September 2025 – Optimal Control with a VSTAR

I have redone the whole empirical analysis with the new time series for prices, PCEPI, the price index of personal consumption expenditure. The excel file AAA\_Dataset has been updated to include the new series.

In the paper, Figure 1 has been updated to include the new inflation series. This is saved as FigXDataPlot.eps.

All results are saved in the excel file AAA\_Results.

The re-estimation of the VSTAR is carried out in two stages with results and codes saved in two folders:

1. Folder ‘Estimation\_LagChoice’ include the results for the VSTAR estimated with GLS using from one to twelve lags. The results and info criteria are separately compared in the end in the excel file JAAE Estimation Results. In this, I compute two sets of info criteria. The first are the standard info criteria. The second are the normalized criteria where both the log-likelihood and the penalty term are scaled by the number of observations, as computed by MATLAB in the file ResultsEstimval.m. The table in the second TAB summarizes the main results which are then reported in the appendix. The selected model is that with one lag. With the GLS the log-lik of the model with one lag is 0.41. However, if I re-estimate the same model with one lag using FIML I get a log-lik of 0.886. Since I am selecting as best model that with one lag, I am using the results from this estimation. All results are saved in AAA\_Results, tab LagSelection.
2. Folder ‘Estimation\_ModelChoice\_OneLag’ includes results for VSTAR estimation with one lag only using full information maximum likelihood as well as all alternative specifications (VAR, VSTARC, VSTARV, etc…). For the model with one lag, the estimation is started using 0.1 everywhere. In the neglog function the last two para are squared otherwise they become negative and the code stops. Once it has converged, I square the last two estimated coefficients and I rev=move the squares from neglog. I have tried to re-estimate the whole model using 1 everywhere, but it stops as the last two para go negative. All results are saved in AAA\_Results, tab ModelComparison. I have also updated the table in the paper.

Having completed the estimation, the next part is the analysis of the results. In the folder CodeLevel3 I have updated Preliminaries.m to include as start the best estimated coefficients for the VSTAR with one lag. Once running main, this returns exactly the same loglikelihood as above. I have then saved the coefficients in CoeffVSTAR.mat and run the file PlotTransitions.m to generate the figure 2 in the paper. This is saved as Fig\_TransProb22.eps and I have imported into Overleaf.

The next task is to compute the IRF. In the paper I have computed the following IRFs: (i) to a credit shock, to a supply shock, to a demand shock, to a credit shock when GDP growth is used instead of industrial production, to a credit shock when debt is used instead on industrial production. The first is reported in the paper, the other four in the Appendix as part of the robustness analysis.

To compute the first three types of IRF I have created three separate m files, called NSIRFComputeCredit, NSIRFComputeSupply and NSIRFComputeDemand. In each of these I first had to recompute the histories, as I have changed the price series. This is done in lines 6 to 9. I have updated with the new results for initial values in Table F1 in Appendix.

In NSIRFComputeCredit I use:

pos = 1; (in line 15);

delta = 1; (in line 40) and

z1=[squeeze(S(2,1,1:ho))'<=0 squeeze(S(3,1,1:ho))'<=0 squeeze(S(4,1,1:ho))'>=0 ]; (in line 79).

The IRFs are saved as

impMFIN = impM;

impSFIN = impS;

save impMFIN.mat impMFIN

save impSFIN.mat impSFIN

In NSIRFComputeDemand I use:

pos = 4; (in line 15);

delta = 1; (in line 40) and

z1=[squeeze(S(2,1,1:ho))'<=0 squeeze(S(3,1,1:ho))'<=0 squeeze(S(4,1,1:ho))'>=0 ];

(in line 79).

The IRFs are saved as

impMDEM = impM;

impSDEM = impS;

save impMDEM.mat impMDEM

save impSDEM.mat impSDEM

In NSIRFComputeSupply I use:

pos = 4; (in line 15);

delta = 1; (in line 40) and

z1=[squeeze(S(2,1,1:ho))'>=0 squeeze(S(3,1,1:ho))'<=0 squeeze(S(4,1,1:ho))'>=0 ];

(in line 79).

