

Research Article

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Effects of local fiber discontinuity on the fatigue strength parameter at the fiber inclusion corner in fiber-reinforced composites

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Abstract: To comprehend the fatigue failure mechanism at the fiber discontinuity in fiber-reinforced composites, it is necessary to evaluate the local mechanical behaviors. The fatigue strength depends on the stress distribution at the fiber inclusion corner. An improved advanced finite element method (IAFEM) is proposed for the stress intensity factor (SIF) analysis at the fiber inclusion corner. In the IAFEM, the element stiffness matrix of singular inclusion corner element (SICE) is obtained, and the singular elastic field at the tip of the fiber inclusion is determined. The effects of load direction, fiber distribution, fiber geometry, and material properties on SIFs are analyzed numerically using the IAFEM. The difference in stress field distribution between two-dimensional and three-dimensional fiber inclusions is discussed. The IAFEM and calculation results can provide reference for fatigue strength analysis and preparation of composite materials.

Keywords: fatigue strength, fiber-reinforced composite, singular inclusion corner element, stress intensity factor

1 Introduction

Fiber-reinforced composites are utilized in automotive, aerospace, and other engineering fields due to their lightweight, low cost, and high-quality processing performance [1–4]. In composite materials subjected to cyclic loading, the high stress zone prone to crack initiation is usually located at geometric discontinuities due to material mismatch. In fiber-reinforced composites, local fiber fracture is a typical defect [5–7]. The criteria for evaluating the fatigue strength of materials mostly depend on assumed structural models. Fatigue behavior can be evaluated in terms of parameters which are meaningfully significant to fatigue phenomena, such as the common S–N curve which combines design stress with the number of cycles. Generally, the fatigue strength S in the S–N curve is related to the definition of nominal stress [8]. If the root radius of the inclusion corner tip is assumed to be zero, stress singularity will appear in linear elastic analysis. Failure prediction based on stress singularity analysis is an active research topic [9]. Seweryn's criterion of brittle failure is based on whether the average stress for crack initiation or propagation to a specific damage stage reaches the critical value [10]. Although the stress near the stress singularity point tends to be infinite, the energy in the small piece of material around the stress singularity point is finite, thus appropriate characterization parameters are a prerequisite for accurate prediction of fatigue failure. Fatigue tests prove that maximum principal stress can be applied to the prediction of fatigue limit, but the scatter of the experimental data was obvious. Relatively speaking, stress intensity factor (SIF) is used as a meaningful parameter for the prediction of fatigue failure [11].

There have been a lot of research works on the singular stress field analysis and SIF of fiber inclusions in composites. Bogy and Wang [12] first studied the stress field at the corner of cylindrical inclusions. Zbib et al. [13] established an analytical expression of SIF for cylindrical cracks located at the fiber-matrix interface. Jiang et al. [14]

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developed an analytical method for periodic cylindrical inclusions under longitudinal shear by using equivalent inclusion technique and combining the results of Riemann boundary value problem. Bian et al. [15] used Eshelby's equivalent inclusion theory and Mori–Tanaka method to derive an analytical formula for stress distribution near fibers. Mantič [16] studied the plane strain problem of a single cylindrical inclusion embedded in an infinite matrix under uniform uniaxial transverse tension based on the stress criterion of coupling nodes and incremental energy criterion. Vable and Maddi [17] effectively applied the boundary element method for stress analysis of polygonal inclusions. Chen [18] introduced the basic density function in the physical force method to characterize the stress singularity at the inclusion corner in numerical analysis. Noda et al. [19–21] systematically calculated the generalized SIF at the inclusion corner with the body force method. Bulling et al. [22] proposed a finite element method (FEM) for automatic enrichment near stress singularities and constructed a semi-analytical singular mode to evaluate the SIFs in the areas of stress singularities.

Due to the complexity of boundary conditions and governing equations, the analytical method is only suitable for certain models which are relatively simple. The goal of fracture analysis using FEM or other numerical methods is to compute the singular stress at the crack tip or inclusion. Tan and Meguid [23] established a new singular FEM by using the complex function method, which could reliably and effectively determine the singular stress field and the related SIFs for various wedge geometry and elastic mismatches. Liou [24] developed a global-local FEM to predict the stress distributions of short fiber-reinforced composite. Madenci et al. [25] developed finite element analysis combining special and conventional elements to provide an accurate description of the stress field at the interface of two materials or at the junction of different materials. Lei et al. [26] evaluated the micromechanical properties of composite materials reinforced with single and clustered polygonal fibers using the FEM. Zhang and Katsube [27] proposed a new mixed FEM for mechanical analysis of heterogeneous materials with randomly dispersed inclusions. Symplectic singular element is a special singular element in the framework of FEM, which can be used to solve the dynamic SIF of bi-material interface cracks [28,29]. Chen et al. [30] developed a singular edge smooth FEM to solve crack problems in anisotropic media.

In order to ensure the compatibility of singular elements, it is usually necessary to use transition elements between singular elements and regular elements. However,

Chen and Ping [31] and Ping et al. [32] developed a kind of super singular corner element method based on the special finite element characteristic analysis method, which achieved the consistency degree of freedom between the singular element and the conventional element. By using the characteristic solutions of numerical stress field and displacement field, the singular stress near the tip of polygonal inclusion embedded in the matrix was analyzed by the super singular corner element. The super singular corner element method was further extended to deal with the 3D singular stress field problems [33–35]. The local fiber discontinuity problem is a typical three-dimensional (3D) inclusion problem in fiber-reinforced composites. It is necessary to propose an effective solution method and do more research on the stress singularity problem at the fiber inclusion corner.

In this work, using an ad hoc finite element eigen-analysis method and based on the Hellinger–Reissner (H–R) variational principle, a singular inclusion corner element (SICE) containing a part of cylindrical inclusion corner front is established. By using the SICE method, the singular stress field at the inclusion corner tip of double cylindrical inclusions under different loading conditions is analyzed. The effects of material mismatches, inclusion size, inclusion spacing, and unit cell size on SIF are analyzed. The differences in singular stress fields solved by two-dimensional (2D) and 3D models are compared. Once the fatigue strength judgment parameters are obtained, the numerical analysis results can be used to predict fatigue damage problems at 3D fiber discontinuity position.

2 Quantitative parameters of fatigue strength

Inclusion is the key factor to cause low-cycle fatigue failure. SIF is an important characterization parameter of progressive stress distribution near inclusion corner. The relationship between the SIF and the stress components at the angle bisector of the matrix corner adjacent to the inclusion corner in asymptotic stress field can be approximated as follows:

$$K_I = \lim_{\rho \rightarrow 0} \sqrt{2} \rho^{-\lambda_I} \sigma_{\phi}(\rho, \phi_b, \theta), \quad (1)$$

$$K_{II} = \lim_{\rho \rightarrow 0} \sqrt{2} \rho^{-\lambda_{II}} \sigma_{\rho\phi}(\rho, \phi_b, \theta), \quad (2)$$

$$K_{III} = \lim_{\rho \rightarrow 0} \sqrt{2} \rho^{-\lambda_{III}} \sigma_{\phi\theta}(\rho, \phi_b, \theta), \quad (3)$$

where ϕ_b is the angle ϕ located at the angle bisector of the matrix corner as shown in Figure 2a. For singular stress field analysis, $\rho = 0.0001R$. The values of characteristic roots λ_I , λ_{II} , and λ_{III} are related to material properties and inclusion shape [36].

