

Research Article

Zhenhao Li, Song Yang, Xiaoning Liu, Guoqing Xiao, Hongzhan San, Yanru Zhang, Wei Wang*, and Zhibo Yang*

Grinding force model for ultrasonic assisted grinding of γ -TiAl intermetallic compounds and experimental validation

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Abstract: The introduction of ultrasonic vibration in the grinding process of γ -TiAl intermetallic compounds can significantly reduce its processing difficulty. It is of great significance to understand the grinding mechanism of γ -TiAl intermetallic compounds and improve the processing efficiency by studying the mechanism of ordinary grinding of abrasive grains. Based on this, this study proposes a grinding force prediction model based on single-grain ultrasonic assisted grinding (UAG) chip formation mechanism. First, the prediction model of grinding force is established based on the chip formation mechanism of abrasive sliding ordinary grinding and the theory of ultrasonic assisted machining, considering the plastic deformation and shear effect in the process of material processing. Second, the UAG experiment of γ -TiAl intermetallic compounds was

carried out by using diamond grinding wheel, and the unknown coefficient in the model was determined. Finally, the predicted values and experimental values of grinding force under different parameters were compared to verify the rationality of the model. It was found that the maximum deviation between the predicted value of tangential force and the actual value is 23%, and the maximum deviation between the predicted value of normal force and the actual value is 21.7%. In addition, by changing the relevant parameters, the model can predict the grinding force of different metal materials under different processing parameters, which is helpful for optimizing the UAG parameters and improving the processing efficiency.

Keywords: γ -TiAl intermetallic compounds, prediction model, mechanism research, ultrasonic assisted grinding, comparative verification

* **Corresponding author: Wei Wang**, School of Emergency Management, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: wangwei@hpu.edu.cn

* **Corresponding author: Zhibo Yang**, Institute of Mechanical and Electrical Engineering, Hebi Polytechnic, Hebi, 458030, China, e-mail: yangzhibo@hpu.edu.cn

Zhenhao Li: School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: zhenhao1128@126.com

Song Yang: School of Software, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: 2682139520@qq.com

Xiaoning Liu: School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: 1143460674@qq.com

Guoqing Xiao: School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: xgq77923@163.com

Hongzhan San: School of Materials Science and Engineering, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: Hongshansan@hpu.edu.cn

Yanru Zhang: School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo, 454000, China, e-mail: zhangyanru@hpu.edu.cn

Nomenclature

a_c	average undeformed chip thickness
a_p	grinding depth
A	ultrasonic amplitudes
b	grinding width
C_1 and C_2	coefficients related to the physical properties of the workpiece material
dA	effective contact area of the workpiece-abrasive
dF_x	component force of F in any direction
E and f	correlation coefficients
F_{ch}	chip formation force
$F_{n,ch}$	normal chip formation force during UAG
$F_{n,pg}$	normal plowing force during UAG
$F'_{n,sl}$	normal sliding force between a single abrasive particle and the workpiece
$F_{n,sl}$	normal sliding force during UAG
F_{pl}	plowing force

F_p	grinding force per unit area in the plowing stage	Γ	grinding area
F_{sl}	sliding force	d_s	radius of the grinding wheel
$F_{t,ch}$	tangential chip formation force during UAG	ξ_{sl}	proportion of dynamic effective grains in the sliding stage
$F_{t,pg}$	tangential plowing force during UAG	ξ_{pl}	proportion of dynamic effective grains in the plowing stage
$F_{t,sl}$	tangential sliding force during UAG	ξ_{ch}	proportion of dynamic effective grains in the chip formation stage
g	correlation coefficient	ς	ratio of grinding wheel speed to vibration speed
$k_1, k_3, k_4,$ and k_5	correlations coefficient	ψ	angle between any direction and the workpiece feed direction
$k_2, k_6, k_7, k_8,$ and k_9	constants	$\alpha, \beta,$ and γ	constants
l_g	busbar of the contact zone between the grain and the workpiece	u	energy consumed to remove the unit volume of material
l_s	grinding arc length	τ	shear flow stress
N'_{dt}	distribution density of dynamic effective abrasive particles	γ'	shear strain rate
N_{dt}	total number of dynamic effective abrasive grains on the grinding wheel surface	γ	shear strain
$N_{d,ch}$	dynamic effective abrasive grain counts in the chip formation stage	ϕ	shear angle
$N_{d,pl}$	dynamic effective abrasive grain counts in the plowing stage	Δs	thickness of shear band
$N_{d,sl}$	dynamic effective abrasive grain counts in sliding stage	γ'_0	constant
P_0	constant		
$\bar{P}$	average contact pressure of a single abrasive particle-workpiece		
r	radius of the S-region		
$\bar{S}$	effective contact area of a single abrasive particle-workpiece		
T	deformation temperature		
UAG	ultrasonic assisted grinding		
u_{ch}	grinding specific energy of single abrasive grain in chip formation stage		
$u_{ch,st}$	static grinding specific energy in chip formation stage		
$u_{ch,dy}$	dynamic grinding specific energy in chip formation stage		
v_s	grinding wheel speed		
v_w	workpiece speed		
μ_{sl}	friction coefficient of abrasive-workpiece contact surface in sliding stage		
μ_{pl}	friction coefficient of abrasive-workpiece contact surface in plowing stage		
μ_{ch}	friction coefficient of abrasive-workpiece contact surface in the chip formation stage		
Δ	deviation between the radius of the grinding wheel and the curvature radius of the grinding path		