The IRFs are saved as

impMSUPPLY = impM;

impSSUPPLY = impS;

save impMSUPPLY.mat impMSUPPLY

save impSSUPPLY.mat impSSUPPLY

With regard to the IRF to a credit shock when using either GDP or debt instead of industrial production, I have created two separate folders, called CodeLeve3Robustness\_DEBT and CodeLeve3Robustness\_GDP and re-estimated the VSTAR using the same methods described above. I have then computed the IRF for both models and saved then into two matlab files called impMNEGFINDEBT.mat, impSNEGFINDEBT.mat, impMNEGFINGDP.mat and impSNEGFINGDP.mat. I have copied these four files into the folder Robustness\_IRF/CodeLevel3 where upon running the file Main one can plot the IRF. These are saved as FIG\_IRF\_Robustness.jpeg and imported into the appendix. I need to check and adjust comments.

Once all IRFs are computed and saved in .mat files, I use the code NSIRFPlot to plot the responses to a credit shock during normal times and during the ZLB in Figure 3 of the paper. PROBLEM. In the code, the figure is saved as FigIRFCredit.eps in line 168. The problem with this is that for some reason the shaded areas overlap with each other and do not display well once I upload the figure in overleaf. This could be a problem with the pdf generator. The solution I found is to generate the figure and then save it using FILE/save as/ and then choosing the format .jpg. need to adjust this.

**OPTIMAL CONTROL**

For the purpose of the optimal control, I need first of all to update the calibration of the rate of inflation target in line 21 of preliminaries. I have recomputed the average inflation using observations from Dec 2007 until the end of the sample. This gives 1.55%. I have also updated the paper with this new value.

Second, I need to re-estimate the linear VAR, save the estimated coefficients and residuals. To this end I use the file LinearEstimation; The estimated coefficients and residuals are transposed and the saves as VARstarS and saveu respectively. In STABLEVAR each row includes the coefficients of one estimated equation. The intercept constants are in the last column.

Just using the two changes above I have run the code up until line 60 to replicate the results in ys1 and ysd. Ys1 includes the one-period ahead forecasts using the actual data and VSTAR. Ysd the dynamic forecasts. If everything until then is correct, then the these should return the data. This is the case, both ys1 and ysd return the actual data from November 2007 to Sep 2018, see AAA\_Results.xlx, ModelSimulationCheck.

**Welfare Gains**

Upon running the code the welfare results are saved in the matrix TAB31, with first row having the results for no QE, second for actual and third for optimal. This gathers the data then reported in table 2 of the paper. The code needs to be executed several times by changing the weights in Preliminaries. The new results are stored in AAA\_Results.xlx, WelfareGains. The first table includes the results based on the subsample 2007:12 - 2016:10, which is the same used in the paper in table 2. The problem with these results is that the volatility of unemployment without QE is smaller than under the actual policy. This is particularly problematic when the inflation weight is reduced to .5. I have therefore tried to change the subsample size to see if I can change this results. The second table extends the subsamnple to 2007:12 - 2018:10. Now the volatility without QE is higher than under the actual policy, but most of the gain is with the actual policy and the contribution of the optimal policy is way smaller. I have then tried to increase the subsample by only two months, 2007:12 - 2016:12. This results are reported in the table starting from column U. I will keep these for now, but need to update the paper to explain the new subsample choice, add the new table and amend comments). To include these results in the paper, I have pasted the table in column AE onward and then I have adjusted the order of the rows in a way compatible with how this is presented in the paper.

**IRF under optimal control.**

I have created four separate files that compute the IRF to a credit shock under control, one file for each history. These files are called NSIRFsControlComputeHistX;X=1,2,3,4. I had to create the four separate because the computation is very slow. For each file the results are saved separately as impMCFINhistX.mat, X=1,2,3,4 and imported into NSIRFsPlot where now Figure 2 plots the IRF to a credit shock under the actual and optimal at the ZLB, while Figure 3 plots the same responses for the post zlb period. Figure 2 is saved as FigIRFCreditOpt.jpg and imported into the paper. The problem with Figure 3 is that the IRFs are all over and show a lot of instability. To deal with this I am trying to show robustness in a different direction. For example, I have prepared a code called NSIRFsControlComputeHist3LONG where the credit shock at the ZLB lasts for 1 quarter, i.e. 3 months, instead of one month. I have also started a simulation of the IRF under control to a supply shock in a file called: NSIRFsControlComputeHist3SUPPLY.

**Counterfactual**

The counterfactual is generated by the file SimulationSub in figure 3. This plots the simulated series with three alternative assumptions about the rates of inflation targeted by the Fed.