For comparative analysis of 2D and 3D models, the dimensionless SIFs are expressed as:

$$F_I = \lim_{\rho \rightarrow 0} \frac{\rho^{-\lambda_I} \sigma_\phi(\rho, \phi_b, \theta)}{\sigma^\infty R^{-\lambda_I} \sqrt{\pi} f_{\phi_b, I}^I}, \quad (4)$$

$$F_{II} = \lim_{\rho \rightarrow 0} \frac{\rho^{-\lambda_{II}} \sigma_\rho(\rho, \phi_b, \theta)}{\sigma^\infty R^{-\lambda_{II}} \sqrt{\pi} f_{\phi_b, I}^{II}}, \quad (5)$$

$$F_{III} = \lim_{\rho \rightarrow 0} \frac{\rho^{-\lambda_{III}} \sigma_\theta(\rho, \phi_b, \theta)}{\sigma^\infty R^{-\lambda_{III}} \sqrt{\pi} f_{\phi_b, I}^{III}}, \quad (6)$$

where specific expressions of $f_{\phi_b, I}^I$, $f_{\phi_b, I}^{II}$, and $f_{\phi_b, I}^{III}$ can be referred from ref. [21].

3 SICE model for a cylindrical inclusion corner

In this section, based on the numerical eigen solutions of singular stress field and the generalized H–R variational principle, an SICE is established, and its validity is verified.

3.1 Establishment of the SICE model

In Figure 1a, we assume that 3D cylindrical inclusions are located in the matrix. To analyze the singular stress field at the corner of the inclusion, a unit cell containing a cylindrical inclusion is established, as shown in Figure 1b. $2H$ is the height of the unit cell, $2W$ is its length, and $2B$ is its width. If there are no special instructions, it is assumed that $H = W = B$.

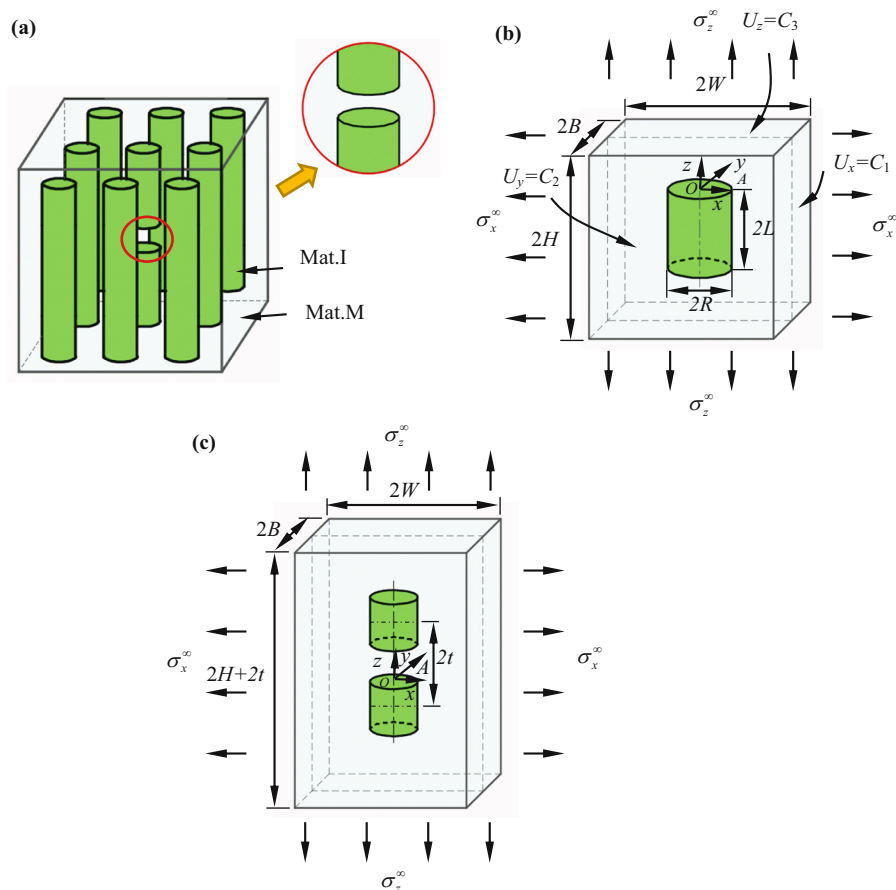


Figure 1: (a) Periodic inclusion model with a local fiber discontinuity; (b) a cylindrical inclusion in a unit cell model; and (c) two cylindrical inclusions aligned in the z -direction.

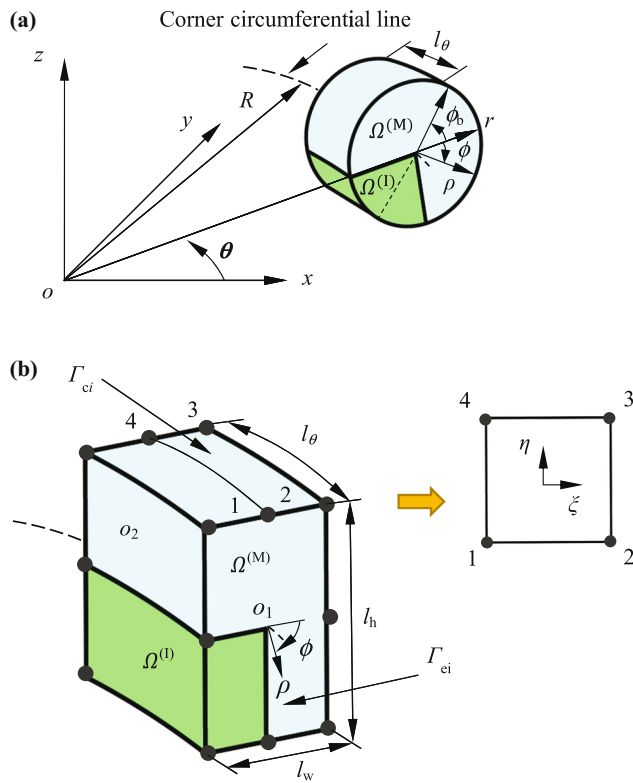


Figure 2: Definition of a SICE along the inclusion corner line: (a) Local curvilinear coordinate in rectangular coordinate and cylindrical coordinate; and (b) SICE containing partial inclusion corner line.

The diameter of the cylindrical inclusion is $2R$ and the height is $2L$. The boundary condition of the unit cell satisfies $U_x = C_1$, $U_y = C_2$, and $U_z = C_3$. That is, the six planes are translational in the loading process. The elasticity modulus of the inclusion and the matrix are E_I and E_M , respectively. The Poisson's ratios are ν_I and ν_M , respectively. Taking the center point O of the top surface of the cylindrical inclusion as the origin point, the Cartesian coordinate system is established to obtain the singular stress field near the inclusion corner line A . As shown in Figure 2a, a local coordinate system (ρ, ϕ, θ) is established with a point on the inclusion corner front line A . In the region where $\rho \ll R$, the asymptotic displacement and stress on the angular bisector of the cylindrical inclusion corner can be expressed as follows:

$$u_i(\rho, \phi, \theta) = \text{Re} \left[\sum_{n=1}^{N+M} \rho^{\lambda_n+1} u_i^{(n)}(\phi, \theta) \beta_n \right], \quad (7)$$

$$\sigma_{ij}(\rho, \phi, \theta) = \mathbf{D} \text{Re} \left\{ \sum_{n=1}^{N+M} \rho^{\lambda_n} \varepsilon_{ij}^{(n)}(\phi, \theta) \beta_n \right\}, \quad (8)$$

where λ is the characteristic root of the characteristic equation [37]. The number of intercepted characteristic

roots may be multiple, which may be real or complex. N is the number of intercepted complex roots, and M is the number of intercepted real roots. Due to the bounded strain energy, only the eigen values within the range of $\text{Re}(\lambda) > -1$ are considered herein. $\mathbf{D}$ is the elastic matrix of the material, and β_n is the stress intensity coefficient to be solved.

