

Research Article

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CSG Elastic Modulus Model Prediction Considering Meso-components and its Effect

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Abstract: Cemented sand gravel (CSG) is different from ordinary concrete in its mechanical properties such as elastic modulus due to the non-sieving of aggregate. On the basis of previous studies, the serial model, parallel model, Hashin-Shtrikman model and the elastic modulus prediction method based on the mesoscopic random aggregate simulation have been analyzed, and the influences of different mesoscopic component parameters on the elastic modulus calculation of the above four methods have been studied. The results show that the elastic modulus and tensile strength of meso component have great influence on the elastic modulus of random aggregate simulation, the elastic modulus of cement mortar has the largest impact on the series model and the Hashin-Shtrikman lower boundary model, and the elastic modulus of the aggregate has the greatest influence on the parallel model and the Hashin-Shtrikman upper boundary model. This paper proposed to determine the meso-component parameters of the elastic modulus prediction model through mesoscopic inversion, studied the correlation between the meso-component volume fraction and the overall elastic modulus of the material, and by adding a correction factor to the correlation, concluded that when the meso-component volume fraction falls between 35% and 55%, the random aggregate model-based prediction and the equivalent theoretical model can better predict the elastic modulus of the CSG. This study may provide theoretical basis for CSG mix design and multi-scale model research.

Keywords: elastic modulus, equivalent model, numerical simulation, sensitivity analysis, mesoscopic inversion

1 Introduction

CSG is composed of water, unscreened aggregate and a few cementing materials. Such composition has a significant impact on the mechanical properties of the CSG. The elastic modulus, as an important parameter describing mechanical behaviors, is usually obtained through experiments, but the data obtained by such means is random and costly, and cannot reflect the effect of the meso-components.

Though CSG is a composite material, the mesomechanics-based elastic modulus prediction is mostly made on the basis of concrete. For example, Li Zongli *et al.* [1], from a mesoscopic perspective, proposed a multiphase mixture inclusion model considering aggregate grading to predict the elastic modulus of concrete, verified the results of the model using the Stock test, and found that this model can fully achieve the prediction results with a higher accuracy; Ying Zongquan *et al.* [2] calculated the effective elastic modulus of the composite sphere between the aggregate and the interface, established an REV using the random aggregate model generated by the composite sphere, and predicted the effective elastic modulus of the REV with the mesomechanical numerical homogenization method; Li Chaohong *et al.* [3], based on the mesoscopic inclusion theory, adopted a multi-step method to establish a concrete mixture inclusion model, and verified the validity of the model through numerical examples. The above studies all considered the impact of the meso-components on the elastic modulus model, but did not go deep into the determination of the meso-component parameters, and there were no researches on the elastic modulus of the CSG.

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With CSG as the research object and for the purpose of determining the macro and meso elastic modulus, this paper quantified the relationship between the macro and meso parameters, used numerical simulation and theoretical analysis methods to analyze different elastic modulus models and the sensitivity of the meso-component parameters to the elastic modulus model, proposed a method for determining the meso-parameters based on inversion analysis, and finally applied the equivalent models of different elastic modulus in actual projects, providing a theoretical basis for the CSG structural design.

2 Prediction of CSG elastic modulus considering meso-components

The calculation of the elastic modulus of a composite material is generally based on the equivalence. At present, the relatively mature analysis methods for elastic modulus prediction include Eshelb equivalent inclusion theory [4], self-consistent theory [5, 6], Mori-Tanaka method [7], and generalized self-consistent method [8], etc. The elastic modulus prediction model considering meso-components is analyzed as follows.

2.1 Series model of elastic modulus considering meso-components

The CSG was regarded as a three-phase composite material mixed of cement mortar (as the matrix), aggregate and interfacial transition zone (ITZ). See the equivalent model in Figure 1. The stresses of the material elements in each phase were assumed to be the same, and all took the average stress value; the average strain of the composite material was expressed by the average stress value; thus, the equivalent elastic modulus of the composite material can be deduced from the following formula:

$$\sigma^G(x) = \sigma^M(x) = \sigma^i(x) = \bar{\sigma} \quad (1)$$

$$\varepsilon^G(x) = \frac{1}{E^G} \times \bar{\sigma} \quad (2)$$

$$\varepsilon^M(x) = \frac{1}{E^M} \times \bar{\sigma} \quad (3)$$

$$\bar{\varepsilon} = f^G \times \varepsilon^G(x) + f^M \times \varepsilon^M(x) + f^i \times \varepsilon^i(x) \quad (4)$$

$$\varepsilon^i(x) = \frac{1}{E^i} \times \bar{\sigma} \quad (5)$$

$$E = \frac{1}{f^G \times \frac{1}{E^G} + f^M \times \frac{1}{E^M} + f^i \times \frac{1}{E^i}} \quad (6)$$

