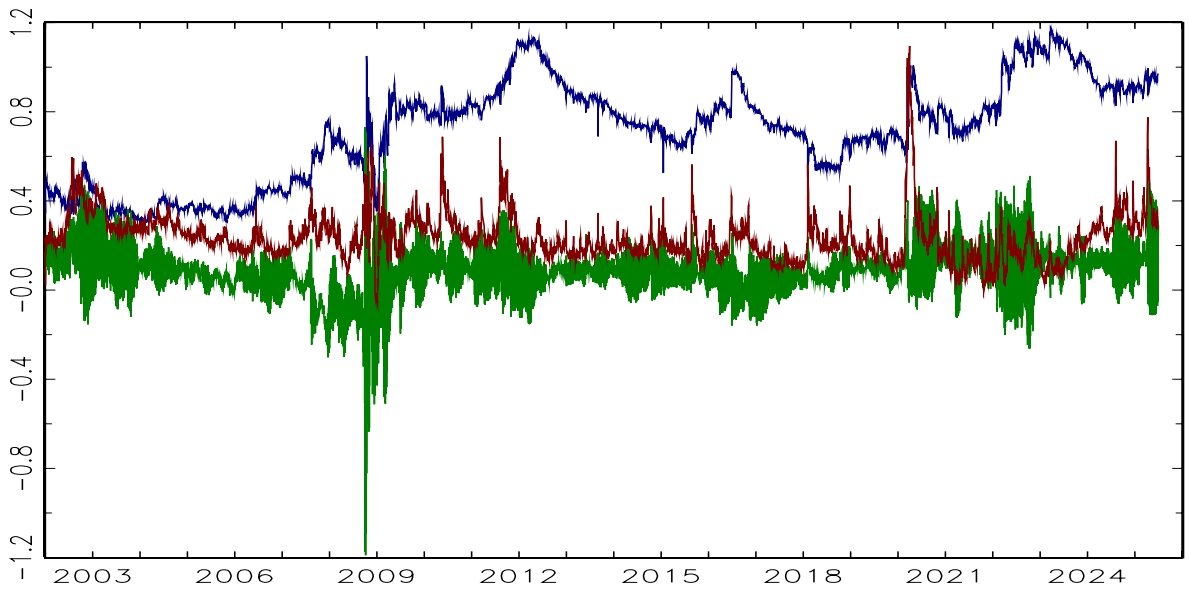

Figure C1: Time-varying beta estimation for LPX 50, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C2: Parameter estimates for the Gaussian-ACB model for LPX Composite (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.2211^{*}	0.1163
	β_2 (SMB)	0.0702	0.0580
	β_3 (HML)	0.0717	0.0732
	λ	-5.8458^{***}	0.0595
Macro sensitivity	$\Gamma_{2,1}$	-0.8096^{***}	0.3076
(Equation 2)	$\Gamma_{2,2}$	0.0669	0.1082
	$\Gamma_{2,3}$	3.3388^{**}	1.3810
	$\Gamma_{3,1}$	0.5661	0.4498
	$\Gamma_{3,2}$	-0.2250^{*}	0.1207
	$\Gamma_{3,3}$	−0.7506	1.4399
	$\Gamma_{4,1}$	0.9443^{**}	0.4385
	$\Gamma_{4,2}$	−0.0716	0.1222
	$\Gamma_{4,3}$	0.1513	1.7738
	$\Gamma_{5,1}$	5.9568^{***}	0.3294
	$\Gamma_{5,2}$	-0.1441^{*}	0.0834
	$\Gamma_{5,3}$	−1.5941	1.1617
Persistence	$\Phi_{2,2}$	0.9997^{***}	0.0002
(Equation 3)	$\Phi_{3,3}$	0.9632^{***}	0.0399
	$\Phi_{4,4}$	0.9931^{***}	0.0037
	$\Phi_{5,5}$	0.9847^{***}	0.0033
Score dynamics	$\Psi_{2,2}$	83.9421^{***}	12.5663
(Equation 3)	$\Psi_{3,3}$	121.9496	91.5178
	$\Psi_{4,4}$	107.4978^{***}	27.3953
	$\Psi_{5,5}$	0.0174^{***}	0.0018

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

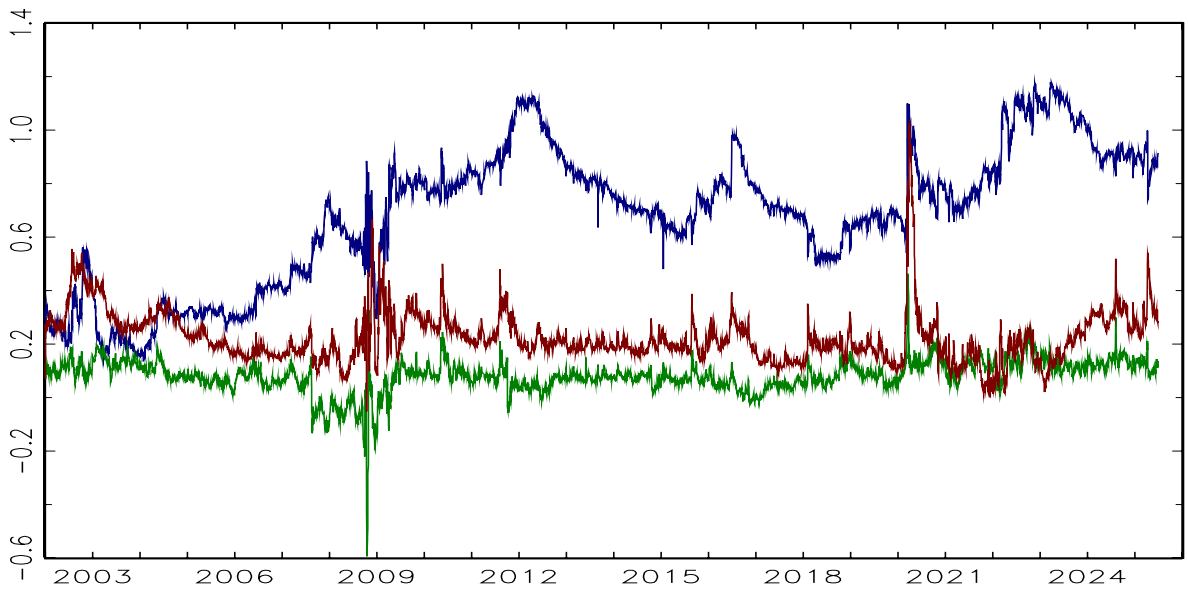

Figure C2: Time-varying beta estimation for LPX Composite, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C3: Parameter estimates for the Gaussian-ACB model for LPX Major Market (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0003^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.4246^{***}	0.0922
	β_2 (SMB)	0.0271	0.0580
	β_3 (HML)	0.0391	0.0741
	λ	-5.7119^{***}	0.0536
Macro sensitivity	$\Gamma_{2,1}$	-0.6000^{*}	0.3371
(Equation 2)	$\Gamma_{2,2}$	0.0625	0.1167
	$\Gamma_{2,3}$	2.5526^{*}	1.4491
	$\Gamma_{3,1}$	0.1471	0.4664
	$\Gamma_{3,2}$	−0.1611	0.1243
	$\Gamma_{3,3}$	0.5234	1.4602
	$\Gamma_{4,1}$	0.8681^{*}	0.4612
	$\Gamma_{4,2}$	−0.0250	0.1226
	$\Gamma_{4,3}$	0.7265	1.7043
	$\Gamma_{5,1}$	5.7131^{***}	0.3257
	$\Gamma_{5,2}$	-0.1225^{+}	0.0850
	$\Gamma_{5,3}$	-1.7670^{+}	1.1823
Persistence	$\Phi_{2,2}$	0.9992^{***}	0.0003
(Equation 3)	$\Phi_{3,3}$	0.9919^{***}	0.0015
	$\Phi_{4,4}$	0.9920^{***}	0.0049
	$\Phi_{5,5}$	0.9849^{***}	0.0033
Score dynamics	$\Psi_{2,2}$	81.1847^{***}	12.9771
(Equation 3)	$\Psi_{3,3}$	0.0049	0.0334
	$\Psi_{4,4}$	83.8499^{***}	27.7256
	$\Psi_{5,5}$	0.0176^{***}	0.0018

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

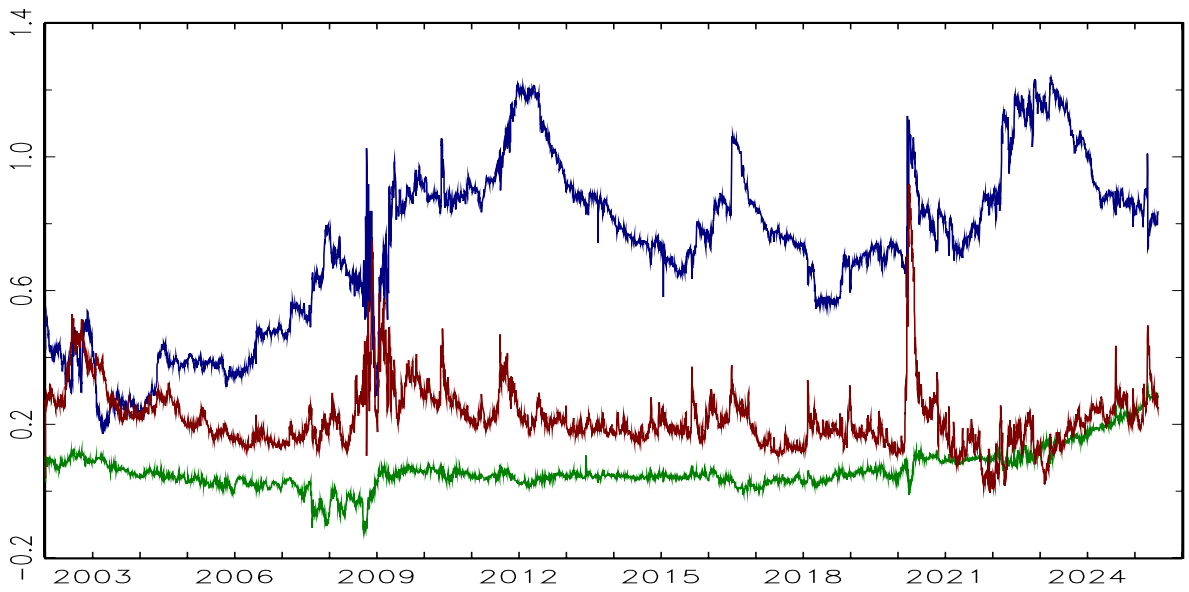

Figure C3: Time-varying beta estimation for LPX Major Market, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C4: Parameter estimates for the Gaussian-ACB model for LPX Buyout (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.2747^{***}	0.0716
	β_2 (SMB)	0.0376	0.0483
	β_3 (HML)	0.0278	0.0481
	λ	-5.9191^{***}	0.0545
Macro sensitivity	$\Gamma_{2,1}$	−0.1094	0.2830
(Equation 2)	$\Gamma_{2,2}$	0.1548^{+}	0.1012
	$\Gamma_{2,3}$	0.7978	1.2332
	$\Gamma_{3,1}$	0.3748	0.3898
	$\Gamma_{3,2}$	−0.1325	0.1218
	$\Gamma_{3,3}$	0.0832	1.2136
	$\Gamma_{4,1}$	1.2777^{***}	0.3267
	$\Gamma_{4,2}$	−0.0394	0.1095
	$\Gamma_{4,3}$	−0.1531	1.1485
	$\Gamma_{5,1}$	5.9815^{***}	0.3164
	$\Gamma_{5,2}$	0.2532^{***}	0.0981
	$\Gamma_{5,3}$	-2.7049^{**}	1.1232
Persistence	$\Phi_{2,2}$	0.9988^{***}	0.0003
(Equation 3)	$\Phi_{3,3}$	0.9675^{***}	0.0228
	$\Phi_{4,4}$	0.4098	0.4355
	$\Phi_{5,5}$	0.9743^{***}	0.0049
Score dynamics	$\Psi_{2,2}$	73.8868^{***}	11.0894
(Equation 3)	$\Psi_{3,3}$	125.6982^{*}	68.3667
	$\Psi_{4,4}$	143.4870^{+}	92.2157
	$\Psi_{5,5}$	0.0238^{***}	0.0021