The displacement and stress field in the global coordinate system (x, y, z) can be obtained by transforming the corresponding parameters in the local coordinate system (ρ, ϕ, θ) , i.e.,

$$\mathbf{u}(x, y, z) = \mathbf{T}_u \mathbf{u}(\rho, \phi, \theta) = \mathbf{T}_u \mathbf{U} \boldsymbol{\beta}, \quad (9)$$

$$\boldsymbol{\sigma}(x, y, z) = \mathbf{D} \mathbf{T}_\varepsilon \boldsymbol{\varepsilon}(\rho, \phi, \theta) = \mathbf{D} \mathbf{T}_\varepsilon \mathbf{E} \boldsymbol{\beta}, \quad (10)$$

in which

$$\mathbf{U} = [\hat{\mathbf{u}}^{(1)} \dots \hat{\mathbf{u}}^{(N)} \mathbf{u}^{(N+1)}(\rho, \phi, \theta) \dots \mathbf{u}^{(N+M)}(\rho, \phi, \theta)], \quad (11)$$

$$\mathbf{E} = [\hat{\boldsymbol{\varepsilon}}^{(1)} \dots \hat{\boldsymbol{\varepsilon}}^{(N)} \boldsymbol{\varepsilon}^{(N+1)}(\rho, \phi, \theta) \dots \boldsymbol{\varepsilon}^{(N+M)}(\rho, \phi, \theta)], \quad (12)$$

$$\boldsymbol{\beta} = [\hat{\boldsymbol{\beta}}_1^T \dots \hat{\boldsymbol{\beta}}_N^T \beta_{N+1} \dots \beta_{N+M}]^T. \quad (13)$$

In Figure 2b, an SICE containing partial inclusion corner line is established. The singular element contains the inclusion domain Ω^I and the matrix domain Ω^M . Γ_{ci} is the end surface of the SICE. Γ_{ei} is the remaining surface of the SICE. In order to solve the unknown parameter β_n according to the H-R variational principle, the variational functional at the inclusion corner is defined as follows:

$$\begin{aligned} \prod_{H-R} = & -\frac{1}{2} \sum_{k=I, M} \int_{\Gamma_c^{(I)}} \sigma_{ij}^{(k)}(x, y, z) n_j u_i^{(k)}(x, y, z) d\Gamma \\ & + \sum_{k=I, M} \int_{\Gamma_c^{(I)}} \sigma_{ij}^{(k)}(x, y, z) n_j \bar{u}_i^{(k)}(x, y, z) d\Gamma, \end{aligned} \quad (14)$$

By solving the steady state value of the functional, the element stiffness matrix of the SICE is established, and the singular stress field at the inclusion corner is obtained.

$$\mathbf{K}_c = \mathbf{G}^T \mathbf{H}^{-1} \mathbf{G}, \quad (15)$$

$$\mathbf{H} = \frac{1}{2} \int_{\Gamma_c} [(\mathbf{n} \mathbf{E}_c)^T \mathbf{U}_c + (\mathbf{U}_c)^T (\mathbf{n} \mathbf{E}_c)] dA, \quad (16)$$

$$\mathbf{G} = \int_{\Gamma_c} (\mathbf{n} \mathbf{E}_c)^T \mathbf{N}_h dA, \quad (17)$$

$$\mathbf{U}_c = [(1 - \eta) \mathbf{T}_u \mathbf{U} / 2 \quad (1 + \eta) \mathbf{T}_u \mathbf{U} / 2], \quad (18)$$

$$\mathbf{E}_c = [(1 - \eta) \mathbf{D} \mathbf{T}_\varepsilon \mathbf{E} / 2 \quad (1 + \eta) \mathbf{D} \mathbf{T}_\varepsilon \mathbf{E} / 2], \quad (19)$$

where Γ_c is the assembly of Γ_{ci} , $\mathbf{n}$ is the unit normal vector, and $\mathbf{N}_h$ is the interpolation function matrix.

When $-1 < \lambda < 1$, the singular stress field at the inclusion corner can be expressed as the sum of the third-order singular stresses:

$$\sigma_{ij}(\rho, \phi_b, \theta) = \frac{(1-\eta)}{2} \cdot \operatorname{Re} \left[\sum_{n=1}^3 \rho^{\lambda_n} D_{ijkl} \varepsilon_{kl}^{(n)}(\phi, \theta) c_1^{(n)} \right] + \frac{(1+\eta)}{2} \cdot \operatorname{Re} \left[\sum_{n=1}^3 \rho^{\lambda_n} D_{ijkl} \varepsilon_{kl}^{(n)}(\phi, \theta) c_2^{(n)} \right]. \quad (20)$$

3.2 Verification of the SICE model

In order to verify the SICE model, the unit cell model as shown in Figure 1b is considered. Under the applied load σ_z^∞ , the stresses $\sigma_\phi(\rho, \phi_b, \theta)/\sigma_z^\infty$ and $\sigma_{\rho\phi}(\rho, \phi_b, \theta)/\sigma_z^\infty$ on the bisector of the corner of the cylindrical inclusion end are analyzed. The value of ρ varies from 0.0001R to 0.001R. The size of the cylindrical inclusion is $L/R = 1$. The size of the unit cell is $H/L = B/R = W/R = 100$. As shown in Figure 3a, due to the symmetry of the structure, a quarter of the model within the range of $x > 0$ and $y > 0$ is taken for analysis. A 16-node SICE is used at the inclusion corner. The circumferential dimensions and sectional dimensions of the element are as follows: $l_\theta = \pi R/80$, $l_w = l_h = 0.02R$. In order to ensure the high accuracy of calculation, 40 SICEs are set up at the corner of the cylindrical inclusion. They are assembled directly with the surrounding conventional 8-node elements. The type of the conventional 8-node elements is C3D8R and the SICEs are user-defined elements. Singular elements have good compatibility with traditional elements due to the consistent degrees of freedom at element nodes. Under the same condition, the traditional FEM is used to analyze the singular stresses at the inclusion corner, as

shown in Figure 3b. In the range of $0.02R \times 0.02R$, 3,362 elements are used in the traditional finite element model. In contrast, traditional finite element modeling is more complex. Table 1 demonstrates the number of elements and CPU running time analyzed by the SICE method and the traditional FEM. The comparison shows that the SICE model has higher computational efficiency. Figure 4a and b shows the variation in singular stress field parameters $\sigma_\phi(0.0001R, \phi_b, \theta)/\sigma_z^\infty$ and $\sigma_{\rho\phi}(0.0001R, \phi_b, \theta)/\sigma_z^\infty$ vs ρ/R in the vicinity of the inclusion corner in the unit cell under tensile loading σ_z^∞ .

Figure 4c and d shows the variation in the singular stress components $\sigma_\phi(0.0001R, \phi_b, \theta)/\sigma_x^\infty$ and $\sigma_{\rho\phi}(0.0001R, \phi_b, \theta)/\sigma_x^\infty$ under tensile loading σ_x^∞ . By comparison, it is found that the stresses calculated by SICE method is consistent with the stress calculated by traditional FEM, and the maximum error is less than 3.130%.

