Online Appendix for: Forecasting transaction counts with integer-valued $GARCH$ models

Abdelhakim Aknouche[*], Bader S. Almohaimeed[**], and Stefanos Dimitrakopoulos[1***]

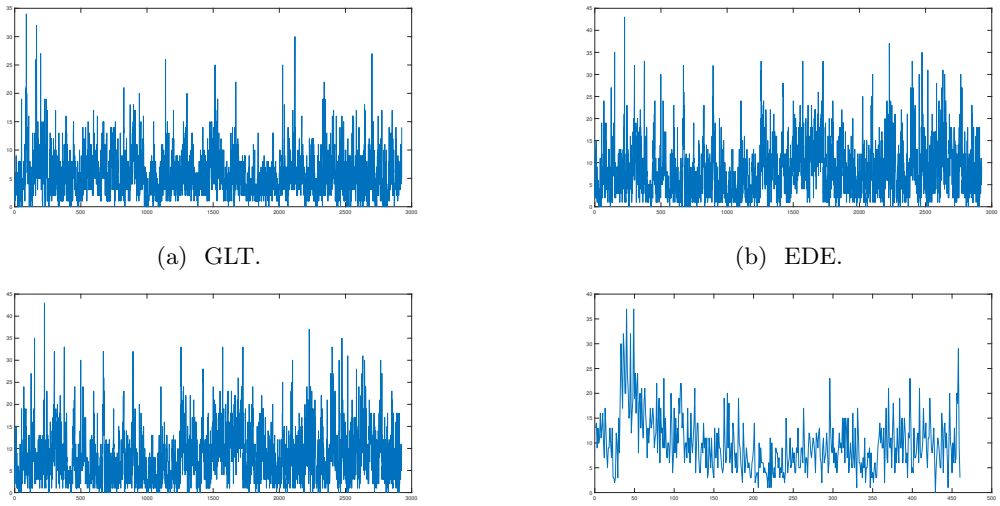
[*] Department of Mathematics, College of Science, Qassim University (51431, P.O. Box 707, Saudi Arabia), & Faculty of Mathematics, University of Science and Technology Houari Boumediene (Algeria), Email: a.aknouche@qu.edu.sa

[**]Department of Mathematics, College of Science, Qassim University (51431, P.O. Box 707, Saudi Arabia), Email: bsmhiemied@qu.edu.sa

[***]Economics Division, Leeds University Business School, University of Leeds, Leeds, LS2 9JT, UK, Email: s.dimitrakopoulos@leeds.ac.uk

1 Empirical analysis

1.1 Descriptive plots

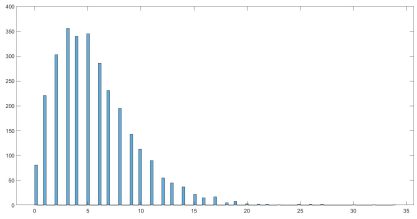


(a) GLT. (b) EDE.

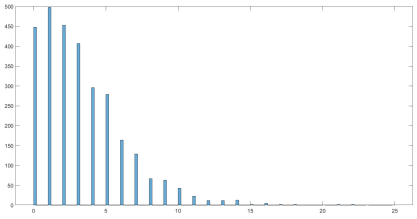
(c) WPP. (d) Ericsson B

Figure 1: Empirical results: Time series plots for the four financial series.

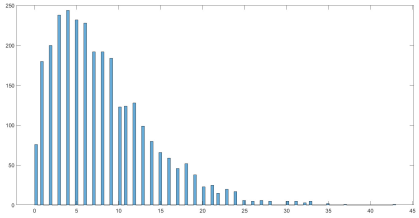
[1]Correspondence to: Stefanos Dimitrakopoulos, s.dimitrakopoulos@leeds.ac.uk.



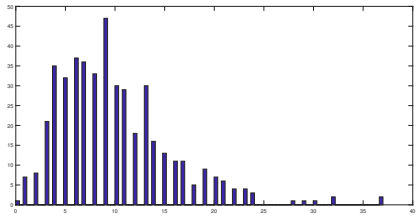
(a) GLT.



(b) EDE.



(c) WPP.



(d) Ericsson B

Figure 2: Empirical results: Histograms for the four financial time series.

1.2 Empirical estimation results

The hyperparameters for the prior distributions of the models in question are similar to those used in the simulation study. We run each algorithm for 5000 iterations after a burn-in period of 10000 cycles.

