

This document presents supplementary results and illustrations for the paper “Estimation of long memory in volatility using wavelets”, which were, due to their extent, omitted in the printed version.

1 FIEGARCH transformation

As discussed in Perez and Zaffaroni (2008) and Zaffaroni (2009), to estimate the FIEGARCH model using FWE, it is necessary to rewrite the model in a signal plus noise form, for which the spectral density can be derived. Let's begin with the original FIEGARCH(1,d,2) model:

$$\varepsilon_t = z_t h_t^{1/2} \quad (1)$$

$$\ln(h_t) = \omega + \Phi(L)g(z_{t-1}) \quad (2)$$

$$g(z_t) = \theta z_t + \gamma[|z_t| - E(|z_t|)] \quad (3)$$

$$\Phi(L) = (1-L)^{-d}[1 + \alpha_{[2]}(L)][\beta(L)]^{-1} \quad (4)$$

where for FIEGARCH(1,d,2) $\alpha_{[2]}(L) = \alpha L$, $\beta(L) = 1 - \beta L$. Following Zaffaroni (2009), this can be rewritten as

$$\varepsilon_t^2 = z_t^2 h_t \quad (5)$$

$$\ln(\varepsilon_t^2) = \ln(z_t^2) + \ln(h_t) \quad (6)$$

$$\ln(h_t) = \omega + \Phi(L)g(z_{t-1}) \quad (7)$$

$$\Phi(L)g(z_{t-1}) = \sum_{s=0}^{\infty} \Phi_s g(z_{t-s-1}) \quad (8)$$

$$g(z_t) = \theta z_t + \gamma[|z_t| - E(|z_t|)], \quad (9)$$

which leads to

$$\ln(\varepsilon_t^2) = \ln(z_t^2) + \omega + \sum_{s=0}^{\infty} \Phi_s g(z_{t-s-1}) \quad (10)$$

$$g(z_t) = \theta z_t + \gamma[|z_t| - E(|z_t|)] \quad (11)$$

$$\Phi(L) = (1-L)^{-d}[1 + \alpha L][1 - \beta L]^{-1}. \quad (12)$$

From Perez and Zaffaroni (2008), the spectral density of this process for $z_t \sim N(0, 1)$, $\omega = 0$ and simple Fourier frequencies $\lambda_j = j/T$; $\lambda_j \in \langle -1/2, 1/2 \rangle$ is

$$\begin{aligned} f(\lambda_j, v) &= \frac{A(\xi)}{2\pi} + \frac{B(v)}{2\pi} \left| \sum_{s=0}^{\infty} \Phi_s(\zeta) e^{2\pi i s \lambda_j} \right|^2 + \\ &+ \frac{C(v)}{2\pi} \left(e^{2\pi i \lambda_j} \sum_{s=0}^{\infty} \Phi_s(\zeta) e^{2\pi i s \lambda_j} \right) + \\ &+ \frac{C(v)}{2\pi} \left(e^{-2\pi i \lambda_j} \sum_{s=0}^{\infty} \Phi_s(\zeta) e^{-2\pi i s \lambda_j} \right) \end{aligned} \quad (13)$$

where $A(\xi) = \text{var}(\ln(z_0^2))$, $B(v) = \text{var}(g(z_0))$, $C(v) = \text{cov}(\ln(z_0^2), g(z_0))$, $v = (\xi', \zeta')'$, and ξ is the vector of parameters in the distribution function of variable z . For $z_t \sim N(0, 1)$:

$$A(\xi) = \Psi\left(\frac{1}{2}\right) \quad (14)$$

$$B(v) = \theta_2 + \delta^2(1 - \mu_{|z|}^2) \quad (15)$$

$$C(v) = \delta\mu_z(\psi(1) - \psi(\frac{1}{2})) \quad (16)$$

$$\mu_{|z|} = E(|z|) = \sqrt{\frac{2}{\pi}}, \quad (17)$$

where $\psi(x)$ and $\Psi(x)$ are digamma and trigamma functions respectively. Evaluated at Fourier frequencies, this spectral density occurs in both terms of the FWE objective function. For a generalization to z_t following GED or Student-t distribution, see Perez and Zaffaroni (2008).

2 Discrete wavelet transform

The core of a wavelet transform is a wavelet system, whose construction, together with means of the projection applied, determine the characteristics of the transformed data. For any $s \in \mathbb{R}$, a basic wavelet system can be defined as a set $\{\{\varphi_{j_0,k}(s)\}, \{\psi_{j,k}(s)\}; k \in \mathbb{Z}, j \in \mathbb{Z}\}$, creating an orthonormal basis in $L^2(\mathbb{R})$; which means that any function $f \in L^2(\mathbb{R})$ can be expressed as

$$f(s) = \sum_k \alpha_{j_0,k} \varphi_{j_0,k}(s) + \sum_{j=-\infty}^{j_0} \sum_k \beta_{j,k} \psi_{j,k}(s), \quad (18)$$

where $\alpha_k = \int f(s) \overline{\varphi_{j_0,k}(s)} ds$, and $\beta_{j,k} = \int f(s) \overline{\psi_{j,k}(s)} ds$, where the elements α_k , $\beta_{j,k}$, $\varphi(s)$ and $\psi(s)$ are called scaling coefficients, detail (wavelet) coefficients, scaling function (father wavelet) and wavelet function (mother wavelet) respectively, and the translated and dilated transformations of the mother wavelet are called daughter wavelets. With increasing j , these daughter wavelets get wider, with $j \leq 0$ they are narrower than the mother wavelet.

The basic conditions for $\psi(s)$ to be a valid mother wavelet are that $\int \psi(s) ds = 0$ and $\int \psi^2(s) ds = 1$, while the usual requirement is also the “admissibility” condition $\int \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty$, where $\hat{\psi}$ is the Fourier transform of ψ . This condition ensures that we can reconstruct the original time series from its transform. For

complete conditions on $\varphi(s)$, $\psi(s)$ to be valid father and mother wavelets in the context various subsets of $L^2(\mathbb{R})$ and for other details concerning construction of wavelet systems see Hardle, Kerkyacharian, Tsybakov, and Picard (1998). In addition, we provide examples in online appendix.

Next, any method that decomposes original data using the wavelet system and expresses them in terms of coefficients $\{\alpha_k, \beta_{j,k}\}$ and functions $\{\varphi(s), \psi(s)\}$ defined above, is a wavelet transform. In case of $j \in \mathbb{Z}$, as applied in our work, we speak about a discrete wavelet transform (DWT), while for $j \in \mathbb{R}$ the transform is continuous (CWT). By tradition, the default choice of scales is $\{2^{1-j}; j \in \mathbb{Z}\}$, thus the standard DWT can be defined in terms of the wavelet expansion (Eq. 18) with scaling defined as $s(j) = 2^{1-j}$ (i.e. “scale j ” refers to the scaling 2^{1-j} , scale 1 refers to $2^0 = 1$). The DWT coefficients are obtained for scales $j_0 = J$, $j_0 - 1 = J - 1$, $j_0 - 2 = J - 2, \dots$, $j_0 - (j_0 - 1) = 1$ using two-channel filter banks and down-sampling, so that at each level of decomposition j of a series of length M we get $M/2^j$ DWT coefficients, see e.g. Jensen (2000). These coefficients can be in turn used for decomposition of the variance σ^2 of the process x_t :

$$\sigma^2 = E(x_t^2) - [E(x_t)]^2 = \frac{\|x_t\|^2}{T} - [E(x_t)]^2 = \frac{\sum_{j=1}^J \|W_j\|^2 + \|V_J\|^2}{T} - [E(x_t)]^2, \quad (19)$$

where W_j ; $j = 1, \dots, J$ and V_J are vectors of wavelet and scaling coefficients respectively and the $[E(x_t)]^2$ can be estimated using the squared sample mean \bar{x}_t^2 , or using the true squared mean whenever known. The spectral density $f(\lambda, \zeta)$ of x_t can be estimated using the relationship:

$$\frac{\|W_j\|^2}{T} = \frac{\sigma_{W,j}^2}{2^j} \approx 2 \int_{1/2^{j+1}}^{1/2^j} f(\lambda, \zeta) d\lambda \quad (20)$$

$$\frac{\|V_J\|^2}{T} - \bar{x}_t^2 = \frac{\sigma_{V,J}^2}{2^J} - \left(1 - \frac{1}{2^J}\right) \bar{x}_t^2 \approx 2 \int_0^{1/2^{J+1}} f(\lambda, \zeta) d\lambda, \quad (21)$$

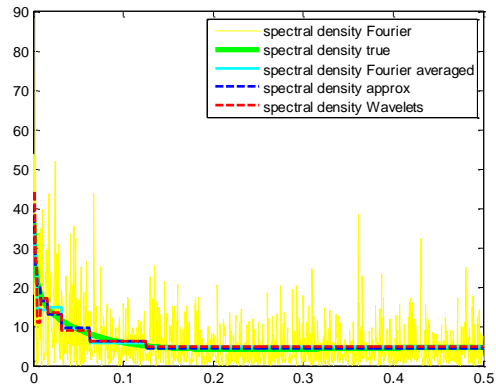
where $\sigma_{W,j}^2$ and $\sigma_{V,J}^2$ are the sample variances of the wavelet and scaling coefficients respectively for $j = 1, 2, \dots, J$.

References

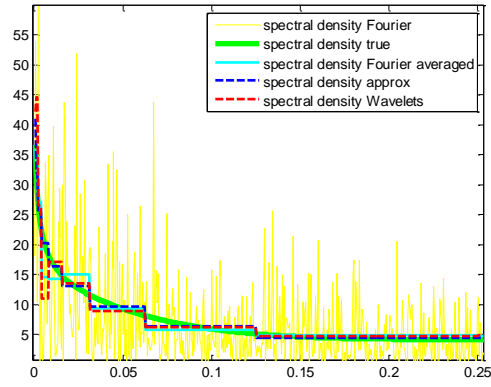
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- Perez, A. and P. Zaffaroni (2008): “Finite-sample properties of maximum likelihood and whittle estimators in egarch and fiegarch models,” *Quantitative and Qualitative Analysis in Social Sciences*, 2, 78–97.
- Zaffaroni, P. (2009): “Whittle estimation of egarch and other exponential volatility models,” *Journal of Econometrics*, 151, 190–200, URL <http://ideas.repec.org/a/eee/econom/v151y2009i2p190-200.html>.

