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Thermal buckling load optimization of laminated general quadrilateral and trapezoidal thin plates

Abstract: This paper deals with thermal buckling load optimization of symmetrically laminated angle-ply general quadrilateral and trapezoidal thin plates. The objective function is to maximize the critical temperature capacity of the quadrilateral and trapezoidal laminated plates and the fiber orientation is considered as a design variable. The mathematical formulation is based on the classical laminated plate theory for the frequency analysis. The modified feasible direction method is used as the optimization routine. Therefore, a program based on FORTRAN is used. Finally, the significant effects of aspect ratios, boundary conditions, taper ratios and unsymmetric trapezoidal plates on the optimal results are investigated and the results are compared.

Keywords: laminated quadrilateral and trapezoidal plates; modified feasible direction method; optimization; thermal buckling.

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1 Introduction

Laminated composites are often employed to replace traditional metal for the skin panels of aircraft wings and fuselage in order to reduce the weight of flight vehicles. In the design of composite skins for aircraft wings, one of the important issues is buckling of the panels. High-speed aircraft structural panels are subjected not only to aerodynamic loading but also to aerodynamic heating. The temperature rise may buckle the plate and exhaust the load-carrying capacity.

A good amount of literature is available based on thermal buckling load optimization of laminated composite flat rectangular plates. For example, Topal and Uzman [1] investigated thermal buckling optimization of laminated plates subjected to a uniformly distributed temperature load using the modified feasible direction (MFD) method. The objective function was to maximize the critical temperature capacity of laminated plates and the fiber

orientation was considered as a design variable. Fares et al. [2] presented a multiobjective optimization problem to determine the optimal layer thickness and optimal closed-loop control function for a symmetric cross-ply laminate subjected to thermomechanical loadings. Spallino and Thierauf [3] investigated thermal buckling optimization of laminated composite plates subjected to thermal loading. The optimal design problem was solved using evolution strategies under strain and ply contiguity constraints. An optimal design of antisymmetric laminates under thermal loads was given by Adali and Duffy [4] for the nonhybrid and hybrid cases. The optimum results were given for simply supported laminates with graphite, boron, and glass layers under a uniform temperature change. Walker et al. [5] investigated the optimal designs of laminated plates subjected to nonuniform temperature distributions for maximum buckling temperature. The golden section method was used as optimization routine. Lee et al. [6] presented the design of a thick laminated composite plate subjected to a thermal buckling load under a uniform temperature distribution. In design procedures of composite laminated plates for a maximum thermal buckling load, the golden section method was used as an optimization routine. Singha et al. [7] maximized buckling temperatures of graphite/epoxy laminated composite plates for a given total thickness. The fiber orientations and thicknesses of layers were adopted as design variables. Genetic algorithm was employed to optimize as many as 10 variables for the five layered plates. Chen et al. [8] investigated design optimization for structural thermal buckling. The optimization model was constructed and solved by the sequential linear programming or sequential quadratic programming algorithm. Fares et al. [9] presented design and control optimization to minimize the thermal post-buckling dynamic response and to maximize the buckling temperature level of composite laminated plates subjected to thermal distribution varying linearly through the thickness and arbitrarily with respect to the in-plane coordinates.

Topal [10] investigated frequency optimization of symmetrically laminated angle-ply general quadrilateral and trapezoidal thin plates. The objective function is to maximize the fundamental frequency of the quadrilateral and trapezoidal laminated plates and the fiber orientation

is considered as design variable. However, there is no research on the thermal buckling load optimization of quadrilateral and trapezoidal laminated plates in the literature. To the best knowledge of the authors, this is the first instance in which optimization has been adopted for the thermal buckling load of laminated quadrilateral and trapezoidal plates. Therefore, in this study, thermal buckling load optimization of quadrilateral and trapezoidal laminated plates is investigated to fill this gap. The objective function is to maximize the critical thermal buckling load of the symmetrically laminated angle-ply quadrilateral and trapezoidal thin plates and the fiber orientation is considered as a design variable. The mathematical formulation is based on the classical laminated plate theory (CLPT) for the frequency analysis. The MFD method is used as the optimization routine. Therefore, a program based on FORTRAN is used. Finally, the significant effects of aspect ratios, boundary conditions, taper ratios, and unsymmetric trapezoidal plates on the optimal results are investigated and the results are compared.