The first is 2%. The second is 6%, in line with the average inflation rate over the sample of 5.8% with the previous data. The third is 4% in line with the average inflation rate from 2008 onward (about 3.4% with the previous data) and with the conjecture of Ball(2014). Now that I have changed the series for inflation I have updated these targets. With the new data, the average inflation rate over the whole sample and from 2008 onward are 2.9% and 1.5%. So I will use as targets 4% to maintain the comparison with Ball. But replace 6% with 3% as an intermediate case.

For these simulations I have replaced the inflation rates of 2,3 and 4 % in Preliminaries and run SimulationSub each time using the one-period ahead and save the resulting simulated series as ys1c2.mat, ys1c3.mat and ys1c4.mat. I have then redone figure 6 in the paper, I need to update comments.

I have also updated the calculations of the ZLB duration reported in table 3 of the paper. These are executed in the file SimulationSub, from line 126 onward using:

% TABLE ZLB duration

sum(ys1(:,7)<.25)

sum(ys1c2(:,7)<.25)

sum(ys1c3(:,7)<.25)

sum(ys1c4(:,7)<.25)

I report the resulting durations manually in the paper. I have also copied the simulated series for the interest rate in the excel file AAA\_Results, under the tab ZLBduration. In these I identify the first and last month of ZLB now reported in the paper.

In the file SimulationSub I also calculate the balancesheet decomposition under the actual data, and the optimal policy for the 2,3 and 4 % inflation target. Results are collected in four separate matrices, QE\_Table\_Actual, etc, and then copied in the excel file AAA\_Results, under the tab BalanceSheetDecomp. I have then copied these manually into the paper.

**ALTERNATIVE OPTIMIZATION**

All these calculations are in the folder CodeLevel3\_AlternativeOptimization. The results are obtained by running main. I have first changed the data to update for the new inflation rate.

I had to update Preliminaries and import in the folder the new estimated coefficients for coeffVSTAR.mat to be used for model predictive control and nonlinear optimization.

In main, the alternative optimization can be carried out with three alternative methods. The first, MPC requires the execution of

% ModelPredOptim;

% ModelPredOptimSub;

The second, nonlinear optimization, requires the execution of

%NonLinearOptim;

% NonLinearOptimSub;

The final, linear VAR requires the execution of

% load coeffVAR.mat

% tic

% [lf,vc] = neglog(bhat,nobs,y1,y2,y3,y4,y5,y6,y7,maxlag,x1,x2,d,treshM1,treshV1);

% disp(' log-like')

% disp(-lf)

% % Forecasts

% Simulation;

% %SimulationSub;

In each of these three cases, the code can be executed over the whole sample or post 2007 (labelled as sub). The inflation rate needs to be reset accordingly in each case. For each method and sample, the optimization needs to be carried out using the alternative policy weights in preliminaries.

For the linear estimation, I had to go back in the folder Estimation\_ModelChoice\_OneLag and re-estimate the VAR. I then had to map the estimated coefficients in a format for the vector bhat compatible with that used in neglogVSTAR. This is saved as coeffVAR.mat and then taken into the folder for the alternative optimization. Upon running the code for alternative optimization based on linear VAR it returns the same log lik of -19.4023, which confirms the mappin is correct.

When I run the optimization using

% ModelPredOptim;

% ModelPredOptimSub;

%NonLinearOptim;

% NonLinearOptimSub;

The simulated series for the rate of inflation appear to be very volatile. For this reason, in each of these files I have replaced the simulated series of the rate of interest with a smoother one, obtained by applying the command smooth.

All results are saved in the excel file AAA\_Results under the tab AlternativeOptimization.

**Professional Forecasts**

In the folder Professional Forecasts I have gathered the Professional Forecasts for key macroeconomic indicators of the US economy. These are included in the excel files Dispersion1 and Dispersion2. The matlab file main, then reads these data, selects the appropriate series and converts the quarterly forecast into monthly data. These are then saved into Data25.mat and Data75.mat, which include the 25th and 75th percentile of the professional forecasts. These mat files are then uploaded into SimulationSub around lines 119 where they are plotted agains the actual and optimal data to generate figure 4 and then used to generate some summary statistics of the deviations in the matrix FORE\_COMP\_TAB. This matrix is then saved in AAA\_Results under the tab ProfFore and then copied in the main paper. In SimulationSub I have also generated a figure that reported the simulated series under control, the actual data and the professional forecasts. This is included in the Appendix.