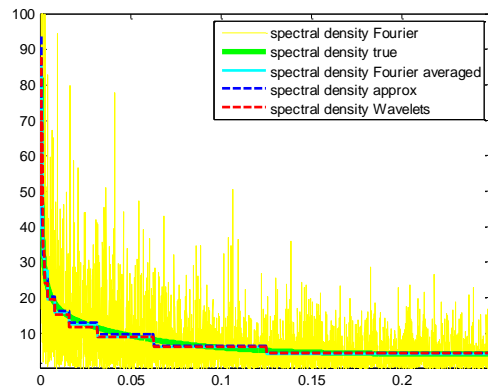
3 Spectral density estimation II



(a) $T=2048$ (2^{11}), level=10

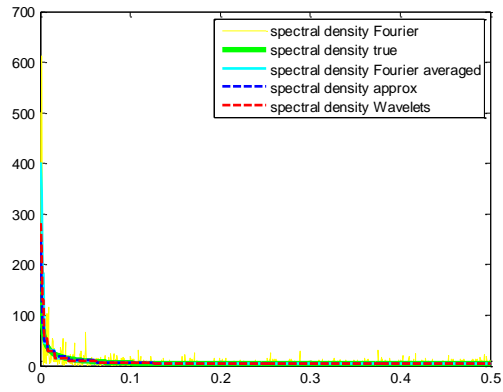


(b) $T=2048$ (2^{11}), level=10, zoom

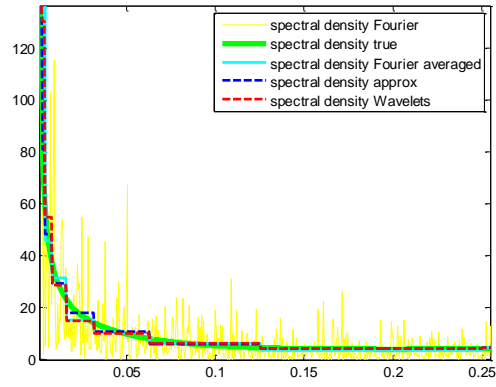


(c) $T=16384$ (2^{14}), level=13, zoom

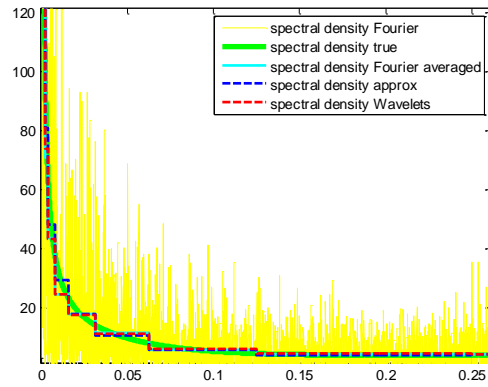
Figure 1: Spectral density estimation ($d=0.25$)



(a) $T=2048$ (2^{11}), level=10

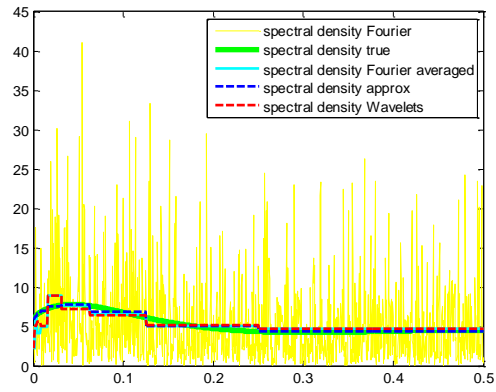


(b) $T=2048$ (2^{11}), level=10, zoom

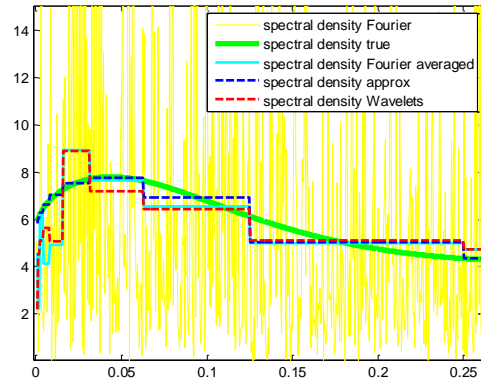


(c) $T=16384$ (2^{14}), level=13, zoom

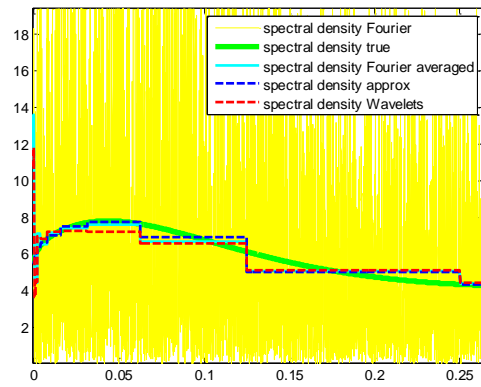
Figure 2: Spectral density estimation ($d=0.45$)



(a) $T=2048$ (2^{11}), level=10



(b) $T=2048$ (2^{11}), level=10, zoom



(c) $T=16384$ (2^{14}), level=13, zoom

Figure 3: Spectral density estimation ($d=-0.25$)

4 Wavelets and Wavelet coefficients analysis

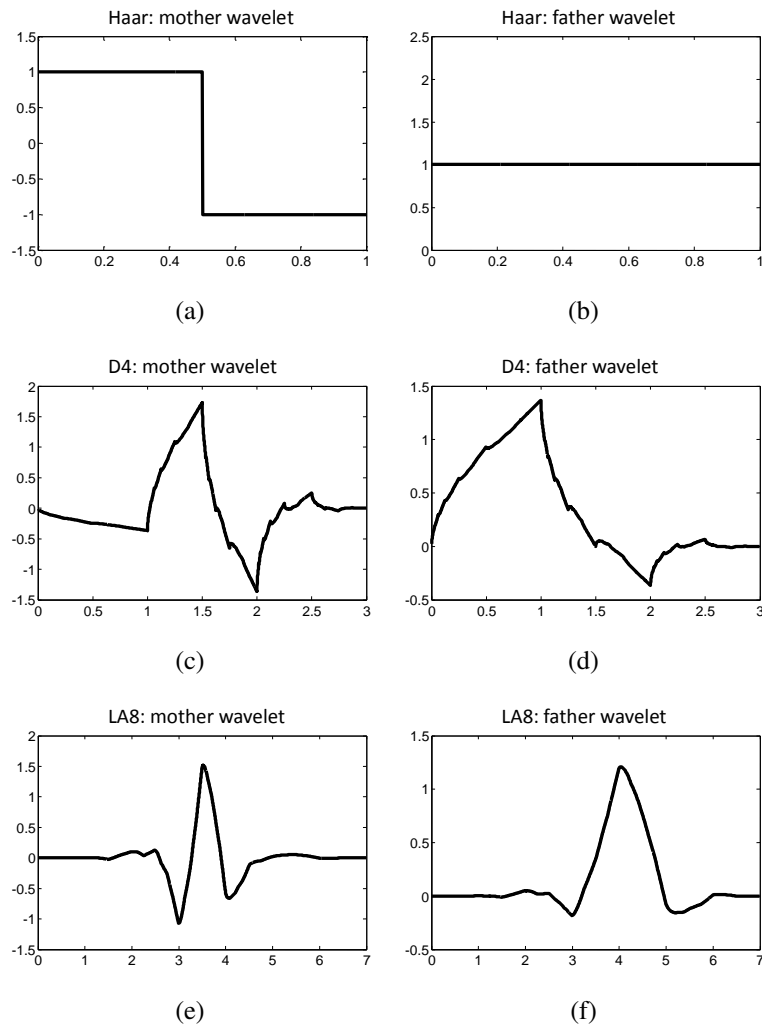
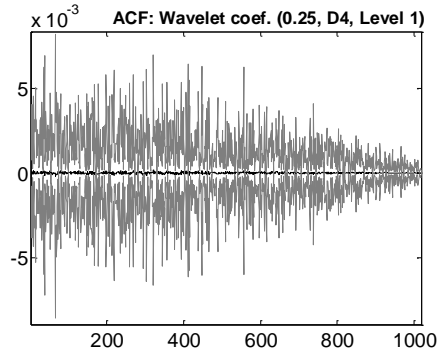
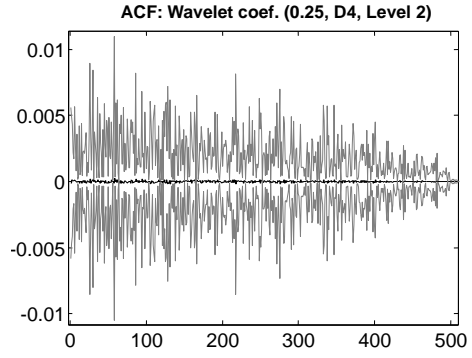


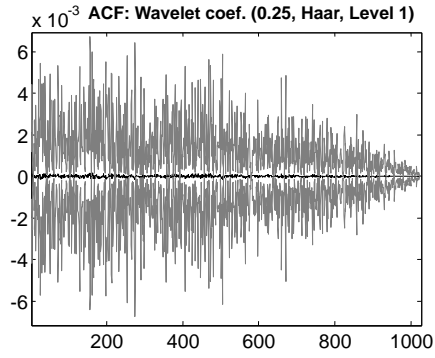
Figure 4: Wavelets: Haar, D4, LA8



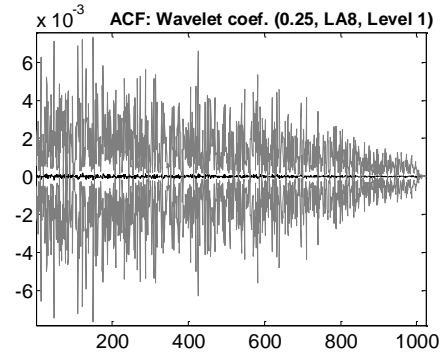
(a)



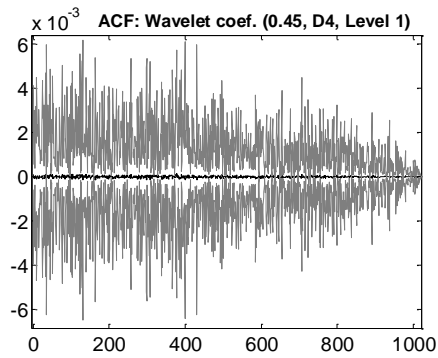
(b)



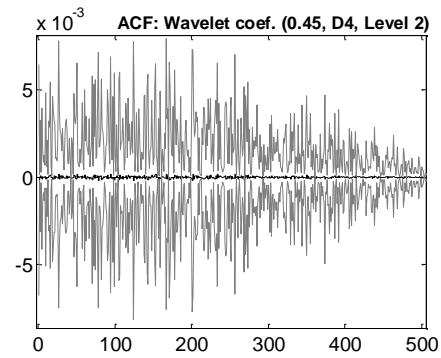
(c)



(d)

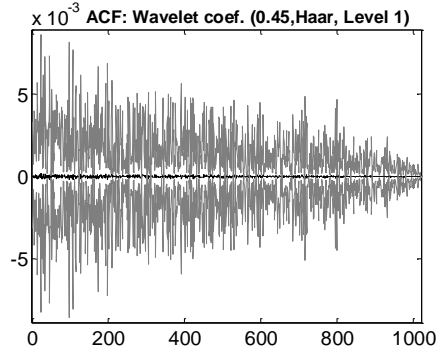


(e)

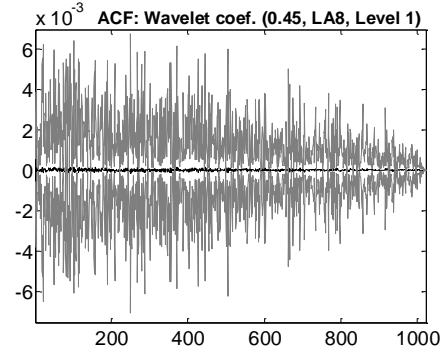


(f)

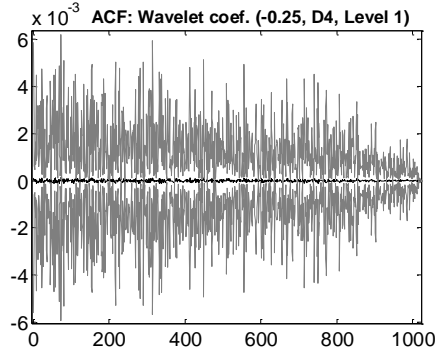
Figure 5: Estimates of autocorrelation functions (ACFs) of wavelet coefficients respective to FIEGARCH processes for $T = 2^{11}$; $d = 0.25$, $d = 0.45$ and filters Haar, D4 and LA8; both sample mean and 95% confidence intervals based on 500 FIEGARCH simulations are provided for each lag available.