Where, G represents the aggregate; M represents the cement mortar; i represents the ITZ; σ represents the stress (unit: MPa); ε represents the strain; E represents the equivalent elastic modulus of the series model (unit: MPa); f represents the volume fraction of the material (unit: %).

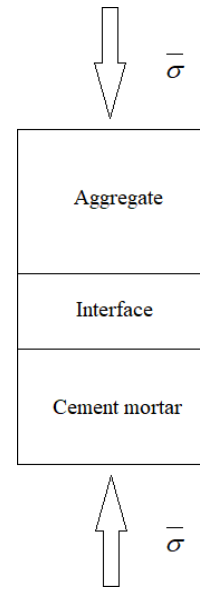


Figure 1: series model considering microscopic components

2.2 Parallel model of elastic modulus considering meso-components

See the calculation model in Figure 2. The strains of the material elements in each phase were assumed to be the same, and all took the average strain value; the average stress of the composite material was expressed by the average strain value; thus, the relationship of the parallel elastic modulus of the composite material was listed as formulas (7)-(10):

$$\varepsilon^G(x) = \varepsilon^M(x) = \varepsilon^i(x) = \bar{\varepsilon} \quad (7)$$

$$\sigma^G(x) = E^G \times \bar{\varepsilon}, \quad \sigma^M(x) = E^M \times \bar{\varepsilon}, \quad \sigma^i(x) = E^i \times \bar{\varepsilon} \quad (8)$$

$$\bar{\sigma} = f^G \times \sigma^G(x) + f^M \times \sigma^M(x) + f^i \times \sigma^i(x) \quad (9)$$

$$E = f_G \times E_G + f_i \times E_i + f_M \times E_M \quad (10)$$

All variables in these formulas were the same as those in the series model.

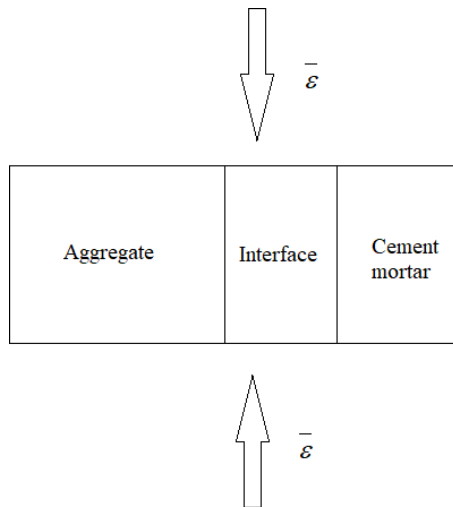


Figure 2: parallel model considering microscopic components

2.3 Hashin-Shtrikman model for elastic modulus considering meso-components

The Hashin-Shtrikman model provided the limits of the effective bulk modulus K and the shear modulus G . The material components met the following properties:

$$(k_1 - k_2)(\mu_1 - \mu_2) \geq 0 \quad (11)$$

The upper limit (L) and the lower limit (U) of the effective bulk modulus and the shear modulus [9] were calculated according to:

$$K^L = K_1 + \frac{V_2}{\frac{1}{K_2 - K_1} + \frac{3V_1}{3K_1 + 4G_1}}; \quad (12)$$

$$K^U = K_2 + \frac{V_1}{\frac{1}{K_1 - K_2} + \frac{3V_2}{3K_2 + 4G_2}}$$

$$G^L = G_1 + \frac{V_2}{\frac{1}{G_2 - G_1} + \frac{6(K_1 + 2G_1)V_1}{5G_2(3K_1 + 4G_1)}}; \quad (13)$$

$$G^U = G_2 + \frac{V_1}{\frac{1}{G_1 - G_2} + \frac{6(K_2 + 2G_2)V_2}{5G_1(3K_2 + 4G_2)}}$$

Where, K and G are the bulk modulus and shear modulus respectively; V is the volume fraction of the material; 1 and 2 are the number of two materials.