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

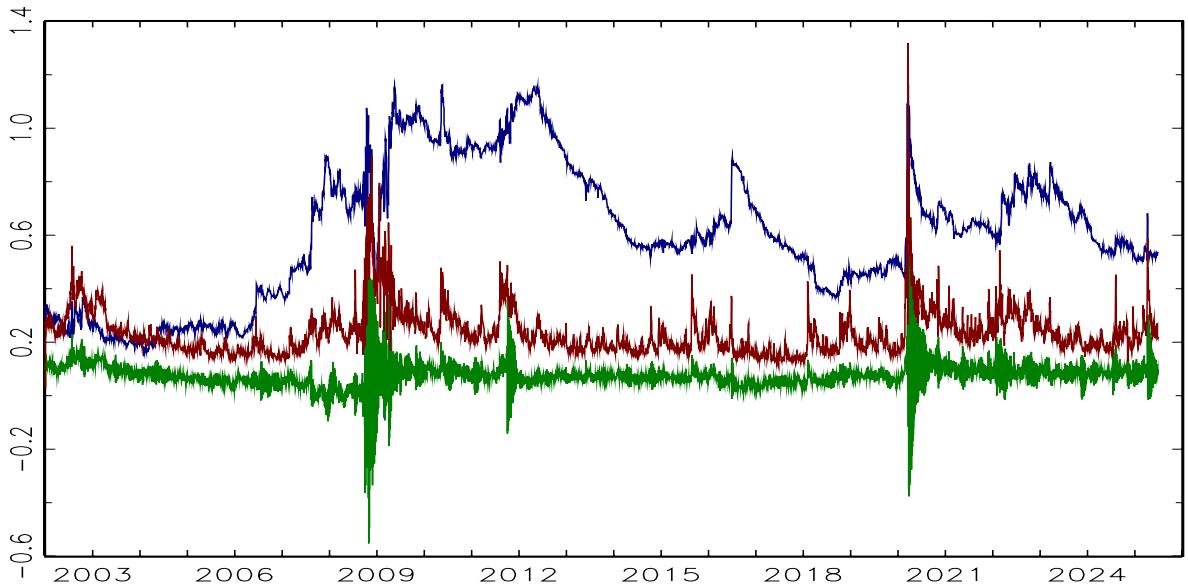

Figure C4: Time-varying beta estimation for LPX Buyout, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C5: Parameter estimates for the Gaussian-ACB model for LPX Venture (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0002^{*}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.4523^{***}	0.0581
	β_2 (SMB)	0.1643^{***}	0.0606
	β_3 (HML)	0.0982^{*}	0.0540
	λ	-5.2312^{***}	0.0435
Macro sensitivity	$\Gamma_{2,1}$	-0.4359^{+}	0.2834
(Equation 2)	$\Gamma_{2,2}$	0.1084	0.0966
	$\Gamma_{2,3}$	1.0170	1.2967
	$\Gamma_{3,1}$	0.7444^{+}	0.4817
	$\Gamma_{3,2}$	-0.4673^{***}	0.1259
	$\Gamma_{3,3}$	0.6112	1.4758
	$\Gamma_{4,1}$	0.3384	0.3799
	$\Gamma_{4,2}$	-0.2293^{*}	0.1209
	$\Gamma_{4,3}$	0.4427	1.2399
	$\Gamma_{5,1}$	4.3983^{***}	0.2839
	$\Gamma_{5,2}$	0.2568^{***}	0.0864
	$\Gamma_{5,3}$	-5.0176^{***}	1.0183
Persistence	$\Phi_{2,2}$	0.9974^{***}	0.0013
(Equation 3)	$\Phi_{3,3}$	0.7101^{*}	0.3958
	$\Phi_{4,4}$	0.9217^{***}	0.0401
	$\Phi_{5,5}$	0.9621^{***}	0.0064
Score dynamics	$\Psi_{2,2}$	45.6429^{***}	11.1893
(Equation 3)	$\Psi_{3,3}$	−218.9708	179.1709
	$\Psi_{4,4}$	99.0747^{**}	44.5950
	$\Psi_{5,5}$	0.0224^{***}	0.0022

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

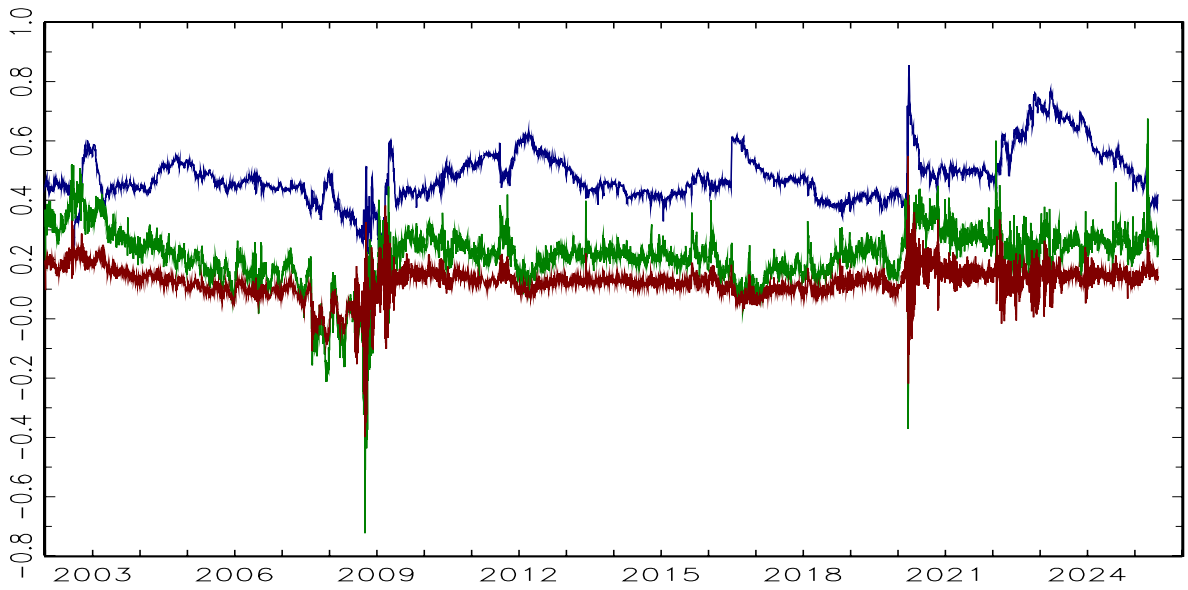

Figure C5: Time-varying beta estimation for LPX Venture, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C6: Parameter estimates for the Gaussian-ACB model for LPX Direct (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0003^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.3737^{***}	0.0587
	β_2 (SMB)	0.0517	0.0570
	β_3 (HML)	0.0465	0.0503
	λ	-5.9323^{***}	0.0450
Macro sensitivity	$\Gamma_{2,1}$	−0.0858	0.2913
(Equation 2)	$\Gamma_{2,2}$	0.0979	0.1053
	$\Gamma_{2,3}$	1.3472	1.2471
	$\Gamma_{3,1}$	0.2672	0.4289
	$\Gamma_{3,2}$	-0.3004^{**}	0.1330
	$\Gamma_{3,3}$	1.1879	1.4022
	$\Gamma_{4,1}$	0.9248^{***}	0.3367
	$\Gamma_{4,2}$	−0.1403	0.1054
	$\Gamma_{4,3}$	1.1015	1.1870
	$\Gamma_{5,1}$	5.7412^{***}	0.2945
	$\Gamma_{5,2}$	0.1950^{**}	0.0829
	$\Gamma_{5,3}$	−0.8000	0.9675
Persistence	$\Phi_{2,2}$	0.9983^{***}	0.0004
(Equation 3)	$\Phi_{3,3}$	0.9423^{***}	0.0534
	$\Phi_{4,4}$	0.3081	0.6594
	$\Phi_{5,5}$	0.9658^{***}	0.0070
Score dynamics	$\Psi_{2,2}$	74.2226^{***}	12.2620
(Equation 3)	$\Psi_{3,3}$	175.8659^{+}	122.0920
	$\Psi_{4,4}$	123.5524	96.3003
	$\Psi_{5,5}$	0.0202^{***}	0.0022

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

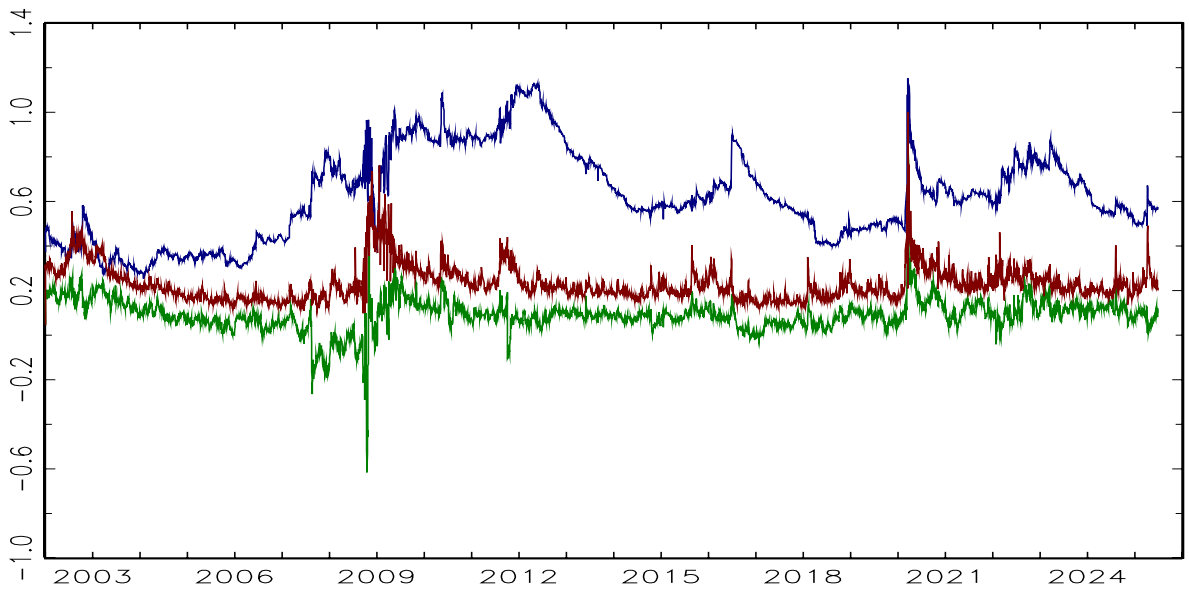

Figure C6: Time-varying beta estimation for LPX Direct, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C7: Parameter estimates for the Gaussian-ACB model for LPX Indirect (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0007^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.0715^{+}	0.0486
	β_2 (SMB)	0.1098^{**}	0.0524
	β_3 (HML)	0.1208^{**}	0.0555
	λ	-5.5823^{***}	0.0284
Macro sensitivity	$\Gamma_{2,1}$	0.0332	0.2216
(Equation 2)	$\Gamma_{2,2}$	0.1444^{**}	0.0694
	$\Gamma_{2,3}$	−0.8035	0.8057
	$\Gamma_{3,1}$	−0.1226	0.4234
	$\Gamma_{3,2}$	−0.1038	0.1111
	$\Gamma_{3,3}$	−1.3016	1.2813
	$\Gamma_{4,1}$	0.5142	0.3599
	$\Gamma_{4,2}$	-0.3960^{***}	0.1132
	$\Gamma_{4,3}$	−0.7980	1.2604
	$\Gamma_{5,1}$	5.2023^{***}	0.2236
	$\Gamma_{5,2}$	0.4035^{***}	0.0574
	$\Gamma_{5,3}$	-6.1354^{***}	0.6289
Persistence	$\Phi_{2,2}$	0.9991^{***}	0.0004
(Equation 3)	$\Phi_{3,3}$	0.9558^{*}	0.5344
	$\Phi_{4,4}$	0.9557^{***}	0.0340
	$\Phi_{5,5}$	0.9869^{***}	0.0014
Score dynamics	$\Psi_{2,2}$	20.6281^{***}	7.1055
(Equation 3)	$\Psi_{3,3}$	9.9223	92.5058
	$\Psi_{4,4}$	73.1719^{+}	44.5731
	$\Psi_{5,5}$	0.0062^{***}	0.0005