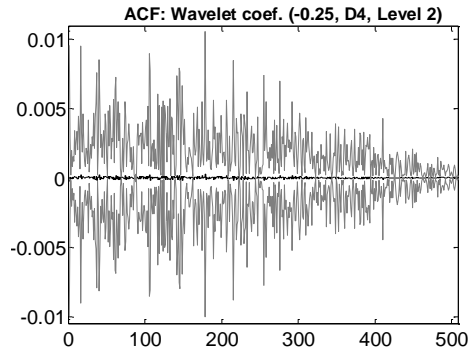
(a)



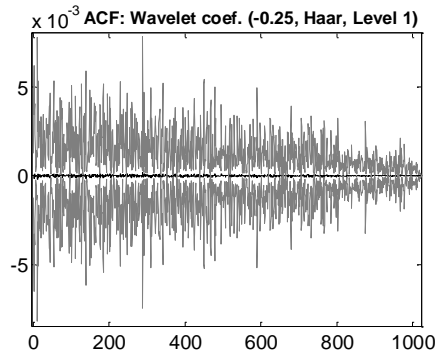
(b)



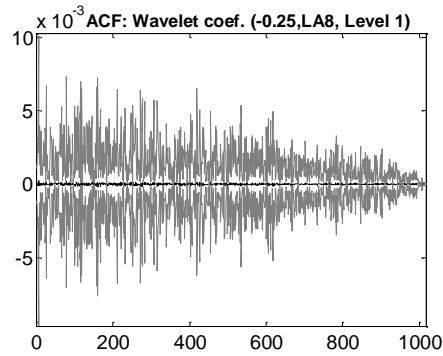
(c)



(d)



(e)



(f)

Figure 6: Estimates of autocorrelation functions (ACFs) of wavelet coefficients respective to FIEGARCH processes for $T = 2^{11}$; $d = 0.45$, $d = -0.25$ and filters Haar, D4 and LA8; both sample mean and 95% confidence intervals based on 500 FIEGARCH simulations are provided for each lag available.

5 Results: Maximal Decomposition

Simulation:

- FIEGARCH(1,d,2)
- Truncation of infinite ARCH representation at lag = 1000
- Discarding first 1000 simulated numbers
- Number of simulations after truncation = 1000
- Sample lengths: $2^9, 2^{10}, 2^{11}, 2^{12}, 2^{13}, 2^{14}$
- Parameters: $(\omega_1, \alpha_1, \alpha_2, \beta_1, \theta, \gamma) = (0, 1, 0.5, 0.5, -0.3, 0.5)$, $d = 0.25, 0.45, -0.25$
- Estimators: MLE, FWE, WWE(Haar), WWE(D4), WWE(LA8)
- Levels of decomposition: Maximal (depending on filter support)

PAR	TRUE	METHOD	N=2 ⁹ (512)				N=2 ¹⁰ (1024)				N=2 ¹¹ (2048)			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	0.250	WWE1	0.109	-0.141	0.309		0.146	-0.104	0.270		0.202	-0.048	0.183	
		WWE2	0.132	-0.118	0.309		0.138	-0.112	0.287		0.171	-0.079	0.232	
		WWE3	0.176	-0.074	0.306		0.135	-0.115	0.298		0.145	-0.105	0.263	
		FWE	0.127	-0.123	0.284		0.172	-0.078	0.224		0.215	-0.035	0.147	
		MLE	0.174	-0.076	0.183		0.221	-0.029	0.111		0.232	-0.018	0.081	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.393	-0.107	0.557		0.419	-0.081	0.522		0.406	-0.094	0.504	
		WWE2	0.434	-0.066	0.580		0.502	0.002	0.499		0.500	-0.000	0.475	
		WWE3	0.439	-0.061	0.584		0.531	0.031	0.508		0.572	0.072	0.426	
		FWE	0.570	0.070	0.460		0.611	0.111	0.402		0.592	0.092	0.348	
		MLE	0.536	0.036	0.244		0.529	0.029	0.174		0.516	0.016	0.123	
$\hat{\beta}_1$	0.500	WWE1	0.542	0.042	0.392		0.529	0.029	0.347		0.509	0.009	0.285	
		WWE2	0.548	0.048	0.371		0.532	0.032	0.329		0.530	0.030	0.288	
		WWE3	0.526	0.026	0.377		0.553	0.053	0.313		0.539	0.039	0.278	
		FWE	0.523	0.023	0.347		0.512	0.012	0.277		0.501	0.001	0.203	
		MLE	0.549	0.049	0.205		0.516	0.016	0.139		0.515	0.015	0.097	
$\hat{\theta}$	-0.300	WWE1	-0.319	-0.019	0.318		-0.310	-0.010	0.250		-0.332	-0.032	0.197	
		WWE2	-0.272	0.028	0.305		-0.260	0.040	0.245		-0.278	0.022	0.195	
		WWE3	-0.256	0.044	0.307		-0.240	0.060	0.236		-0.252	0.048	0.194	
		FWE	-0.258	0.042	0.289		-0.224	0.076	0.221		-0.237	0.063	0.178	
		MLE	-0.308	-0.008	0.054		-0.302	-0.002	0.037		-0.300	-0.000	0.026	
$\hat{\gamma}$	0.500	WWE1	0.463	-0.037	0.354		0.467	-0.033	0.259		0.458	-0.042	0.209	
		WWE2	0.521	0.021	0.357		0.515	0.015	0.246		0.496	-0.004	0.191	
		WWE3	0.526	0.026	0.363		0.512	0.012	0.240		0.499	-0.001	0.180	
		FWE	0.490	-0.010	0.301		0.502	0.002	0.214		0.498	-0.002	0.172	
		MLE	0.499	-0.001	0.088		0.500	0.000	0.063		0.501	0.001	0.044	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 1: Monte Carlo 1a: d=0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ¹² (4096)				N=2 ¹³ (8192)				N=2 ¹⁴ (16384)			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	0.250	WWE1	0.242	-0.008	0.103		0.252	0.002	0.058		0.257	0.007	0.041	
		WWE2	0.216	-0.034	0.148		0.244	-0.006	0.070		0.255	0.005	0.044	
		WWE3	0.199	-0.051	0.186		0.237	-0.013	0.083		0.250	-0.000	0.047	
		FWE	0.240	-0.010	0.086		0.248	-0.002	0.049		0.251	0.001	0.035	
		MLE	0.244	-0.006	0.041		-	-	-		-	-	-	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.350	-0.150	0.466		0.278	-0.222	0.411		0.240	-0.260	0.358	
		WWE2	0.444	-0.056	0.411		0.392	-0.108	0.356		0.358	-0.142	0.291	
		WWE3	0.534	0.034	0.366		0.462	-0.038	0.296		0.441	-0.059	0.241	
		FWE	0.559	0.059	0.288		0.533	0.033	0.230		0.519	0.019	0.165	
		MLE	0.503	0.003	0.086		-	-	-		-	-	-	
$\hat{\beta}_1$	0.500	WWE1	0.495	-0.005	0.226		0.512	0.012	0.161		0.518	0.018	0.110	
		WWE2	0.522	0.022	0.212		0.517	0.017	0.147		0.516	0.016	0.103	
		WWE3	0.509	0.009	0.234		0.513	0.013	0.146		0.507	0.007	0.104	
		FWE	0.495	-0.005	0.136		0.497	-0.003	0.094		0.496	-0.004	0.066	
		MLE	0.505	0.005	0.058		-	-	-		-	-	-	
$\hat{\theta}$	-0.300	WWE1	-0.348	-0.048	0.165		-0.384	-0.084	0.130		-0.396	-0.096	0.118	
		WWE2	-0.286	0.014	0.158		-0.321	-0.021	0.108		-0.334	-0.034	0.079	
		WWE3	-0.260	0.040	0.157		-0.292	0.008	0.107		-0.305	-0.005	0.073	
		FWE	-0.242	0.058	0.152		-0.268	0.032	0.105		-0.279	0.021	0.074	
		MLE	-0.301	-0.001	0.018		-	-	-		-	-	-	
$\hat{\gamma}$	0.500	WWE1	0.460	-0.040	0.160		0.455	-0.045	0.123		0.454	-0.046	0.098	
		WWE2	0.496	-0.004	0.158		0.491	-0.009	0.118		0.490	-0.010	0.091	
		WWE3	0.502	0.002	0.148		0.500	-0.000	0.112		0.500	0.000	0.086	
		FWE	0.503	0.003	0.139		0.502	0.002	0.107		0.504	0.004	0.082	
		MLE	0.501	0.001	0.029		-	-	-		-	-	-	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 2: Monte Carlo 1b: d=0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ⁹ (512)				N=2 ¹⁰ (1024)				N=2 ¹¹ (2048)			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	0.450	WWE1	0.272	-0.178	0.301		0.343	-0.107	0.225		0.411	-0.039	0.120	
		WWE2	0.243	-0.207	0.322		0.294	-0.156	0.271		0.367	-0.083	0.191	
		WWE3	0.262	-0.188	0.310		0.281	-0.169	0.280		0.322	-0.128	0.246	
		FWE	0.316	-0.134	0.254		0.360	-0.090	0.201		0.425	-0.025	0.103	
		MLE	0.389	-0.061	0.152		0.424	-0.026	0.085		0.446	-0.004	0.042	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.341	-0.159	0.593		0.341	-0.159	0.573		0.287	-0.213	0.537	
		WWE2	0.414	-0.086	0.583		0.431	-0.069	0.528		0.413	-0.087	0.484	
		WWE3	0.434	-0.066	0.571		0.478	-0.022	0.513		0.494	-0.006	0.419	
		FWE	0.510	0.010	0.492		0.525	0.025	0.438		0.523	0.023	0.366	
		MLE	0.521	0.021	0.257		0.500	0.000	0.185		0.496	-0.004	0.124	
$\hat{\beta}_1$	0.500	WWE1	0.618	0.118	0.330		0.579	0.079	0.277		0.539	0.039	0.211	
		WWE2	0.628	0.128	0.323		0.622	0.122	0.267		0.574	0.074	0.214	
		WWE3	0.631	0.131	0.311		0.631	0.131	0.262		0.599	0.099	0.223	
		FWE	0.595	0.095	0.281		0.582	0.082	0.219		0.529	0.029	0.147	
		MLE	0.552	0.052	0.161		0.529	0.029	0.104		0.508	0.008	0.066	
$\hat{\theta}$	-0.300	WWE1	-0.342	-0.042	0.280		-0.351	-0.051	0.231		-0.386	-0.086	0.183	
		WWE2	-0.277	0.023	0.271		-0.276	0.024	0.220		-0.298	0.002	0.170	
		WWE3	-0.243	0.057	0.266		-0.245	0.055	0.219		-0.260	0.040	0.161	
		FWE	-0.247	0.053	0.245		-0.233	0.067	0.194		-0.245	0.055	0.147	
		MLE	-0.304	-0.004	0.050		-0.304	-0.004	0.036		-0.302	-0.002	0.024	
$\hat{\gamma}$	0.500	WWE1	0.498	-0.002	0.367		0.499	-0.001	0.291		0.501	0.001	0.226	
		WWE2	0.538	0.038	0.355		0.525	0.025	0.279		0.531	0.031	0.218	
		WWE3	0.535	0.035	0.340		0.524	0.024	0.259		0.524	0.024	0.203	
		FWE	0.511	0.011	0.315		0.515	0.015	0.245		0.521	0.021	0.190	
		MLE	0.514	0.014	0.090		0.513	0.013	0.064		0.510	0.010	0.042	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 3: Monte Carlo 2a: d=0.45; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ¹² (4096)				N=2 ¹³ (8192)				N=2 ¹⁴ (16384)			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	0.450	WWE1	0.439	-0.011	0.064		0.450	0.000	0.039		0.456	0.006	0.028	
		WWE2	0.422	-0.028	0.097		0.449	-0.001	0.043		0.458	0.008	0.031	
		WWE3	0.401	-0.049	0.137		0.437	-0.013	0.058		0.451	0.001	0.033	
		FWE	0.445	-0.005	0.050		0.454	0.004	0.034		0.456	0.006	0.026	
		MLE	0.450	0.000	0.031		-	-	-		-	-	-	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.274	-0.226	0.476		0.214	-0.286	0.423		0.211	-0.289	0.366	
		WWE2	0.374	-0.126	0.425		0.363	-0.137	0.352		0.340	-0.160	0.297	
		WWE3	0.452	-0.048	0.373		0.431	-0.069	0.304		0.421	-0.079	0.246	
		FWE	0.543	0.043	0.309		0.537	0.037	0.244		0.525	0.025	0.182	
		MLE	0.498	-0.002	0.091		-	-	-		-	-	-	
$\hat{\beta}_1$	0.500	WWE1	0.515	0.015	0.158		0.513	0.013	0.117		0.502	0.002	0.087	
		WWE2	0.539	0.039	0.153		0.513	0.013	0.108		0.504	0.004	0.081	
		WWE3	0.549	0.049	0.161		0.524	0.024	0.106		0.508	0.008	0.078	
		FWE	0.508	0.008	0.098		0.495	-0.005	0.075		0.492	-0.008	0.056	
		MLE	0.502	0.002	0.047		-	-	-		-	-	-	
$\hat{\theta}$	-0.300	WWE1	-0.410	-0.110	0.154		-0.432	-0.132	0.150		-0.437	-0.137	0.146	
		WWE2	-0.327	-0.027	0.120		-0.346	-0.046	0.086		-0.355	-0.055	0.076	
		WWE3	-0.290	0.010	0.117		-0.309	-0.009	0.072		-0.319	-0.019	0.051	
		FWE	-0.261	0.039	0.109		-0.278	0.022	0.071		-0.288	0.012	0.043	
		MLE	-0.301	-0.001	0.017		-	-	-		-	-	-	
$\hat{\gamma}$	0.500	WWE1	0.484	-0.016	0.178		0.479	-0.021	0.129		0.476	-0.024	0.096	
		WWE2	0.513	0.013	0.171		0.506	0.006	0.128		0.504	0.004	0.096	
		WWE3	0.512	0.012	0.164		0.508	0.008	0.124		0.507	0.007	0.092	
		FWE	0.508	0.008	0.154		0.505	0.005	0.116		0.504	0.004	0.088	
		MLE	0.506	0.006	0.030		-	-	-		-	-	-	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 4: Monte Carlo 2b: d=0.45; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ⁹ (512)			N=2 ¹⁰ (1024)			N=2 ¹¹ (2048)		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	-0.250	WWE1	-0.213	0.037	0.321	-0.219	0.031	0.285	-0.243	0.007	0.246
		WWE2	-0.153	0.097	0.366	-0.222	0.028	0.301	-0.255	-0.005	0.247
		WWE3	-0.077	0.173	0.422	-0.170	0.080	0.345	-0.275	-0.025	0.250
		FWE	-0.249	0.001	0.263	-0.259	-0.009	0.233	-0.275	-0.025	0.202
		MLE	-0.115	0.135	0.200	-0.141	0.109	0.158	-0.165	0.085	0.124
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.423	-0.077	0.595	0.432	-0.068	0.588	0.449	-0.051	0.541
		WWE2	0.426	-0.074	0.607	0.521	0.021	0.540	0.523	0.023	0.503
		WWE3	0.396	-0.104	0.622	0.474	-0.026	0.574	0.548	0.048	0.492
		FWE	0.563	0.063	0.474	0.597	0.097	0.403	0.609	0.109	0.329
		MLE	0.550	0.050	0.248	0.539	0.039	0.173	0.520	0.020	0.118
$\hat{\beta}_1$	0.500	WWE1	0.472	-0.028	0.425	0.492	-0.008	0.394	0.513	0.013	0.363
		WWE2	0.424	-0.076	0.436	0.455	-0.045	0.407	0.519	0.019	0.322
		WWE3	0.396	-0.104	0.438	0.437	-0.063	0.406	0.528	0.028	0.313
		FWE	0.430	-0.070	0.404	0.435	-0.065	0.365	0.469	-0.031	0.297
		MLE	0.327	-0.173	0.291	0.356	-0.144	0.224	0.400	-0.100	0.158
$\hat{\theta}$	-0.300	WWE1	-0.267	0.033	0.320	-0.268	0.032	0.270	-0.272	0.028	0.236
		WWE2	-0.241	0.059	0.315	-0.233	0.067	0.265	-0.240	0.060	0.237
		WWE3	-0.236	0.064	0.315	-0.231	0.069	0.264	-0.231	0.069	0.231
		FWE	-0.255	0.045	0.308	-0.247	0.053	0.259	-0.228	0.072	0.224
		MLE	-0.297	0.003	0.059	-0.298	0.002	0.040	-0.299	0.001	0.029
$\hat{\gamma}$	0.500	WWE1	0.418	-0.082	0.295	0.398	-0.102	0.236	0.399	-0.101	0.184
		WWE2	0.496	-0.004	0.317	0.466	-0.034	0.241	0.449	-0.051	0.178
		WWE3	0.540	0.040	0.335	0.504	0.004	0.248	0.471	-0.029	0.163
		FWE	0.501	0.001	0.244	0.489	-0.011	0.172	0.485	-0.015	0.133
		MLE	0.488	-0.012	0.098	0.487	-0.013	0.068	0.493	-0.007	0.049
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 5: Monte Carlo 3a: d=-0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ¹² (4096)				N=2 ¹³ (8192)				N=2 ¹⁴ (16384)			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	-0.250	WWE1	-0.246	0.004	0.212		-0.232	0.018	0.176		-0.202	0.048	0.147	
		WWE2	-0.274	-0.024	0.193		-0.269	-0.019	0.164		-0.253	-0.003	0.137	
		WWE3	-0.284	-0.034	0.197		-0.280	-0.030	0.166		-0.267	-0.017	0.133	
		FWE	-0.272	-0.022	0.168		-0.277	-0.027	0.144		-0.269	-0.019	0.119	
		MLE	-0.183	0.067	0.095		-	-	-		-	-	-	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.465	-0.035	0.481		0.441	-0.059	0.419		0.430	-0.070	0.370	
		WWE2	0.527	0.027	0.440		0.528	0.028	0.368		0.478	-0.022	0.311	
		WWE3	0.576	0.076	0.410		0.565	0.065	0.359		0.546	0.046	0.292	
		FWE	0.586	0.086	0.256		0.559	0.059	0.189		0.530	0.030	0.134	
		MLE	0.512	0.012	0.077		-	-	-		-	-	-	
$\hat{\beta}_1$	0.500	WWE1	0.522	0.022	0.303		0.507	0.007	0.268		0.486	-0.014	0.224	
		WWE2	0.528	0.028	0.284		0.521	0.021	0.236		0.526	0.026	0.195	
		WWE3	0.514	0.014	0.268		0.515	0.015	0.223		0.508	0.008	0.187	
		FWE	0.479	-0.021	0.240		0.499	-0.001	0.168		0.504	0.004	0.133	
		MLE	0.427	-0.073	0.115		-	-	-		-	-	-	
$\hat{\theta}$	-0.300	WWE1	-0.302	-0.002	0.196		-0.325	-0.025	0.161		-0.345	-0.045	0.124	
		WWE2	-0.269	0.031	0.199		-0.283	0.017	0.162		-0.307	-0.007	0.123	
		WWE3	-0.254	0.046	0.201		-0.263	0.037	0.170		-0.282	0.018	0.126	
		FWE	-0.252	0.048	0.193		-0.257	0.043	0.164		-0.274	0.026	0.125	
		MLE	-0.299	0.001	0.021		-	-	-		-	-	-	
$\hat{\gamma}$	0.500	WWE1	0.393	-0.107	0.161		0.395	-0.105	0.137		0.392	-0.108	0.126	
		WWE2	0.449	-0.051	0.134		0.446	-0.054	0.106		0.446	-0.054	0.089	
		WWE3	0.472	-0.028	0.117		0.472	-0.028	0.090		0.472	-0.028	0.071	
		FWE	0.487	-0.013	0.101		0.493	-0.007	0.078		0.496	-0.004	0.057	
		MLE	0.499	-0.001	0.033		-	-	-		-	-	-	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 6: Monte Carlo 3b: d=-0.25; MLE, FWE, Haar, D4, LA8