In the stress intensity coefficient expression, the effect of $\operatorname{Dim}(\beta) = (2N + M)$ on the singular stress field of a cylindrical inclusion is shown in Table 2. When $\operatorname{Dim}(\beta) = 10-26$, the errors of $\sigma_\phi(0.0001R, \phi_b, \theta)/\sigma_z^\infty$ and $\sigma_{\rho\phi}(0.0001R, \phi_b, \theta)/\sigma_z^\infty$ are less than 3.128% compared with those obtained by Noda (19), indicating that the SICE method has good convergence.

Table 1: The number of elements and CPU running time for SICE method and traditional FEM (Case in Figure 1b)

L/H	Num. of elements		CPU time	
	SICE + FEM	FEM	SICE + FEM	FEM
L/H = 0.8	61,688	167,442	0:01:48	0:05:53
L/H = 0.67	66,350	196,361	0:02:05	0:07:03
L/H = 0.5	80,864	254,736	0:02:21	0:08:07
L/H = 0.33	89,027	267,255	0:02:31	0:08:13
L/H = 0.1	105,094	282,590	0:02:58	0:10:13

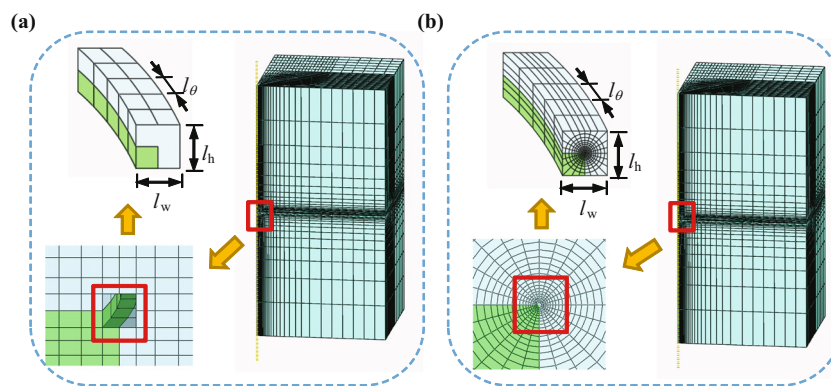


Figure 3: The meshes at the corner of 3D cylindrical inclusion as shown in Figure 1b ($L/R = 1$, $H/L = 100$): (a) the mesh using traditional elements and SICEs and (b) the mesh using traditional elements.

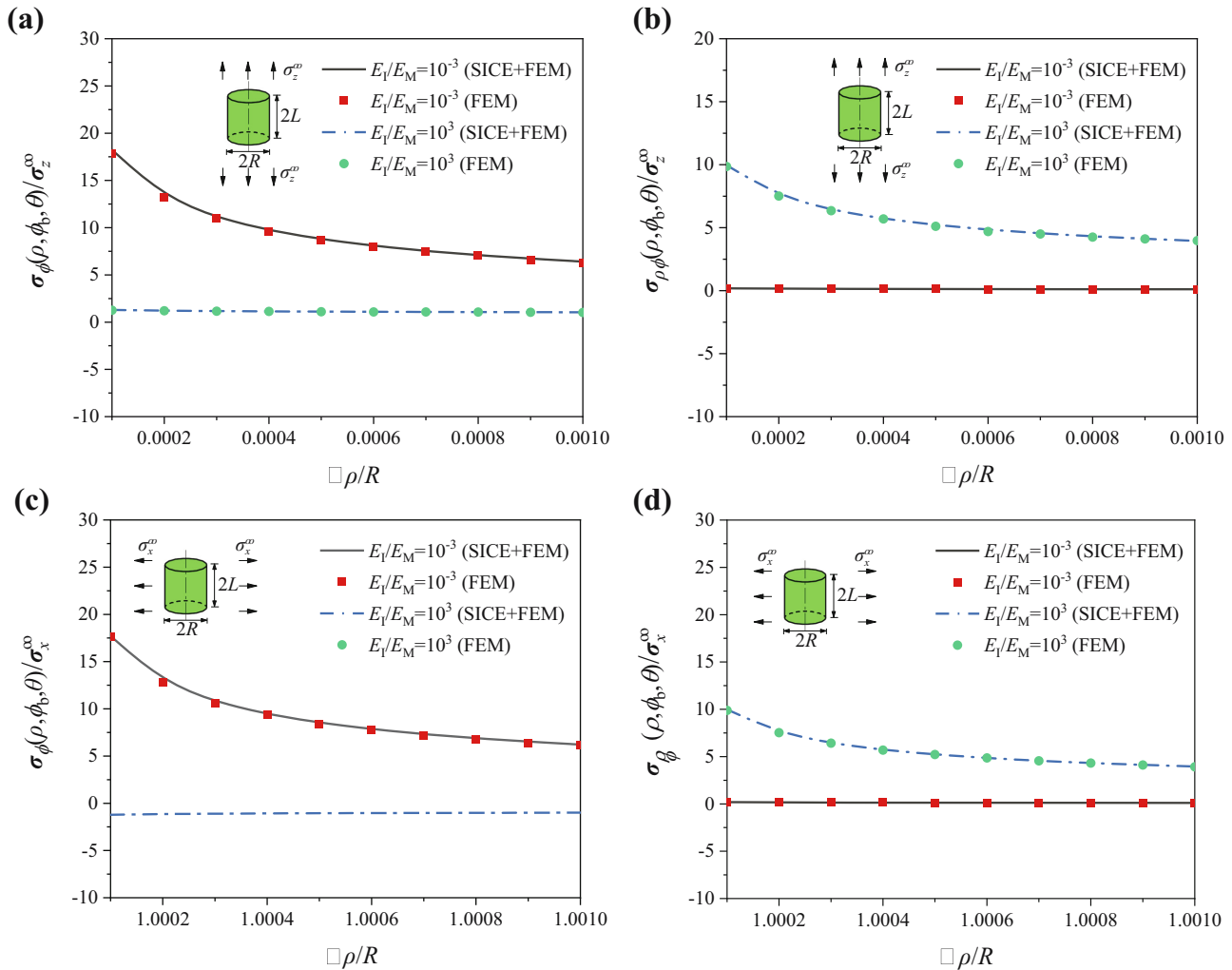


Figure 4: Comparison of stress results between the SICE method and the traditional FEM ($L/R = 1$, $L/H = 100$): (a) and (b) results under tensile loading σ_z^∞ and (c) and (d) results under tensile loading σ_x^∞ .

4 Discussion of results

This section applies the SICE model to calculate the fiber inclusion problem under the loading as shown in Figure 1.

Table 2: Effect of $\text{Dim}(\beta) = (2N + M)$ on singular stress field under tensile loading σ_z^∞ ($H/L = 100$, $L/R = 1$)

$\text{Dim}(\beta) = 2N + M$	$\sigma_\phi(0.0001R, \phi_b, \theta)/\sigma_z^\infty$		$\sigma_{\rho\phi}(0.0001R, \phi_b, \theta)/\sigma_z^\infty$	
	$E_I/E_M = 10^{-2}$	$E_I/E_M = 10^2$	$E_I/E_M = 10^{-2}$	$E_I/E_M = 10^2$
10	15.827	0.193	1.177	8.028
14	15.892	0.193	1.179	8.092
18	15.986	0.195	1.183	8.092
22	15.988	0.195	1.184	8.093
26	15.987	0.195	1.185	8.090

The effects of load direction, fiber spacing, fiber geometry, and material properties on SIFs at the inclusion corner are analyzed numerically.

