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Optimization of thermal conductivity in composites loaded with the solid-solid phase-change materials

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Abstract: This work focuses on identifying the thermal conductivity of composites loaded with phase-change materials (PCMs). Three configurations are studied: (1) the PCMs are divided into identical spherical inclusions arranged in one plane, (2) the PCMs are inserted into the matrix as a plate on the level of the same plane of arrangement, and (3) the PCMs are divided into identical spherical inclusions arranged periodically in the whole matrix. The percentage PCM/matrix is fixed for all cases. A comparison among the various situations is made for the first time, thus providing a new idea on how to insert PCMs into composite matrices. The results show that the composite conductivity is the most important consideration in the first case, precisely when the arrangement plane is parallel with the flux and diagonal to the entry face. In the present work, we are interested in exploring the solid-solid PCMs. The PCM polyurethane and a wood matrix are particularly studied.

Keywords: arrangement; polyurethane-wood composite; solid-solid phase change material; thermal conductivity.

1 Introduction

The use of phase-change materials (PCMs) has become an important research topic because of two main reasons: energy and environmental issues are becoming increasingly sensitive and the requirement of realizing interior comfort in the field of thermal regulation. These smart materials have appeared on the construction market as a first step to reduce the need for air conditioning during hot periods [1–3]. PCMs can improve the energy performance of the envelope while increasing thermal inertia inside buildings and in products based on polymer, plasters, or concrete. The market development of phase change composites is promising because of their thermo-physical properties, which can be further explored for the storage/retrieval of energy [4, 5]. Therefore, it is essential to have theoretical, numerical, and experimental tools to characterize these materials. The thermophysical properties of these composites depend on many factors, such as the conductivities of the constituents, load rate, microstructure, and so on [6–8]. The control of these properties should allow the determination of optimum conditions for manufacturing and analyzing heat transfer in various industrial applications (electronic circuits).

This work focuses on the thermal analysis and identification of the thermophysical parameters of the phase change composites, thus gaining better insights on the thermal behavior and the mechanism of the conductive transfer of such materials. A numerical simulation of the thermal behavior of the PCMs is conducted and then subected to analytical analysis. We are particularly interested in the configuration geometry of PCMs in a cubic matrix. Two- and three-dimensional configurations are studied. In the first case, four arrangements are envisaged: two in the frontal planes with respect to conductive heat flux and two others on parallel planes to this flux. The PCMs are distributed as identical spheres centered on the same plane of arrangement. In each case, a compact limit case is examined where the PCM is spread in the shape of a plate. In the 3D case, several theoretical and empirical approaches have been employed to analyze the thermal conductivity of composites, as described in the literature [9, 10]. Here, a periodic arrangement of the PCM spheres is considered. A new simple method of calculating is proposed. A comparison among the different configurations is performed. This is achieved by keeping the same boundary conditions adopted for the composite material and maintaining the same volume percentage PCM-composite at 20%.

In the present work, we are interested in the solid-solid PCMs, which according to recent studies, may have a higher latent heat than that of solid-fluid PCMs [1, 11–15]. Aside from this capital property, the mechanical problem called “material fatigue,” which is caused by the alternating volume change of the solid-fluid PCM type during the

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melting and solidification processes added to the supercooling phenomenon, make the study of solid-solid PCMs primordial and their use in electrotechnical field possible. The PCM hyperbranched polyurethane copolymer (HB-PUPCM) with a latent heat equal to 138.2 kJ/kg [11, 16] and wood matrix are studied. The conductivity of this PCM is conserved during its phase change. The numerical calculation of the thermal conductivity is obtained using CFD code Fluent [17]. Thus, the variation of conductivity is studied versus several parameters, such as the plane of arrangement (parallel or frontal) to the heat flux, geometry of the PCM (sphere, plate), and finally, the volumetric distribution of the spheres within the matrix.

2 Mathematical formulation of the problem

The composite material consists of PCM distributed as identical spheres centered on the same plane of arrangement and inserted into a matrix. By way of simplification, the matrix has a rectangular parallelepiped shape (cube), and it is assumed that the PCM represents the volume fraction f_v of the composite material while the matrix occupies the rest.

