DE GRUYTER DOI 10.1515/secm-2012-0120 — Sci Eng Compos Mater 2013; 20(2): 179–185

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Effects of cutouts on natural frequency of laminated composite plates

Abstract: In this study, the effects of cutouts (circular, square, triangular and elliptical), cutout position and fiber orientations of laminated composite plates on natural frequencies are investigated. In order to study the effects of cutouts on natural frequencies of the composite plates, experiments have been carried out. Finite element analysis is also performed to predict the effects of different cutouts, cutout position, fiber orientation and length of the plate on natural frequencies for E-glass/polyester. Several outcomes and behavioral characteristics are discussed. These outcomes include the cutout shape, cutout orientation and fiber orientation angle. It has been seen that all of the experimental and finite element results are very close to each other. It has also been concluded that the natural frequency of composite plate is affected by the fiber orientation and cutout location but not considerably affected by cutout shape. The results show that fiber orientation angle and cutout location are the most important parameters on the natural frequency. When the fiber orientation angle increases, both bending and twisting natural frequencies decrease. Maximum natural frequency occurs on $[(0/90)_4]_S$ laminated plate.

Keywords: cutout; laminated composite plates; natural frequency.

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1 Introduction

Fiber reinforced composite materials are very important in engineering applications due to their excellent features such as high strength-to-weight and high stiffness-to-weight ratios. Thus, they are mainly used in civil, marine and the aerospace industry. In a structural design, elimination of resonant frequency is very important as resonance effects can induce failure of constructed structures. Prediction of natural frequency and working optimum frequency in structures is therefore required.

Cutouts are mainly used in practical applications such as damage inspection, altering the natural frequency, and access ports for mechanical and electrical systems. Natural frequency and damping properties of composite structures have been investigated by many researchers [1–16]. Chandrashekhara et al. [1] presented free vibration of symmetrically laminated composite beams and obtained exact solutions for free vibration. In calculations, rotary inertia and first-order shear deformation effects were considered for the analysis. Qatu [2] studied symmetrically laminated composite plates and performed effects of various parameters such as material, fiber orientation and boundary conditions on the natural frequencies and mode shapes. Narita and Leissa [3] performed free vibration of cantilever and rectangular angle-ply and cross-ply laminated composite plates. Natural frequencies were calculated using Ritz method, and different parameters such as material and fiber angle were considered. Huang and Sakiyama [4] developed a function for free vibration of rectangular plates with different cutout shapes such as circular, rectangular and triangular. In order to calculate free vibration of rectangular plates with an arbitrarily located hole with different shapes, discrete solution was proposed. In discrete solution, the Green functions were used to transform a free vibration problem into the eigenvalue problem. Khdeir and Reddy [5] developed a complete set of linear equations for free vibration of cross-ply and angle-ply composite plates using second-order theory and obtained exact analytical solutions for moderately thick and thin plates. Turvey et al. [6] investigated effects of anisotropy, hole size ratio and boundary conditions on the natural frequency of square pultruded glass reinforced plastic plates. Clamped, simply supported and free edge supports were considered as boundary conditions. Vibration experiments were done on plates with central circular cutouts. Experiment and finite element mode shape results were in accordance with each other. Liew et al. [7] analyzed free vibration of rectangular plates with central rectangular cutouts. Combinations of simply supported and clamped boundary conditions were used. Won and Sung [8] presented free and forced vibration analysis of laminated composites. Assumed strain method and constitutive equation in the finite element of composite plates were used in the study. Exact solutions were presented to show application of formulation to rectangular isotropic and rectangular composite plates for free vibration. Aydogdu [9] studied vibration analysis of angle-ply laminated beams with different boundary conditions. Combinations of free, clamped and simply supported edge conditions were considered. Free vibration frequencies were obtained using the Ritz method with a 3 degrees of freedom shear deformable beam theory. Moon and Sangbo [10] analyzed vibration analysis of rectangular plates using the coordinate coupling method. Analytical results were compared with finite element and experimental results for rectangular plates. Mohammed et al. [11] investigated the dynamic behavior of composite beams using the finite element method.