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

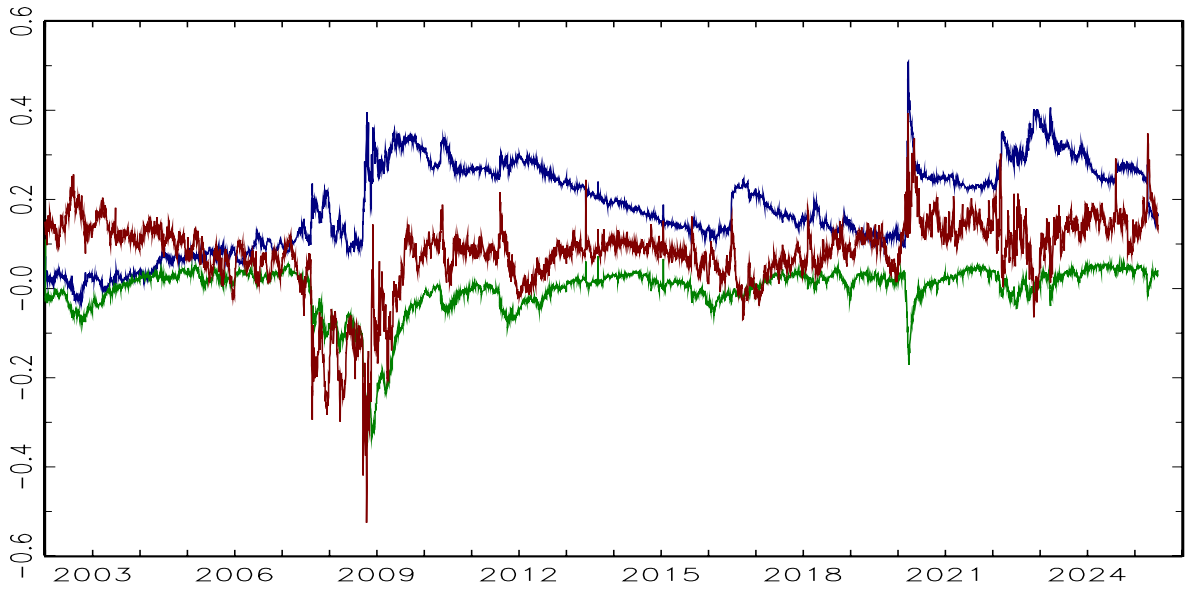

Figure C7: Time-varying beta estimation for LPX Indirect, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C8: Parameter estimates for the Gaussian-ACB model for LPX America (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0001^{+}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.7704^{***}	0.0555
	β_2 (SMB)	0.2471^{***}	0.0754
	β_3 (HML)	-0.0266	0.0435
	λ	-5.9816^{***}	0.0389
Macro sensitivity	$\Gamma_{2,1}$	0.8846^{***}	0.2719
(Equation 2)	$\Gamma_{2,2}$	-0.1426^{+}	0.0903
	$\Gamma_{2,3}$	-1.0075	1.5515
	$\Gamma_{3,1}$	0.7352^{*}	0.3836
	$\Gamma_{3,2}$	0.1250	0.0898
	$\Gamma_{3,3}$	2.8710^{**}	1.3260
	$\Gamma_{4,1}$	0.3399	0.2572
	$\Gamma_{4,2}$	0.1101	0.0868
	$\Gamma_{4,3}$	5.1212^{***}	1.1033
	$\Gamma_{5,1}$	4.3494^{***}	0.2747
	$\Gamma_{5,2}$	0.0120	0.0687
	$\Gamma_{5,3}$	3.6115^{***}	0.7776
Persistence	$\Phi_{2,2}$	0.9966^{***}	0.0011
(Equation 3)	$\Phi_{3,3}$	0.9902^{***}	0.0001
	$\Phi_{4,4}$	0.6713^{***}	0.1930
	$\Phi_{5,5}$	0.9612^{***}	0.0062
Score dynamics	$\Psi_{2,2}$	135.4943^{***}	14.6656
(Equation 3)	$\Psi_{3,3}$	36.7383^{***}	12.5381
	$\Psi_{4,4}$	155.0465^{***}	52.0631
	$\Psi_{5,5}$	0.0212^{***}	0.0016

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

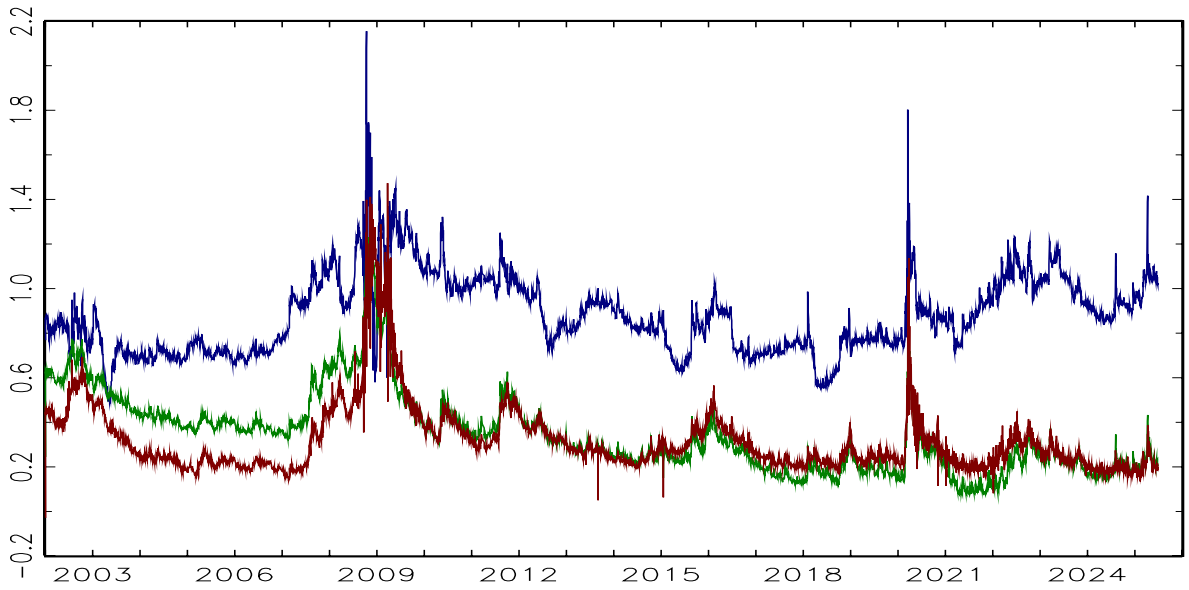

Figure C8: Time-varying beta estimation for LPX America, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C9: Parameter estimates for the Gaussian-ACB model for LPX Europe (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0006^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.2389^{**}	0.0948
	β_2 (SMB)	0.0037	0.0742
	β_3 (HML)	0.0564	0.0642
	λ	-5.5518^{***}	0.0481
Macro sensitivity	$\Gamma_{2,1}$	-1.3714^{***}	0.3744
(Equation 2)	$\Gamma_{2,2}$	0.2324^{*}	0.1216
	$\Gamma_{2,3}$	2.5664^{*}	1.4919
	$\Gamma_{3,1}$	−0.0560	0.5667
	$\Gamma_{3,2}$	-0.2596^{+}	0.1614
	$\Gamma_{3,3}$	−0.3272	1.7691
	$\Gamma_{4,1}$	0.9621^{**}	0.4256
	$\Gamma_{4,2}$	-0.2204^{*}	0.1248
	$\Gamma_{4,3}$	−0.9900	1.4443
	$\Gamma_{5,1}$	6.0440^{***}	0.3180
	$\Gamma_{5,2}$	0.2333^{***}	0.0860
	$\Gamma_{5,3}$	-3.9389^{***}	1.0468
Persistence	$\Phi_{2,2}$	0.9991^{***}	0.0003
(Equation 3)	$\Phi_{3,3}$	0.9917^{***}	0.0007
	$\Phi_{4,4}$	0.9998^{***}	0.0003
	$\Phi_{5,5}$	0.9631^{***}	0.0060
Score dynamics	$\Psi_{2,2}$	59.0200^{***}	9.6373
(Equation 3)	$\Psi_{3,3}$	0.0156	0.0519
	$\Psi_{4,4}$	18.1151^{***}	6.7146
	$\Psi_{5,5}$	0.0252^{***}	0.0020

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

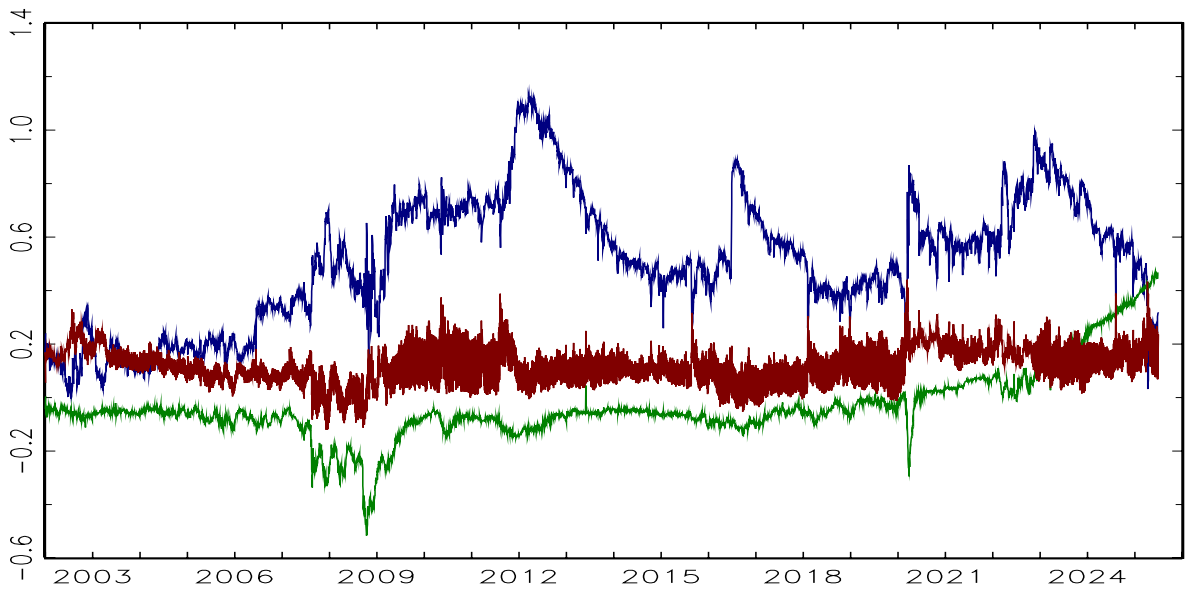

Figure C9: Time-varying beta estimation for LPX Europe, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Table C10: Parameter estimates for the Gaussian-ACB model for LPX UK (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0006^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.1645^{**}	0.0666
	β_2 (SMB)	0.1136	0.0898
	β_3 (HML)	0.1738^{***}	0.0667
	λ	-5.5323^{***}	0.0429
Macro sensitivity	$\Gamma_{2,1}$	-1.1029^{***}	0.2830
(Equation 2)	$\Gamma_{2,2}$	0.2929^{***}	0.0826
	$\Gamma_{2,3}$	1.2583	1.0245
	$\Gamma_{3,1}$	-0.9189^{*}	0.5251
	$\Gamma_{3,2}$	-0.1853^{+}	0.1229
	$\Gamma_{3,3}$	1.8219	1.7008
	$\Gamma_{4,1}$	0.6884^{+}	0.4399
	$\Gamma_{4,2}$	-0.2319^{*}	0.1307
	$\Gamma_{4,3}$	-2.6110^{*}	1.5767
	$\Gamma_{5,1}$	5.1398^{***}	0.2761
	$\Gamma_{5,2}$	0.2544^{***}	0.0778
	$\Gamma_{5,3}$	-2.7300^{***}	0.9201
Persistence	$\Phi_{2,2}$	0.9995^{***}	0.0002
(Equation 3)	$\Phi_{3,3}$	0.9905^{***}	0.0006
	$\Phi_{4,4}$	0.9585^{***}	0.0232
	$\Phi_{5,5}$	0.9621^{***}	0.0044
Score dynamics	$\Psi_{2,2}$	20.7199^{***}	5.3889
(Equation 3)	$\Psi_{3,3}$	26.9444^{***}	9.1280
	$\Psi_{4,4}$	70.5882^{*}	38.1118
	$\Psi_{5,5}$	0.0232^{***}	0.0013

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

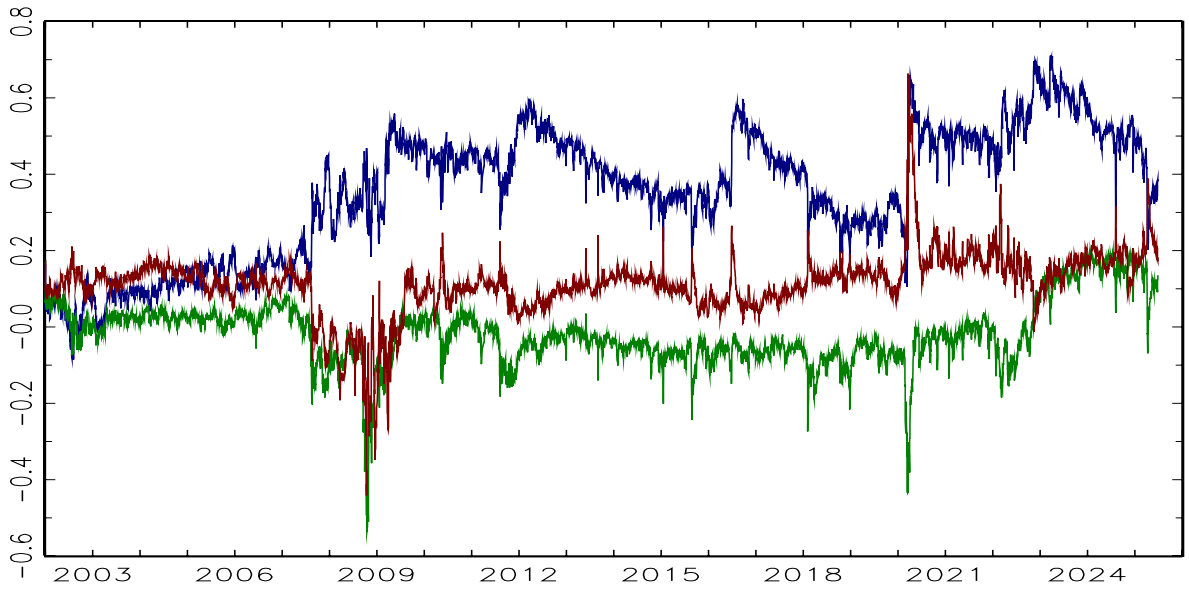

Figure C10: Time-varying beta estimation for LPX UK, Gaussian-ACB model (January 2, 2002 to June 30, 2025).