2.1 Preliminary study

First we start by calculating the radius of these spheres. The condition on the composition in volume led to the relationship (Eq. 1) shown below.

$$V_2 = f_v V_1 \tag{1}$$

This relationship gives us the radius of the equal-sized spheres that have been inserted.

$$r_n = \sqrt[3]{\frac{3\times f_v}{4\pi n}} \times a \tag{2}$$

2.1.1 Two-dimensional arrangement

The PCM spheres are centered on a same single insertion plane. To prevent the spheres from overlapping, the representative surface of their intersections with the insertion plane (cross section) must be smaller than the total surface of this plane (Figure 1). This relationship is expressed mathematically as follows (Eqs. 3a and b) :

$$\begin{cases} n\pi r_n^2 \le a^2 & \text{for mediator plane} \\ n\pi r_n^2 \le \sqrt{2}a^2 & \text{for tilted plane} \end{cases} \tag{3a, b}$$

This condition is necessary but not sufficient. Although it provides information on the maximum number of spheres that can be inserted, it should be checked further whether the diameters of the spheres do not exceed the edge or the diagonal of the insertion plane cited above. This last condition cannot be predicted but must be checked case by case in accordance with the number of spheres.

Cubic matrix is of edge $a = 10$ cm. The radius of the integrated spheres is a function of their number n. This number n is not infinitely possible, but its maximum value obviously varies with the percentage and according to the position of the plane of arrangement, either a mediator of one of the edges (Figure 1A and B), or it contains the diagonal of the entry face (Figure 1C and D). For the sake of comparison, n is limited to number four; maximum number available without overlapping spheres for the mediator planes of edges for these geometrical forms of the matrix and the PCM and the percentage of composition 20%.

2.1.2 Three-dimensional arrangement

The volumetric distribution of spheres in the composite is assumed to be uniform, and thus, periodic (Figure 2). Hence, we only studied the simple cubic lattice structure. Conductivity is an intensive property; the calculation of

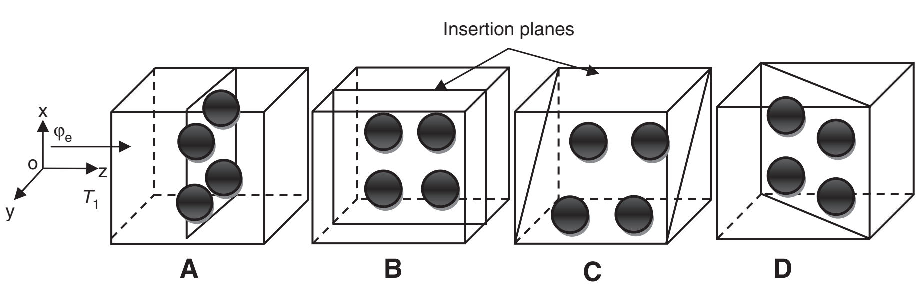

Figure 1: Configuration of spheres. (A) perpendicular plane frontal to the flux, (B) plane parallel to the flux, (C) tilted plane parallel to the flux, (D) tilted plane frontal to the flux.

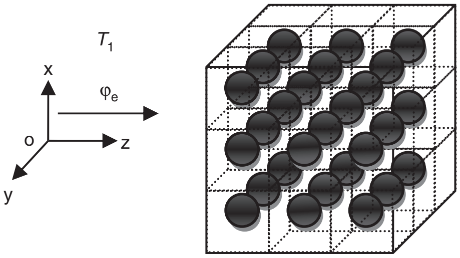

Figure 2: Three-dimensional periodic arrangement of the spheres in the composite.

equivalent thermal conductivity is restricted, by reason of symmetry, to that on an edifice containing only one sphere (elementary cell). This cubic elementary cell is of elementary volume V_{elem} (Eq. 4), whose edge a^* (Eq. 5) should not be exceeded by the diameter of the sphere, which is allowed as much as the percentage PCM/material does not rise above 52.3%. The possible number n is unlimited in this case (e.g. 1, 8, 27, 64…etc).

$$V_{\text{elem}} = a^3 / n \tag{4}$$

$$a^* = a / n^{1/3} \tag{5}$$

2.2 Equation of heat transfer in the composite

Our system consists of a material of important thermal inertia, where a second material inserted can change its physical status in the temperature range studied. The composite material is subjected to a heat flux through one of its faces. The study of heat transfer of a physical problem requires the resolution of the energy equation which makes it possible to obtain the space-time evolution of the temperature and heat flux. In this work, the study is limited to the steady case and to the solid-solid PCM. The energy balance (Eq. 6) allows us to consider the equation of heat (Eq. 7) valid for each phase [18].

$$\varphi_e + \varphi_g = \varphi_s + \varphi_{st} \tag{6}$$

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right) + \dot{q} = \rho C \frac{\partial T}{\partial t} \tag{7}$$

In each presumedly isotropic component i of the composite, this equation can be simplified in equation (Eq. 8) by considering that there is no generation of energy within the material while being interested in the steady state.