2 Theoretical formulation

The geometric configurations of flat, thin, fiber-reinforced laminated plates consisting of a general quadrilateral and symmetric trapezoidal planform ($\gamma_1 = \gamma_2$) are shown in Figure 1A and B, respectively.

In the present study, the CLPT is employed to analyze the problem and the following displacement field is assumed:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \frac{\partial w}{\partial x}, & v(x, y, z) &= v_0(x, y) \\ & - z \frac{\partial w}{\partial y}, & w(x, y, z) &= w_0(x, y) \end{aligned} \quad (1)$$

where (u_0, v_0, w_0) are the displacement components along the (x, y, z) coordinate directions of a point on the mid-plane, respectively. The membrane strains ($\varepsilon_x, \varepsilon_y, \gamma_{xy}$) and the bending curvatures ($\kappa_x, \kappa_y, \kappa_{xy}$) are defined as follows:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix} \quad (2)$$

$$\{\kappa\} = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{0,xx} \\ -w_{0,yy} \\ -2w_{0,xy} \end{Bmatrix} \quad (3)$$

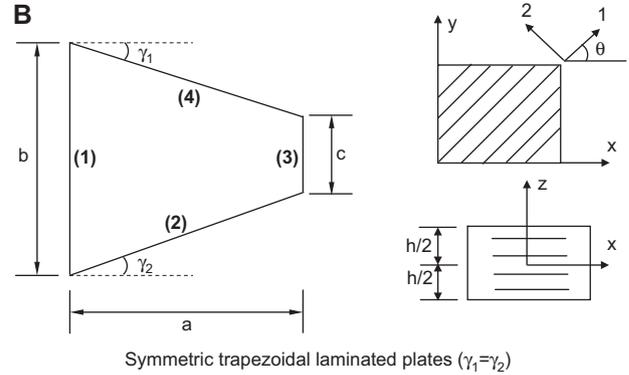
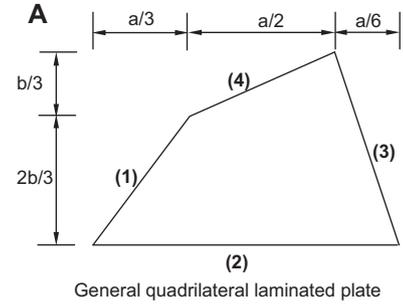


Figure 1 Laminated plates of (A) general quadrilateral planform and (B) symmetric trapezoidal planform.

Here, a subscript after the comma denotes differentiation with respect to the variable following the comma. The stress resultants (N_x, N_y, N_{xy}) and the moment resultants (M_x, M_y, M_{xy}) are defined as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ & & A_{66} & B_{16} & B_{26} & B_{66} \\ \text{sym} & & & D_{11} & D_{12} & D_{16} \\ & & & & D_{22} & D_{26} \\ & & & & & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (4)$$

where A_{ij} , B_{ij} and D_{ij} denote extensional, coupling and bending stiffness coefficients of the laminates, respectively. A_{ij} , B_{ij} and D_{ij} can be calculated as follows

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(1, z, z^2) dz \quad (i, j=1, 2, 6) \quad (5)$$

3 Finite element formulation

In this study, four-noded Lagrangian rectangular elements having three degrees of freedom per node are used for the

thermal buckling load of the quadrilateral and trapezoidal laminated plates. Using the same shape functions associated with node i ($i=1, 2, \dots, n$), Φ_i , for interpolating the variables in each element, the displacement vectors can be written as

$$u = \sum_{i=1}^n \Phi_i(x, y) u_i \quad (6)$$

where u_i is the value of the displacement vector corresponding to the node i , and is given by

$$u = \{u_o^{(i)}, v_o^{(i)}, w_o^{(i)}\}^T \quad (7)$$

The discrete eigenvalue equation of the static buckling problem of laminates can be derived as

$$([K_b + K_s] - \lambda [K_{og}]) \{u\} = 0 \quad (8)$$

The product of λ and the initial guess value Δ is the critical buckling temperature T_{cr} , that is,

$$T_{cr} = \lambda \Delta T \quad (9)$$

Equation (8) is a set of homogeneous linear equations in the unknown displacements $\{u\}$. For nontrivial solutions, the determinant is equal to zero and the eigenvalues correspond to the thermal buckling load of the laminated plates. The subspace iteration method is used for the frequency analysis of the quadrilateral and trapezoidal laminated plates.