4.1 Effects of inclusion size and spacing on the fatigue strength parameters

In the unit cell model as shown in Figure 1b, the effects of inclusion size and spacing on the SIF at corner line A under tensile loading σ_z^∞ are analyzed. A unit cell is considered to be an approximate simulation of periodic inclusions distribution. Let L/H vary from 0.1 to 0.8, variation in SIFs at the inclusion corner with the geometrical parameter L/H is shown in Figure 5. When $E_I/E_M < 1$ (soft inclusion), K_I decreases with L/H by 22.466%.

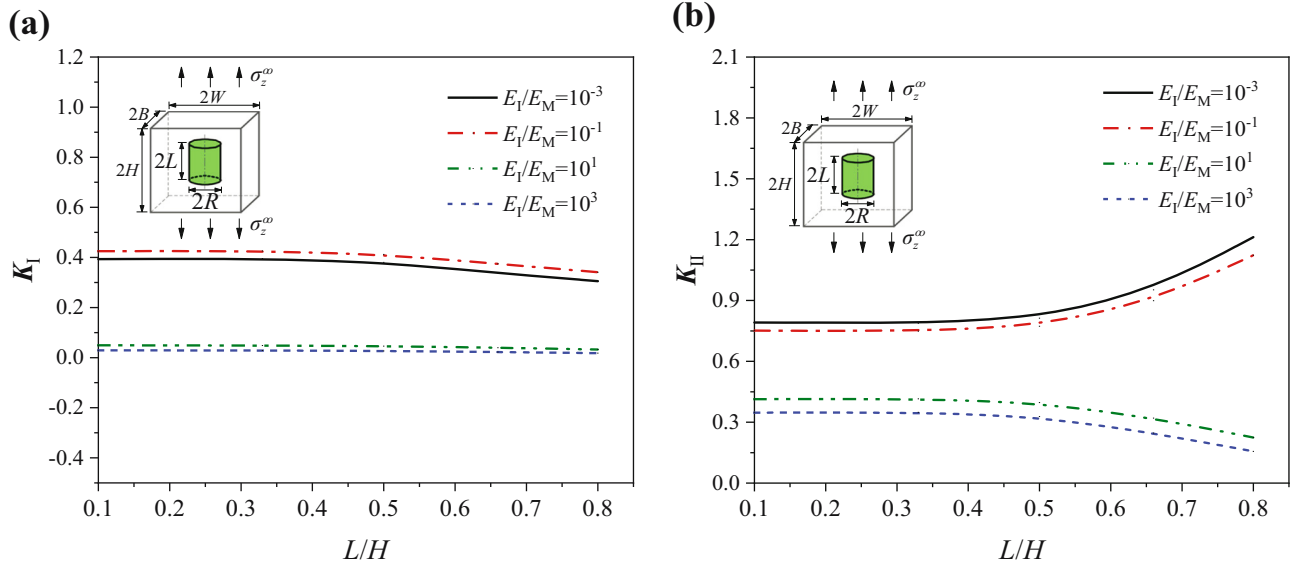


Figure 5: Variation in SIFs at the inclusion corner with the geometrical parameter L/H ($L/R = 1$) in (a) K_I and (b) K_{II} .

K_{II} increases with L/H by 53.176%. When $E_I/E_M > 1$ (hard inclusion), K_I remains unchanged with L/H . K_{II} decreases with L/H by 54.988%. The variation in the analysis results is considered to be caused by the influence of the nearby inclusion.

In order to analysis the influence of fiber fracture, the model of double inclusions aligned in the z -direction is established, as shown in Figure 1c. Let $L/H = 0.01$, which makes the model approximate to an inclusion in an infinite solid. The z -direction distance between the two inclusions is represented by $2t$. As shown in Figure 6, K_I

decreases with L/t for both the soft and hard inclusions. For soft inclusions, K_{II} increases with the increase in the distance $2t$ between the two inclusions. While for hard inclusions, K_{II} is basically stable after a little decrease with increase in the distance $2t$. The extrema of K_{II} when $E_I/E_M = 10^1$ and $E_I/E_M = 10^{-1}$ both appear at $L/t = 0.7$, their values are 0.463 and 0.402, respectively. The results show that the interference of inclusions aligned in the z -direction reduces the tendency of opening fracture at the corner front of the inclusion under tensile loading σ_z^0 . The increase in spacing parameter L/t leads to a

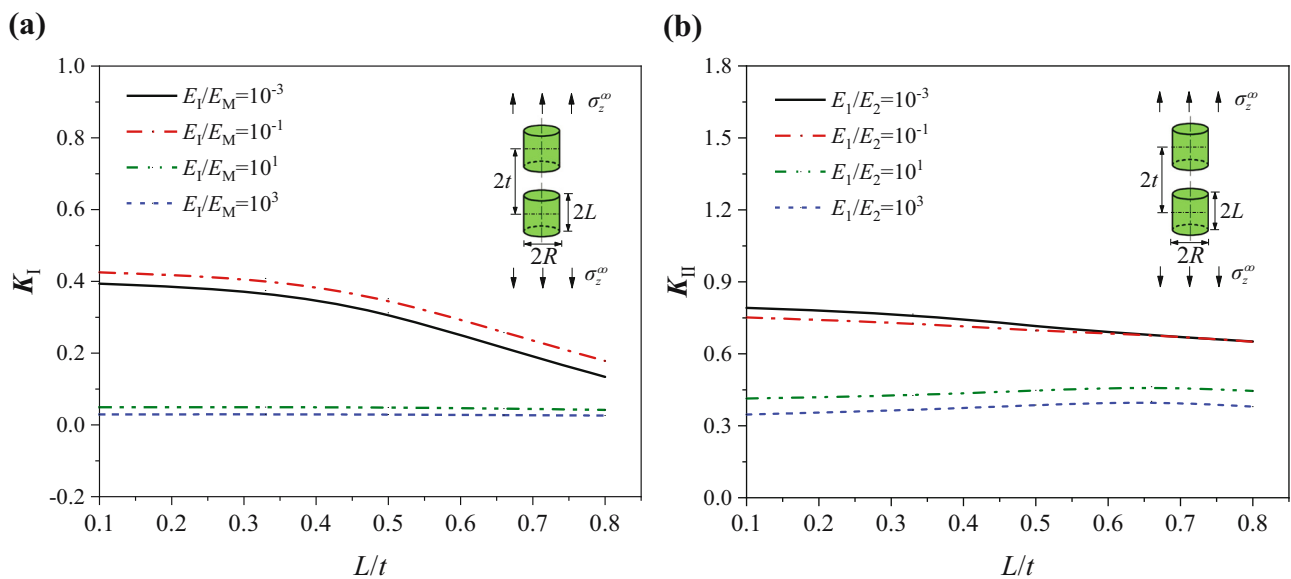


Figure 6: (a) K_I and (b) K_{II} in the interaction of double cylindrical inclusions aligned in the z -direction ($L/R = 1$).

decreasing trend of sliding fracture mode for soft inclusions, while for hard inclusions, the situation is just opposite.

Figure 7 shows K_I and K_{II} in the interaction of double inclusions aligned in the z -direction and at the corner of an inclusion in a unit cell when $L/R = 10$. By comparing Figure 7 with Figures 5 and 6, K_I is basically the same under tensile loading σ_z^∞ as L/t or L/H changes when $L/R > 10$, as opening fracture mode can be observed.