$$\vec{\nabla}(\lambda_i \vec{\nabla} T) = 0 \tag{8}$$

2.3 Boundary conditions

The conductivity of a material is an intrinsic characteristic of that material and the function of its temperature. Thus, it is necessary to limit the interval of temperature in the vicinity of that state change in order to consider constant conductivity. The latter is independent of the nature of the conditions imposed on the material surfaces, such as the flux, the temperature, or both. In our study, the following assumptions of simplification are considered: the thermophysical characteristics are constant; the entry face and the exit face of the cube are maintained at temperatures T_1 and T_2, respectively, and assumd to be uniform and constant; and the side faces are imposed on a null flux. Inside the composite material, a perfect contact between the two solids is assured.

2.4 Calculation of equivalent conductivity

Generally, the analytical calculation of the conductivity of composite materials is not possible because of its complex geometry. Hence, it must be calculated numerically. In this work, we are interested in the materials that undergo a solid-solid transition; therefore, the transfer is purely conductive. The difference between the thermal expansion coefficients of wood and polyurethane is very weak and of a few tens of μ°C^{-1}. In addition, the study interval ΔT (≈25°C) on both sides of the phase transition temperature is small, thus making the difference in expanded volumes negligible. Consequently, the interface thermal resistance is neglected. The heat transfer with outside through side surfaces is maintained null. The heat density flux is proportional to the thermal conductivity of the medium and the temperature gradient according to the Fourier analysis (9), which gives the expression of conductivity (10) below.

$$\psi = -\lambda \frac{\partial T}{\partial z} \tag{9}$$

$$\lambda = \frac{\varphi a}{(T_1 - T_2) S} \tag{10}$$

3 Numerical resolution

The resolution of the equations governing the field of temperature is performed using the finite volume method. The grid used is not uniform and is tight at the level of spheres so that their low sizes can be accommodated when n is high;

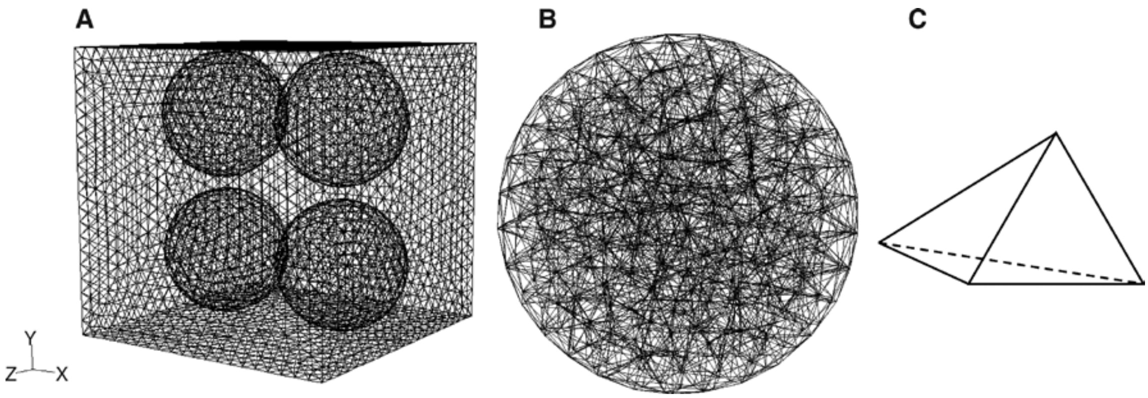

Figure 3: Grid of the composite material. (A) grid of the matrix faces and the interfaces of the PCM spheres for a mediator plane, (B) grid 3D of a PCM sphere, (C) tetrahedral cell.

it is also tight at the level of the matrix when the spheres are near the external faces of the matrix (walls). The equations were integrated on a volume of control $\Delta x \times \Delta y \times \Delta z$. To solve the problem, Fluent CFD code was used.

The grid, carried out on Gambit [17], consisted of two zones (Figure 3A and B, respectively):

– Zone 1, material portion matrix cubic hollow with a 10-cm edge, and is represented by an unstructured grid formed by tetrahedral cells with a distance of 5 mm between nodes and diameter of spherical cavities given by r_n (Eq. 2).
– Zone 2, the material portion of the PCM; n spheres of diameter r_n, represented by an unstructured grid formed by tetrahedral cells also, with a distance of 1–5 mm between nodes.

In this study, the complexity of the geometry prevented us from using a structured grid. We used only one type of unstructured grid where the cells are tetrahedral (Figure 3C). The grid is cubic and has a 100-mm edge. For example, the grid is made up of 56,432 cells and 12,029 nodes in the case of four spheres located on a mediator plane.