As seen from the above literature, free vibration analysis of composite plates has been studied by many researchers using the finite element method. Furthermore, circular central cutout effect and the fiber orientation angle have been studied, but the combination of variables considered during previous studies is still limited. In this study, effects of fiber orientation, different cutout shapes (circular, square, triangular and elliptical), cutout orientations, cutout sizes and length of the plate on the natural frequency of laminated composite plates are taken into consideration. Cantilever boundary condition is considered. ANSYS package program (ANSYS, Inc., Berkeley, USA) is used in the numerical studies.

2 Mathematical formulation

If the length of the plate is so much greater than other dimensions, the composite plate can be considered as a thin beam [12]. In this study, dimensions of plates are considered as 150 mm long (L), 20 mm wide (W) and 1.6 mm thick (t) with eight layers as shown in Figure 1.

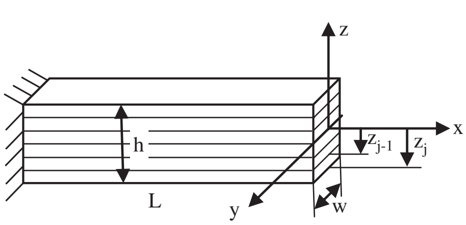

Figure 1 Cantilever laminated composite beam.

The Euler-Bernoulli beam theory is used in formulations and calculations. It is assumed that plane cross sections remain plane and normal to the centerline after the deformation and both rotary inertia. Shear deformation effects are neglected.

The Euler equation for a beam can be written as follows [12].

$$EI\frac{\partial^4 w}{\partial x^4}-\rho A\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

$w(x,t)$ can be written as another form by separation of the variable

$$w(x,t)=X\phi \tag{2}$$

where $X=X(x)$ and $\phi=\phi(t)$. Substitution of Eq. (2) into Eq. (1) yields

$$EIX^{IV}\phi+\rho AX\phi'' \tag{3}$$

Eq. (3) can be written as

$$\frac{X^{IV}}{X}=-\left(\frac{\rho A}{EI}\right)\frac{\phi''}{\phi} \tag{4}$$

When Eq. (4) is separated by two parts

$$X^{IV}-\alpha^4 X=0 \tag{5}$$

$$\phi''+\omega^2\phi=0 \tag{6}$$

where $\omega^2=\frac{\rho A\alpha^4}{EI}$ and α is a constant.

2.1 Boundary conditions of cantilever beam

For clamped sides:

$$\text{At } x=0,\ w=0,\quad \frac{\partial w}{\partial x}=0 \tag{7}$$

For free sides:

$$\text{At } x=L,\quad \frac{\partial^2 w}{\partial x^2}=0 \quad\text{and}\quad \frac{\partial^3 w}{\partial x^3}=0 \tag{8}$$

Eqs. (7) and (8) boundary conditions subjected to Eq. (2) yield

$$\beta^2=(\alpha L)^2=\omega L^2\sqrt{\frac{\rho A}{E_x I_{yy}}} \tag{9}$$

and

$$\cos(\alpha L)\cosh(\alpha L)+1=0,\ \beta=\lambda^4 \tag{10}$$

Eq. (10) is solved using Mathematica, and non-dimensional frequency parameters are obtained (Table 1).

Beam	$\alpha_1 L$	$\alpha_2 L$	$\alpha_3 L$
Clamped-free	1.875	4.694	7.855

Table 1 Non-dimensional frequency parameters (β) of cantilever beam.