In the current analysis, when $E_I/E_M = 10^{-3}$, the length-diameter ratio (L/R) of the inclusion in the unit cell increases, a decreasing trend corresponding to the two cases $L/R = 1$ and 10, K_I decreased by 24.834 and 2.969%, respectively.

4.2 Effects of length-diameter ratio and size of a unit cell on fatigue strength parameters

Figure 8a and b shows the variation in K_I and K_{II} at the corner A with the length-diameter ratio and size of unit cell under the loading σ_z^∞ . When $E_I/E_M = 10^3$ and $L/R = 10$, K_I and K_{II} increase with W/R , and in most cases decrease with H/L . When $H/L > 10$, the analysis results tend to be consistent, which can be considered as the approximate situation of $H/L \rightarrow \infty$. Figure 8c and d shows K_I and K_{II} at the corner A of the inclusion when $E_I/E_M = 10^{-3}$. K_I and K_{II} decrease with W/R , and in most cases increase with

H/L . This variation trend is contrary to that when $E_I/E_M = 10^3$.

4.3 The fatigue strength parameter results under loading σ_x^∞

The variation in K_I and K_{II} with L/H of the cylindrical inclusion in a unit cell under tensile loading σ_x^∞ is shown in Figure 9. Compared with K_{II} , the size parameter L/H has less effect on K_I . The maximum change rate of K_I is 10.183%. For soft inclusions, K_{II} increases with L/H by 63.119%. For hard inclusions, K_{II} decreases with L/H by 46.001%. As shown in Figure 10, the variation trend of K_I and K_{II} with L/t at the corner of double cylindrical inclusions aligned in the z -direction under σ_x^∞ is almost opposite to the analysis results under tensile loading σ_z^∞ . An increasing L/t leads to a decreasing trend of sliding fracture mode for hard inclusions, while for soft inclusions, the situation is just opposite.

As shown in Figure 11, under σ_x^∞ and when $B = W$, K_I at the hard inclusion corner in the unit cell is generally small, which means that the possibility of opening fracture mode is small. K_{II} increases with W/R , and the possibility of sliding fracture mode increases. The K_I and K_{II} at soft inclusion corner in a unit cell increase with W/R in most cases, and the possibility of crack initiation increases. Therefore, the SIF at the corner front of the unit cell is affected by the inclusion material, length-diameter ratio of the inclusion, and size of the unit cell.

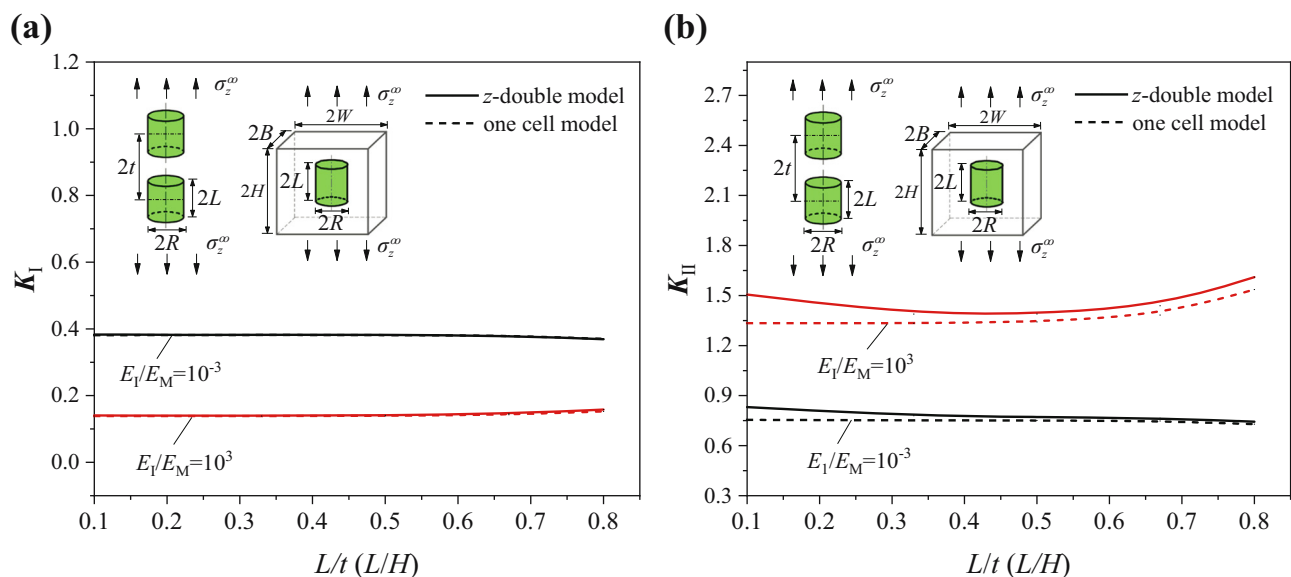


Figure 7: (a) K_I and (b) K_{II} at the corner of double cylindrical inclusions aligned in the z -direction and inclusion in unit cell ($L/R = 10$).