4 Results and discussion

4.1 Two-dimensional case

With a percentage of 20%, a matrix of wood whose thermal conductivity is equal to $\lambda_b = 0.173$ W/m°C and a PCM of polyurethane of thermal conductivity $\lambda_p = 0.032$ W/m°C are examined. The latter is putrefied in the form of spheres arranged in only one plane of insertion (Figure 1). Two cases of planes are studied: frontal planes to the heat flux along axis (Oz); [plane ($z=0$) and plane ($y=z$)], and planes parallel to this flux [plane ($y=0$) and plane ($x=y$)]. We impose a uniform inlet temperature, constant and equal to 70°C and a temperature equal to 20°C to the face of exit. The heat transfer with outside through the side surfaces is supposed to be null $\varphi = 0$. We handled a numerical simulation code to solve the heat transfer equation. Doing so enabled us to determine the evolution of the temperature and of the heat flux and thus deduce thermal conductivity.

Figure 4 describes the variation of thermal conductivity versus the number of spheres. This figure shows that thermal conductivity is a decreasing function with the number of spheres in the case of the frontal plane, whereas it is increasing in the case of the parallel plane. This figure also shows that the distribution of the PCM

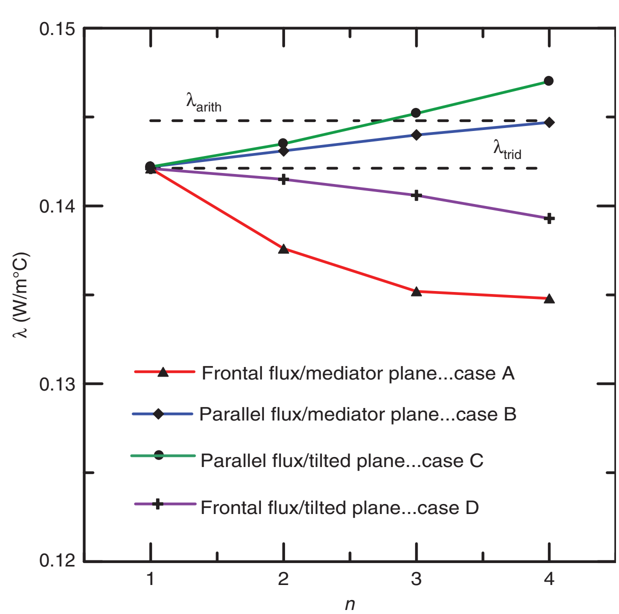

Figure 4: Thermal conductivity versus the number of spheres.

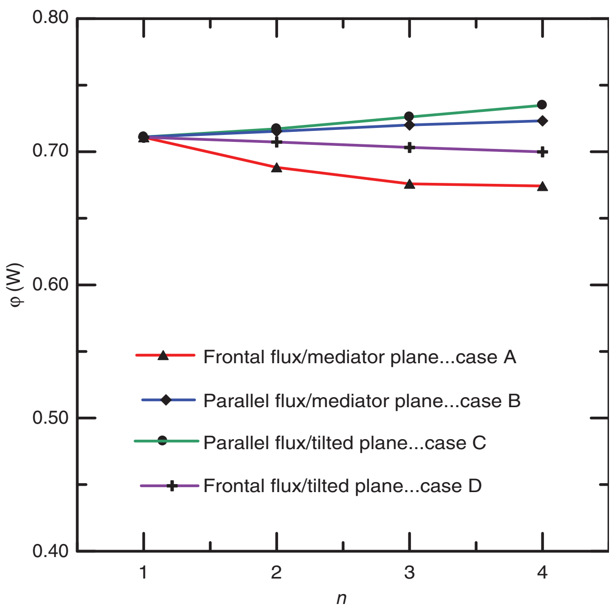

Figure 5: Thermal flux versus the number of spheres.

spheres according to a plane of insertion is tilted exposed in a side way to the heat flux, thus having the advantage of improving the equivalent thermal conductivity of the composite. In comparison, this conductivity decreases quickly when the distribution is according to a mediator plane of edges exposed perpendicular to the heat flux.