1 Introduction

γ -TiAl intermetallic compounds have the advantages of low density, high specific strength, strong creep resistance, and good high-temperature stability [1]. γ -TiAl intermetallic compounds can still work normally at 900°C, and its density is only about 50% of the nickel-based alloy. It is widely considered to be a high-temperature alloy that can replace the nickel-based alloy in the future, and has a good application prospect in high-end manufacturing fields such as aviation and automobile [2,3]. However, γ -TiAl intermetallic compounds are typical difficult-to-machine materials due to their high strength, poor plasticity, and high hardness [4,5].

Ultrasonic assisted grinding (UAG) is a high-performance hybrid machining technology [6–8]. As an advanced processing method, UAG combines conventional grinding with ultrasonic vibration, which is an effective method to realize the precision machining of difficult-to-machine materials such as titanium alloy [9–11]. At the same time, UAG can effectively reduce the grinding force, grinding temperature, and surface roughness with the significant improvement of the surface residual compressive stress [12–14]. Therefore, UAG has been widely studied. Wang *et al.* [15] carried out ultrasonic assisted filing and traditional filing experiments, comparing the force, surface quality, and surface roughness generated during the processing of the two. The results showed that the processing effect is better after applying ultrasonic vibration. Dong

et al. [16] designed a single-factor experiment to study the effect of ultrasonic vibration on the grinding edge size of deep holes. The results show that the application of ultrasonic vibration improves the quality of the hole. It can be found that the applied ultrasonic vibration provides better machining results compared to conventional machining.

As an important factor to evaluate the machining effect, grinding force plays an important role in the study of the grinding mechanism [17–20]. Studies have shown that the magnitude of the grinding force directly affects the machining accuracy of the workpiece and the quality of the machined surface [21–23]. Therefore, scholars studied the grinding force when machining γ -TiAl intermetallic compounds. Studies have shown that there are some differences in grinding force when using different types of grinding wheels to carry out grinding experiments, which is mainly due to the different levels of wear of the grinding wheel during the grinding process [24,25]. Xi *et al.* used different kinds of grinding wheels to grind γ -TiAl intermetallic compounds and found that the diamond grinding wheel has the best grinding effect, followed by the CBN grinding wheel [26,27]. In addition, Hood *et al.* [28,29] have reported the grinding results of γ -TiAl intermetallic compounds: when using SiC grinding wheel to grind the workpiece surface, the cutting depth is the most important factor affecting the grinding force, rather than the grinding wheel size, grinding wheel grade, and grinding wheel structure. Moreover, under the same grinding conditions, the normal force, size, and change trend of the workpiece are the same as those of Ti–6Al–4V alloy, while the tangential force is only about half of that of Ti–6Al–4V alloy. Chen *et al.* [30] used the grey correlation analysis method to obtain the influencing factors in the grinding process of γ -TiAl intermetallic compounds. It was found that the grinding depth has the greatest influence on the normal grinding force and grinding temperature of γ -TiAl intermetallic compounds, while the wheel speed has the greatest influence on the tangential grinding force. In addition, Chen *et al.* also obtained the optimal parameter combination for γ -TiAl intermetallic compounds grinding: $v_s = 90 \text{ m}\cdot\text{s}^{-1}$, $v_w = 0.5 \text{ m}\cdot\text{min}^{-1}$, $a_q = 0.1 \text{ mm}$.