Supplementary Material D

Table D1: Parameter estimates for the t-ACB model for LPX 50 (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.3860^{***}	0.0947
	β_2 (SMB)	0.0115	0.0541
	β_3 (HML)	0.0846^{*}	0.0474
	λ	-5.9842^{***}	0.0613
Macro sensitivity	$\Gamma_{2,1}$	-0.7469^{**}	0.2983
(Equation 2)	$\Gamma_{2,2}$	0.0204	0.0995
	$\Gamma_{2,3}$	3.0087^{**}	1.2655
	$\Gamma_{3,1}$	0.6730^{+}	0.4317
	$\Gamma_{3,2}$	-0.1971^{*}	0.1119
	$\Gamma_{3,3}$	−0.2335	1.3790
	$\Gamma_{4,1}$	0.3120	0.3200
	$\Gamma_{4,2}$	-0.1808^{*}	0.0948
	$\Gamma_{4,3}$	1.6794^{+}	1.1392
	$\Gamma_{5,1}$	5.6896^{***}	0.4004
	$\Gamma_{5,2}$	0.0968	0.1032
	$\Gamma_{5,3}$	−1.3483	1.2878
Persistence	$\Phi_{2,2}$	0.9993^{***}	0.0003
(Equation 3)	$\Phi_{3,3}$	0.9924^{***}	0.0015
	$\Phi_{4,4}$	0.9992^{***}	0.0004
	$\Phi_{5,5}$	0.9613^{***}	0.0097
Score dynamics	$\Psi_{2,2}$	100.6091^{***}	16.1393
(Equation 3)	$\Psi_{3,3}$	0.0005	0.0038
	$\Psi_{4,4}$	38.8717^{**}	16.5739
	$\Psi_{5,5}$	0.0289^{***}	0.0039
Degrees of freedom	ν	12.0088^{***}	1.7099

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

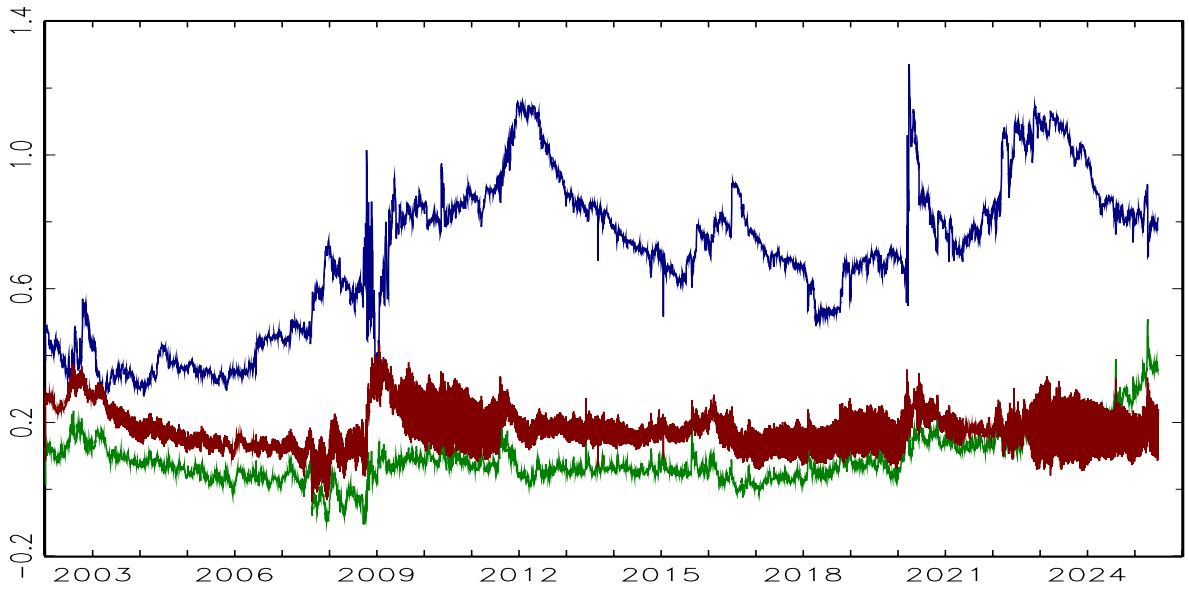

Figure D1: Time-varying beta estimation for LPX 50, t-ACB model (January 2, 2002 to June 30, 2025).

Table D2: Parameter estimates for the t-ACB model for LPX Composite (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.2310^{*}	0.1184
	β_2 (SMB)	0.0021	0.0555
	β_3 (HML)	0.0651	0.0483
	λ	-5.9403^{***}	0.0761
Macro sensitivity	$\Gamma_{2,1}$	-0.7690^{**}	0.3021
(Equation 2)	$\Gamma_{2,2}$	0.1342	0.1021
	$\Gamma_{2,3}$	2.4546^{*}	1.3403
	$\Gamma_{3,1}$	0.4154	0.4315
	$\Gamma_{3,2}$	−0.1336	0.1132
	$\Gamma_{3,3}$	0.4356	1.4637
	$\Gamma_{4,1}$	0.4133	0.3259
	$\Gamma_{4,2}$	−0.1142	0.0954
	$\Gamma_{4,3}$	1.5329	1.1874
	$\Gamma_{5,1}$	5.9564^{***}	0.4247
	$\Gamma_{5,2}$	−0.1550	0.1157
	$\Gamma_{5,3}$	−1.2530	1.5843
Persistence	$\Phi_{2,2}$	0.9996^{***}	0.0002
(Equation 3)	$\Phi_{3,3}$	0.9914^{***}	0.0010
	$\Phi_{4,4}$	0.5132	44.5994
	$\Phi_{5,5}$	0.9817^{***}	0.0048
Score dynamics	$\Psi_{2,2}$	103.2187^{***}	16.1785
(Equation 3)	$\Psi_{3,3}$	2.5950	16.6743
	$\Psi_{4,4}$	1.8308	121.0112
	$\Psi_{5,5}$	0.0251^{***}	0.0032
Degrees of freedom	ν	12.0377^{***}	1.6873

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

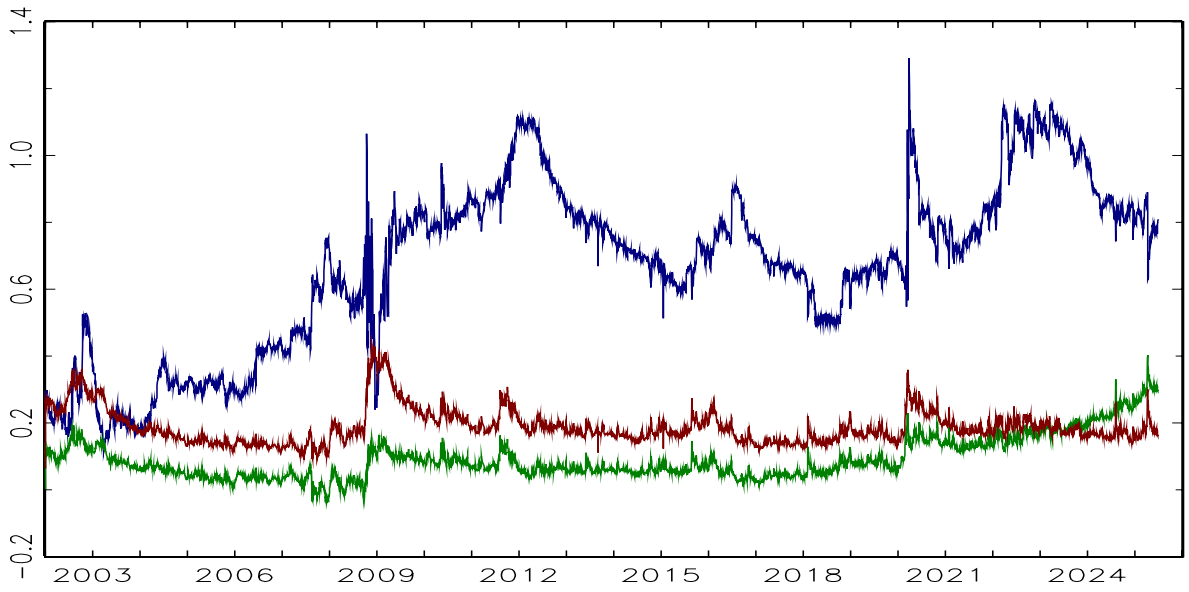

Figure D2: Time-varying beta estimation for LPX Composite, t-ACB model (January 2, 2002 to June 30, 2025).

Table D3: Parameter estimates for the t-ACB model for LPX Major Market (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.5483^{***}	0.0794
	β_2 (SMB)	0.0215	0.0593
	β_3 (HML)	0.0385	0.0615
	λ	-5.8365^{***}	0.0693
Macro sensitivity	$\Gamma_{2,1}$	-0.6752^{**}	0.3330
(Equation 2)	$\Gamma_{2,2}$	0.0738	0.1149
	$\Gamma_{2,3}$	1.8151	1.4587
	$\Gamma_{3,1}$	0.4297	0.4753
	$\Gamma_{3,2}$	−0.1706	0.1200
	$\Gamma_{3,3}$	−0.3998	1.5506
	$\Gamma_{4,1}$	0.2795	0.3922
	$\Gamma_{4,2}$	−0.1669	0.1214
	$\Gamma_{4,3}$	3.0013^{**}	1.5199
	$\Gamma_{5,1}$	5.6299^{***}	0.4153
	$\Gamma_{5,2}$	−0.1285	0.1164
	$\Gamma_{5,3}$	−0.8939	1.5362
Persistence	$\Phi_{2,2}$	0.9988^{***}	0.0004
(Equation 3)	$\Phi_{3,3}$	0.9920^{***}	0.0014
	$\Phi_{4,4}$	0.9579^{***}	0.0285
	$\Phi_{5,5}$	0.9808^{***}	0.0051
Score dynamics	$\Psi_{2,2}$	105.1098^{***}	17.5617
(Equation 3)	$\Psi_{3,3}$	0.0044	0.0285
	$\Psi_{4,4}$	98.5703^{*}	50.9942
	$\Psi_{5,5}$	0.0251^{***}	0.0033
Degrees of freedom	ν	11.7513^{***}	1.6660

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

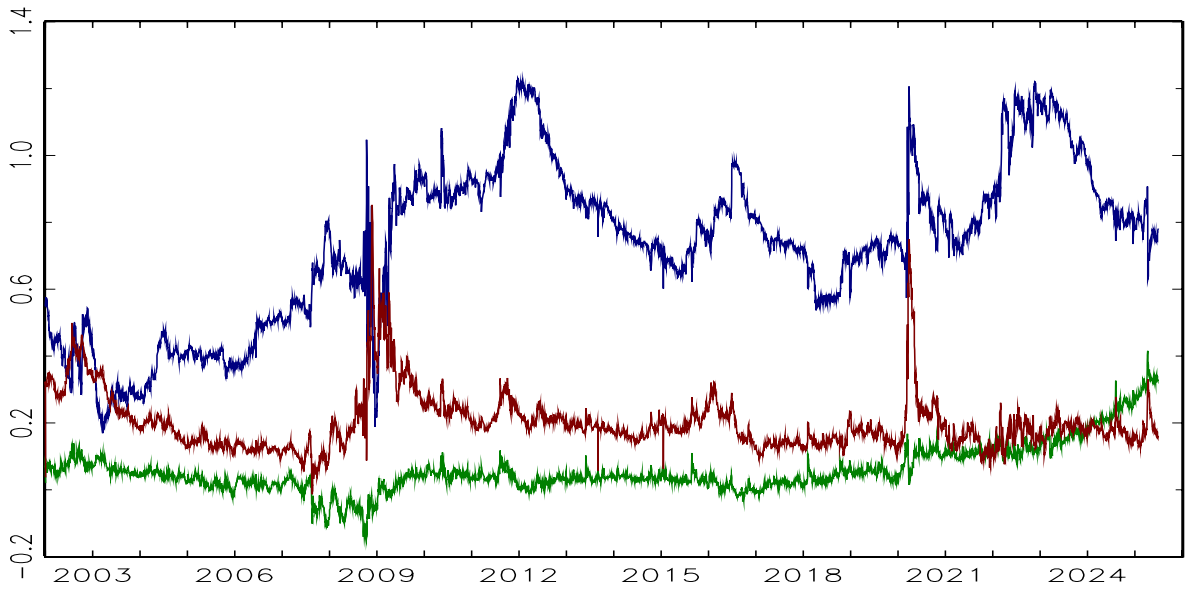

Figure D3: Time-varying beta estimation for LPX Major Market, t-ACB model (January 2, 2002 to June 30, 2025).