Thermal conductivity is more important in the case of flux parallel and precisely when the plane of insertion is tilted – the only case where it exceeds the arithmetic conductivity λ_{arith} of composite material, which is equal to 0.1448 W/m°C (Eq. 11). This arrangement is thus the most effective and most useful in maintaining a material composite of conductivity close to its matrix while preserving its equivalent heat capacity.

$$\lambda_{\text{arith}} = (1-f_v)\lambda_1 + f_v\lambda_2 \tag{11}$$

These results remain valid as long as the PCM has a thermal conductivity that is weaker than that of the matrix. This situation is the most widespread in nature, while the weakness of the thermal conductivity of the PCMs materials is one of the handicaps of their industrial use [19–25]. Figure 5 shows that thermal conductivity and the heat flux evolve identically towards the rise of the number of spheres. These curves also indicate the anisotropy of the composite material obtained; consequently, crossing flux is no longer the same one and now varies according to the choice of the entry face.

The borderline case of the distributions of spheres, which is a plate of PCM inserted in the matrix into the level of the plane of arrangement, is studied (Figure 6). The numerical results are shown in Table 1. Otherwise, by electric analogy, the thermal conductivity of this composite material is analytically obtained (Eqs. 12–14). They correspond to the cases A, B and C, respectively, as illustrated in Figure 6:

$$\lambda = \frac{a\lambda_1\lambda_2}{2L_1\lambda_2 + L_2\lambda_1}, \tag{12}$$

$$\lambda = \frac{2L_1\lambda_1 + L_2\lambda_2}{a}, \tag{13}$$

$$\lambda = \frac{2\lambda_1\left(a-(d-b)/\sqrt{2}\right)^2 + \lambda_2(d^2-b^2)}{2a^2}, \tag{14}$$

with

$$\begin{cases} b = \sqrt{1.6a} \\ d = \sqrt{2a} \end{cases}, \tag{15a, b}$$

which is simplified as follows:

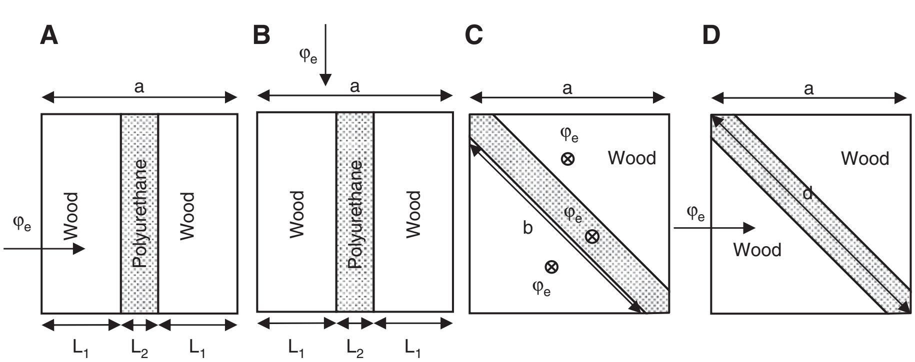

Figure 6: Different configurations of the PCM plate. (A) perpendicular plane frontal to the flux, (B) plane parallel to the flux, (C) tilted plane parallel to the flux, (D) tilted plane frontal to the flux.

$$\lambda = 0.8\lambda_1 + 0.2\lambda_2. \tag{16}$$

For the situation (D), the analytical calculation of λ based on the electric analogy is no longer valid, and the concept of association in derivation and series is lost. As the medium is not homogeneous neither on the level of all materials nor on the level of each crossed section, the applicable formula is as follows:

$$R_{th} = \int_0^a \frac{dz}{\lambda(z)S(z)} \tag{17}$$

The calculation of this situation (D) lead us to the following expression:

$$\lambda = 1\Bigg/\left(\frac{2}{\lambda_1-\lambda_2}\ln\left|\frac{\lambda_1(a-c)+\lambda_2 c}{\lambda_1(a-2c)+2\lambda_2 c}\right| + \frac{a-2c}{\lambda_1(a-2c)+2\lambda_2 c}\right), \tag{18}$$

Table 1: Comparison between the numerical model and the analytical calculation.

		Case A	Case B	Case C	Case D
Numerical results	φ (W)	0.47	0.6936	0.7262	0.5586
	λ (W/m°C)	0.0940	0.1387	0.1456	0.1117
Analytical results	λ (W/m°C)	0.0919	0.1448	0.1448	0.1191
Relative error on λ (%)		2	4	0.3	6

with

$$c = (d-b)\sqrt{2}. \tag{19}$$

This last result is valid for $c < a/2$ (i.e. $f_v < 0.75$). If $c > a/2$, the calculation of this situation (D) leads us to the following modified expression:

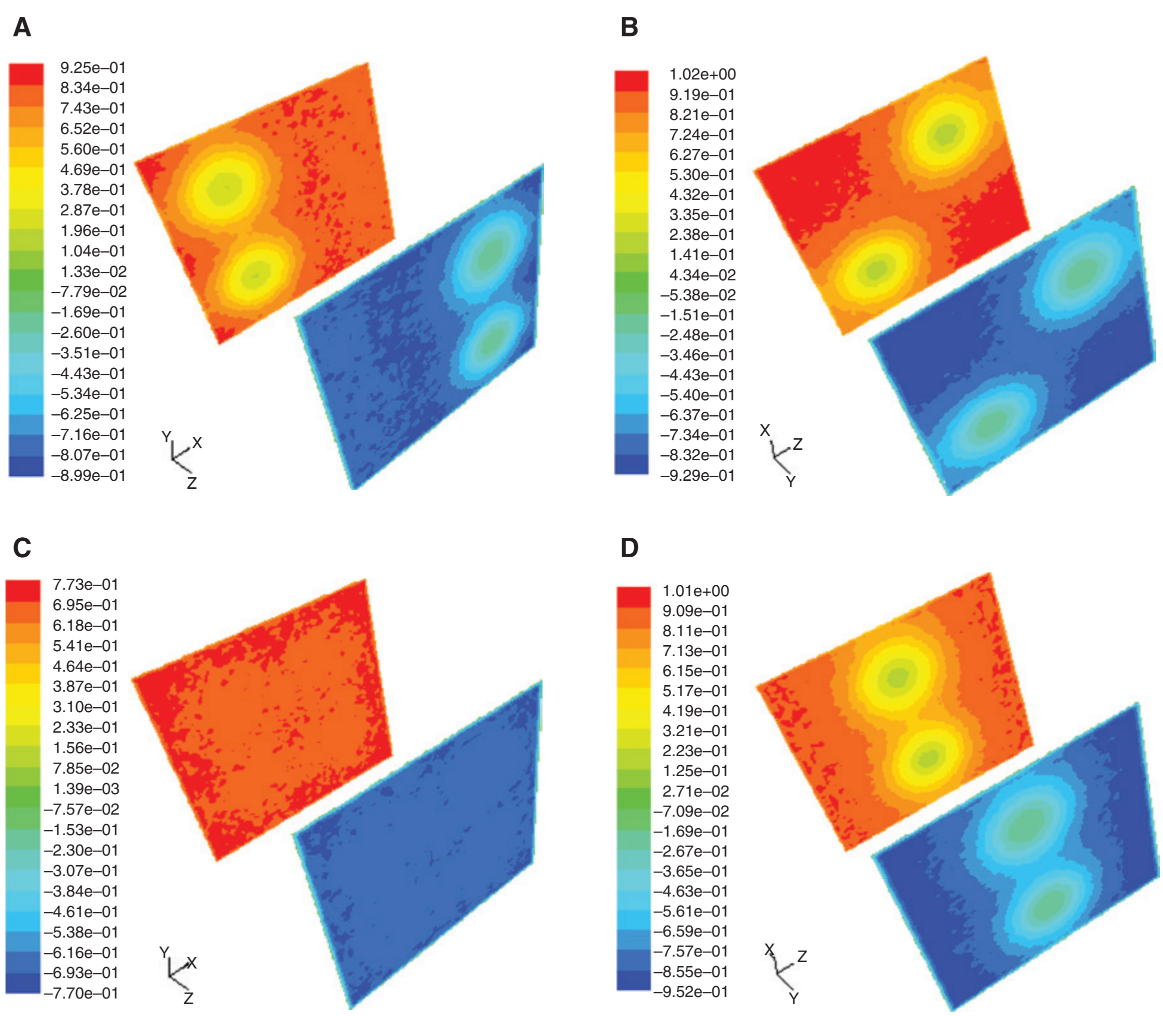

Figure 7: Flux density contours of composite wood/polyurethane on the levels of the entry and exit faces. Tilted insertion plane ($x=z$) case crossed by: (A) frontal heat flux along (Oz), (B) parallel heat flux along (Oy); mediator plane ($z=0$) case crossed by: (C) frontal heat flux along (Oz), (D) parallel heat flux along (Oy).

$$\lambda = 1\Big/\left(\frac{2}{\lambda_1-\lambda_2}\ln\left|\frac{\lambda_1(a-c)+\lambda_2 c}{\lambda_1(a-2c)+2\lambda_2 c}\right| + \frac{2c-a}{\lambda_2 a^2}\right). \quad (20)$$

These thermal conductivities, which are analytically calculated, are very close to the numerical results with relative errors less than 6% made by numerical modeling (Table 1). The limits of these functions (in kind of materials and in volume fractions), which are obtained analytically, are checked for each situation quoted above.