Considering that the CSG is composed of cement mortar, interface and aggregate, the ITZ and the cement mortar were equivalent first to form a cement equivalent matrix, and then the aggregate was equivalent to the cement equivalent matrix to form a CSG equivalent matrix. The CSG equivalent elastic modulus was calculated according to the formula (14):

$$E = \frac{9K}{1 + \frac{3K}{G}} \quad (14)$$

2.4 Prediction of elastic modulus based on mesoscopic random aggregate model

From a mesoscopic perspective, the CSG is a three-phase composite material composed of aggregate, mortar, and interface between the aggregate and the mortar. Based on this, a random aggregate model was established, and the elastic modulus of concrete was predicted using the finite element method.

2.4.1 Random aggregate model

Analyzed from a mesoscopic perspective, the CSG consists of 2 materials (aggregate and cement mortar) and 1 interface (ITZ), and its polygonal random aggregate model is shown in Figure 3, which contains 3 elements, namely aggregate element, mortar element and interface element. During numerical simulation, the mesoscopic constitutive model adopted a linear elastic model [10–17], and the failure criterion was based on the theory of maximum tensile stress, that is, when the maximum tensile stress exceeds the tensile strength, the material cracks [18, 19].

2.4.2 Calculation of elastic modulus

By definition, the elastic modulus was calculated using displacement. According to Hooke's law, the elastic modulus E was calculated in the following formula:

$$E = \frac{Nl}{\Delta l A} = \frac{ql}{\Delta l} = \sigma/\varepsilon \quad (15)$$

Where, q is the area load imposed onto the top surface of the test piece, expressed by MPa.

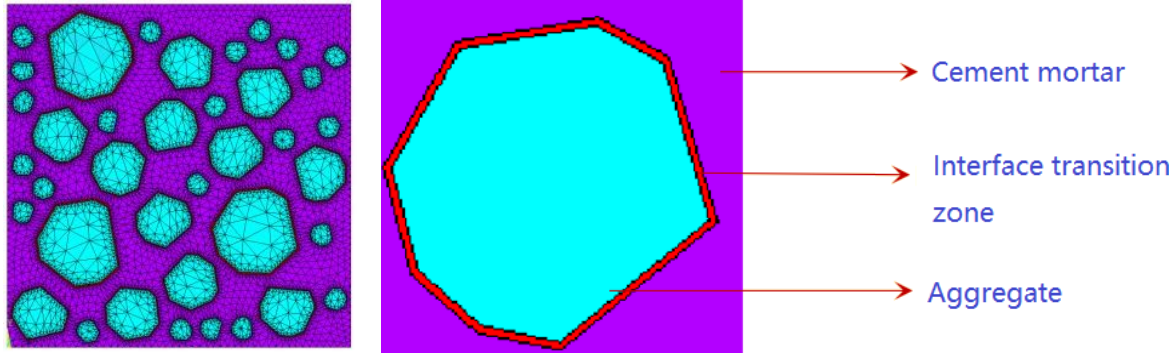


Figure 3: CSG polygon random aggregate model

2.4.3 Implementation of finite element for elastic modulus prediction

According to (15), the implementation of finite element for elastic modulus prediction based on mesoscopic numerical simulation was as follows:

1. Determine the number of aggregate particles according to the mix proportion;
2. Randomly place the aggregate into the test block to form a random aggregate model, see the literature [20, 21];
3. Conduct the numerical simulation of tension and compression based on test loading, and draw a stress-strain curve, see the Figure 4;
4. Calculate the slope of the elastic stage according to the formula (15), which was the elastic modulus. As shown in Figure 4, the ab slope of the stress-strain curve was the elastic modulus.