Table D4: Parameter estimates for the t-ACB model for LPX Buyout (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0004^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.2881^{***}	0.0737
	β_2 (SMB)	-0.0033	0.0584
	β_3 (HML)	0.0234	0.0531
	λ	-6.0118^{***}	0.0711
Macro sensitivity	$\Gamma_{2,1}$	-0.2355	0.2912
(Equation 2)	$\Gamma_{2,2}$	0.1779^{*}	0.1016
	$\Gamma_{2,3}$	0.5770	1.2345
	$\Gamma_{3,1}$	-0.1621	0.4474
	$\Gamma_{3,2}$	0.0278	0.1074
	$\Gamma_{3,3}$	1.6257	1.4425
	$\Gamma_{4,1}$	1.1822^{***}	0.3761
	$\Gamma_{4,2}$	0.0405	0.1125
	$\Gamma_{4,3}$	-0.4039	1.2266
	$\Gamma_{5,1}$	6.0683^{***}	0.3988
	$\Gamma_{5,2}$	0.2152^{*}	0.1238
	$\Gamma_{5,3}$	-2.6852^{*}	1.5091
Persistence	$\Phi_{2,2}$	0.9988^{***}	0.0003
(Equation 3)	$\Phi_{3,3}$	0.9909^{***}	0.0008
	$\Phi_{4,4}$	0.9913^{***}	0.0189
	$\Phi_{5,5}$	0.9756^{***}	0.0058
Score dynamics	$\Psi_{2,2}$	94.3681^{***}	14.3843
(Equation 3)	$\Psi_{3,3}$	11.5149^{+}	7.7800
	$\Psi_{4,4}$	9.6700	16.0140
	$\Psi_{5,5}$	0.0300^{***}	0.0034
Degrees of freedom	ν	12.7093^{***}	1.9202

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

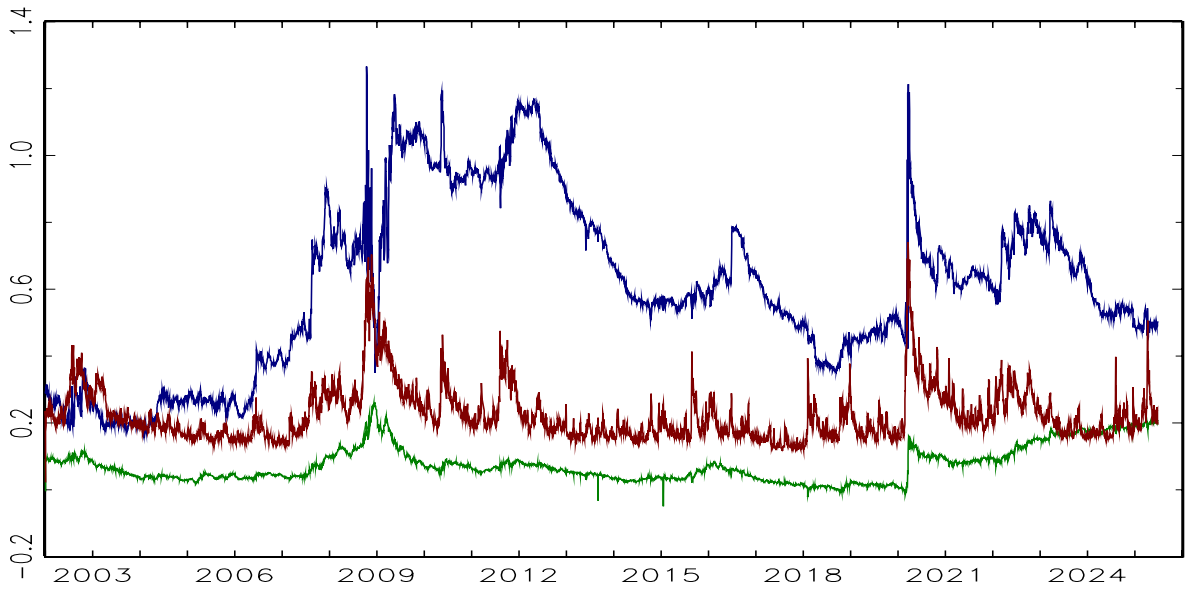

Figure D4: Time-varying beta estimation for LPX Buyout, t-ACB model (January 2, 2002 to June 30, 2025).

Table D5: Parameter estimates for the t-ACB model for LPX Venture (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0002^{*}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.4390^{***}	0.0528
	β_2 (SMB)	0.1696^{+}	0.1128
	β_3 (HML)	0.0782	0.0670
	λ	-5.3307^{***}	0.0633
Macro sensitivity	$\Gamma_{2,1}$	-0.5506^{*}	0.2968
(Equation 2)	$\Gamma_{2,2}$	−0.0262	0.0947
	$\Gamma_{2,3}$	2.2868^{*}	1.2486
	$\Gamma_{3,1}$	1.9802^{***}	0.6250
	$\Gamma_{3,2}$	-0.3900^{**}	0.1685
	$\Gamma_{3,3}$	-4.8388^{**}	2.1754
	$\Gamma_{4,1}$	0.5898	0.4527
	$\Gamma_{4,2}$	-0.2458^{*}	0.1449
	$\Gamma_{4,3}$	0.2322	1.5437
	$\Gamma_{5,1}$	4.4180^{***}	0.3784
	$\Gamma_{5,2}$	0.1910^{*}	0.1155
	$\Gamma_{5,3}$	-4.8691^{***}	1.3995
Persistence	$\Phi_{2,2}$	0.9969^{***}	0.0015
(Equation 3)	$\Phi_{3,3}$	0.9989^{***}	0.0011
	$\Phi_{4,4}$	0.9440^{***}	0.0381
	$\Phi_{5,5}$	0.9760^{***}	0.0063
Score dynamics	$\Psi_{2,2}$	47.1614^{***}	14.7202
(Equation 3)	$\Psi_{3,3}$	143.9289^{***}	44.5021
	$\Psi_{4,4}$	162.6280^{**}	76.1515
	$\Psi_{5,5}$	0.0244^{***}	0.0032
Degrees of freedom	ν	10.7486^{***}	1.3112

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

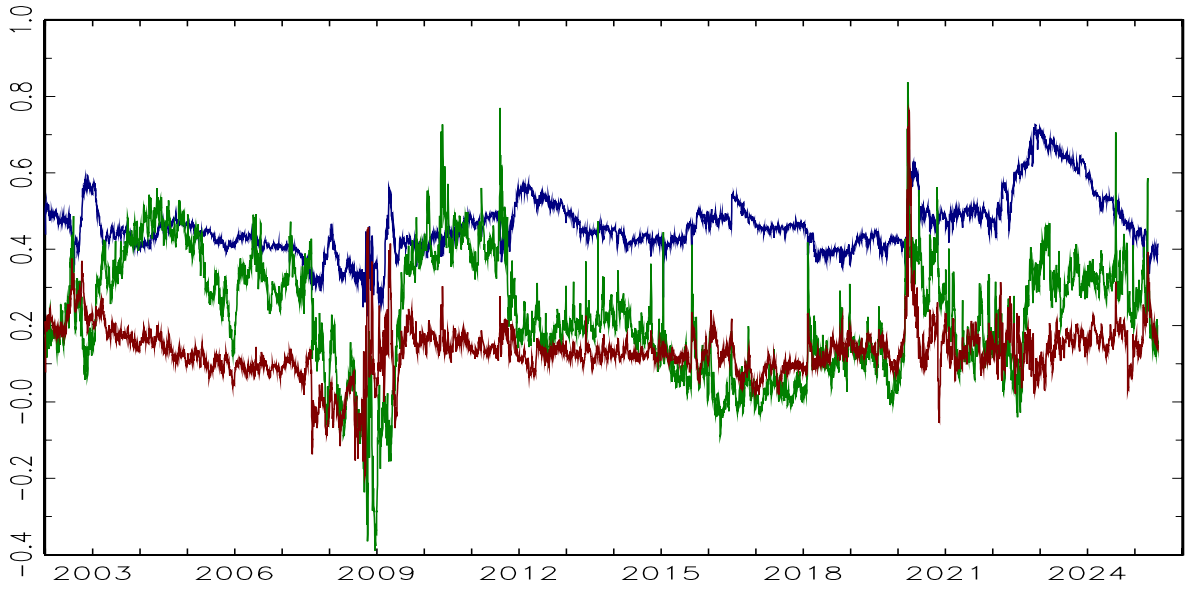

Figure D5: Time-varying beta estimation for LPX Venture, t-ACB model (January 2, 2002 to June 30, 2025).

Table D6: Parameter estimates for the t-ACB model for LPX Direct (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0003^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.3771^{***}	0.0608
	β_2 (SMB)	0.0467	0.0520
	β_3 (HML)	0.0406	0.0483
	λ	-6.0148^{***}	0.0593
Macro sensitivity	$\Gamma_{2,1}$	−0.0640	0.2942
(Equation 2)	$\Gamma_{2,2}$	0.0964	0.0978
	$\Gamma_{2,3}$	0.9340	1.1850
	$\Gamma_{3,1}$	0.1066	0.4323
	$\Gamma_{3,2}$	-0.2784^{**}	0.1186
	$\Gamma_{3,3}$	1.7451	1.3449
	$\Gamma_{4,1}$	1.0193^{***}	0.3224
	$\Gamma_{4,2}$	−0.1241	0.1011
	$\Gamma_{4,3}$	0.6577	1.1583
	$\Gamma_{5,1}$	5.8795^{***}	0.3743
	$\Gamma_{5,2}$	0.1180	0.1076
	$\Gamma_{5,3}$	−0.8516	1.2610
Persistence	$\Phi_{2,2}$	0.9984^{***}	0.0004
(Equation 3)	$\Phi_{3,3}$	0.9735^{***}	0.2835
	$\Phi_{4,4}$	0.9901^{***}	0.0002
	$\Phi_{5,5}$	0.9692^{***}	0.0081
Score dynamics	$\Psi_{2,2}$	79.0755^{***}	13.7292
(Equation 3)	$\Psi_{3,3}$	8.1230	73.3296
	$\Psi_{4,4}$	20.7259^{***}	7.9501
	$\Psi_{5,5}$	0.0246^{***}	0.0033
Degrees of freedom	ν	13.6498^{***}	2.1673

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

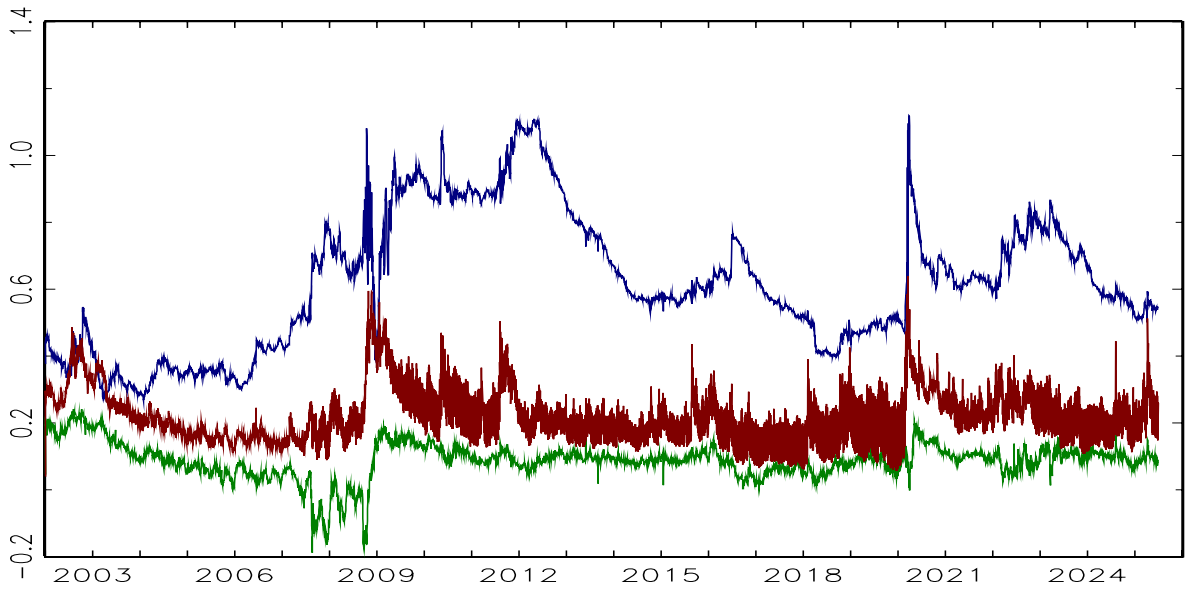

Figure D6: Time-varying beta estimation for LPX Direct, t-ACB model (January 2, 2002 to June 30, 2025).