4 Optimization problem

In this study, the optimization problem is the maximization of the critical thermal buckling load by designing the fiber orientations in the stacking sequence of the quadrilateral and trapezoidal laminated plates. The optimal design problem can be stated mathematically as follows:

Find: θ ,

$$\text{Maximize } (T_{cr})_{\max} = \max_{\theta} T_{cr}(\theta) \quad (10)$$

$$\text{Subject to } 0^\circ \leq \theta_k \leq 90^\circ, \Delta\theta = 1^\circ$$

The thermal buckling load T_{cr} for a given fiber orientation is determined from the finite element solution of the eigenvalue problems given by Eq. (8). The optimization procedure involves the stages of evaluating thermal buckling load and improving the fiber orientation θ to maximise T_{cr} . Thus, the computational solution consists of successive stages of analysis and optimization until

a convergence is obtained and the optimal angle θ_{opt} is determined within a specified accuracy. The MFD method is used in the optimization stage [1, 10]

5 Numerical results and discussion

In this study, four-layered, symmetric, angle-ply $\theta/-\theta/\theta/\theta$ quadrilateral and trapezoidal laminated plates subjected to a uniform temperature load are investigated for optimization problem. Each of the lamina is assumed to be same thickness. All of the numerical results presented are for a T300/5208 graphite/epoxy material whose properties are as follows: $E_1=181$ GPa, $E_2=10.3$ GPa, $G_{12}=7.17$ GPa, $\nu_{12}=0.28$, $\alpha_1=0.02 \times 10^{-6} \text{C}^{-1}$, $\alpha_2=2.25 \times 10^{-6} \text{C}^{-1}$.

The nondimensional thermal buckling load parameter is defined as

$$\bar{T}_{cr} = \alpha_o T_{cr} \times 10^3 \quad (11)$$

where $\alpha_o = 10^{-6} / ^\circ\text{C}$.

The effects of plate aspect ratios (a/b) and four different boundary conditions on the optimal design are investigated for symmetrically laminated four-layered angle-ply quadrilateral and symmetric trapezoidal plates ($c/b=0.25$, $\gamma_1=\gamma_2$) for $b/h=100$. In Figure 2A and B, the effects of a/b ratios and boundary conditions on the

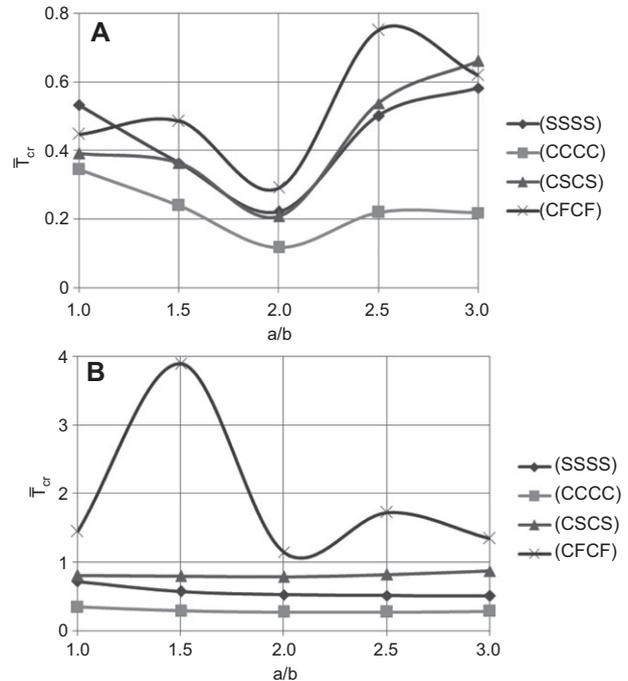


Figure 2 Effects of plate aspect ratios and boundary conditions on critical thermal buckling load.

Boundary conditions	$\theta_{opt} (^\circ)$				
	a/b				
	1.0	1.5	2.0	2.5	3.0
(SSSS)	46	44	42	40	39
(CCCC)	51	54	54	56	56
(CSCS)	37	42	41	39	37
(CFFF)	36	39	40	41	41

Table 1 Effects of a/b ratios and boundary conditions on the optimum fiber orientations for quadrilateral laminated plates.