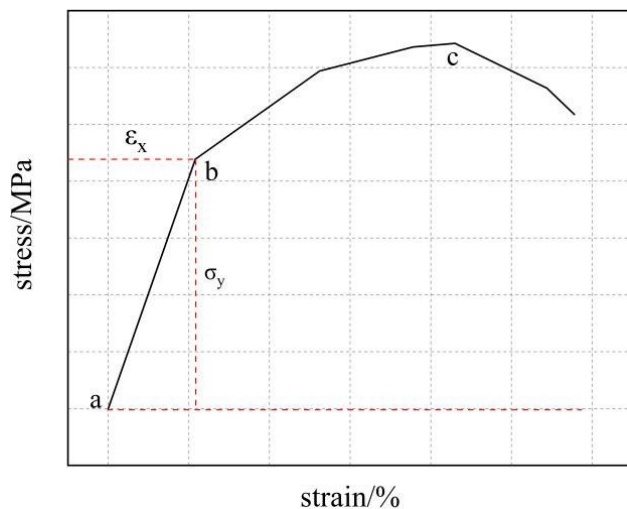


Figure 4: Schematic diagram of stress-strain curve

3 Sensitivity analysis of parameters of elastic modulus model considering meso-components

The meso-parameters are the key to calculating the equivalent elastic modulus, but they are difficult to obtain by experiment. Generally, the meso-parameters adopt a set of parameters that approximate the laboratory test results after continuous trial-and-error. In order to determine the parameters reasonably, the sensitivities of each meso-parameter value to the theoretical model of equivalent elastic modulus were investigated.

3.1 Sensitivity analysis method

According to literature [22], the ratio of the relative error ($\delta_P = |\Delta P|/P$) of the research question characteristics P to the relative error ($\delta_{a_k} = |\Delta a_k|/a_k$) of the parameter a_k was defined as the sensitivity function $S_k(a_k)$ of the parameter a_k .

$$S_k(a_k) \triangleq \left(\frac{|\Delta P|}{P} \right) / \left(\frac{|\Delta a_k|}{a_k} \right) = \left| \frac{\Delta P}{\Delta a_k} \right| \frac{a_k}{P} \quad (16)$$

$$k = 1, 2, \dots, n$$

Where: S_k^* , $k = 1, 2, \dots, n$ is dimensionless, rather than negative real numbers. The larger the S_k^* value is, the higher the sensitivity of P to a_k is in the normal state. By comparing S_k^* , the sensitivities of system characteristics to various factors were obtained.

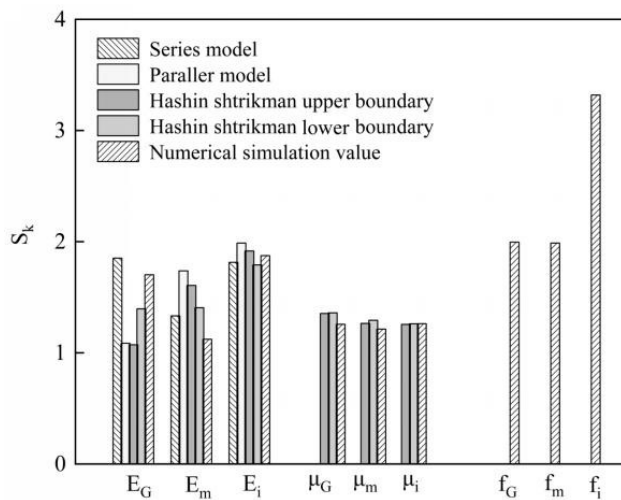
Table 1: Basic value of parameter sensitivity analysis

Meso component	Modulus of elasticity/MPa	Poisson's ratio	tensile strength/MPa
Aggregate	500-1000	0.2-0.25	4-10
Cement mortar	100-200	0.15-0.1875	1-2
Interface	40-80	0.15-0.1875	0.5-1

3.2 Sensitivity analysis of different models

The basic meso-parameters required for the elastic modulus prediction model were aggregate, cement mortar, elastic modulus of ITZ and Poisson's ratio, represented by symbols $E_g/E_m/E_i/\mu_g/\mu_m/\mu_i$ respectively. The mesoscopic numerical simulation added the tensile strengths of the aggregate, cement mortar and ITZ, expressed by symbols $f_g/f_m/f_i$ respectively. The parameters used in the calculation were shown in Table 1, among which, the aggregate volume fraction was 55%.

The results of sensitivity analysis were shown in Figure 5.

**Figure 5:** Sensitivity Analysis of model parameters

From Figure 5, the elastic modulus of the aggregate showed the highest sensitivity in the series model, while in the parallel model, the elastic modulus of the interface showed the highest sensitivity; as for the mesoscopic numerical simulation, the tensile strength of the interface showed the highest sensitivity. Such conclusions are consistent with the literature [16], and also verify the rationality of this study. It is the small amount of cement in the CSG that led to a larger water-binder ration on the interface under the action of water film around the aggregate,

and the aggregate performance became poor, generating a great impact on the mechanical properties of the CSG.