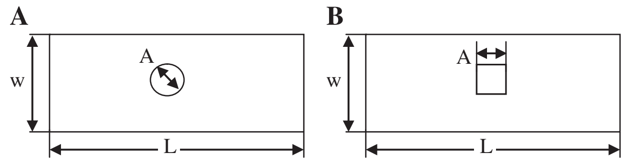

Figure 3 Laminated composite plate with cutout. (A) Circular cutout, (B) rectangular cutout.

3 Material properties

In this study, woven glass polyester was used to produce laminated composite plates. Mechanical properties of the plates were obtained using Shimadzu AG-X series tensile test machine (Shimadzu Corporation, Kyoto, Japan) as listed in Table 2. The composite plies were laid up to form 8-ply laminates having $[\theta]_8$ stacking sequences, and dimensions of plates were taken as 150 mm×20 mm×1.6 mm (Figure 2). Fiber orientations were chosen as $[(0/90)_4]_S$, $[(45/-45)_4]_S$ and $[(30/-60)_4]_S$. Circular and rectangular types of cutout were prepared for the experiments. For each fiber orientation, nine test samples were prepared for each cutout type; one out of three of them had a central circular cutout of 10 mm diameter (A) and another one out of three had a square cutout of 10 mm side length (A) as shown in Figure 3. In total, 27 specimens were prepared for the experiments.

4 Experimental modal analysis

An accelerometer (Brüel & Kjær Sound & Vibration Measurement A/S, Nærum, Denmark) and data acquisition card (NI USB-6009, National Instruments Corporation, Austin, USA) were used in the experimental setup as shown in Figure 4. One end of the specimen was fixed in a rigid support while the other end was free to vibrate. The 20 mm initial displacements were given as an input and signals were captured by an accelerometer. Captured voltage signals were transformed to the frequency signals using fast Fourier transform using the MATLAB program. $[(0/90)_4]_S$, $[(30/-60)_4]_S$ and $[(45/-45)_4]_S$ cross-ply laminated composite plates without cutout (w-c), with square cutout (s-c) and circular cutout (c-c) were considered in the modal analysis.

E_1=E_2 (GPa)	ν_{12}=ν_{21}	G_{12}=G_{21} (GPa)
20.50	0.21	3.56

Table 2 Mechanical properties of specimens.

Figure 2 Laminated glass-polyester composite plates.

5 Numerical studies

Natural frequencies of the composite plates were performed using the ANSYS12.0 finite element analysis program. During the analyses, the effects of circular, triangular, square and elliptical cutouts on natural frequency were investigated. In the analysis, five different fiber orientation angles, ten different widths over cutout length ratio, seven lengths over width ratio and five modes of frequency were used. Length (L) and width (w) of the plate were taken as 150 mm and 20 mm, respectively. The thickness of the plate was taken as 1.6 mm and the number of layers was taken as 8.

In order to inspect the twisting frequency modes of the composite plate, SHELL91 element type with 250/30 division is selected. SHELL91 [17] element geometry illustrated in Figure 5 can be used for layered applications to

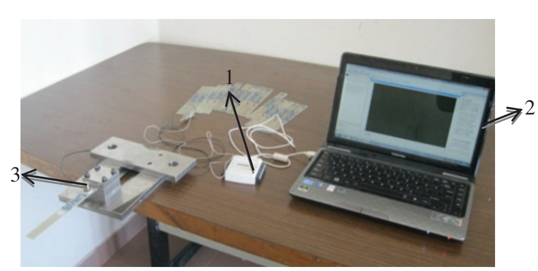

Figure 4 Experimental setup. (1) Data acquisition card, (2) computer, (3) accelerometer.