It is noted that for the situations (B) and (C) of Figure 6 (analoguous with the parallel resistances) λ is the same regardless of the configuration of the plate of PCM. Moreover, it is maximal and equal to arithmetic conductivity λ_{arith} (Eqs. 13 and 16 and Table 1). This is coherent with the result found for the discontinuous distribution of the PCM spheres, as illustrated in Figure 1.

For calculation with the Fluent code, in order to pass from the case of the frontal flux to the parallel flux case, the position of the insertion planes is preserved whereas the flux is carried by axis (Oy) instead of axis (Oz). We represent below, for example, the contours of the flux in the case of PCM distributed in four spheres. Figure 7 shows that the propagation of heat is more important when the flux is parallel (Figure 1A and B) than in the case of a frontal flux (Figure 1C and D) compared with the distribution plane of the PCM spheres, thus reflecting obviously on the thermal conductivity in a proportional way. We also note that the flux density is not uniform and is more intense in the area devoid of the PCM spheres.

4.2 Three-dimensional case

Here, we consider a composite material consisting of a periodic array of identical spheres of radius r_n and conductivity λ_2 embedded in a uniform matrix of conductivity λ_1, in which no thermal resistance exists at the interface between the two phases.

Let

$$R^*_{th} = 2R^*_1 + R^*_2 \quad (21)$$

be the thermal resistance of the composite material. Below, R^*_1 is the thermal resistance of the matrix of thickness L^*_1, and R^*_2 is the thermal resistance of the part of the composite material of thickness $2r_n$ (Figure 8), including the PCM sphere.

$$R^*_1 = \int_{r_n}^{a^*/2} \frac{dz}{\lambda_1 a^{*2}} \quad (22)$$

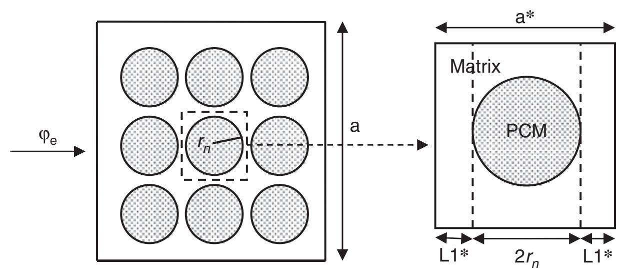

Figure 8: Elementary cell.

$$R^*_1 = \frac{a^* - 2r_n}{\lambda_1 a^{*2}} \quad (23)$$

$$R^*_2 = \int_{-r_n}^{r_n} \frac{dz}{\sum_{i=1}^{i=2} \lambda_i(z) S_i(z)} \quad (24)$$

$$R^*_2 = \int_0^{\pi} \frac{r_n \sin\theta d\theta}{\lambda_1 a^{*2} + \pi(\lambda_2-\lambda_1) r_n^2 \sin^2\theta} \quad (25)$$

$$R^*_2 = \int_{-1}^{1} \frac{r_n du}{\pi(\lambda_1-\lambda_2)\left(\frac{\lambda_1 a^{*2}}{\pi(\lambda_1-\lambda_2)} - r_n^2 + u^2\right)} \quad (26)$$

Let us suppose

$$A = \left|\frac{\lambda_1 a^{*2}}{\pi(\lambda_1-\lambda_2) r_n^2} - 1\right|^{1/2}. \quad (27)$$

Thus,

$$R^*_2 = \begin{cases} \dfrac{2}{\pi(\lambda_1-\lambda_2) A r_n} arctg(1/A) & \text{si } \lambda_2 < \lambda_1 \\ \dfrac{1}{\pi(\lambda_2-\lambda_1) A r_n} \ln\left|\dfrac{1+A}{1-A}\right| & \text{si } \lambda_2 > \lambda_1 \end{cases}, \quad (28a, b)$$

This calculation remains valid when the volume fraction does not exceed the limiting value corresponding to $2r_n \le a$ and results in $f_v \le 0.52$.

Finally, we arrive at

$$\lambda = \frac{1}{a^*(2R^*_1 + R^*_2)}. \quad (29)$$

Given that A is independent of n, and R^*_1 and R^*_2 are proportional to $n^{1/3}$, whereas a^* varies as a function of $n^{-1/3}$, therefore, λ is independent of n. Thus, for conductivity, the distribution of PCM either in a single sphere or in a periodic distribution of multiple spheres is similar.