Table D7: Parameter estimates for the t-ACB model for LPX Indirect (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0006^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.1074^{**}	0.0467
	β_2 (SMB)	0.0661	0.0579
	β_3 (HML)	0.0712^{+}	0.0453
	λ	-5.8789^{***}	0.0930
Macro sensitivity	$\Gamma_{2,1}$	−0.1088	0.2385
(Equation 2)	$\Gamma_{2,2}$	0.1712^{***}	0.0596
	$\Gamma_{2,3}$	−1.1765	0.8511
	$\Gamma_{3,1}$	−0.4218	0.4366
	$\Gamma_{3,2}$	−0.1292	0.0921
	$\Gamma_{3,3}$	0.1386	1.4127
	$\Gamma_{4,1}$	0.5470^{*}	0.3100
	$\Gamma_{4,2}$	-0.2727^{***}	0.0783
	$\Gamma_{4,3}$	−0.6182	1.0446
	$\Gamma_{5,1}$	5.4812^{***}	0.4288
	$\Gamma_{5,2}$	0.0285	0.1470
	$\Gamma_{5,3}$	-3.5228^{*}	1.8399
Persistence	$\Phi_{2,2}$	0.9987^{***}	0.0004
(Equation 3)	$\Phi_{3,3}$	0.9912^{***}	0.0012
	$\Phi_{4,4}$	0.9958^{***}	0.0060
	$\Phi_{5,5}$	0.9876^{***}	0.0031
Score dynamics	$\Psi_{2,2}$	34.7319^{***}	11.8356
(Equation 3)	$\Psi_{3,3}$	0.1423	0.8428
	$\Psi_{4,4}$	19.2683	14.5496
	$\Psi_{5,5}$	0.0311^{***}	0.0033
Degrees of freedom	ν	7.0351^{***}	0.4326

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

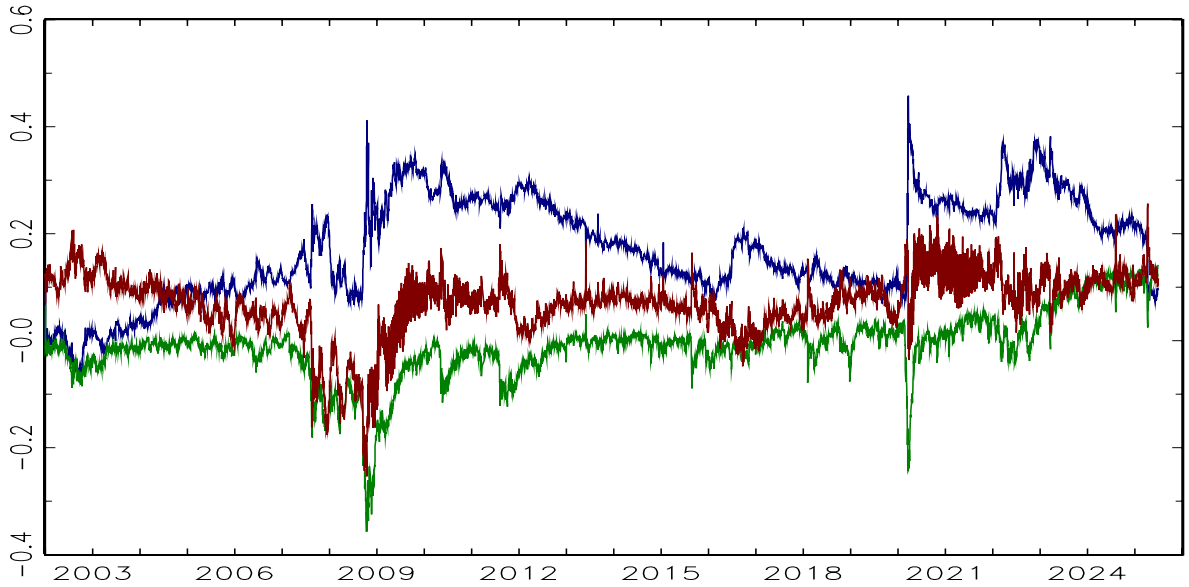

Figure D7: Time-varying beta estimation for LPX Indirect, t-ACB model (January 2, 2002 to June 30, 2025).

Table D8: Parameter estimates for the t-ACB model for LPX America (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0002^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.8221^{***}	0.0588
	β_2 (SMB)	-0.0173	0.0628
	β_3 (HML)	-0.0169	0.0427
	λ	-6.0799^{***}	0.0618
Macro sensitivity	$\Gamma_{2,1}$	0.6051^{**}	0.2612
(Equation 2)	$\Gamma_{2,2}$	-0.1543^{+}	0.0942
	$\Gamma_{2,3}$	-1.6702	1.5348
	$\Gamma_{3,1}$	1.2936^{***}	0.4177
	$\Gamma_{3,2}$	0.2524^{**}	0.1224
	$\Gamma_{3,3}$	1.3128	1.7455
	$\Gamma_{4,1}$	0.5991^{**}	0.2596
	$\Gamma_{4,2}$	0.1045	0.0847
	$\Gamma_{4,3}$	4.0986^{***}	1.0875
	$\Gamma_{5,1}$	3.9154^{***}	0.4110
	$\Gamma_{5,2}$	0.1320	0.1090
	$\Gamma_{5,3}$	3.2268^{**}	1.3319
Persistence	$\Phi_{2,2}$	0.9963^{***}	0.0012
(Equation 3)	$\Phi_{3,3}$	0.9937^{***}	0.0034
	$\Phi_{4,4}$	0.9991^{***}	0.0006
	$\Phi_{5,5}$	0.9644^{***}	0.0084
Score dynamics	$\Psi_{2,2}$	228.1608^{***}	23.1866
(Equation 3)	$\Psi_{3,3}$	200.6939^{***}	65.5986
	$\Psi_{4,4}$	56.4004^{***}	11.3598
	$\Psi_{5,5}$	0.0339^{***}	0.0042
Degrees of freedom	ν	7.0898^{***}	0.6289

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

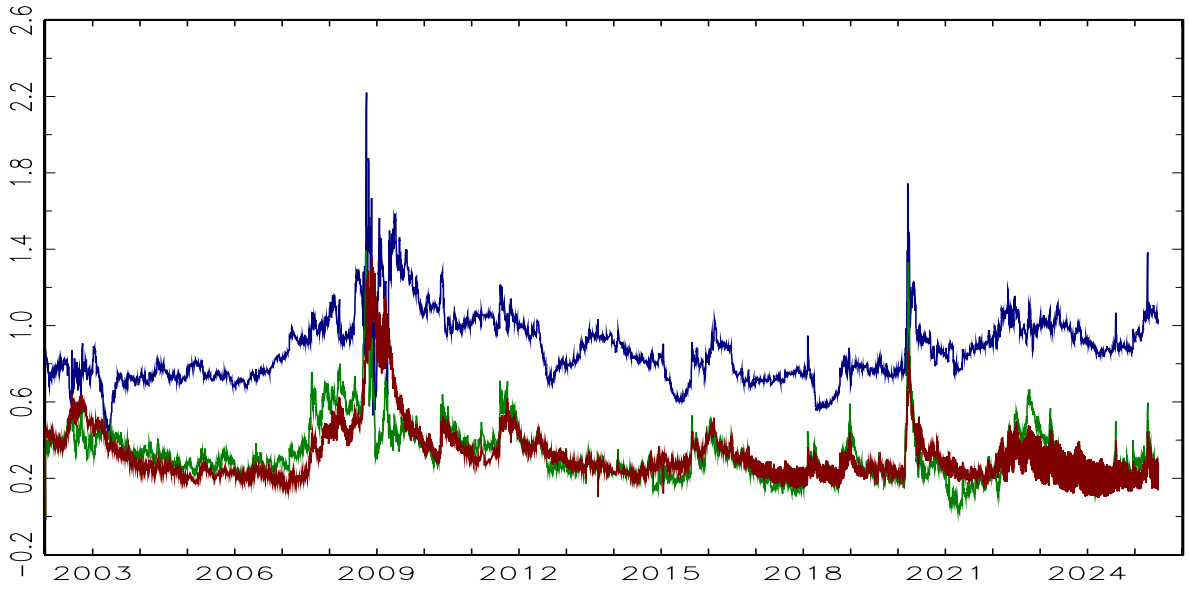

Figure D8: Time-varying beta estimation for LPX America, t-ACB model (January 2, 2002 to June 30, 2025).

Table D9: Parameter estimates for the t-ACB model for LPX Europe (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0006^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.2692^{***}	0.0942
	β_2 (SMB)	0.1368^{*}	0.0703
	β_3 (HML)	0.0280	0.0726
	λ	-5.6692^{***}	0.0684
Macro sensitivity	$\Gamma_{2,1}$	-1.3268^{***}	0.3949
(Equation 2)	$\Gamma_{2,2}$	0.2618^{**}	0.1225
	$\Gamma_{2,3}$	1.7461	1.4989
	$\Gamma_{3,1}$	0.6654	0.6003
	$\Gamma_{3,2}$	-0.4202^{***}	0.1543
	$\Gamma_{3,3}$	-3.8951^{**}	1.7957
	$\Gamma_{4,1}$	0.9519^{**}	0.4823
	$\Gamma_{4,2}$	-0.2068^{+}	0.1427
	$\Gamma_{4,3}$	−0.4678	1.6991
	$\Gamma_{5,1}$	6.3502^{***}	0.4227
	$\Gamma_{5,2}$	0.1351	0.1160
	$\Gamma_{5,3}$	-4.3250^{***}	1.4184
Persistence	$\Phi_{2,2}$	0.9990^{***}	0.0003
(Equation 3)	$\Phi_{3,3}$	0.3865	0.5060
	$\Phi_{4,4}$	0.9750^{***}	0.0372
	$\Phi_{5,5}$	0.9707^{***}	0.0067
Score dynamics	$\Psi_{2,2}$	80.3052^{***}	14.5755
(Equation 3)	$\Psi_{3,3}$	502.6106^{*}	300.1483
	$\Psi_{4,4}$	37.3812	44.4220
	$\Psi_{5,5}$	0.0309^{***}	0.0035
Degrees of freedom	ν	10.2993^{***}	1.1533

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

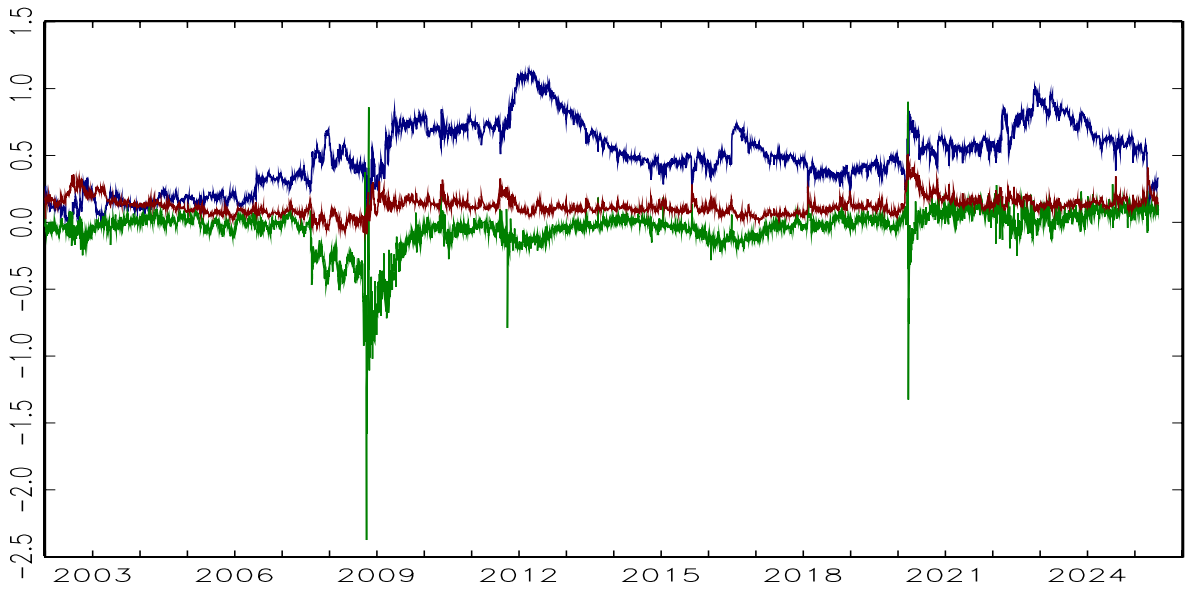

Figure D9: Time-varying beta estimation for LPX Europe, t-ACB model (January 2, 2002 to June 30, 2025).