6 Partial Decomposition II

PAR	TRUE	METHOD	N=2 ⁹ (512); level 4			N=2 ¹⁰ (1024); level 4			N=2 ¹¹ (2048); level 4		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.250	WWE1	0.268	0.018	0.279	0.270	0.020	0.245	0.275	0.025	0.210
		WWE2	0.293	0.043	0.267	0.304	0.054	0.228	0.309	0.059	0.196
		WWE3	0.306	0.056	0.252	0.301	0.051	0.226	0.307	0.057	0.189
		FWE	0.131	-0.119	0.287	0.163	-0.087	0.236	0.214	-0.036	0.145
		MLE	0.177	-0.073	0.176	0.212	-0.038	0.124	0.236	-0.014	0.065
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.289	-0.211	0.649	0.334	-0.166	0.610	0.302	-0.198	0.603
		WWE2	0.328	-0.172	0.637	0.351	-0.149	0.620	0.386	-0.114	0.568
		WWE3	0.360	-0.140	0.636	0.409	-0.091	0.592	0.431	-0.069	0.523
		FWE	0.572	0.072	0.471	0.564	0.064	0.401	0.571	0.071	0.348
		MLE	0.552	0.052	0.251	0.509	0.009	0.187	0.507	0.007	0.127
$\hat{\beta}_1$	0.500	WWE1	0.525	0.025	0.420	0.504	0.004	0.365	0.528	0.028	0.279
		WWE2	0.521	0.021	0.417	0.522	0.022	0.344	0.504	0.004	0.266
		WWE3	0.530	0.030	0.411	0.524	0.024	0.320	0.505	0.005	0.252
		FWE	0.525	0.025	0.350	0.522	0.022	0.292	0.512	0.012	0.199
		MLE	0.547	0.047	0.198	0.530	0.030	0.141	0.511	0.011	0.089
$\hat{\theta}$	-0.300	WWE1	-0.307	-0.007	0.299	-0.326	-0.026	0.273	-0.337	-0.037	0.233
		WWE2	-0.270	0.030	0.293	-0.279	0.021	0.260	-0.285	0.015	0.217
		WWE3	-0.252	0.048	0.284	-0.261	0.039	0.260	-0.263	0.037	0.210
		FWE	-0.236	0.064	0.272	-0.229	0.071	0.226	-0.225	0.075	0.179
		MLE	-0.302	-0.002	0.053	-0.305	-0.005	0.038	-0.302	-0.002	0.025
$\hat{\gamma}$	0.500	WWE1	0.539	0.039	0.420	0.515	0.015	0.314	0.487	-0.013	0.229
		WWE2	0.612	0.112	0.434	0.565	0.065	0.340	0.527	0.027	0.231
		WWE3	0.613	0.113	0.444	0.567	0.067	0.327	0.533	0.033	0.228
		FWE	0.514	0.014	0.269	0.513	0.013	0.214	0.505	0.005	0.170
		MLE	0.496	-0.004	0.089	0.503	0.003	0.061	0.502	0.002	0.042
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 7: Monte Carlo 2a (Level 4): d=0.25; MLE, FWE, Haar, D4, LA8