4 Determination of CSG meso-parameters based on inverse analysis

The parameters in the aggregate phase and in the cement mortar phase of the CSG obtained through experimental methods were greatly different, and the parameters in the interface phase were invalid, therefore, this paper adopted the parametric inversion to obtain the meso-component parameters.

4.1 Inversion method

This paper inverted the meso-parameters based on the laboratory test results, and optimized them using the genetic algorithm, which was realized by data transfer between MATLAB and finite element software. The realization processes were as follows: ① generating the population of initial parameters through MATLAB, namely the parameters required for mesoscopic numerical simulation; ② uploading the generated data file to the finite element software for experimental simulation analysis, and outputting the numerical simulation results; ③ MATLAB read the numerical simulation result file, and compared it with the laboratory test results. The discriminant equation was:

$$\min J = \sum_{k=1}^N [\sigma_k^j - \sigma_k^m]^2 \quad (17)$$

Where: σ_k^j is the calculated stress value at the k-th load step, σ_k^m is the experimental stress value at the k-th load step, and N is the total load step controlled by the total displacement.

The ending of the iteration depended on whether $\min J$ was infinitely small. If the accuracy met the requirements, the algorithm ended. If failed, re-select and cross-generate a new population, repeat steps ②~③ until the optimal solution is obtained.

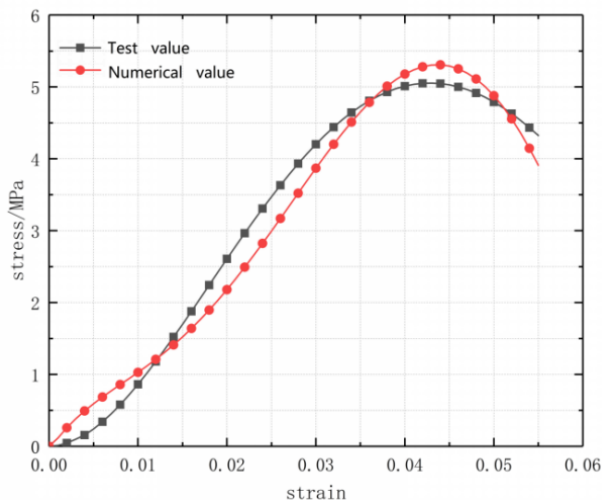
Table 2: CSG micro component parameter table

Meso component	Modulus of elasticity/Mpa	Poisson's ratio	tensile strength/MPa
Aggregate	800	0.15	4.8
Cement mortar	100	0.25	2.8
Interface	50	0.2	1.5

4.2 Verification analysis

This paper used the laboratory uniaxial compression test for verification. In the test, the cement content in the CSG was 70kg/m^3 , the water-binder ratio was 1.0, the aggregate grading was 2, the size of the test block was $100\text{mm}\times 100\text{mm}\times 100\text{mm}$, a universal testing machine and a displacement control method were used, the total displacement was xx, which was loaded by xx steps, and a stress-strain curve was obtained by actual measurement. During analysis of the numerical simulation, the boundary conditions were fully constrained at the bottom and free at the top. When using the genetic algorithm for inverse analysis, the crossover probability was taken as 70%, and the mutation probability 10%.

After 16 iterations, the numerical simulation results of uniaxial compression and laboratory test results were shown in Figure 6. The inverse CSG meso-parameter results were shown in Table 2.

**Figure 6:** Analysis of parameters inversion results

It was found that the meso-parameter values of the CSG were significantly different from the concrete parameter values, and the former was lower, which may generate a great impact on the overall mechanical properties of the

CSG. Therefore, the CSG parameters cannot be speculated according to the concrete parameters.

5 Relationship between meso-components and elastic modulus of CSG and its correction

5.1 Calculation conditions

The effect of meso-components mainly refers to the effect of meso-component parameters and meso-component volumes. Due to the limited test conditions, this paper borrowed the above inversion parameters, used different models to calculate the compressive elastic modulus, and analyzed the mechanism by controlling the volume of meso-components. Because there were less interface volumes, the impact of aggregate volume (aggregate occupancy) was mainly investigated, and the meso-parameters obtained through inversion in 3.2 were substituted into the series model, parallel model, Hashin-Shtrikman upper and lower boundaries, and mesoscopic numerical random aggregate model. The calculation results were shown in Figure 7(a). The values of the aggregate volume ratio of 45% in the results of the four equivalent models were corrected according to the laboratory test values, and the correction coefficient of each model was calculated. The elastic modulus values corresponding to the other aggregate volume values were multiplied by each correction coefficient respectively, the corrected results were shown in Figure 7(b).