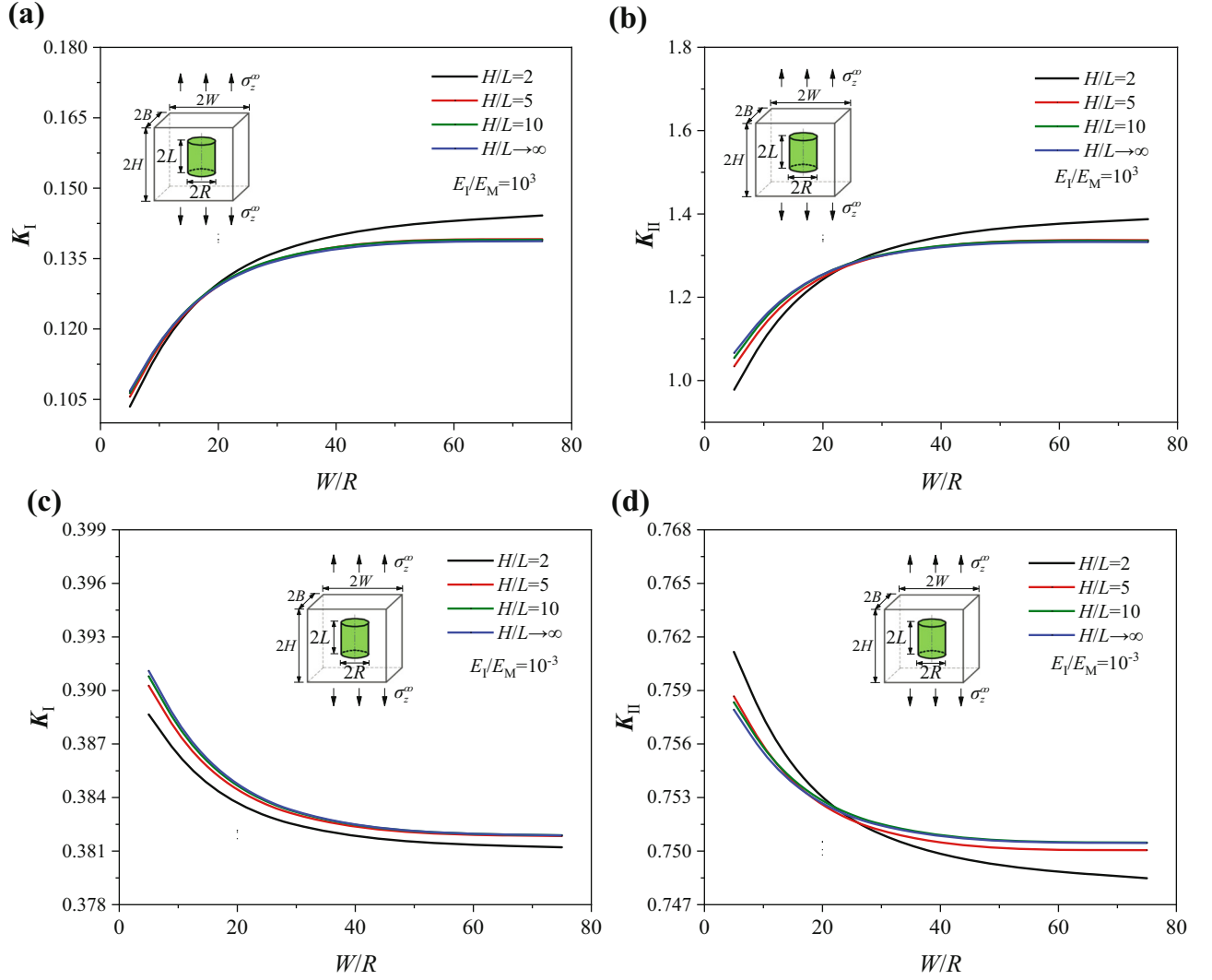


Figure 8: Effects of the unit cell size parameter W/R and the length-width ratio H/L on K_I and K_{II} under σ_z^∞ : (a and b) $E_I/E_M = 10^3$ and (c and d) $E_I/E_M = 10^{-3}$, when $L/R = 10$; $B = W$.

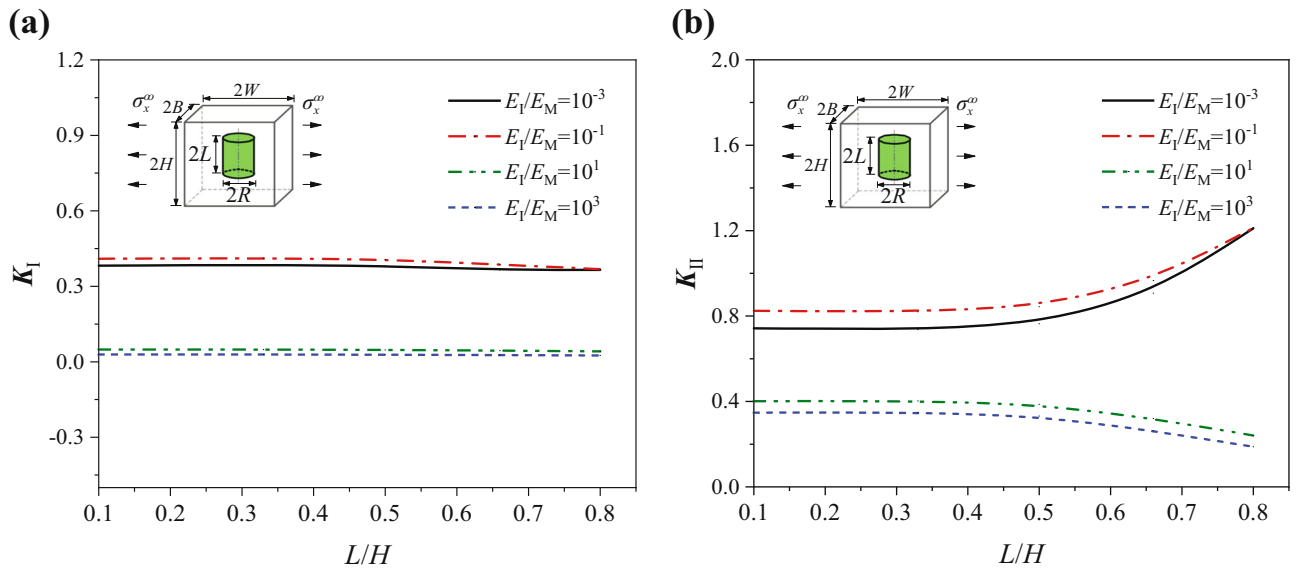


Figure 9: Variation in (a) K_I and (b) K_{II} with L/H of the unit cell ($L/R = 1$).

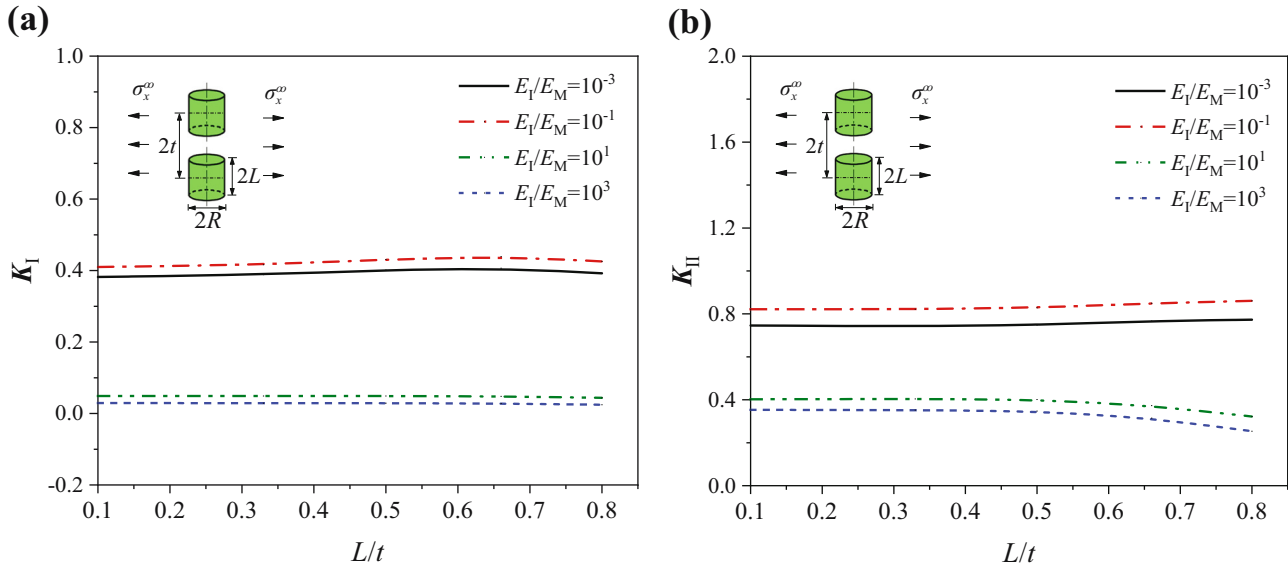


Figure 10: Variation in (a) K_I and (b) K_{II} with position parameter of the cylindrical inclusions ($L/R = 1$).