PAR	TRUE	METHOD	N=2 ¹² (4096); level 4			N=2 ¹³ (8192); level 4			N=2 ¹⁴ (16384); level 4		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.250	WWE1	0.268	0.018	0.182	0.275	0.025	0.159	0.296	0.046	0.136
		WWE2	0.290	0.040	0.163	0.281	0.031	0.142	0.276	0.026	0.132
		WWE3	0.295	0.045	0.160	0.270	0.020	0.145	0.274	0.024	0.126
		FWE	0.235	-0.015	0.091	0.251	0.001	0.050	0.250	0.000	0.035
		MLE	0.241	-0.009	0.046	-	-	-	-	-	-
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.348	-0.152	0.495	0.310	-0.190	0.420	0.290	-0.210	0.351
		WWE2	0.397	-0.103	0.482	0.393	-0.107	0.375	0.361	-0.139	0.313
		WWE3	0.463	-0.037	0.429	0.469	-0.031	0.342	0.431	-0.069	0.260
		FWE	0.555	0.055	0.287	0.547	0.047	0.228	0.516	0.016	0.168
		MLE	0.502	0.002	0.087	-	-	-	-	-	-
$\hat{\beta}_1$	0.500	WWE1	0.489	-0.011	0.221	0.480	-0.020	0.165	0.463	-0.037	0.135
		WWE2	0.501	0.001	0.202	0.485	-0.015	0.152	0.492	-0.008	0.127
		WWE3	0.484	-0.016	0.195	0.483	-0.017	0.144	0.487	-0.013	0.111
		FWE	0.505	0.005	0.130	0.488	-0.012	0.097	0.498	-0.002	0.064
		MLE	0.509	0.009	0.063	-	-	-	-	-	-
$\hat{\theta}$	-0.300	WWE1	-0.357	-0.057	0.181	-0.383	-0.083	0.142	-0.393	-0.093	0.125
		WWE2	-0.311	-0.011	0.173	-0.327	-0.027	0.121	-0.337	-0.037	0.099
		WWE3	-0.285	0.015	0.165	-0.297	0.003	0.114	-0.309	-0.009	0.089
		FWE	-0.246	0.054	0.147	-0.269	0.031	0.101	-0.279	0.021	0.076
		MLE	-0.301	-0.001	0.018	-	-	-	-	-	-
$\hat{\gamma}$	0.500	WWE1	0.461	-0.039	0.171	0.453	-0.047	0.128	0.454	-0.046	0.101
		WWE2	0.505	0.005	0.168	0.492	-0.008	0.122	0.494	-0.006	0.092
		WWE3	0.511	0.011	0.163	0.503	0.003	0.118	0.504	0.004	0.089
		FWE	0.500	0.000	0.139	0.501	0.001	0.107	0.504	0.004	0.083
		MLE	0.501	0.001	0.030	-	-	-	-	-	-
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 8: Monte Carlo 2b (Level 4): d=0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ⁹ (512); level 5				N=2 ¹⁰ (1024); level 5				N=2 ¹¹ (2048); level 5			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	0.250	WWE1	0.184	-0.066	0.299		0.206	-0.044	0.276		0.190	-0.060	0.258	
		WWE2	0.215	-0.035	0.289		0.214	-0.036	0.261		0.203	-0.047	0.245	
		WWE3	0.221	-0.029	0.287		0.212	-0.038	0.260		0.213	-0.037	0.243	
		FWE	0.127	-0.123	0.284		0.172	-0.078	0.224		0.212	-0.038	0.156	
		MLE	0.174	-0.076	0.183		0.221	-0.029	0.111		0.232	-0.018	0.072	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.337	-0.163	0.627		0.399	-0.101	0.597		0.455	-0.045	0.529	
		WWE2	0.375	-0.125	0.629		0.450	-0.050	0.563		0.495	-0.005	0.494	
		WWE3	0.422	-0.078	0.590		0.484	-0.016	0.532		0.557	0.057	0.459	
		FWE	0.570	0.070	0.460		0.611	0.111	0.402		0.583	0.083	0.341	
		MLE	0.536	0.036	0.244		0.529	0.029	0.174		0.507	0.007	0.123	
$\hat{\beta}_1$	0.500	WWE1	0.528	0.028	0.396		0.498	-0.002	0.365		0.494	-0.006	0.323	
		WWE2	0.516	0.016	0.374		0.508	0.008	0.327		0.500	-0.000	0.298	
		WWE3	0.506	0.006	0.385		0.514	0.014	0.313		0.487	-0.013	0.282	
		FWE	0.523	0.023	0.347		0.512	0.012	0.277		0.502	0.002	0.202	
		MLE	0.549	0.049	0.205		0.516	0.016	0.139		0.515	0.015	0.092	
$\hat{\theta}$	-0.300	WWE1	-0.308	-0.008	0.316		-0.306	-0.006	0.253		-0.313	-0.013	0.198	
		WWE2	-0.269	0.031	0.308		-0.257	0.043	0.244		-0.265	0.035	0.199	
		WWE3	-0.251	0.049	0.301		-0.238	0.062	0.238		-0.238	0.062	0.192	
		FWE	-0.258	0.042	0.289		-0.224	0.076	0.221		-0.230	0.070	0.180	
		MLE	-0.308	-0.008	0.054		-0.302	-0.002	0.037		-0.301	-0.001	0.025	
$\hat{\gamma}$	0.500	WWE1	0.487	-0.013	0.379		0.480	-0.020	0.275		0.465	-0.035	0.208	
		WWE2	0.548	0.048	0.386		0.526	0.026	0.271		0.508	0.008	0.200	
		WWE3	0.540	0.040	0.380		0.530	0.030	0.260		0.516	0.016	0.194	
		FWE	0.490	-0.010	0.301		0.502	0.002	0.214		0.505	0.005	0.167	
		MLE	0.499	-0.001	0.088		0.500	0.000	0.063		0.502	0.002	0.042	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 9: Monte Carlo 2c (Level 5): d=0.25; MLE, FWE, Haar, D4, LA8