Table D10: Parameter estimates for the t-ACB model for LPX UK (January 2, 2002 to June 30, 2025).

Parameter group	Parameter	Estimate (θ)	Std. Error (SE)
Constant parameters	α (Intercept)	0.0006^{***}	0.0001
(Equation 2)	β_1 (Mkt − RF)	0.1389^{*}	0.0723
	β_2 (SMB)	0.0953^{+}	0.0648
	β_3 (HML)	0.1070^{*}	0.0617
	λ	-5.7400^{***}	0.0828
Macro sensitivity	$\Gamma_{2,1}$	-1.1039^{***}	0.3176
(Equation 2)	$\Gamma_{2,2}$	0.3529^{***}	0.0893
	$\Gamma_{2,3}$	1.4128	1.1252
	$\Gamma_{3,1}$	−0.3038	0.5620
	$\Gamma_{3,2}$	-0.2691^{**}	0.1327
	$\Gamma_{3,3}$	−0.6554	1.7773
	$\Gamma_{4,1}$	0.6641^{+}	0.4162
	$\Gamma_{4,2}$	-0.1925^{*}	0.1162
	$\Gamma_{4,3}$	−1.3643	1.4114
	$\Gamma_{5,1}$	5.8599^{***}	0.4147
	$\Gamma_{5,2}$	0.1339	0.1356
	$\Gamma_{5,3}$	-3.7150^{**}	1.7234
Persistence	$\Phi_{2,2}$	0.9993^{***}	0.0002
(Equation 3)	$\Phi_{3,3}$	0.9920^{***}	0.0012
	$\Phi_{4,4}$	0.9692^{***}	0.0823
	$\Phi_{5,5}$	0.9849^{***}	0.0040
Score dynamics	$\Psi_{2,2}$	46.0784^{***}	12.2063
(Equation 3)	$\Psi_{3,3}$	0.0039	0.0241
	$\Psi_{4,4}$	13.1253	34.3460
	$\Psi_{5,5}$	0.0269^{***}	0.0030
Degrees of freedom	ν	7.9099^{***}	0.5582

Notes: $p < 0.10$ ($^{+}$), $p < 0.05$ (*), $p < 0.01$ (**), $p < 0.001$ (***).

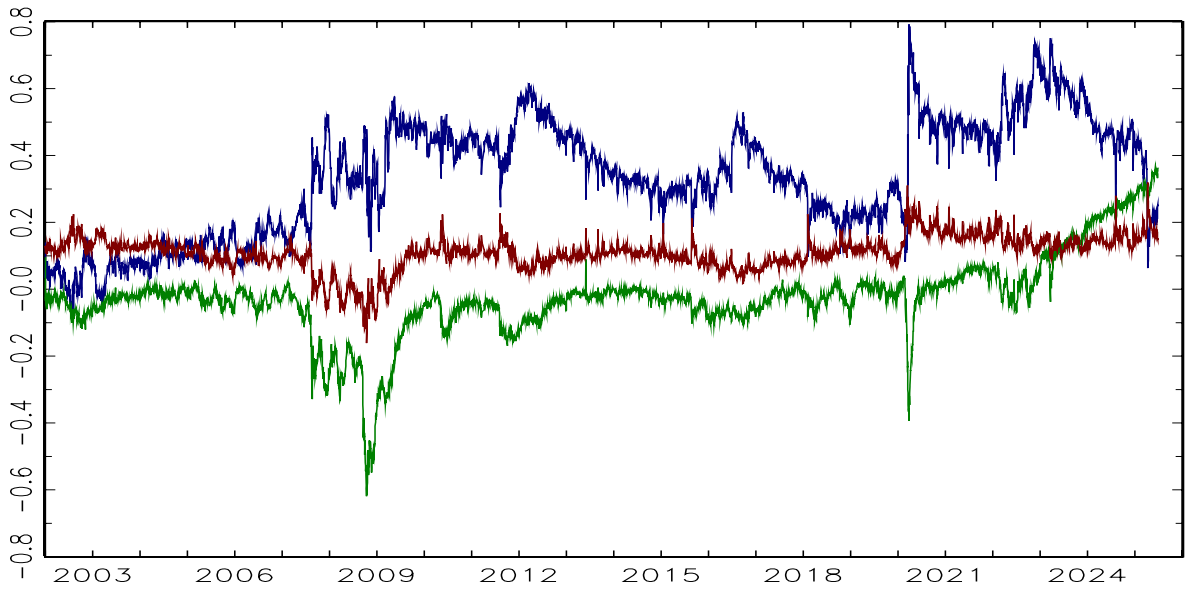

Figure D10: Time-varying beta estimation for LPX UK, t-ACB model (January 2, 2002 to June 30, 2025).

Supplementary Material E

Table E1: Correlation matrix of $\beta_{1,t}$ estimates across the LPX indices (January 2, 2002 to June 30, 2025).

	LPX 50	LPX Composite	LPX Major Market	LPX Buyout	LPX Venture	LPX Direct	LPX Indirect	LPX America	LPX Europe	LPX UK
LPX 50	1.0000	0.9905	0.9890	0.8326	0.5937	0.8555	0.8305	0.5248	0.9353	0.9180
LPX Composite	0.9905	1.0000	0.9884	0.8201	0.5641	0.8349	0.8473	0.5203	0.9306	0.9309
LPX Major Market	0.9890	0.9884	1.0000	0.8451	0.5710	0.8627	0.8364	0.5265	0.9423	0.9135
LPX Buyout	0.8326	0.8201	0.8451	1.0000	0.2413	0.9893	0.8301	0.6842	0.8924	0.8169
LPX Venture	0.5937	0.5641	0.5710	0.2413	1.0000	0.3210	0.4321	0.0729	0.5238	0.5506
LPX Direct	0.8555	0.8349	0.8627	0.9893	0.3210	1.0000	0.8193	0.6710	0.9021	0.8137
LPX Indirect	0.8305	0.8473	0.8364	0.8301	0.4321	0.8193	1.0000	0.6246	0.8644	0.9186
LPX America	0.5248	0.5203	0.5265	0.6842	0.0729	0.6710	0.6246	1.0000	0.4726	0.5103
LPX Europe	0.9353	0.9306	0.9423	0.8924	0.5238	0.9021	0.8644	0.4726	1.0000	0.9267
LPX UK	0.9180	0.9309	0.9135	0.8169	0.5506	0.8137	0.9186	0.5103	0.9267	1.0000

Notes: The table shows pairwise Pearson correlations based on daily $\beta_{1,t}$ estimates from the t-ACB model.

Table E2: Correlation matrix of $\beta_{2,t}$ estimates across the LPX indices (January 2, 2002 to June 30, 2025).

	LPX 50	LPX Composite	LPX Major Market	LPX Buyout	LPX Venture	LPX Direct	LPX Indirect	LPX America	LPX Europe	LPX UK
LPX 50	1.0000	0.9727	0.9686	0.5917	0.3789	0.5524	0.5412	−0.0993	0.4642	0.6795
LPX Composite	0.9727	1.0000	0.9362	0.7337	0.2571	0.4977	0.4807	−0.0230	0.3345	0.6032
LPX Major Market	0.9686	0.9362	1.0000	0.5301	0.3635	0.4507	0.7193	−0.2637	0.5830	0.8300
LPX Buyout	0.5917	0.7337	0.5301	1.0000	−0.0023	0.1566	0.0723	0.3360	−0.2123	0.1616
LPX Venture	0.3789	0.2571	0.3635	−0.0023	1.0000	0.3091	0.2102	0.0557	0.4735	0.3184
LPX Direct	0.5524	0.4977	0.4507	0.1566	0.3091	1.0000	0.2556	−0.1986	0.2992	0.2908
LPX Indirect	0.5412	0.4807	0.7193	0.0723	0.2102	0.2556	1.0000	−0.7445	0.7594	0.9759
LPX America	−0.0993	−0.0230	−0.2637	0.3360	0.0557	−0.1986	−0.7445	1.0000	−0.5912	−0.6464
LPX Europe	0.4642	0.3345	0.5830	−0.2123	0.4735	0.2992	0.7594	−0.5912	1.0000	0.7748
LPX UK	0.6795	0.6032	0.8300	0.1616	0.3184	0.2908	0.9759	−0.6464	0.7748	1.0000

Notes: The table shows pairwise Pearson correlations based on daily $\beta_{2,t}$ estimates from the t-ACB model.

Table E3: Correlation matrix of $\beta_{3,t}$ estimates across the LPX indices (January 2, 2002 to June 30, 2025).

	LPX 50	LPX Composite	LPX Major Market	LPX Buyout	LPX Venture	LPX Direct	LPX Indirect	LPX America	LPX Europe	LPX UK
LPX 50	1.0000	0.6768	0.6143	0.4359	0.4444	0.8657	0.3497	0.5782	0.5404	0.2266
LPX Composite	0.6768	1.0000	0.9103	0.8024	0.5279	0.8169	0.1735	0.8136	0.6822	0.1213
LPX Major Market	0.6143	0.9103	1.0000	0.7173	0.6062	0.7146	0.0832	0.7723	0.6456	0.0585
LPX Buyout	0.4359	0.8024	0.7173	1.0000	0.3812	0.7845	−0.0695	0.8125	0.5948	0.0512
LPX Venture	0.4444	0.5279	0.6062	0.3812	1.0000	0.4901	0.5902	0.1737	0.8118	0.6387
LPX Direct	0.8657	0.8169	0.7146	0.7845	0.4901	1.0000	0.2389	0.7390	0.6655	0.2221
LPX Indirect	0.3497	0.1735	0.0832	−0.0695	0.5902	0.2389	1.0000	−0.3419	0.6936	0.9221
LPX America	0.5782	0.8136	0.7723	0.8125	0.1737	0.7390	−0.3419	1.0000	0.2738	−0.3581
LPX Europe	0.5404	0.6822	0.6456	0.5948	0.8118	0.6655	0.6936	0.2738	1.0000	0.7702
LPX UK	0.2266	0.1213	0.0585	0.0512	0.6387	0.2221	0.9221	−0.3581	0.7702	1.0000

Notes: The table shows pairwise Pearson correlations based on daily $\beta_{3,t}$ estimates from the t-ACB model.

Figure E1(a): LPX 50 $\beta_{2,t}$ (blue); LPX Composite $\beta_{2,t}$ (green); LPX Major Market $\beta_{2,t}$ (red).