Table 1: Empirical results for P-INGARCH model

	GLT			EDE			WPP			Ericsson B		
	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD
ω	0.517	11.793	0.369	0.530	10.24	-0.165	0.859	9.515	-0.977	1.0957	24.324	-0.718
	(0.052)			(0.052)			(0.062)			(0.217)		
α	0.190	15.806	-1.365	0.224	16.537	-0.071	0.270	11.985	-1.204	0.214	17.649	-0.999
	(0.010)			(0.0117)			(0.009)			(0.021)		
β	0.718	13.634	0.344	0.615	11.385	-0.036	0.624	10.43	0.964	0.677	22.354	0.825
	(0.016)			(0.023)			(0.014)			(0.036)		

Standard deviation in parentheses. CD stands for Converge Diagnostics and IF stands for Inefficiency Factor.

Table 2: Empirical results for NB1-INGARCH model

	GLT			EDE			WPP			Ericsson B		
	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD
ω	0.529	17.129	0.020	0.482	21.237	1.406	0.881	12.884	-1.551	0.708	32.748	1.315
	(0.084)			(0.070)			(0.118)			(0.290)		
α	0.180	18.272	-0.282	0.214	19.306	1.124	0.260	13.246	-0.304	0.194	29.208	0.305
	(0.014)			(0.017)			(0.017)			(0.031)		
β	0.724	18.053	0.014	0.639	23.909	-1.186	0.629	13.57	0.836	0.736	34.128	-0.637
	(0.025)			(0.032)			(0.027)			(0.050)		
r_1	0.950	1.181	3.068	0.747	1.895	0.419	0.467	1.815	2.856	0.706	1.296	-0.314
	(0.053)			(0.038)			(0.020)			(0.085)		

Standard deviation in parentheses. CD stands for Converge Diagnostics and IF stands for Inefficiency Factor.

Table 3: Empirical results for NB2-INGARCH model

	GLT			EDE			WPP			Ericsson B		
	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD
ω	0.518	15.815	-0.625	0.519	20.959	-0.606	0.833	12.682	1.694	0.722	79.684	-0.319
	(0.081)			(0.088)			(0.111)			(0.321)		
α	0.193	18.281	0.433	0.224	15.846	-0.150	0.271	13.071	1.820	0.198	61.863	-1.165
	(0.015)			(0.019)			(0.017)			(0.030)		
β	0.713	18.408	0.027	0.618	19.732	0.554	0.623	12.053	-2.036	0.730	70.747	1.129
	(0.026)			(0.040)			(0.026)			(0.049)		
r_2	5.042	2.937	0.729	2.529	17.432	-2.338	3.565	294.78	5.597	6.504	1.038	-0.849
	(0.201)			(0.208)			(0.093)			(0.741)		

Standard deviation in parentheses. CD stands for Converge Diagnostics and IF stands for Inefficiency Factor.

Table 4: Empirical results for DP-INGARCH model

	GLT			EDE			WPP			Ericsson B		
	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD
ω	0.533	30.103	-0.266	0.536	19.439	-0.784	0.868	13.941	0.846	1.126	50.444	0.602
	(0.093)			(0.094)			(0.123)			(0.343)		
α	0.192	15.253	-0.531	0.225	21.537	0.009	0.270	17.535	0.229	0.215	30.978	0.764
	(0.016)			(0.018)			(0.018)			(0.0347)		
β	0.713	25.247	0.293	0.612	20.531	0.514	0.622	15.855	-0.634	0.673	53.553	-0.819
	(0.028)			(0.041)			(0.029)			(0.057)		
γ	0.467	1.927	0.667	0.427	1.712	-0.723	0.318	1.598	0.323	0.403	1.010	-0.286
	(0.012)			(0.011)			(0.008)			(0.026)		

Standard deviation in parentheses. CD stands for Converge Diagnostics and IF stands for Inefficiency Factor.

Table 5: Empirical results for GP-INGARCH model

	GLT			EDE			WPP			Ericsson B		
	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD	Mean	IF	CD
ω	0.3577	40.5632	-0.9355	0.3197	62.1411	-0.4070	1.3346	61.5803	0.9183	0.4850	51.5858	0.1120
	(0.0551)			(0.0479)			(0.0671)			(0.2016)		
α	0.1250	13.4330	-0.5888	0.1398	91.9817	-0.3501	0.2339	42.1838	1.0198	0.1261	77.9150	1.0581
	(0.0094)			(0.0102)			(0.0104)			(0.0212)		
β	0.7248	21.0878	1.0617	0.6347	56.7566	0.3232	0.0161	7.5167	-0.0873	0.7309	59.0310	-0.3922
	(0.0222)			(0.0319)			(0.0006)			(0.0527)		
τ	0.3169	16.7528	-0.5020	0.427	1.712	-0.723	0.4704	51.5128	-2.3814	0.3539	21.3495	0.9529
	(0.0099)			(0.0108)			(0.0088)			(0.0236)		

Standard deviation in parentheses. CD stands for Converge Diagnostics and IF stands for Inefficiency Factor.