5.2 Calculation result and its correction

It can be seen from Figure 7 (a) and (b): (1) The CSG elastic modulus increases with the increase of the aggregate volume fraction. According to the *Technical Guideline for Cemented Granular Material Dams*, this study can provide a basis for the CSG mix design. The reduced elastic modulus of the composite material is beneficial to its structure. Combined with the equivalent theoretical model, the aggregate volume fraction can be reduced, and the optimal value is

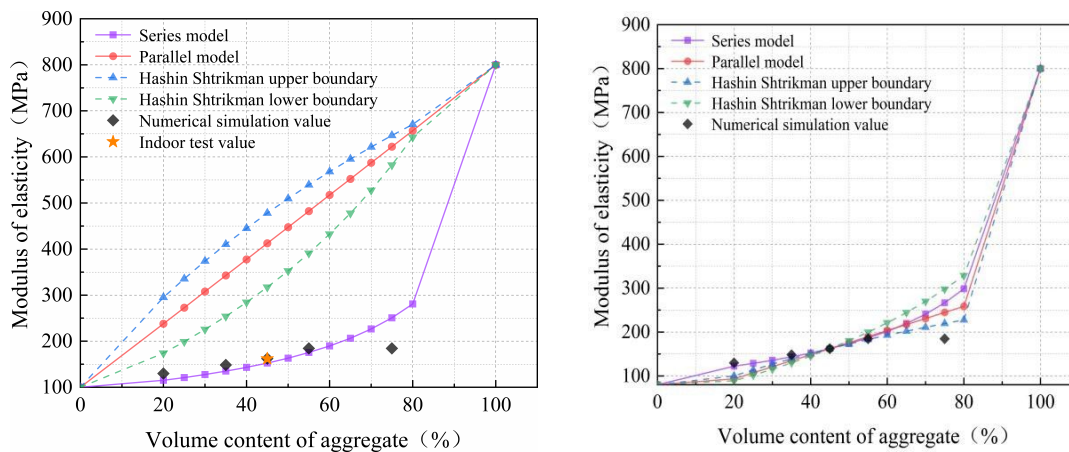


Figure 7: Prediction value of effective elastic modulus of different aggregate volume ratio and its correction

between 35% and 55%. In addition, the non-screening of CSG aggregate causes the aggregate grading less standardized, therefore, the selection of a suitable aggregate volume fraction is also crucial to the study of the aggregate grading; (2) The calculation results of the numerical model are close to the experimental values, which verifies the rationality of the numerical model, and at the same time, they also approximate the results of the series model, indicating that the series model can better analyze and predict the elastic modulus of the CSG and the H-S model [2] is applicable to ordinary concrete. After correction, when the aggregate volume fraction is between 35% and 55%, the results of the prediction model are more consistent; (3) In the calculation of the numerical model, if the aggregate volume fraction exceeds 55%, the overall elastic modulus of the material may change little. As for a composite material, when the aggregate volume fraction reaches the critical value, the material will show the aggregate properties overall.

6 Conclusion

In this study, a CSG macro-mechanical property prediction model considering the impact of meso-components was established. With meso-components taken into account, this paper analyzed the series-parallel model, the Hashin-Shtrikman model and the elastic modulus prediction method based on the mesoscopic random aggregate simulation, as well as the sensitivity of the relevant component parameters, adopted the macro-meso parametric inversion analysis method to determine the relevant component parameters, made predictive analysis and correction

of the elastic modulus at different aggregate occupancies, and drew the following conclusions:

1. The elastic modulus of the aggregate has the greatest impact on the results of the series model, and the elastic modulus of the interface has a greater impact on the results of the parallel model. As for mesoscopic numerical simulation, the interface shows the highest sensitivity to tensile strength.
2. The meso-component parameters of the composite material can be obtained by inversion analysis which, upon experimental verification, provides a theoretical basis for the determination of the theoretical model parameters.
3. The calculation results of the series model and numerical simulation based on the inversion of meso-parameters are close to the test results, and the modified equivalent theoretical model is more consistent with the numerical results when the aggregate volume fraction is between 35% and 55%; as the aggregate occupancy increases, the elastic modulus becomes larger, which can provide both theoretical basis and calculation basis for the mix design.

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