Boundary conditions	$\theta_{opt} (^\circ)$				
	a/b				
	1.0	1.5	2.0	2.5	3.0
(SSSS)	42	42	42	41	41
(CCCC)	51	54	56	59	62
(CSCS)	38	36	35	35	36
(CFFF)	40	42	41	42	41

Table 2 Effects of a/b ratios and boundary conditions on the optimum fiber orientations for trapezoidal laminated plates ($c/b=0.25$).

critical thermal buckling loads are given for quadrilateral and trapezoidal laminated plates, respectively. Normally, as a/b ratio increases, the critical thermal buckling load decreases for flat rectangular laminated plates [1]. As seen from Figure 2A, the maximum and minimum

critical thermal buckling loads occur for $a/b=2.5$ and $a/b=2$ for (CFCF) and (CCCC) boundary conditions, respectively, for quadrilateral laminated plates. As seen from Figure 2B, the maximum and minimum critical thermal buckling loads occur for $a/b=1.5$ and $a/b=2$

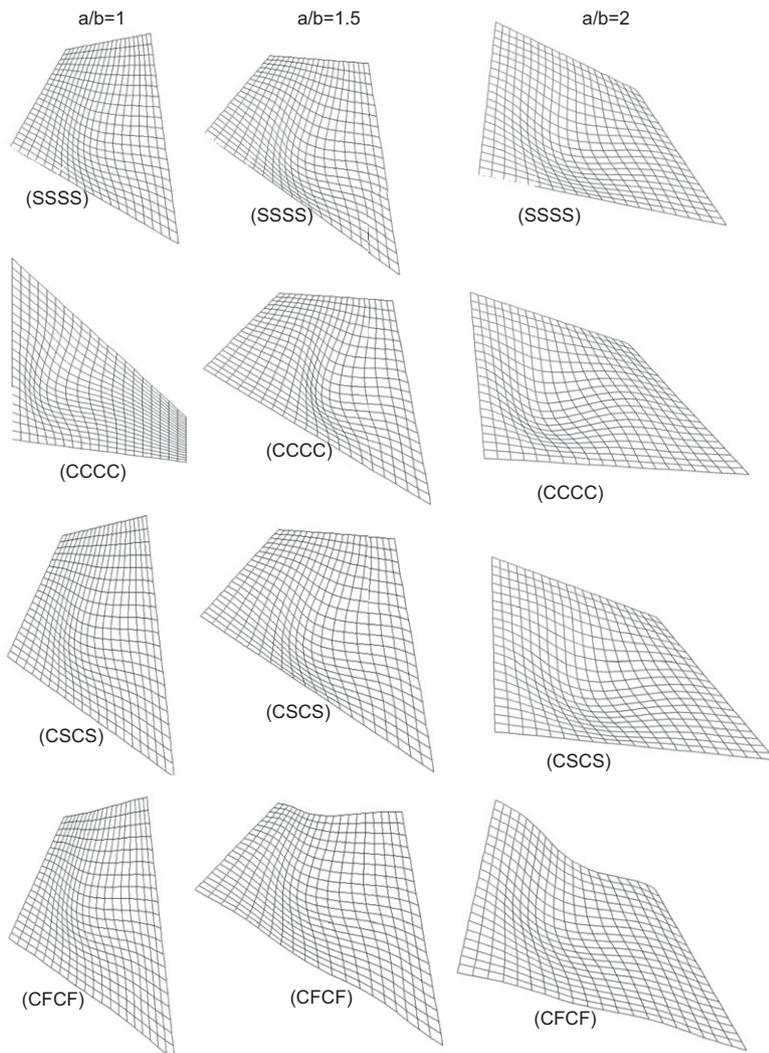


Figure 3 continued

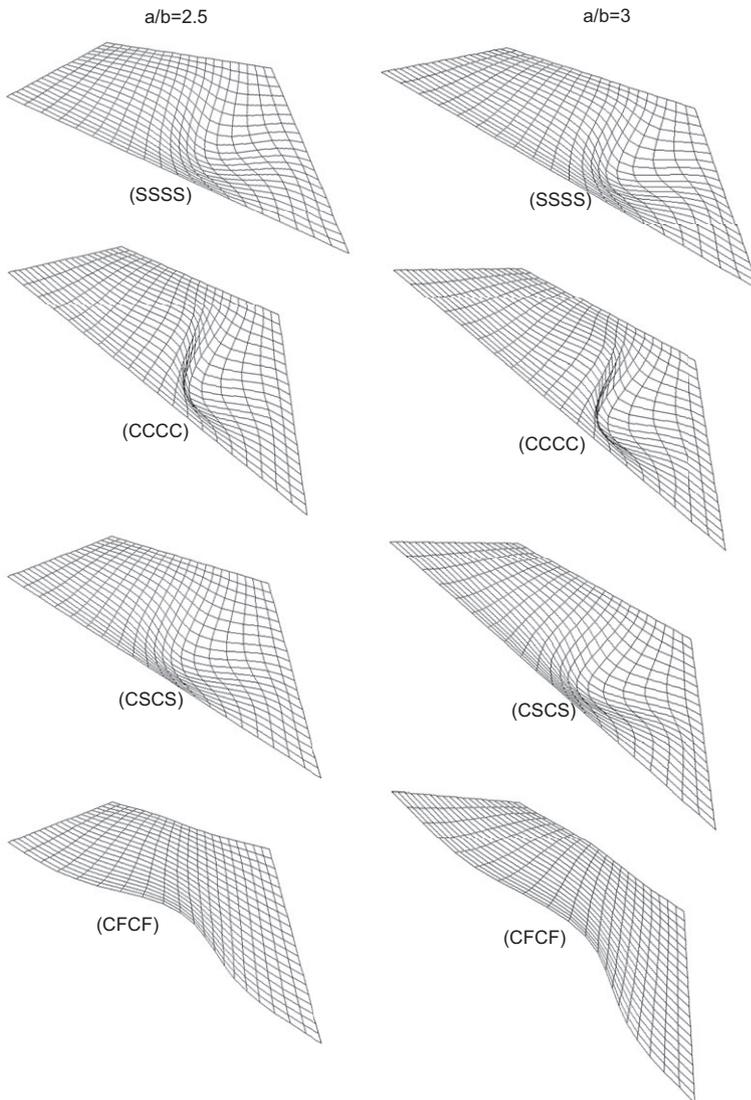


Figure 3 Mode shapes for aspect ratios and boundary conditions for general quadrilateral laminated plates.

for (CFCF) and (CCCC) boundary conditions, respectively, for trapezoidal laminated plates. It is obvious that quadrilateral and trapezoidal laminated plates exhibit more different behavior than flat rectangular laminated plates. However, as seen from Figure 2, the maximum thermal critical buckling load occurs at (CFCF) boundary condition, whereas the minimum thermal critical buckling load occurs at (CCCC) boundary condition. This can be explained by the fact that the free edges provide more degrees of freedom and allow the laminated plate to buckle higher temperatures.

In Tables 1 and 2, the effects of a/b ratios and boundary conditions on the optimum fiber orientations are given for quadrilateral and trapezoidal laminated plates, respectively. As seen from Table 1, as the a/b ratio increases, the optimum fiber orientation decreases