1.3 Forecasting results

We report the ratio of the LPS value of the baseline model to that of a given model, with the baseline model being the P-INGARCH model. Hence, ratios greater than one indicate better density forecasts than the baseline model. Moreover, we subtract the sMSE value of a given model from that of the baseline model. So, positive values indicate better point forecasts.

We also calculated the Diebold and Mariano (1995) statistics, accounting also for the Harvey et al. (1997) finite-sample adjustment. The Diebold and Mariano (1995) approach is a test for equal predictive accuracy. Therefore, we tested whether the forecasting values (point and density forecasts) produced by the models are significantly different from those produced by the baseline model. The asterisk next to the reported density and point forecast value indicates that the respective model shows superior forecast performance relative to the baseline model.

1.3.1 Density forecasts

Table 6: LPS results (GLT).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	1	1	1
NB1-INGARCH	0.8743	1.0871*	1.0833*
NB2-INGARCH	1.1118*	1.1110*	1.1091*
DP-INGARCH	1.0816*	1.0786*	1.0755*
GP-INGARCH	0.7465*	1.0478*	1.0315*

Table 7: LPS results (WPP).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	1	1	1
NB1-INGARCH	0.8744	1.0533*	1.0575*
NB2-INGARCH	1.0690*	1.0619*	1.0659*
DP-INGARCH	1.0611*	1.0489*	1.0520*
GP-INGARCH	0.8111*	1.0386*	1.0255*

Table 8: LPS results (EDE).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	1	1	1
NB1-INGARCH	1.4299*	1.1295*	1.1402*
NB2-INGARCH	1.1464*	1.1503*	1.1594*
DP-INGARCH	1.1104*	1.1136*	1.1254*
GP-INGARCH	1.0816*	1.0757*	1.0723*

Table 9: LPS results (Ericsson B).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	1	1	1
NB1-INGARCH	1.3312*	1.1469*	1.1854*
NB2-INGARCH	1.1294*	1.0254 *	1.169*
DP-INGARCH	1.2290*	1.0478*	1.0256*
GP-INGARCH	1.1154*	1.0133*	1.0134*

1.3.2 Point forecasts

Table 10: sMSE results (GLT).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	0	0	0
NB1-INGARCH	0.0156*	0.0242*	0.0236*
NB2-INGARCH	0.0487*	0.0626*	0.0853*
DP-INGARCH	-0.0006*	-0.0009*	-0.0007*
GP-INGARCH	-0.0016*	-0.0001*	-0.0002*

Table 11: sMSE results (WPP).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	0	0	0
NB1-INGARCH	0.4763*	0.3512*	0.1294*
NB2-INGARCH	0.0087*	0.0094*	0.0095*
DP-INGARCH	0.0001*	0.0004*	0.0008*
GP-INGARCH	-.0047*	-0.0051*	-0.0077*

Table 12: sMSE results (EDE).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	0	0	0
NB1-INGARCH	0.0325*	0.0487*	0.0623*
NB2-INGARCH	0.0222*	0.0148*	0.0356*
DP-INGARCH	0.0126*	0.0059*	0.0085*
GP-INGARCH	0.0004*	0.0023*	0.0036*

Table 13: sMSE results (Ericsson B).

Model	$s = 1$	$s = 4$	$s = 8$
P-INGARCH	0	0	0
NB1-INGARCH	0.1398*	0.1111*	0.0856*
NB2-INGARCH	0.2584*	0.3266*	0.2265*
DP-INGARCH	0.0367*	0.0144*	0.0658*
GP-INGARCH	-0.0008*	-0.0013*	-0.0023*

References

Atchadé, YF. (2006). An adaptive version for the Metropolis adjusted Langevin algorithm with a truncated drift. *Methodology and Computing in Applied Probability* 8: 235–254.

Chib, S. (2001). Markov chain Monte Carlo methods: computation and inference. In: *Handbook of Econometrics*, Volume 5. North Holland, Amsterdam, pp. 3569-3649.

Diebold, F., Mariano, R.S. (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics*, 13: 253–263.

Geweke, J. (1992). Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. In: Bayesian Statistics 4, Oxford: Clarendon Press, pp. 641-649.

Harvey, D., Leybourne, S., Newbold, P. (1997). Testing the equality of prediction mean squared errors. *International Journal of Forecasting*, 13: 281–291.

Martino, L., Yang, H., Luengo, D., Kanniainen, J., Corande, J. (2015). A fast universal self-tuned sampler within Gibbs sampling. *Digital Signal Processing* 47: 68–83.