PAR	TRUE	METHOD	N=2 ¹² (4096); level 5			N=2 ¹³ (8192); level 5			N=2 ¹⁴ (16384); level 5		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.250	WWE1	0.210	-0.040	0.240	0.219	-0.031	0.229	0.236	-0.014	0.204
		WWE2	0.213	-0.037	0.229	0.208	-0.042	0.229	0.224	-0.026	0.212
		WWE3	0.211	-0.039	0.234	0.208	-0.042	0.228	0.216	-0.034	0.205
		FWE	0.240	-0.010	0.086	0.250	0.000	0.049	0.251	0.001	0.035
		MLE	0.244	-0.006	0.041	-	-	-	-	-	-
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.472	-0.028	0.451	0.453	-0.047	0.377	0.399	-0.101	0.337
		WWE2	0.510	0.010	0.430	0.509	0.009	0.365	0.483	-0.017	0.295
		WWE3	0.574	0.074	0.387	0.584	0.084	0.329	0.547	0.047	0.268
		FWE	0.559	0.059	0.288	0.539	0.039	0.223	0.519	0.019	0.165
		MLE	0.503	0.003	0.086	-	-	-	-	-	-
$\hat{\beta}_1$	0.500	WWE1	0.450	-0.050	0.324	0.444	-0.056	0.304	0.442	-0.058	0.279
		WWE2	0.486	-0.014	0.274	0.477	-0.023	0.277	0.469	-0.031	0.259
		WWE3	0.475	-0.025	0.271	0.468	-0.032	0.265	0.474	-0.026	0.244
		FWE	0.495	-0.005	0.136	0.493	-0.007	0.091	0.496	-0.004	0.066
		MLE	0.505	0.005	0.058	-	-	-	-	-	-
$\hat{\theta}$	-0.300	WWE1	-0.335	-0.035	0.162	-0.365	-0.065	0.122	-0.383	-0.083	0.110
		WWE2	-0.279	0.021	0.160	-0.308	-0.008	0.118	-0.325	-0.025	0.080
		WWE3	-0.255	0.045	0.158	-0.279	0.021	0.115	-0.295	0.005	0.081
		FWE	-0.242	0.058	0.152	-0.264	0.036	0.112	-0.279	0.021	0.074
		MLE	-0.301	-0.001	0.018	-	-	-	-	-	-
$\hat{\gamma}$	0.500	WWE1	0.457	-0.043	0.161	0.448	-0.052	0.126	0.452	-0.048	0.097
		WWE2	0.498	-0.002	0.160	0.489	-0.011	0.118	0.488	-0.012	0.090
		WWE3	0.505	0.005	0.152	0.499	-0.001	0.112	0.499	-0.001	0.086
		FWE	0.503	0.003	0.139	0.501	0.001	0.107	0.504	0.004	0.082
		MLE	0.501	0.001	0.029	-	-	-	-	-	-
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 10: Monte Carlo 2d (Level 5): d=0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ⁹ (512); level 6			N=2 ¹⁰ (1024); level 6			N=2 ¹¹ (2048); level 6		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.250	WWE1	0.150	-0.100	0.314	0.143	-0.107	0.307	0.148	-0.102	0.279
		WWE2	0.144	-0.106	0.302	0.167	-0.083	0.279	0.162	-0.088	0.268
		WWE3	0.186	-0.064	0.298	0.152	-0.098	0.283	0.157	-0.093	0.269
		FWE	0.114	-0.136	0.297	0.174	-0.076	0.226	0.211	-0.039	0.157
		MLE	0.170	-0.080	0.188	0.208	-0.042	0.123	0.231	-0.019	0.075
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.413	-0.087	0.567	0.449	-0.051	0.550	0.451	-0.049	0.485
		WWE2	0.458	-0.042	0.566	0.498	-0.002	0.532	0.523	0.023	0.466
		WWE3	0.457	-0.043	0.570	0.531	0.031	0.509	0.559	0.059	0.440
		FWE	0.574	0.074	0.453	0.611	0.111	0.396	0.576	0.076	0.351
		MLE	0.555	0.055	0.256	0.523	0.023	0.179	0.508	0.008	0.126
$\hat{\beta}_1$	0.500	WWE1	0.528	0.028	0.375	0.520	0.020	0.369	0.526	0.026	0.328
		WWE2	0.550	0.050	0.350	0.523	0.023	0.345	0.523	0.023	0.311
		WWE3	0.541	0.041	0.324	0.548	0.048	0.301	0.534	0.034	0.286
		FWE	0.543	0.043	0.340	0.506	0.006	0.290	0.510	0.010	0.211
		MLE	0.554	0.054	0.197	0.529	0.029	0.144	0.517	0.017	0.096
$\hat{\theta}$	-0.300	WWE1	-0.265	0.035	0.297	-0.307	-0.007	0.243	-0.321	-0.021	0.198
		WWE2	-0.225	0.075	0.287	-0.258	0.042	0.239	-0.270	0.030	0.195
		WWE3	-0.221	0.079	0.289	-0.243	0.057	0.233	-0.250	0.050	0.193
		FWE	-0.215	0.085	0.272	-0.230	0.070	0.216	-0.236	0.064	0.178
		MLE	-0.302	-0.002	0.051	-0.303	-0.003	0.037	-0.302	-0.002	0.026
$\hat{\gamma}$	0.500	WWE1	0.474	-0.026	0.329	0.473	-0.027	0.254	0.459	-0.041	0.204
		WWE2	0.520	0.020	0.327	0.516	0.016	0.245	0.496	-0.004	0.197
		WWE3	0.528	0.028	0.318	0.511	0.011	0.232	0.503	0.003	0.189
		FWE	0.519	0.019	0.275	0.504	0.004	0.209	0.499	-0.001	0.177
		MLE	0.491	-0.009	0.091	0.500	0.000	0.064	0.501	0.001	0.044
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 11: Monte Carlo 2e (Level 6): d=0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ¹² (4096); level 6			N=2 ¹³ (8192); level 6			N=2 ¹⁴ (16384); level 6		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.250	WWE1	0.194	-0.056	0.251	0.208	-0.042	0.230	0.243	-0.007	0.170
		WWE2	0.171	-0.079	0.255	0.211	-0.039	0.210	0.223	-0.027	0.177
		WWE3	0.170	-0.080	0.248	0.205	-0.045	0.214	0.215	-0.035	0.174
		FWE	0.240	-0.010	0.079	0.250	0.000	0.049	0.250	0.000	0.035
		MLE	0.243	-0.007	0.041	-	-	-	-	-	-
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.459	-0.041	0.431	0.448	-0.052	0.373	0.358	-0.142	0.330
		WWE2	0.539	0.039	0.395	0.489	-0.011	0.349	0.424	-0.076	0.286
		WWE3	0.583	0.083	0.364	0.568	0.068	0.319	0.488	-0.012	0.247
		FWE	0.556	0.056	0.284	0.539	0.039	0.223	0.516	0.016	0.168
		MLE	0.502	0.002	0.090	-	-	-	-	-	-
$\hat{\beta}_1$	0.500	WWE1	0.472	-0.028	0.317	0.457	-0.043	0.301	0.465	-0.035	0.233
		WWE2	0.507	0.007	0.292	0.487	-0.013	0.257	0.499	-0.001	0.209
		WWE3	0.512	0.012	0.269	0.482	-0.018	0.247	0.504	0.004	0.195
		FWE	0.500	-0.000	0.132	0.493	-0.007	0.091	0.498	-0.002	0.064
		MLE	0.506	0.006	0.061	-	-	-	-	-	-
$\hat{\theta}$	-0.300	WWE1	-0.347	-0.047	0.150	-0.367	-0.067	0.120	-0.385	-0.085	0.110
		WWE2	-0.285	0.015	0.146	-0.311	-0.011	0.113	-0.329	-0.029	0.084
		WWE3	-0.260	0.040	0.151	-0.281	0.019	0.114	-0.300	-0.000	0.080
		FWE	-0.248	0.052	0.140	-0.264	0.036	0.112	-0.279	0.021	0.076
		MLE	-0.302	-0.002	0.018	-	-	-	-	-	-
$\hat{\gamma}$	0.500	WWE1	0.455	-0.045	0.156	0.449	-0.051	0.125	0.451	-0.049	0.099
		WWE2	0.491	-0.009	0.149	0.489	-0.011	0.117	0.492	-0.008	0.090
		WWE3	0.499	-0.001	0.141	0.499	-0.001	0.111	0.502	0.002	0.086
		FWE	0.501	0.001	0.135	0.501	0.001	0.107	0.504	0.004	0.083
		MLE	0.501	0.001	0.031	-	-	-	-	-	-
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 12: Monte Carlo 2f (Level 6): d=0.25; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ⁹ (512); level 4			N=2 ¹⁰ (1024); level 4			N=2 ¹¹ (2048); level 4		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.450	WWE1	0.294	-0.156	0.272	0.359	-0.091	0.201	0.368	-0.082	0.167
		WWE2	0.309	-0.141	0.257	0.384	-0.066	0.168	0.394	-0.056	0.142
		WWE3	0.328	-0.122	0.240	0.389	-0.061	0.167	0.405	-0.045	0.122
		FWE	0.320	-0.130	0.250	0.371	-0.079	0.189	0.428	-0.022	0.093
		MLE	0.388	-0.062	0.144	0.431	-0.019	0.074	0.449	-0.001	0.041
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.289	-0.211	0.643	0.296	-0.204	0.614	0.317	-0.183	0.550
		WWE2	0.355	-0.145	0.620	0.344	-0.156	0.600	0.386	-0.114	0.515
		WWE3	0.406	-0.094	0.601	0.409	-0.091	0.568	0.453	-0.047	0.488
		FWE	0.507	0.007	0.499	0.502	0.002	0.424	0.542	0.042	0.368
		MLE	0.523	0.023	0.258	0.492	-0.008	0.191	0.496	-0.004	0.129
$\hat{\beta}_1$	0.500	WWE1	0.608	0.108	0.350	0.579	0.079	0.308	0.566	0.066	0.236
		WWE2	0.616	0.116	0.327	0.597	0.097	0.306	0.571	0.071	0.228
		WWE3	0.598	0.098	0.324	0.588	0.088	0.292	0.563	0.063	0.218
		FWE	0.592	0.092	0.278	0.566	0.066	0.221	0.526	0.026	0.142
		MLE	0.559	0.059	0.151	0.526	0.026	0.097	0.503	0.003	0.063
$\hat{\theta}$	-0.300	WWE1	-0.321	-0.021	0.288	-0.362	-0.062	0.270	-0.376	-0.076	0.211
		WWE2	-0.259	0.041	0.274	-0.288	0.012	0.246	-0.305	-0.005	0.202
		WWE3	-0.232	0.068	0.261	-0.259	0.041	0.244	-0.265	0.035	0.192
		FWE	-0.232	0.068	0.247	-0.235	0.065	0.196	-0.244	0.056	0.147
		MLE	-0.303	-0.003	0.049	-0.306	-0.006	0.036	-0.303	-0.003	0.024
$\hat{\gamma}$	0.500	WWE1	0.535	0.035	0.378	0.539	0.039	0.327	0.493	-0.007	0.236
		WWE2	0.577	0.077	0.371	0.580	0.080	0.348	0.532	0.032	0.231
		WWE3	0.566	0.066	0.371	0.568	0.068	0.319	0.527	0.027	0.224
		FWE	0.534	0.034	0.297	0.529	0.029	0.240	0.511	0.011	0.194
		MLE	0.514	0.014	0.088	0.515	0.015	0.062	0.511	0.011	0.043
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 13: Monte Carlo 3a (Level 4): d=0.45; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ¹² (4096); level 4			N=2 ¹³ (8192); level 4			N=2 ¹⁴ (16384); level 4		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.450	WWE1	0.372	-0.078	0.153	0.380	-0.070	0.126	0.388	-0.062	0.110
		WWE2	0.396	-0.054	0.128	0.400	-0.050	0.106	0.406	-0.044	0.088
		WWE3	0.405	-0.045	0.116	0.404	-0.046	0.105	0.409	-0.041	0.091
		FWE	0.447	-0.003	0.053	0.454	0.004	0.034	0.456	0.006	0.026
		MLE	0.453	0.003	0.031	-	-	-	-	-	-
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.320	-0.180	0.468	0.268	-0.232	0.416	0.243	-0.257	0.355
		WWE2	0.374	-0.126	0.452	0.375	-0.125	0.372	0.350	-0.150	0.300
		WWE3	0.437	-0.063	0.401	0.428	-0.072	0.334	0.433	-0.067	0.246
		FWE	0.531	0.031	0.297	0.523	0.023	0.249	0.523	0.023	0.183
		MLE	0.503	0.003	0.090	-	-	-	-	-	-
$\hat{\beta}_1$	0.500	WWE1	0.536	0.036	0.189	0.543	0.043	0.153	0.536	0.036	0.116
		WWE2	0.554	0.054	0.183	0.542	0.042	0.137	0.540	0.040	0.108
		WWE3	0.545	0.045	0.157	0.546	0.046	0.121	0.534	0.034	0.094
		FWE	0.503	0.003	0.102	0.496	-0.004	0.074	0.491	-0.009	0.056
		MLE	0.498	-0.002	0.048	-	-	-	-	-	-
$\hat{\theta}$	-0.300	WWE1	-0.392	-0.092	0.170	-0.412	-0.112	0.144	-0.424	-0.124	0.137
		WWE2	-0.321	-0.021	0.149	-0.331	-0.031	0.098	-0.345	-0.045	0.076
		WWE3	-0.285	0.015	0.136	-0.297	0.003	0.090	-0.309	-0.009	0.061
		FWE	-0.259	0.041	0.112	-0.275	0.025	0.075	-0.288	0.012	0.044
		MLE	-0.301	-0.001	0.017	-	-	-	-	-	-
$\hat{\gamma}$	0.500	WWE1	0.494	-0.006	0.173	0.483	-0.017	0.133	0.474	-0.026	0.098
		WWE2	0.530	0.030	0.176	0.514	0.014	0.134	0.501	0.001	0.096
		WWE3	0.528	0.028	0.165	0.515	0.015	0.131	0.505	0.005	0.092
		FWE	0.517	0.017	0.150	0.513	0.013	0.122	0.505	0.005	0.088
		MLE	0.506	0.006	0.030	-	-	-	-	-	-
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 14: Monte Carlo 3b (Level 4): d=0.45; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ⁹ (512); level 5				N=2 ¹⁰ (1024); level 5				N=2 ¹¹ (2048); level 5			
			MEAN	BIAS	RMSE		MEAN	BIAS	RMSE		MEAN	BIAS	RMSE	
\hat{d}	0.450	WWE1	0.272	-0.178	0.287		0.295	-0.155	0.258		0.312	-0.138	0.240	
		WWE2	0.301	-0.149	0.268		0.309	-0.141	0.243		0.328	-0.122	0.216	
		WWE3	0.313	-0.137	0.259		0.319	-0.131	0.229		0.341	-0.109	0.208	
		FWE	0.310	-0.140	0.255		0.382	-0.068	0.174		0.426	-0.024	0.105	
		MLE	0.384	-0.066	0.155		0.433	-0.017	0.073		0.445	-0.005	0.047	
$\hat{\omega}$	0.000	ALL	-	-	-		-	-	-		-	-	-	
$\hat{\alpha}_2$	0.500	WWE1	0.284	-0.216	0.642		0.334	-0.166	0.579		0.355	-0.145	0.525	
		WWE2	0.361	-0.139	0.614		0.393	-0.107	0.565		0.421	-0.079	0.487	
		WWE3	0.389	-0.111	0.599		0.440	-0.060	0.529		0.491	-0.009	0.449	
		FWE	0.506	0.006	0.496		0.559	0.059	0.429		0.548	0.048	0.367	
		MLE	0.511	0.011	0.252		0.511	0.011	0.179		0.500	-0.000	0.128	
$\hat{\beta}_1$	0.500	WWE1	0.625	0.125	0.339		0.603	0.103	0.282		0.584	0.084	0.246	
		WWE2	0.606	0.106	0.334		0.611	0.111	0.264		0.593	0.093	0.224	
		WWE3	0.613	0.113	0.320		0.612	0.112	0.238		0.579	0.079	0.204	
		FWE	0.593	0.093	0.286		0.552	0.052	0.204		0.520	0.020	0.147	
		MLE	0.561	0.061	0.164		0.518	0.018	0.096		0.508	0.008	0.068	
$\hat{\theta}$	-0.300	WWE1	-0.336	-0.036	0.304		-0.340	-0.040	0.241		-0.361	-0.061	0.185	
		WWE2	-0.270	0.030	0.288		-0.272	0.028	0.228		-0.285	0.015	0.175	
		WWE3	-0.250	0.050	0.281		-0.245	0.055	0.214		-0.252	0.048	0.165	
		FWE	-0.253	0.047	0.266		-0.229	0.071	0.189		-0.242	0.058	0.149	
		MLE	-0.308	-0.008	0.051		-0.302	-0.002	0.035		-0.302	-0.002	0.024	
$\hat{\gamma}$	0.500	WWE1	0.518	0.018	0.412		0.513	0.013	0.303		0.497	-0.003	0.228	
		WWE2	0.561	0.061	0.403		0.547	0.047	0.289		0.527	0.027	0.218	
		WWE3	0.540	0.040	0.396		0.536	0.036	0.278		0.527	0.027	0.208	
		FWE	0.507	0.007	0.325		0.518	0.018	0.242		0.517	0.017	0.190	
		MLE	0.517	0.017	0.089		0.513	0.013	0.063		0.510	0.010	0.043	
WWE1(Haar)	WWE2(D4)	WWE3(LA8)												