gradually for the (SSSS) boundary condition. On the other hand, as the a/b ratio increases, the optimum fiber orientation generally increases for (CCCC) and (CFCF) boundary conditions. As the a/b ratio increases, the optimum fiber orientation generally decreases for the (CSCS) boundary condition. As seen from Table 2, the a/b ratio almost has no any effect on the optimum fiber orientation for (SSSS) and (CFFF) boundary conditions. On the other hand, as the a/b ratio increases, the optimum fiber orientation increases gradually for the (CCCC) boundary condition. As the a/b ratio increases, optimum fiber orientation generally decreases for the (CSCC) boundary condition. In Figures 3 and 4, the mode shapes for critical thermal buckling load are given for a/b ratios and boundary conditions for quadrilateral and trapezoidal laminated plates, respectively.

Here, the effects of taper ratio (c/b) on the optimal design are investigated for simply supported laminated symmetric trapezoidal plates ($a/b=1, \gamma_1=\gamma_2, b/h=100$). As seen from Figure 5, as the c/b ratio increases, the critical thermal buckling load decreases. The effect of the c/b ratio on the optimum fiber orientation is given in Table 3. It can be seen that as the taper ratio increases, the optimum fiber orientation generally increases gradually.

The effects of the unsymmetric trapezoidal planform $\gamma_1 \neq \gamma_2$ on the optimal design are investigated for simply supported laminated trapezoidal plates ($c/b=0.25,$

$a/b=1, b/h=100$). As seen from Figure 6, the critical thermal buckling load increases as γ_1 increases. However, as seen from Table 4, the optimum fiber orientation generally decreases gradually as γ_1 increases.