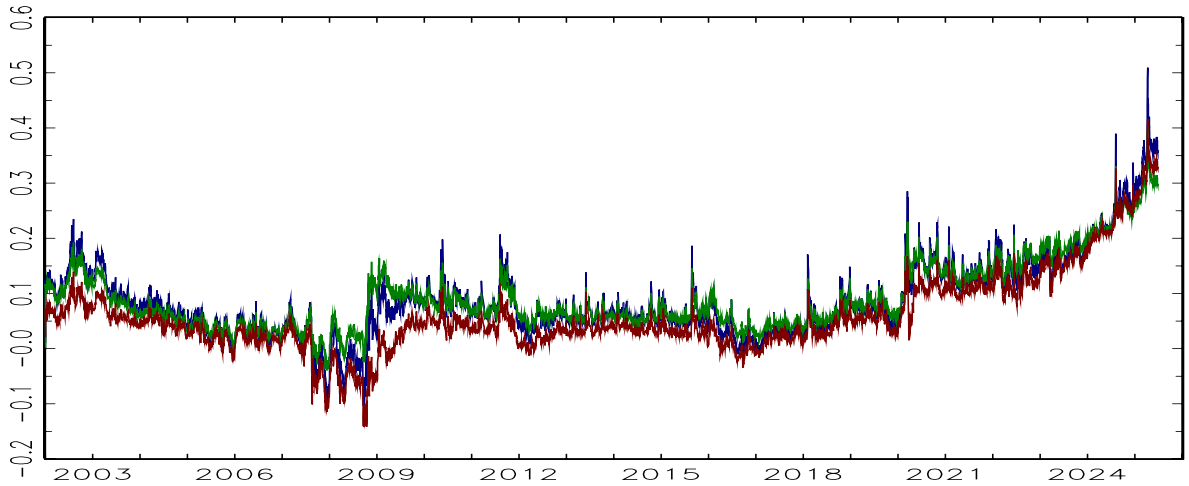

Figure E1(b): LPX Buyout $\beta_{2,t}$ (blue); LPX Venture $\beta_{2,t}$ (green); LPX Direct $\beta_{2,t}$ (red); LPX Indirect $\beta_{2,t}$ (black).

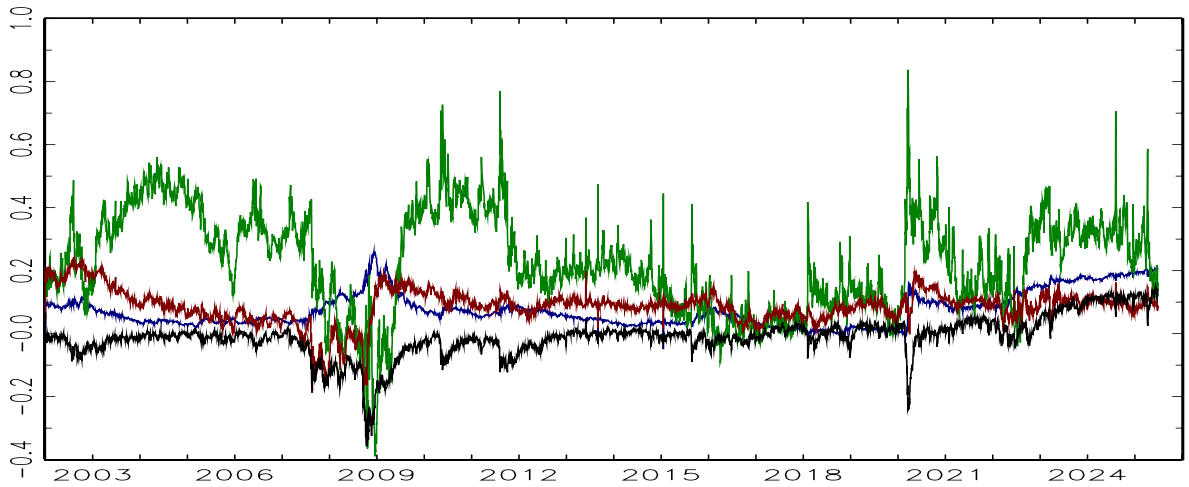

Figure E1(c): LPX America $\beta_{2,t}$ (blue); LPX Europe $\beta_{2,t}$ (green); LPX UK $\beta_{2,t}$ (red).

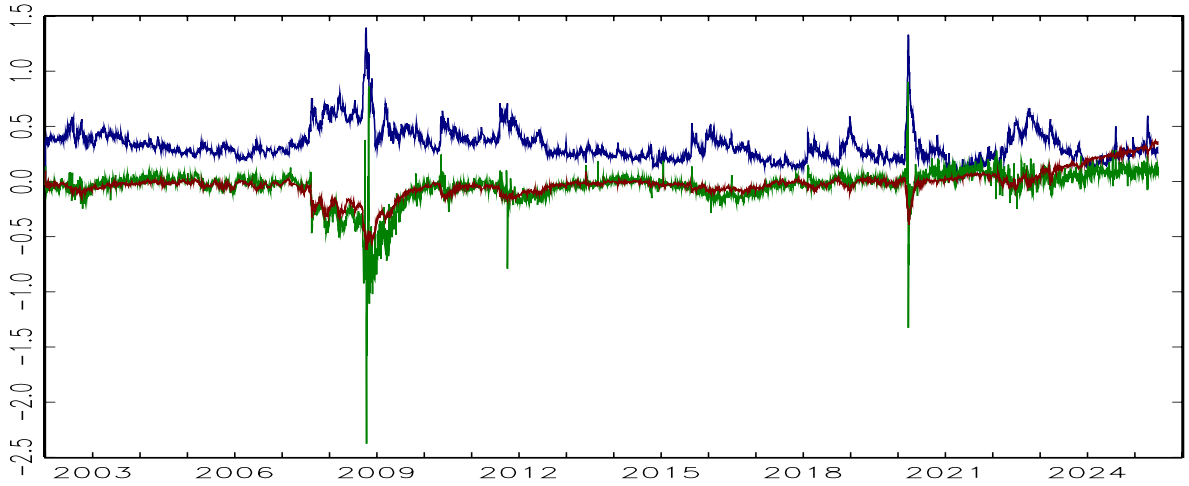

Figure E1: Time-varying beta $\beta_{2,t}$ (size) of the t-ACB model for LPX indices from January 2, 2002, to June 30, 2025. *Notes:* The $\beta_{2,t}$ parameter reflects exposure to the size factor (SMB).

Figure E2(a): LPX 50 $\beta_{3,t}$ (blue); LPX Composite $\beta_{3,t}$ (green); LPX Major Market $\beta_{3,t}$ (red).

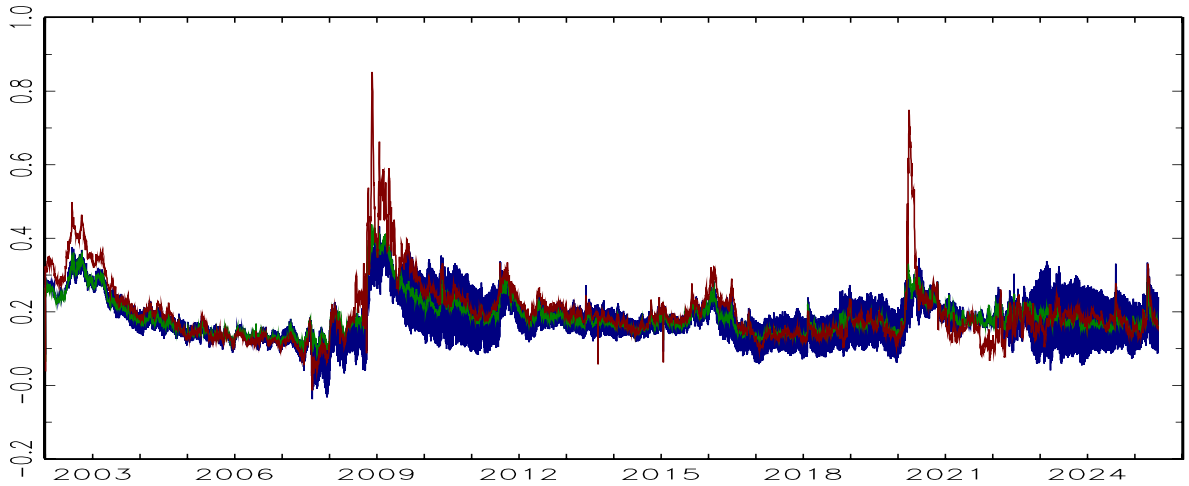

Figure E2(b): LPX Buyout $\beta_{3,t}$ (blue); LPX Venture $\beta_{3,t}$ (green); LPX Direct $\beta_{3,t}$ (red); LPX Indirect $\beta_{3,t}$ (black).

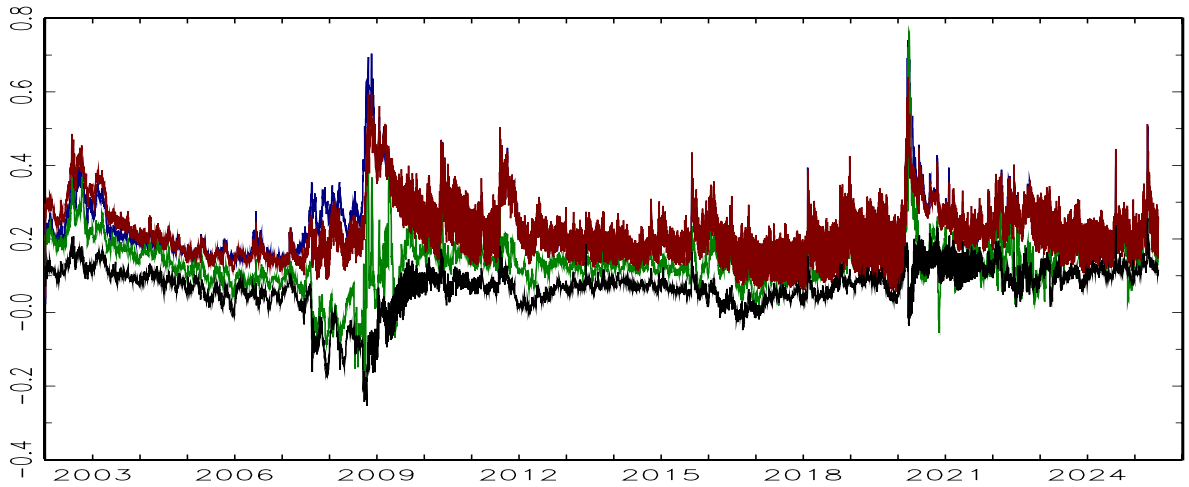

Figure E2(c): LPX America $\beta_{3,t}$ (blue); LPX Europe $\beta_{3,t}$ (green); LPX UK $\beta_{3,t}$ (red).

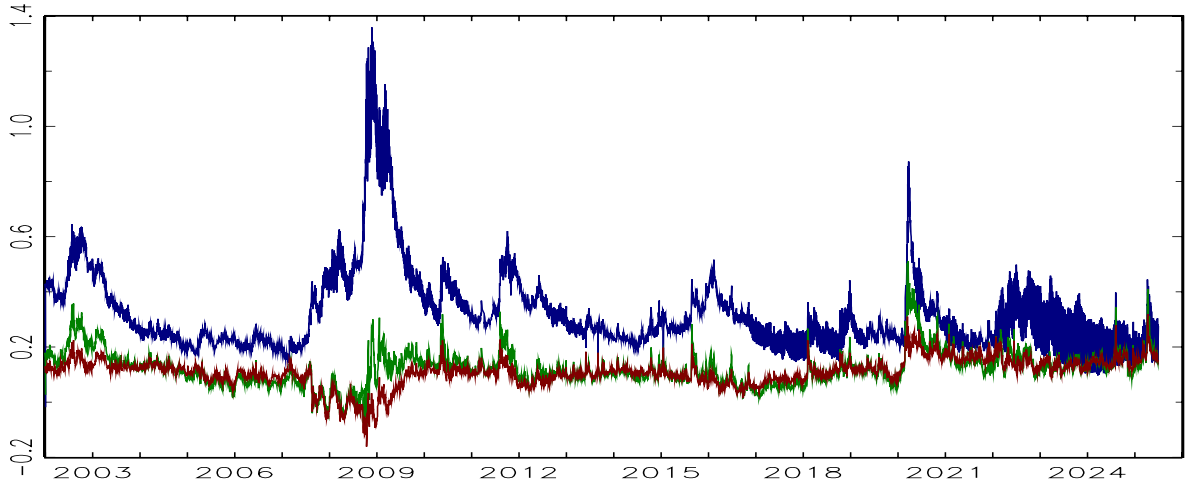

Figure E2: Time-varying beta $\beta_{3,t}$ (value) of the t-ACB model for LPX indices from January 2, 2002, to June 30, 2025. *Notes:* The $\beta_{3,t}$ coefficients capture exposure to the Fama–French HML factor, representing sensitivities to value-versus-growth characteristics.