Table 15: Monte Carlo 3c (Level 5): d=0.45; MLE, FWE, Haar, D4, LA8

PARAM	TRUE	METHOD	N=2 ¹² (4096); level 5			N=2 ¹³ (8192); level 5			N=2 ¹⁴ (16384); level 5		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.450	WWE1	0.336	-0.114	0.209	0.354	-0.096	0.191	0.388	-0.062	0.154
		WWE2	0.337	-0.113	0.204	0.350	-0.100	0.186	0.376	-0.074	0.163
		WWE3	0.346	-0.104	0.193	0.356	-0.094	0.180	0.377	-0.073	0.156
		FWE	0.447	-0.003	0.048	0.454	0.004	0.034	0.456	0.006	0.026
		MLE	0.453	0.003	0.030	-	-	-	-	-	-
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.331	-0.169	0.443	0.291	-0.209	0.385	0.241	-0.259	0.349
		WWE2	0.421	-0.079	0.410	0.390	-0.110	0.344	0.359	-0.141	0.281
		WWE3	0.482	-0.018	0.363	0.474	-0.026	0.295	0.445	-0.055	0.230
		FWE	0.547	0.047	0.307	0.537	0.037	0.243	0.523	0.023	0.183
		MLE	0.500	0.000	0.089	-	-	-	-	-	-
$\hat{\beta}_1$	0.500	WWE1	0.565	0.065	0.210	0.559	0.059	0.177	0.523	0.023	0.183
		WWE2	0.582	0.082	0.193	0.577	0.077	0.168	0.562	0.062	0.144
		WWE3	0.575	0.075	0.175	0.568	0.068	0.155	0.556	0.056	0.136
		FWE	0.501	0.001	0.100	0.494	-0.006	0.072	0.491	-0.009	0.056
		MLE	0.499	-0.001	0.046	-	-	-	-	-	-
$\hat{\theta}$	-0.300	WWE1	-0.389	-0.089	0.153	-0.414	-0.114	0.139	-0.429	-0.129	0.140
		WWE2	-0.305	-0.005	0.130	-0.333	-0.033	0.094	-0.346	-0.046	0.072
		WWE3	-0.269	0.031	0.128	-0.295	0.005	0.084	-0.309	-0.009	0.056
		FWE	-0.257	0.043	0.115	-0.279	0.021	0.071	-0.288	0.012	0.044
		MLE	-0.301	-0.001	0.017	-	-	-	-	-	-
$\hat{\gamma}$	0.500	WWE1	0.483	-0.017	0.176	0.473	-0.027	0.130	0.476	-0.024	0.097
		WWE2	0.515	0.015	0.175	0.501	0.001	0.127	0.501	0.001	0.096
		WWE3	0.513	0.013	0.166	0.505	0.005	0.121	0.504	0.004	0.092
		FWE	0.511	0.011	0.156	0.504	0.004	0.116	0.505	0.005	0.088
		MLE	0.506	0.006	0.028	-	-	-	-	-	-
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 16: Monte Carlo 3d (Level 5): d=0.45; MLE, FWE, Haar, D4, LA8

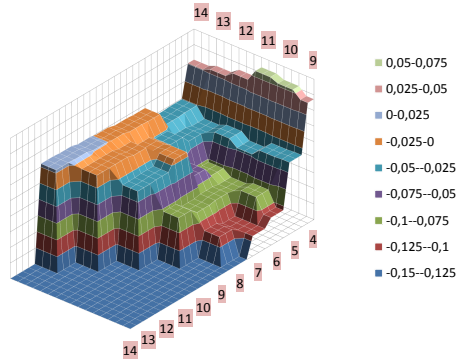
PARAM	TRUE	METHOD	N=2 ⁹ (512); level 6			N=2 ¹⁰ (1024); level 6			N=2 ¹¹ (2048); level 6		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.450	WWE1	0.259	-0.191	0.314	0.278	-0.172	0.288	0.314	-0.136	0.251
		WWE2	0.271	-0.179	0.297	0.300	-0.150	0.257	0.307	-0.143	0.249
		WWE3	0.291	-0.159	0.282	0.291	-0.159	0.267	0.307	-0.143	0.250
		FWE	0.302	-0.148	0.269	0.381	-0.069	0.172	0.421	-0.029	0.107
		MLE	0.377	-0.073	0.164	0.426	-0.024	0.083	0.445	-0.005	0.043
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.341	-0.159	0.599	0.358	-0.142	0.562	0.330	-0.170	0.507
		WWE2	0.412	-0.088	0.581	0.409	-0.091	0.557	0.474	-0.026	0.468
		WWE3	0.433	-0.067	0.574	0.486	-0.014	0.495	0.506	0.006	0.442
		FWE	0.510	0.010	0.498	0.532	0.032	0.425	0.551	0.051	0.380
		MLE	0.533	0.033	0.259	0.508	0.008	0.182	0.505	0.005	0.128
$\hat{\beta}_1$	0.500	WWE1	0.632	0.132	0.318	0.608	0.108	0.288	0.590	0.090	0.244
		WWE2	0.624	0.124	0.306	0.623	0.123	0.261	0.601	0.101	0.233
		WWE3	0.625	0.125	0.289	0.617	0.117	0.254	0.609	0.109	0.223
		FWE	0.612	0.112	0.275	0.561	0.061	0.203	0.528	0.028	0.148
		MLE	0.564	0.064	0.166	0.525	0.025	0.102	0.507	0.007	0.064
$\hat{\theta}$	-0.300	WWE1	-0.297	0.003	0.277	-0.352	-0.052	0.233	-0.382	-0.082	0.192
		WWE2	-0.238	0.062	0.277	-0.278	0.022	0.224	-0.288	0.012	0.166
		WWE3	-0.217	0.083	0.268	-0.248	0.052	0.208	-0.260	0.040	0.161
		FWE	-0.212	0.088	0.247	-0.238	0.062	0.186	-0.248	0.052	0.145
		MLE	-0.303	-0.003	0.049	-0.304	-0.004	0.035	-0.301	-0.001	0.024
$\hat{\gamma}$	0.500	WWE1	0.518	0.018	0.370	0.510	0.010	0.279	0.490	-0.010	0.226
		WWE2	0.561	0.061	0.337	0.541	0.041	0.265	0.514	0.014	0.217
		WWE3	0.549	0.049	0.337	0.526	0.026	0.252	0.514	0.014	0.203
		FWE	0.539	0.039	0.309	0.518	0.018	0.236	0.511	0.011	0.190
		MLE	0.509	0.009	0.091	0.513	0.013	0.064	0.508	0.008	0.042
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 17: Monte Carlo 3e (Level 6): d=0.45; MLE, FWE, Haar, D4, LA8

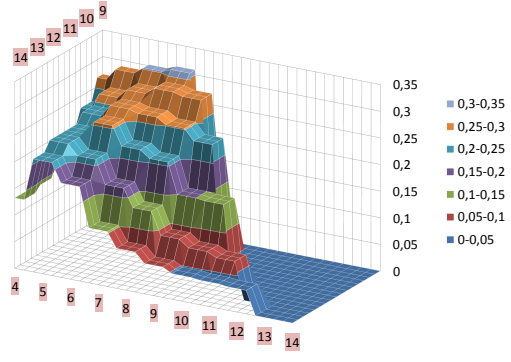
PARAM	TRUE	METHOD	N=2 ¹² (4096); level 6			N=2 ¹³ (8192); level 6			N=2 ¹⁴ (16384); level 6		
			MEAN	BIAS	RMSE	MEAN	BIAS	RMSE	MEAN	BIAS	RMSE
\hat{d}	0.450	WWE1	0.348	-0.102	0.222	0.373	-0.077	0.191	0.422	-0.028	0.119
		WWE2	0.341	-0.109	0.219	0.378	-0.072	0.178	0.417	-0.033	0.124
		WWE3	0.334	-0.116	0.228	0.383	-0.067	0.166	0.410	-0.040	0.123
		FWE	0.446	-0.004	0.048	0.453	0.003	0.035	0.456	0.006	0.027
		MLE	0.452	0.002	0.029	-	-	-	-	-	-
$\hat{\omega}$	0.000	ALL	-	-	-	-	-	-	-	-	-
$\hat{\alpha}_2$	0.500	WWE1	0.311	-0.189	0.444	0.268	-0.232	0.394	0.224	-0.276	0.361
		WWE2	0.422	-0.078	0.398	0.379	-0.121	0.351	0.354	-0.146	0.288
		WWE3	0.491	-0.009	0.355	0.465	-0.035	0.301	0.425	-0.075	0.238
		FWE	0.544	0.044	0.301	0.530	0.030	0.249	0.526	0.026	0.184
		MLE	0.498	-0.002	0.093	-	-	-	-	-	-
$\hat{\beta}_1$	0.500	WWE1	0.569	0.069	0.207	0.553	0.053	0.172	0.522	0.022	0.125
		WWE2	0.582	0.082	0.201	0.561	0.061	0.164	0.532	0.032	0.121
		WWE3	0.587	0.087	0.196	0.553	0.053	0.152	0.540	0.040	0.120
		FWE	0.505	0.005	0.100	0.497	-0.003	0.076	0.492	-0.008	0.058
		MLE	0.500	-0.000	0.048	-	-	-	-	-	-
$\hat{\theta}$	-0.300	WWE1	-0.402	-0.102	0.152	-0.420	-0.120	0.142	-0.435	-0.135	0.144
		WWE2	-0.319	-0.019	0.119	-0.338	-0.038	0.089	-0.353	-0.053	0.075
		WWE3	-0.282	0.018	0.118	-0.300	0.000	0.079	-0.318	-0.018	0.053
		FWE	-0.264	0.036	0.104	-0.276	0.024	0.076	-0.289	0.011	0.045
		MLE	-0.302	-0.002	0.017	-	-	-	-	-	-
$\hat{\gamma}$	0.500	WWE1	0.482	-0.018	0.169	0.483	-0.017	0.131	0.475	-0.025	0.095
		WWE2	0.510	0.010	0.164	0.507	0.007	0.133	0.501	0.001	0.094
		WWE3	0.511	0.011	0.155	0.511	0.011	0.127	0.503	0.003	0.090
		FWE	0.507	0.007	0.148	0.509	0.009	0.119	0.502	0.002	0.085
		MLE	0.506	0.006	0.031	-	-	-	-	-	-
WWE1(Haar)	WWE2(D4)	WWE3(LA8)									

Table 18: Monte Carlo 3f (Level 6): d=0.45; MLE, FWE, Haar, D4, LA8

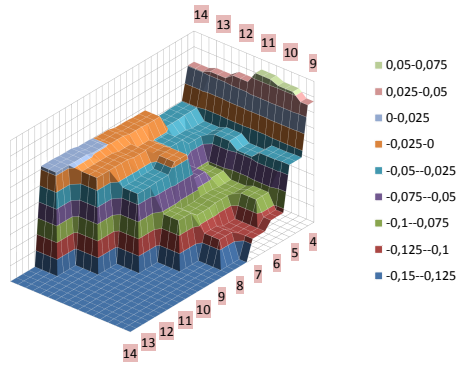
7 Convergence analysis II



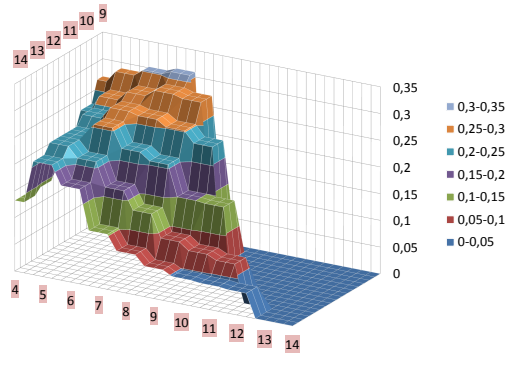
(a) Bias of \hat{d} (D4, d=0.25)



(b) RMSE of \hat{d} (D4, d=0.25)



