This document presents supplementary results and illustrations for the paper Testing for Misspecification in the short-run component of GARCH-type models, which were, due to their extent, omitted in the printed version.

A Supplementary Monte-Carlo simulations

1. Size and power with different sample sizes. Results are presented in Table 1 for the following DGPs:
   - GARCH: $h_t = 0.01 + 0.1\varepsilon_{t-1}^2 + 0.7h_{t-1}$
   - MS: $\begin{cases} \text{Regime 1:} & h_{1,t} = 0.01 + 0.1\varepsilon_{t-1}^2 + 0.7h_{1,t-1} \\ \text{Regime 2:} & h_{2,t} = 0.0001 + 0.2\varepsilon_{t-1}^2 + 0.4h_{2,t-1} \end{cases}$
   - GJR: $h_t = 0.01 + \Big(0.1 + 0.2\,\mathbb{1}(\varepsilon_{t-1} > 0)\Big)\varepsilon_{t-1}^2 + 0.7h_{t-1}$

	GARCH			MS			GJR		
T	500	1000	2000	500	1000	2000	500	1000	2000
$LM_{(1,1,2)}$	0.022	0.032	0.036	0.068	0.042	0.034	0.870	1.000	1.000
$LM_{P,(1,1,3)}$	0.040	0.041	0.043	0.152	0.163	0.248	0.952	1.000	1.000
$LM_{P,(2,2,3)}$	0.034	0.041	0.043	0.572	0.860	0.944	0.900	1.000	1.000
$L\&T$	0.034	0.042	0.043	0.104	0.092	0.080	0.598	1.000	0.986
$H\&O$	0.044	0.047	0.049	0.152	0.130	0.144	0.904	1.000	1.000

Table 1 Rejection frequencies of LT, HO and LM tests, for testing a GARCH(1,1) specification, at nominal level $\alpha = 0.05$ with different sample sizes.

2. Size and power with $k = 2, \ldots, 5$. Results are presented in Table 2 for the following DGPs:
   - GARCH: $h_t = 0.01 + 0.1\varepsilon_{t-1}^2 + 0.7h_{t-1}$
   - GJR: $h_t = 0.01 + \Big(0.1 + 0.1\,\mathbb{1}(\varepsilon_{t-1} > 0)\Big)\varepsilon_{t-1}^2 + 0.7h_{t-1}$

B Estimation of the three exchange rate returns: USD-JPY, EUR-USD, USD-CLP

In this appendix, we propose some paths to go further to model the three exchange rate series. We estimate for each model three models: the standard

k	GARCH	GJR
$LM_{(1,1,2)}$	0.032	0.666
$LM_{P,(1,1,3)}$	0.041	0.792
$LM_{P,(1,1,4)}$	0.042	0.792
$LM_{P,(1,1,5)}$	0.041	0.792

Table 2 Rejection frequencies of LM tests, for testing a GARCH(1,1) specification, at nominal level $\alpha = 0.05$ with different order expansions.

GARCH, the GJR-GARCH and the MS-GARCH. For the standard GARCH and the GJR-GARCH, we estimate both the Gaussian and Student models. Tables 3, 4 and 5 give the estimation results for the three series, respectively for the USD-JPY, the EUR-USD and the USD-CLP exchange rate returns. First, for the USD-JPY returns (Table 3), the GJR-t model outperforms all the others, with both AIC and BIC lower for this model. GJR-t model selection means that there is a leverage effect such that negative returns (i.e. when the US dollar depreciates against the Yen) have a stronger impact on conditional variance than positive shocks. This results can be counter-intuitive since the foreign exchange market is two-sided: for bilateral exchange rates, positive returns for one currency correspond to negative returns for the other, i.e. we can not disentangle "bad news" and "good news". However, we can argue for the presence of asymmetry in the foreign exchange rates market and give two reasons. First, some currencies have greater importance from an economic point of view. Secondly, central bank interventions lead to higher volatility, for example, to depreciate its currency, a central bank would buy its domestic currency and sell USD. Wang and Yang (2009) find a similar result for a longer period between 1996 and 2004 using realized volatility measure. For the EUR-USD (Table 4), GARCH-t and GJR-t perform quite similarly. The conditional volatility is linear according to misspecification tests. Both central banks let their currency float without any intervention. Thus, either an appreciation or a depreciation in the dollar would have the same impact on volatility which is linearly time varying. The MS-t estimation results are not reliable for this serie and thus there is no real switching and we exclude this model. Finally, the MS-GARCH Student model is selected according to the AIC to model the USD-CLP (Table 5) exchange rate returns. The smoothed probability of the turbulent regime is given in Figure 1. It shows that the exchange rate returns were in a low volatility regime period from the beginning of 2009 to mid 2011. By the end of 2008, a dramatic fall in copper prices negatively impacted the Chilean economy, which is very dependent on the price of this commodity price. To counter the global financial crisis and this price drop, the Central Bank of Chile launched a monetary intervention: they purchased a large amount of dollars. Thus, the 2008 intervention by the Central Bank of Chile may be one determinant of this low volatility regime, in contrast to the 2011 intervention which seems not to have any impact on exchange rate returns. To conclude, when all the tests do not reject the null hypothesis, the GARCH model seems to be the best one. When all the tests reject the null hypothesis, the GJR-GARCH outperforms the other models. Finaly, when only the proposed LM test rejects the null hypothesis, MS-GARCH model seems better than the GJR-GARCH and the GARCH models. However, other models should be considered like IGARCH or GAS models for example.