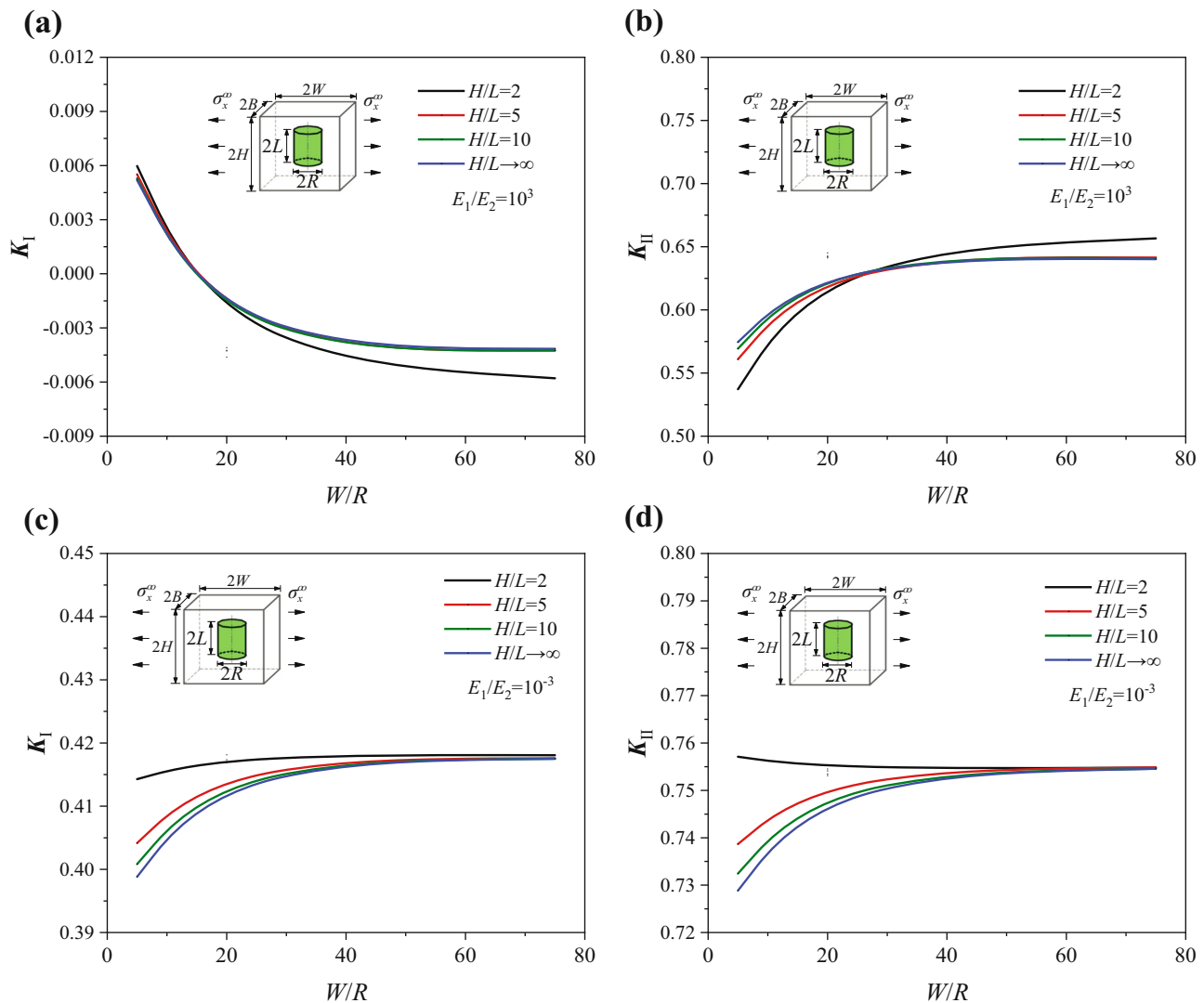


Figure 11: Effects of the unit cell size parameter W/R and the length-width ratio H/L on K_I and K_{II} under σ_x^∞ : (a and b) $E_I/E_M = 10^3$ and (c and d) $E_I/E_M = 10^{-3}$, when $L/R = 10$.

Table 3: Comparison of SIFs between a cylindrical inclusion (3D) and a square inclusion (2D) under loading σ_z^∞ ($L/R = 1$)

E_I/E_M	F_I			Change (%)	F_{II}		Change (%)
	L/H (3D), l_1/l_2 (2D)	1/10	2/3		1/10	2/3	
10^{-1}	3D	0.288	0.253	-12.153	1.904	2.276	19.538
10^{-1}	2D (20)	0.371	0.426	14.825	2.402	3.492	45.379
10	3D	0.295	0.238	-19.322	0.616	0.477	-22.565
10	2D (20)	0.229	0.182	-20.524	0.532	0.355	-33.271

Table 4: Comparison of SIFs between double cylindrical inclusion (3D) and double square inclusion (2D) aligned in the z -direction under loading σ_z^∞ ($L/R = 1$)

E_I/E_M	F_I			Change (%)	F_{II}		Change (%)
	L/t (3D), l_1/l_2 (2D)	1/10	2/3		1/10	2/3	
10^{-1}	3D	0.275	0.174	-36.727	1.843	1.719	-6.728
10^{-1}	2D (20)	0.324	0.179	-44.753	2.210	2.082	-5.792
10	3D	0.297	0.274	-7.744	0.635	0.689	8.504
10	2D (20)	0.226	0.192	-15.044	0.554	0.522	-5.776

5 Comparison between singular stress fields of the 2D and 3D models

The difference between the results of 3D cylindrical inclusion and 2D plane square inclusion is analyzed. Table 3 shows the dimensionless SIFs at the inclusion corner in a unit cell with different material properties and geometric sizes. The variation in the dimensionless SIFs at the 3D cylindrical inclusion corner and the 2D plane square inclusion corner with L/H (l_1/l_2) is basically the same, while there is a special case. In the range of $1/10 \leq L/H$ (l_1/l_2) $\leq 2/3$ and when $E_I/E_M = 10^{-1}$, F_I at the cylindrical inclusion corner decreased by 12.153%, while F_I at planar square inclusion corner increased by 14.825%. Table 4 shows the dimensionless SIFs at the double square inclusion aligned in the z -direction. It is found that the variation trends of F_{II} in 3D hard inclusions with L/t (l_1/l_2) are different from that of 2D hard inclusions. In the 3D model, F_{II} goes up by 8.504% with increase in L/H . On the other hand, in the 2D model, F_{II} goes down by 5.766% with increase in l_1/l_2 . As shown in Table 3, F_I and F_{II} for 3D cylindrical inclusions are larger than that of 2D plane square inclusions when $E_I/E_M = 10^{-1}$. F_I and F_{II} for 3D cylindrical inclusions are smaller than that of 2D plane square inclusions when $E_I/E_M = 10^1$. This same rule applies to the double cylindrical inclusions aligned in the z -direction as shown in Table 4.

If 2D models are used to evaluate 3D results, their differences should be realized. The analysis results are conservative when $E_I/E_M = 10^{-1}$, and the analysis results are dangerous when $E_I/E_M = 10^1$.

6 Conclusion

The SICE model is established to analyze the fatigue strength parameters at the cylindrical inclusion corner in fiber-reinforced composites with local fiber discontinuity. The SIFs of a cylindrical inclusion and in the interaction of double inclusions aligned in the z -direction under tensile loadings in different directions in the unit cell are investigated. The effects of material properties, inclusion geometry and spacing, and unit cell size on SIFs were systematically discussed.

The present discussions show that the effect of the variation in volume ratio on the fracture mechanics properties of soft inclusions and hard inclusions is basically opposite. The sliding fracture effect of the distance between two inclusions on soft inclusions and hard inclusions is different.

The variations in unit cell size parameter and length-width ratio have an effect on SIFs. But when length-width ratio $H/L > 10$, the analysis results tend to be consistent.

Compared with the 2D models, the present analysis results are conservative for soft inclusions, and the analysis results are dangerous for hard inclusions.

The SICE method is effective to analyze the singular stress field at the cylindrical inclusion corner and provide references for the fatigue strength prediction and preparation of fiber-reinforced composites.

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