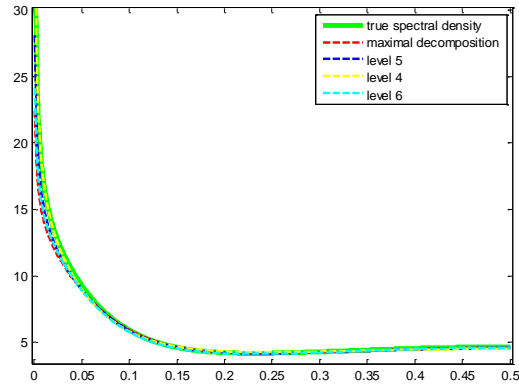
(c) Bias of $\hat{\alpha}$ (D4, d=0.25)



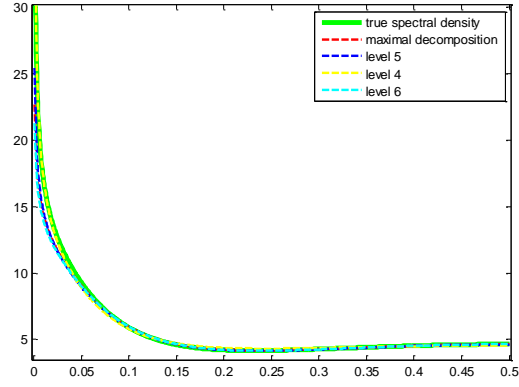
(d) RMSE of $\hat{\alpha}$ (D4, d=0.25)

Figure 7: 3D Plots: Partial decomposition: \hat{d} , $\hat{\alpha}$: Bias, RMSE

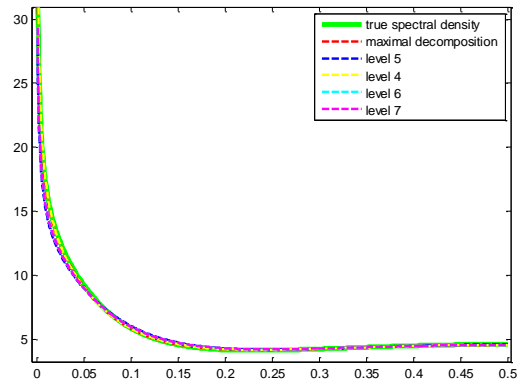
8 Relative Performance



(a) $T=512$ (2^9), level= max(9), 4(Haar),5(D4),6(LA8)

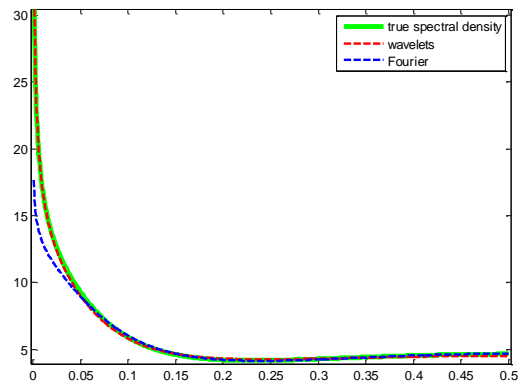


(b) $T=2048$ (2^{11}), level= max(11)(LA8), 4(Haar), 5(D4), 6(D4)

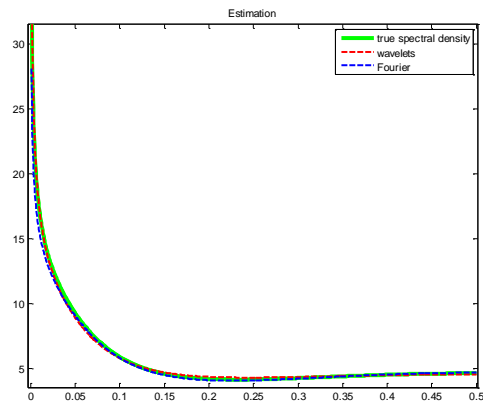


(c) $T=16384$ (2^{14}), level= max(14)(LA8), 4(LA8), 5(D4), 6(LA8), 7(LA8)

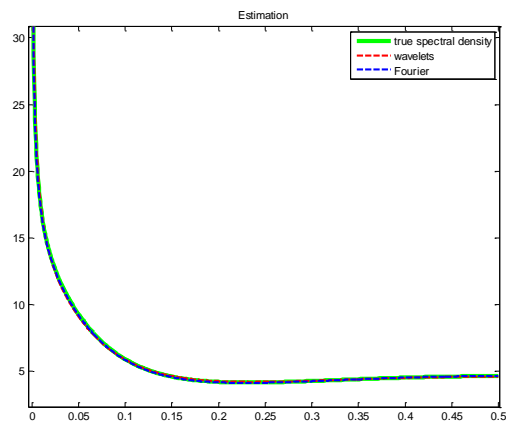
Figure 8: Spectral density estimation: Partial decomposition 0.25



(a) $T=512$ (2^9), level= 4(Haar)

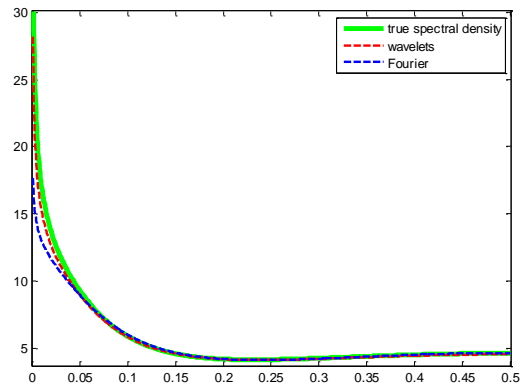


(b) $T=2048$ (2^{11}), level= 4(Haar)

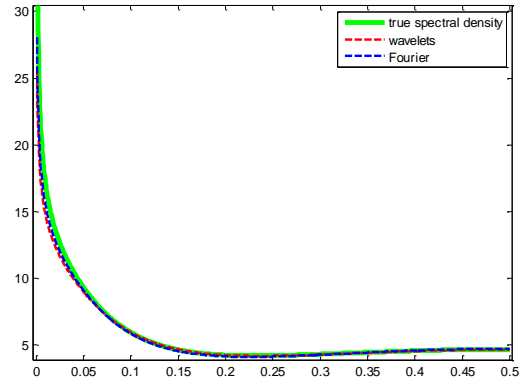


(c) $T=16384$ (2^{14}), level=4(Haar)

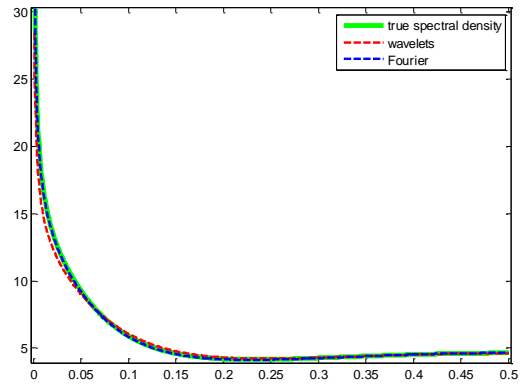
Figure 9: Spectral density estimation: Wavelets (Level 4) vs Fourier



(a) $T=512$ (2^9), level=5(D4)

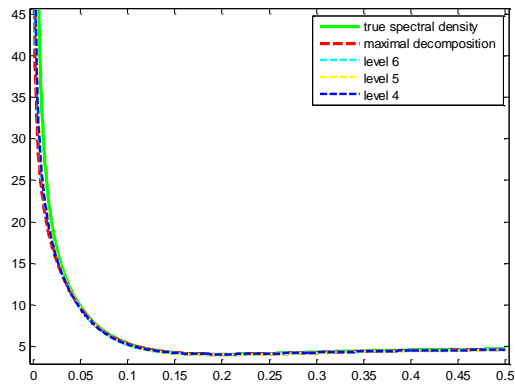


(b) $T=2048$ (2^{11}), level=5(D4)

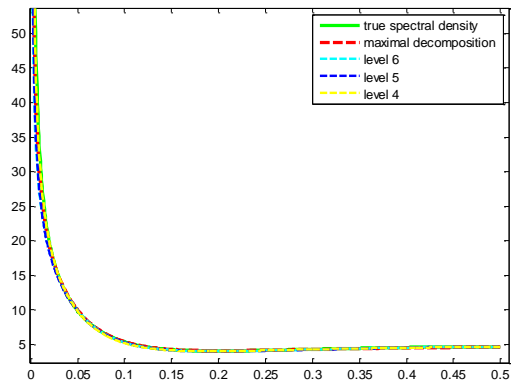


(c) $T=16384$ (2^{14}), level=5(D4)

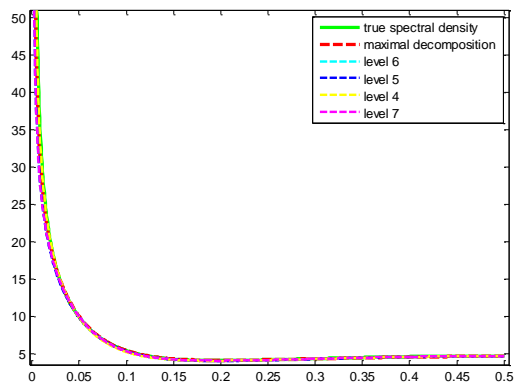
Figure 10: Spectral density estimation: Wavelets (Level 5) vs Fourier



(a) $T=512$ (2^9), level= max(9)(LA8), 4(Haar), 5(D4), 6(LA8)

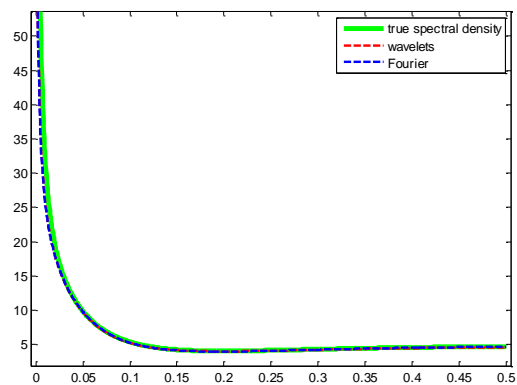


(b) $T=2048$ (2^{11}), level= max(11)(D4), 4(Haar), 5(D4), 6(D4)

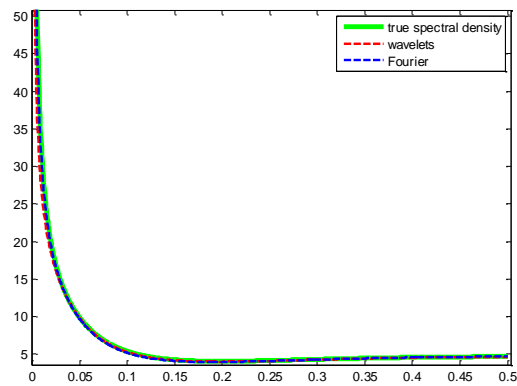


(c) $T=16384$ (2^{14}), level= max(14)(LA8), 4(LA8), 5(D4), 6(LA8), 7(LA8)

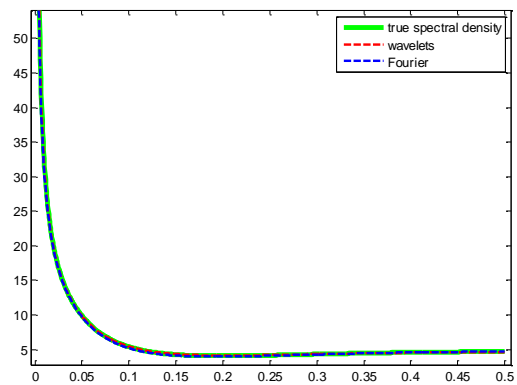
Figure 11: Spectral density estimation: Partial decomposition 0.45



(a) $T=512$ (2^9), level= 4(Haar)



(b) $T=2048$ (2^{11}), level= 5(D4)



(c) $T=16384$ (2^{14}), level= 7(LA8)

Figure 12: Spectral density estimation: Wavelets vs Fourier