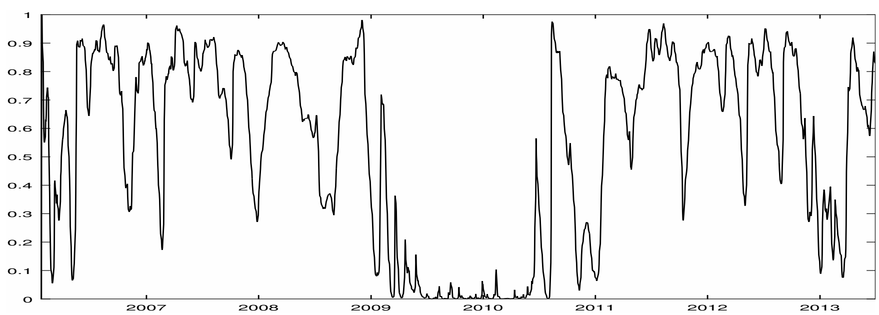

Figure 1: Smoothed inference probabilities for high volatility regime over time of the USD-CLP exchange rate returns.

	GARCH	GARCH-t	GJR	GJR-t	MS-t
$\alpha_0(1)$	0.012 (0.003)	0.006 (0.003)	0.015 (0.004)	0.008 (0.003)	0.005 (0.003)
$\alpha_1(1)$	0.069 (0.013)	0.066 (0.015)	0.024 (0.009)	0.039 (0.012)	0.098 (0.019)
$\gamma(1)$	0.909 (0.017)	0.924 (0.017)	0.887 (0.016)	0.915 (0.017)	0.904 (0.018)
ω	-	-	-0.122 (0.025)	-0.065 (0.023)	-
$\alpha_0(2)$	-	-	-	-	0.174 (0.09)
$\alpha_1(2)$	-	-	-	-	0.045 (0.038)
$\gamma(2)$	-	-	-	-	0.450 (0.273)
dof	-	5.26 (0.615)	-	5.55 (0.470)	5.513 (0.669)
P	-	-	-	-	$\begin{bmatrix} 0.998 & 0.003 \\ 0.002 & 0.997 \end{bmatrix}$
AIC	3.853.8	3686.8	3813.0	3677.0	3682.2
BIC	3870.5	3709.1	3835.3	3704.9	3732.3
LLF	-1923.9	-1839.4	-1902.5	-1833.5	-1832.1

Table 3 Estimation results for the USD-JPY exchange rate returns.
Note: the estimated models are considered with normal and Student distributions: "GARCH" and "GARCH-t" for GARCH(1,1), "GJR" and "GJR-t" for GJR and only "MS-t" for the Markov-Switching model. dof is the degrees of freedom of the Student distribution, P is the transition matrix for the Markov-Switching model, LLF is the value of the log-likelihood function. The parenthesis of each parameter correspond to the two regimes, a (-) means that the parameter is not estimated. We report the standard deviation between parentheses.

	GARCH	GARCH-t	GJR	GJR-t	MS-t
$\alpha_0(1)$	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)
$\alpha_1(1)$	0.038 (0.006)	0.039 (0.008)	0.021 (0.008)	0.022 (0.010)	0.041 (0.007)
$\gamma(1)$	0.960 (0.006)	0.958 (0.008)	0.964 (0.006)	0.962 (0.008)	0.957 (0.007)
ω	-	-	-0.025 (0.009)	-0.025 (0.011)	-
$\alpha_0(2)$	-	-	-	-	0.310 (0.036)
$\alpha_1(2)$	-	-	-	-	0.000 (0.078)
$\gamma(2)$	-	-	-	-	0.000 (0.067)
dof	-	16.27 (5.51)	-	16.94 (6.04)	17.50 (6.50)
P	-	-	-	-	$\begin{bmatrix} 0.997 & 0.011 \\ 0.003 & 0.989 \end{bmatrix}$
AIC	3531.4	3522.0	3527.6	3519.2	3540.4
BIC	3548.1	3544.3	3549.9	3547.1	3590.5
LLF	-1762.7	-1757.0	-1759.8	-1754.6	-1761.2

Table 4 Estimation results for the EUR-USD exchange rate returns.
Note: see note of Table 3

	GARCH	GARCH-t	GJR	GJR-t	MS-t
$\alpha_0(1)$	0.002 (0.001)	0.003 (0.001)	0.002 (0.001)	0.003 (0.001)	0.002 (0.001)
$\alpha_1(1)$	0.058 (0.008)	0.101 (0.015)	0.062 (0.004)	0.112 (0.019)	0.126 (0.024)
$\gamma(1)$	0.942 (0.008)	0.899 (0.013)	0.940 (0.004)	0.898 (0.013)	0.883 (0.020)
ω	-	-	-0.004 (0.009)	-0.021 (0.021)	-
$\alpha_0(2)$	-	-	-	-	0.084 (0.043)
$\alpha_1(2)$	-	-	-	-	0.078 (0.041)
$\gamma(2)$	-	-	-	-	0.730 (0.130)
dof	-	5.51 (0.599)	-	5.50 (0.598)	5.581 (0.956)
P	-	-	-	-	$\begin{bmatrix} 0.997 & 0.004 \\ 0.003 & 0.996 \end{bmatrix}$
AIC	3478.8	3293.8	3492.4	3294.8	3276.2
BIC	3495.5	3316.1	3514.7	3322.7	3326.3
LLF	-1736.4	-1642.9	-1742.2	-1642.4	-1629.1

Table 5 Estimation results for the USD-CLP exchange rate returns.
Note: see note of Table 3

References

Wang, J. and M. Yang (2009): "Asymmetric volatility in the foreign exchange markets," Journal of International Financial Markets, Institutions and Money, 19, 597–615.