Furthermore, the establishment of grinding force prediction model is of great significance for optimizing grinding process parameters and improving processing efficiency and quality. For the effective help for the study of grinding process, scholars have conducted a lot of research on grinding force prediction model [31,32]. Generally speaking, the abrasive grain goes through three stages from cutting in to cutting out, and the grinding force can be considered as the sum of the grinding force of the abrasive grain at different stages in the grinding area. On

this basis, researchers have proposed some more accurate grinding force prediction models based on the mechanism of abrasive scratches. Through experimental analysis, Hou and Komanduri [33] found that the critical cutting depth of the plow and chip formation transition is related to the abrasive particle size, which can be approximately expressed as 0.025 times the abrasive particle size. Durgumahanti *et al.* [34] proposed a method for calculating the total normal force and tangential force of the three stages of sliding, plowing, and chip formation by using the grinding process parameters, and determined the undetermined coefficients in the expression through experiments. The results showed that the predicted values obtained by this method are in good agreement with the experimental values. Based on Hertz's contact theory, Jiang *et al.* [35] further determined the critical depth of friction and tillage transition. Li *et al.* [36] proposed a new method to predict the grinding force, which provides detailed information, including three parts: sliding, plowing, and chip formation. In addition, Li proposed a new strategy to determine the grain-workpiece interaction at each grinding moment and a novel grinding kinematics derivation method considering different grain protrusions. However, the above methods for determining the critical cutting depth are derived based on simplifying the shape of the abrasive particles, so there are some limitations.

Besides, the critical cutting depth is a quantity related to the material properties, and the results of theoretical derivation are not accurate. Based on the research results of Li *et al.*, Zhang *et al.* [37] introduced the plastic accumulation theory and cutting efficiency into the single-grain scratch test, which determined the critical cutting depth of plowing and cutting transition, and optimized the grinding force model. From the three stages of metal material removal, Li *et al.* [38] established a grinding force model for nickel-based high-temperature alloy FGH96 in combination with the theoretical derivation and empirical formulas, taking into account the contact sliding between abrasive grains and the workpiece, the plastic deformation of the material at the plowing stage, and the shear strain effect of the abrasive chip formation on the grinding process. The results showed that the experimental values match the predicted values quite well. The errors of tangential grinding force and normal grinding force are 9.8 and 13.6%, respectively. It is found that the sliding force generated in the sliding friction stage is the main source of grinding force. Ma *et al.* [39] divided the contact area into micro-elements and established a dynamic grinding force model for face gear grinding. The results showed that the agreement degree of prediction values and experimental values is high, and the errors of normal force and tangential force are within ± 0.3 and $\pm 0.1 \text{ N}$, respectively. Based on

Benkai's research, Yi *et al.* [40] established a calculation model of grinding force in the grinding process of a straight groove structure grinding wheel. Through the comparison of the results, it is found that the theoretical value is in good agreement with the experimental value, and the maximum calculation errors of tangential grinding force and normal grinding force are 14.5 and 11.8%, respectively. In addition, when the intermittent ratio of the structured grinding wheel is constant, the groove width has little effect on the grinding force. When the groove width is constant, the intermittent ratio has a great influence on the grinding force. With the increase in the intermittent ratio, the grinding force decreases obviously. Based on the double mechanism of the grinding effect of the rake face and the cutting effect of the cutter surface, Duan *et al.* established the instantaneous milling force model of the integral end mill under the condition of dry nanofluid micro-lubrication. The average absolute error of the force in the x , y , and z directions of the prediction model is 13.3, 2.3, and 7.6%, respectively [41]. Liu proposed an improved grinding force model based on the geometric characteristics of random grains, and verified the model by TC4 dry grinding test. The experimental results showed that the numerical calculation and experimental measurement results are in good agreement under different processing parameters, and the minimum error value is only 1.2022%, which indicates that the calculation accuracy of the grinding force model meets the requirements and is feasible [42,43].