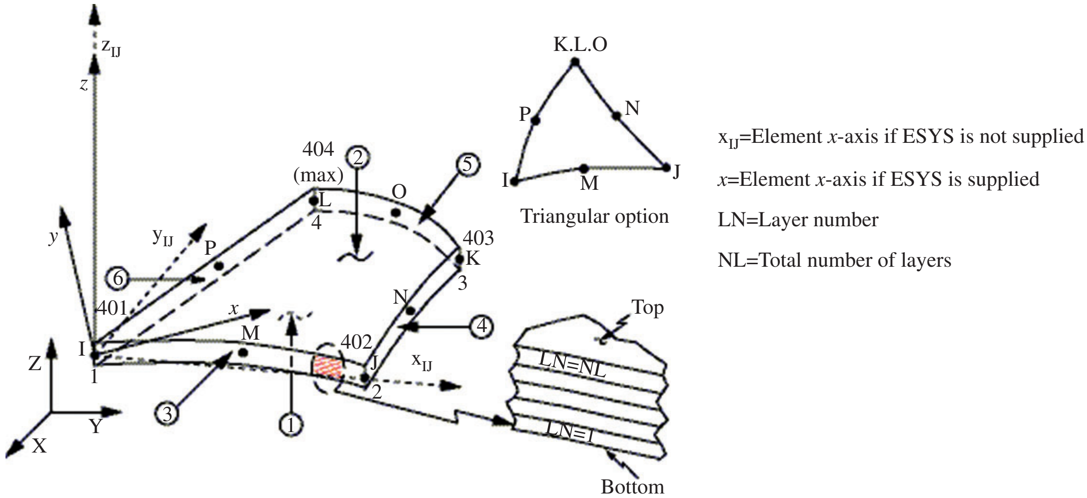

Figure 5 SHELL91 geometry [17].

produce mesh structure. The element has six degrees of freedom at each node: translations in the nodal x, y and z directions and rotations about the nodal x, y and z axes.

Natural frequencies of cantilever plate with (c-c), (s-c) and (w-c) were calculated using ANSYS in terms of fiber orientation angle and compared with the experimental results as shown in Table 3. As seen in Table 3, numerical and experimental results are in close agreement with each other.

5.1 Effect of fiber orientation angle

In this study, $[(0/90)_4]_S$, $[(15/\text{-}75)_4]_S$, $[(30/\text{-}60)_4]_S$ and $[(45/\text{-}45)_4]_S$ fiber orientations were used in order to investigate the effect of fiber orientation angle on the natural frequencies. The first five modes of frequency were considered in the numerical studies. Figure 6 shows the effects of the fiber orientation angle on natural frequencies. It shows that the laminate with fibers $[(45/\text{-}45)_4]_S$ angles has lower natural frequencies than other angles. The laminate with fibers $[(0/90)_4]_S$ angles has a greater natural frequency in bending modes (modes 1, 2 and 5) because 50% of the fibers are oriented at 0° in $[(0/90)_4]_S$, and thus are appropriate for bending.

5.2 Effect of cutout position

In order to investigate the effect of cutout position (CP) on natural frequencies of the composite plates, circular CP is changed from -40 mm to +40 mm on the x-axis as shown in Figure 7.

Only the first two vibration mode frequencies were investigated for square, circular, triangular and elliptical cutouts for the $[(0/90)_4]_S$ fiber orientation. The effect of CP on the natural frequency is shown in Figure 8.

As shown in Figure 8A, when cutout approaches the clamped edge, the natural frequency of the first mode is decreased. In Figure 8B, the natural frequency of the second mode at the center of the plate is lower than any other CP.

Fiber orientation angle	Mode 1						Mode 2					
	ANSYS			Exp.			ANSYS			Exp.		
	w-c	s-c	c-c	w-c	s-c	c-c	w-c	s-c	c-c	w-c	s-c	c-c
0	40.5	38.7	39.7	40	38	39	253.4	244.0	240.5	253	243	244
30	32.8	34.2	33.5	35	33	32	205.1	203.3	210.1	218	200	209
45	31.1	31.2	31.4	29	30	29	194.6	197.4	199.7	201	196	202

Table 3 Natural frequencies of laminated composite plate.