6 Conclusions

This paper deals with thermal buckling load optimization of symmetrically laminated angle-ply general quadrilateral and trapezoidal thin plates. The objective function is to maximize the critical temperature capacity

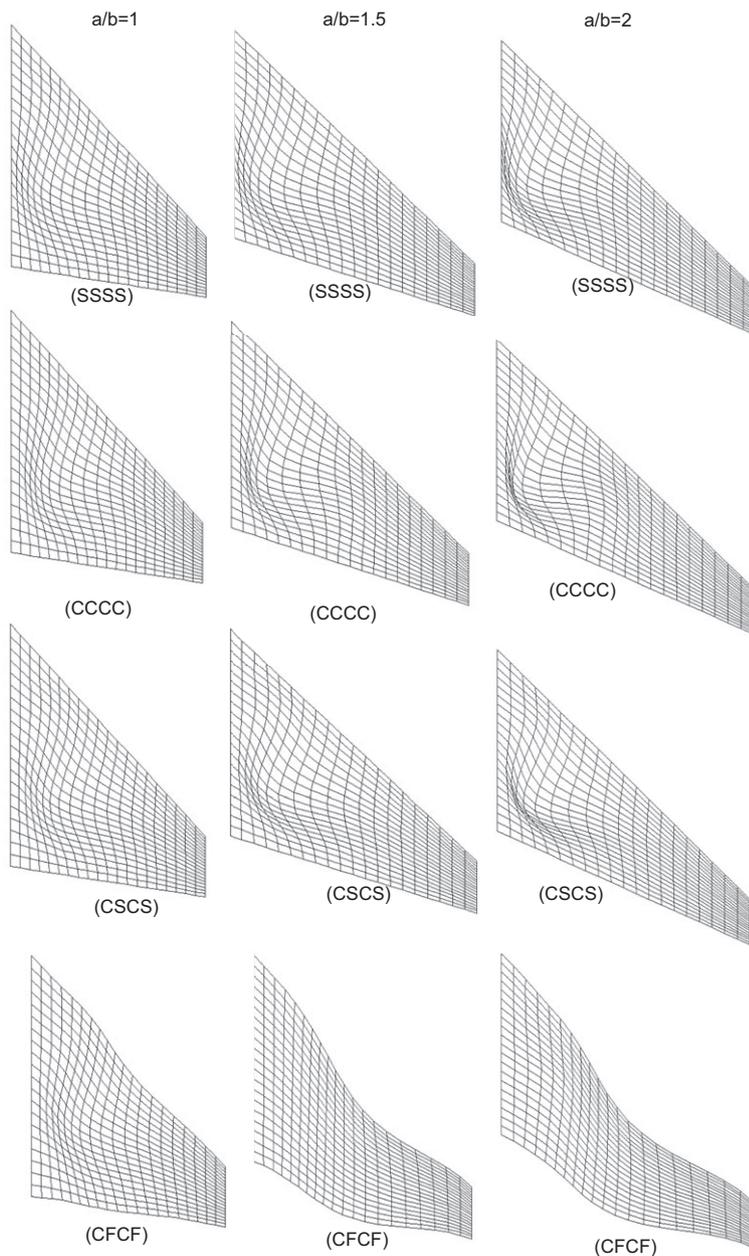


Figure 4 continued

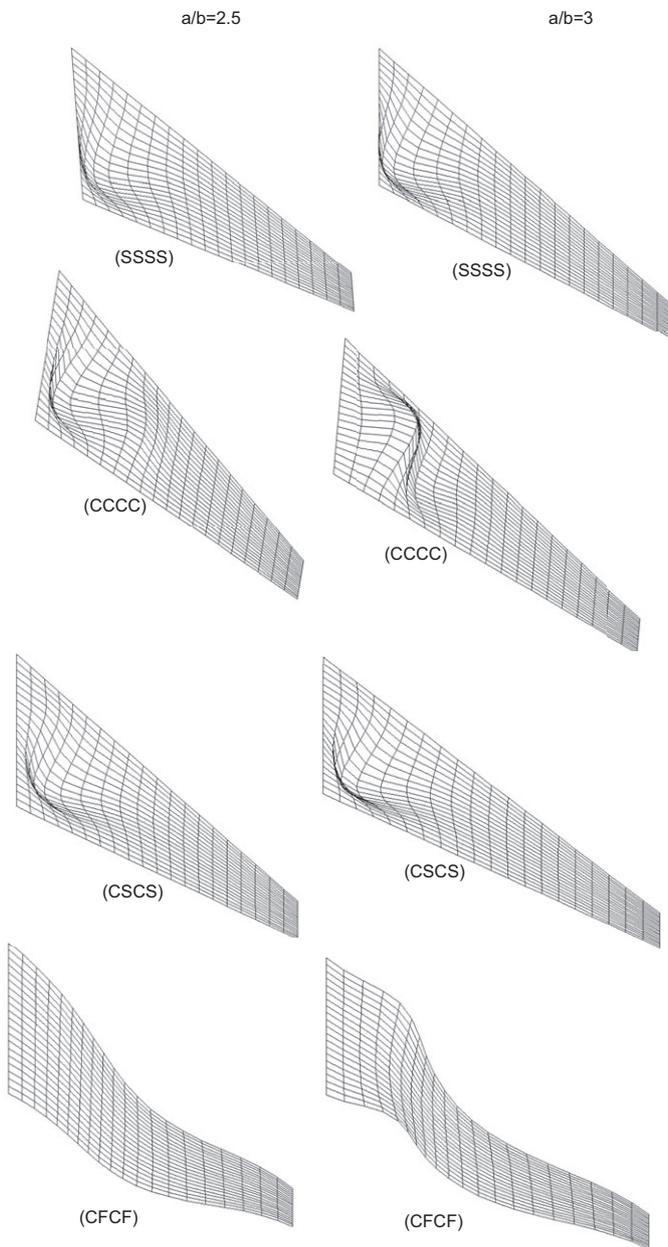


Figure 4 Mode shapes for aspect ratios and boundary conditions for trapezoidal laminated plates.

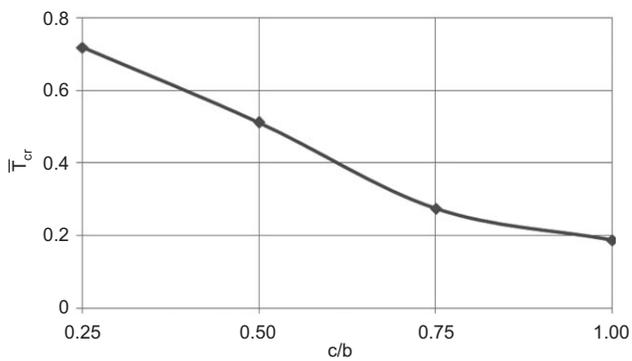


Figure 5 Effect of taper ratio on critical thermal buckling load for trapezoidal plate.

c/b	θ_{opt} (°)
0.25	42
0.50	42
0.75	44
1.00	45

Table 3 Effect of taper ratio on the optimum fiber orientation for trapezoidal plate.

of the quadrilateral and trapezoidal laminated plates and the fiber orientation is considered as a design variable. It can be concluded from the results that maximum and minimum critical thermal buckling loads occur for

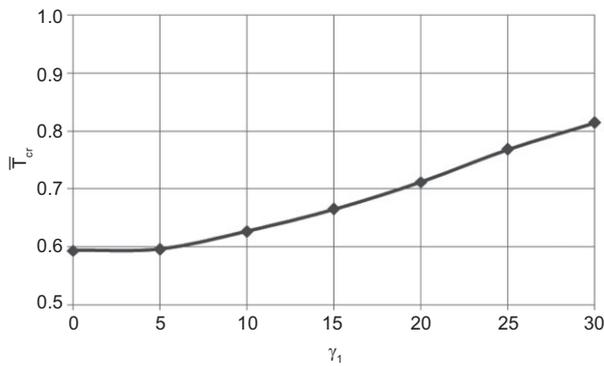


Figure 6 Effect of unsymmetric trapezoidal laminated plates on critical thermal buckling load.

different plate aspect ratios for (CFCF) and (CCCC) boundary conditions, respectively, for quadrilateral laminated plates. It is obvious that quadrilateral and trapezoidal laminated plates exhibit more different behavior than flat rectangular laminated plates. Free edges provide more degrees of freedom and allow the laminated plate to

γ_1	θ_{opt} (°)
0	45
5	44
10	44
15	43
20	42
25	40
30	40

Table 4 Effect of unsymmetric trapezoidal laminated plates on the optimum fiber orientation.

buckle higher temperatures. As the taper ratio increases, the critical thermal buckling load decreases and the optimum fiber orientation generally increases gradually. As γ_1 increases, the critical thermal buckling load increases and the optimum fiber orientation generally decreases gradually.

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