If this result is applied to the percentage studied (20%), we can find that the equivalent thermal conductivity of the composite material wood-polyurethane when it is modelled into a 3D periodic sample is $\lambda = 0.1412$ W/m°C. This result is in good agreement with that found by numerical calcul ation $\lambda_{num} = 0.1422$ W/m°C with a negligible relative error of 0.7%. By comparison with the models treating the composites with isotropic components and the random distribution of spherical inclusions proposed by Eshelby [26] (Eqs. 30–32), we have

$$\lambda = \lambda_1 + f_v(\lambda_2 - \lambda_1)A_f, \tag{30}$$

where

$$A_f = 1/(1+(\lambda_2 - \lambda_1)P), \tag{31}$$

and

$$P = 1/3\lambda_1, \tag{32}$$

which predict $\lambda_{Esh} = 0.1343$ W/m°C and that of Nan [27]. In addition, $\lambda_{Nan} = 0.1369$ W/m°C, which proposes the expression given by the equation (Eqs. 33 and 34), where the interfacial thermal resistance R is zero:

$$\lambda = \lambda_1 \frac{1+2A'f_v}{1-A'f_v}, \tag{33}$$

where

$$A' = \lambda_1 \frac{1-(\lambda_1/\lambda_2 + R\lambda_1/r_n)}{1+2(\lambda_1/\lambda_2 + R\lambda_1/r_n)}. \tag{34}$$

Compared with the random distribution, the periodic distribution with a simple cubic structure improves the conductivity of the composite by almost 4.9%. Moreover, Figure 4 and Table 1 show that the 3D configuration of the PCM spheres has a thermal conductivity whose intermediate value is between those of the parallel planes and those of the planes frontal to flux. Otherwise, it is noted that, in practice, the equivalent conductivity of a composite material can be assimilated to the arithmetic conductivity (Figure 4) with a relative error of 2.54%.

5 Conclusion

In the present work, a theoretical and numerical thermal analysis enabled us to investigate the thermal behavior and conductive transfer mechanism of the composites. The relations giving thermal conductivity in 2D and 3D arrangements are established and are deemed valid for any biphasic solid-solid composite. The variation of conductivity according to several parameters, such as the arrangement plane (parallel or frontal) to the heat flux, the PCM geometry (sphere, plate), is identified. The results highlight the influence of the number of the spheres introduced into the studied composite. Meanwhile, we also studied the influence of the incidence angle of flux, compared with these planes, on the thermal behavior of composite. The results obtained indicate that the ideal composite conductivity is obtained if the load PCM is divided into multiple spheres arranged in a plane parallel with the flux and, particularly, if it contains the diagonal of the entry face. In addition, for conductivity, the PCM distribution is similar either in a single sphere or in a periodic distribution of multiple spheres. A good agreement between the theoretical results and the numerical results is found for all the cases where theoretical analysis is possible. Similarly, a coherence is also observed between the results of the 3D configuration and previous works.

Nomenclature

a	cube edge, m
a^*	edge of elementary cell, m
C	thermal capacity, J/kg°C
f_v	volume fraction of PCM, $f_v = V_2/V_1$, dimensionless
n	number of spheres, dimensionless
L_i	length of part i of the matrix, m
L_i^*	length of part i of the elementary cell, m
$\dot{q}$	density of energy generated, W/m^3
r_n	radius of spheres, m
R_i^*	thermal resistance of part i of the elementary cell, °C/W
R_{th}	thermal resistance of composite, °C/W
S	surface crossed by the heat flux, m^2
t	time, s
T	temperature on point $\vec{r}(x, y, z)$ at instant t, °C
T_1	temperature of the entry face, °C
T_2	temperature of the exit face, °C
V_1	total volume of the composite, m^3
V_2	total volume of PCM, m^3
V_{elem}	volume of elementary cell, m^3
x, y, z	space variables, m

Greek symbols

λ	equivalent thermal conductivity of the composite material, W/m°C
λ_1	thermal conductivity of matrix, W/m°C
λ_2	thermal conductivity of PCM, W/m°C
λ_{arith}	arithmetic conductivity of composite, W/m°C

λ_b	thermal conductivity of the wood, W/m°C
λ_p	thermal conductivity of polyurethane, W/ m°C
$\lambda_x, \lambda_x, \lambda_z$	thermal conductivity components, W/m°C
φ or φ_e	incoming heat flux, W
φ_g	heat flux generated, W
φ_s	Leaving heat flux, W
φ_{st}	heat flux stored, W
ρ	density, kg/m^3
ψ	flux density, W/m^2
θ	spherical coordinate, radian

Subscript

arith	arithmetic
Esh	Eshelby
Nan	Nan
Num	Numerical
trid	tridimensional

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