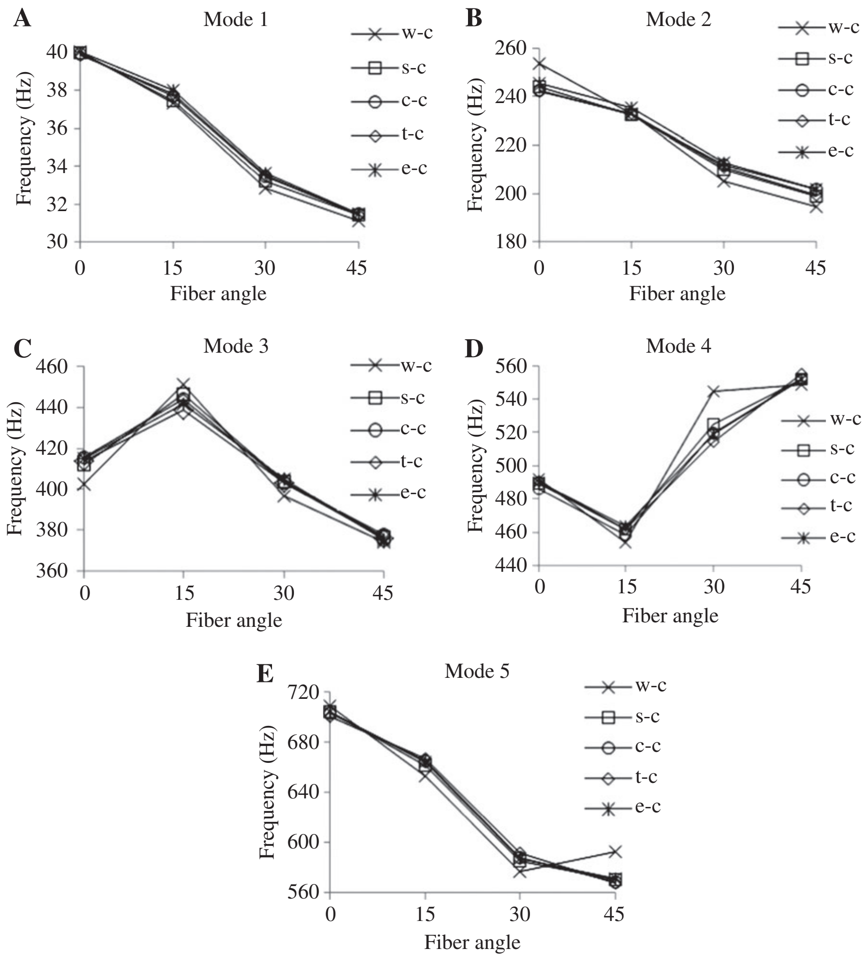

Figure 6 Effects of fiber orientation on natural frequencies.

5.3 Effect of cutout size

The effect of cutout size on natural frequencies was investigated by increasing cutout size while the width of the plate is taken as a constant. Circular and square type of cutouts were preferred and located at the center of the plate for the analysis. As can be seen in Figure 9, the natural frequencies of composite plates with circular and square cutout decrease while the width of plate over cutout size (w/A) ratio increases.

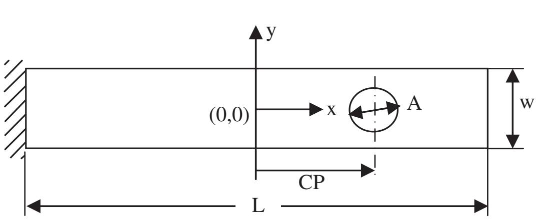

Figure 7 Cutout position of laminated composite plate.

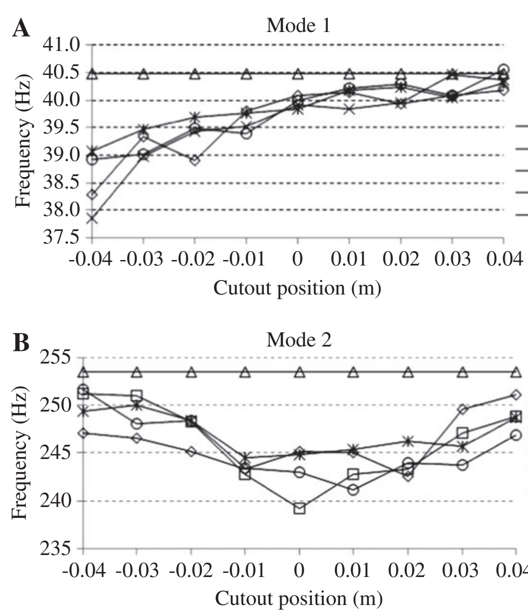

Figure 8 Change in cutout position for (A) first mode and (B) second mode.

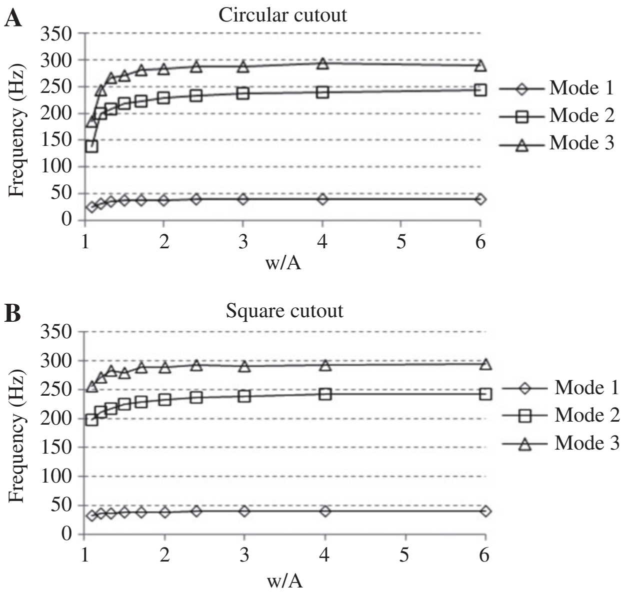

Figure 9 Cutout size for (A) circular cutout and (B) square cutout.

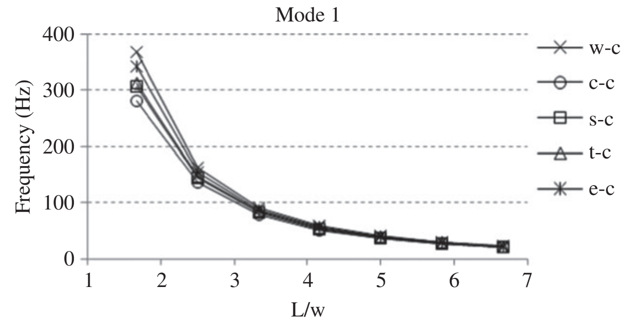

Figure 10 Effects of length of plate on natural frequencies.

5.4 Effect of length of plate

Change of natural frequency for different length over width ratios is shown in Figure 10. The central cutout is used. Its position and diameter are chosen as constant. Natural frequency decreases with increasing plate length

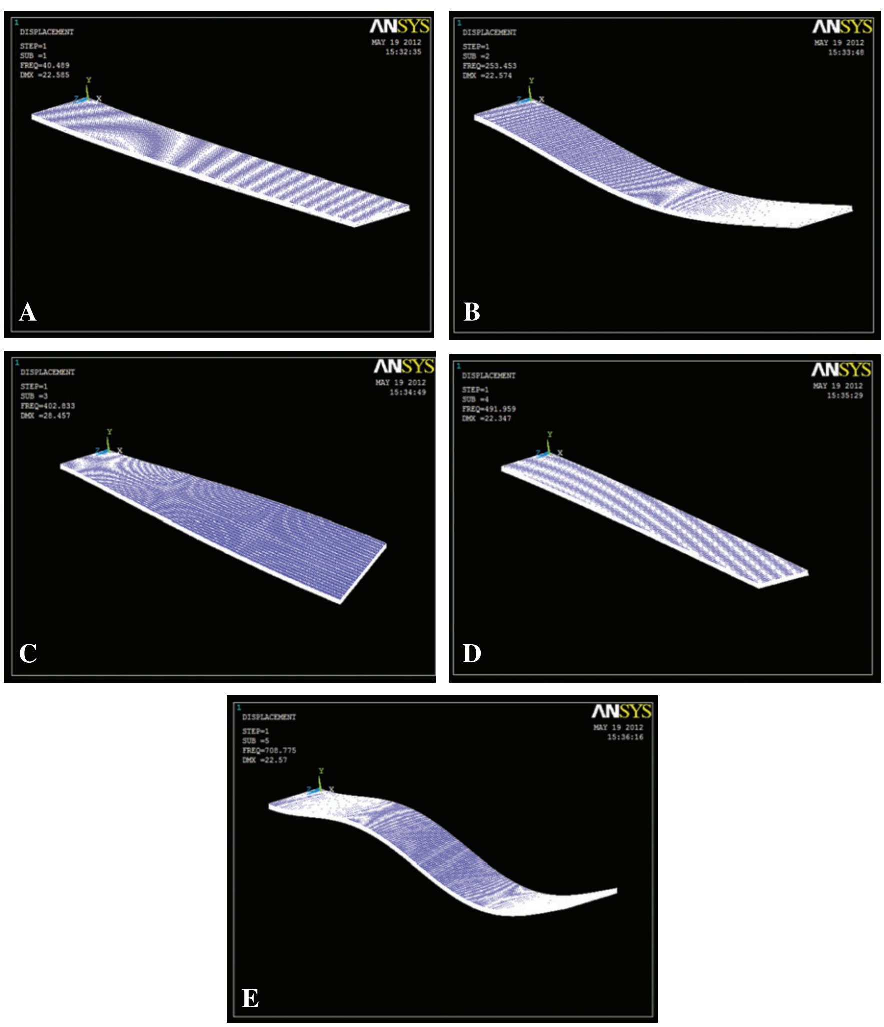

Figure 11 Mode shapes of composite beam (w-c). (A) Mode 1, (B) mode 2, (C) mode 3, (D) mode 4, (E) mode 5.

over width ratio, but natural frequency behavior generally stays the same for each cutout.

Mode shapes of cantilever laminated composite plate are given in Figure 11. As can be seen from the figure, first, second and fifth modes of frequencies are bending, whereas the third and fourth modes of frequencies are torsional vibration frequencies.

6 Conclusions

In this study, detailed parametric studies have been carried out to reveal the effects of cutout size, cutout type, cutout position, length of the plate and fiber orientation angle on the natural frequencies of the laminated composite plates. The numerical results were compared with experimental results and theoretical results. Numerical results are in close agreement with experimental and theoretical results. The main conclusions that can be drawn from this investigation are:

- Natural frequencies of the laminated composite plate are affected by the stacking sequence, and it allows to achieve the desired natural frequencies without changing its geometry or without changing its weight.
- When the fiber angle increases, the natural frequencies decrease. Maximum natural frequency occurs on $[(0/90)_4]_S$ laminated plate.
- A rectangular plate without cutout has greater natural frequencies than plates with the square and circular cutouts.
- Effects of cutout location were also investigated. Natural frequency for the first mode is decreased while cutout approaches the clamped edge. Natural frequency for the second mode is decreased while cutout approaches the center of the plate.
- Natural frequencies for the first three modes and circular and square cutouts are decreased by increasing the w/A ratio.
- Natural frequency decreases with increasing plate length over width ratio, but natural frequency behavior generally stays the same for each cutout.

Received October 8, 2012; accepted November 23, 2012; previously published online December 21, 2012

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