In addition, the researchers predicted the UAG force. Based on the different characteristics of materials, Abdelkawy *et al.* [44] studied the change in UAG force based on plastic-brittle transition. To explore the influence of UAG parameters on grinding force, Lei *et al.* [45] established a dynamic model of instantaneous grinding force in UAG, analyzed the wear forms of grinding wheel abrasive grains through experiments, and discussed the reasons for the formation of different wear forms. Based on the study of a single abrasive grain, Zhang *et al.* [46] proposed an analytical model of grinding force considering the ductile-brittle transition, and established a final model of the number of active abrasive grains in the cutting area, and verified the rationality of the model through experiments. Based on the law of conservation of pulse, Vickers hardness theory, and indentation fracture mechanics theory, Liu *et al.* [47] established a prediction model of ZrO_2 ceramic UAG force considering plastic removal and brittle fracture material removal mechanism. The correctness of the theoretical model was verified by experiments, and the average error between the theoretical value and the experimental value was 22.87%. In addition, Liu *et al.* also found that the rotary UAG force can be reduced by up to 66.76%. Lu *et al.* [48] established a key undeformed

cutting depth prediction model for elliptical vibration assisted cutting of BK7 optical glass. By comparing the experimental results with the predicted results, the average error between the two is only 12%, which verifies the accuracy of the model.

There is no research on the application of chip formation mechanism based on single abrasive grain grinding in ultrasonic assisted machining. Therefore, based on the combination of single abrasive grain grinding chip formation mechanism and ultrasonic assisted machining theory, this study proposes a single abrasive grain UAG chip formation mechanism, and establishes a grinding force prediction model based on this mechanism. The model considers three key factors: the friction between the abrasive and the workpiece, the plastic deformation of the material during the abrasive plowing process, and the shear strain effect of the material during the chip formation process. By modifying the relevant parameters in the ultrasonic assisted grinding prediction force model, the ultrasonic assisted grinding force of other workpieces can be predicted. At the same time, the rationality of the model was verified by the UAG experiment of γ -TiAl intermetallic compounds with diamond grinding wheel.

2 Establishment of predictive force model for UAG

The grinding thickness of a single abrasive grain formed by the interference between any abrasive grain on the grinding wheel and the workpiece gradually increases, and the change in the grinding thickness leads to the change in the interaction mechanism between the single abrasive grain and the workpiece material. Therefore, the chip formation mechanism based on single abrasive grain grinding is formed. The application of ultrasonic vibration during the grinding process changes the trajectory of the abrasive particles, The chip formation mechanism based on single abrasive UAG is formed.

Since the grinding force in the grinding process is the resultant force of the grinding force generated by the abrasive grains participating in the grinding of the whole grinding wheel surface, it is necessary to study the grinding force of a single abrasive grain for studying the grinding force in the whole grinding process. Studies have shown that grain shape during grinding has an important effect on the chip formation mechanism and grinding force [20]. In this study, a conical equivalent abrasive grain model with a top angle of 2θ , is used to better explain the mechanism of grinding force generation.

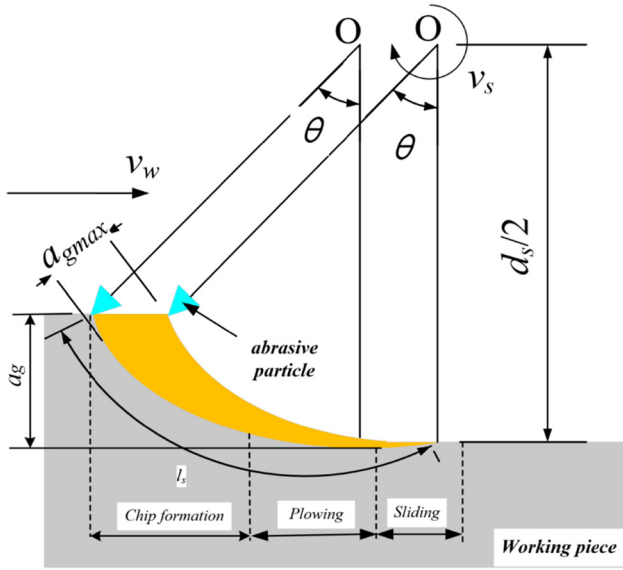


Figure 1: Abrasive particle trajectory diagram.

During UAG, the trajectory of abrasive grain movement is shown in Figure 1. As can be seen from Figure 1, the ultrasound-assisted grinding process has three stages: sliding, plowing, and chip formation. Therefore, the grinding force during the grinding process can be regarded as the superposition of the grinding force in the three stages of sliding friction, plowing, and chip formation:

$$F = F_{sl} + F_{pl} + F_{ch}, \quad (1)$$

where F_{sl} is the sliding force, F_{pl} is the plowing force, and F_{ch} is the chip formation force.

The calculation of tangential grinding force and normal grinding force is shown in formula (2).

$$\begin{cases} F_t = F_{t,sl} + F_{t,pl} + F_{t,ch} \\ F_n = F_{n,sl} + F_{n,pl} + F_{n,ch} \end{cases} \quad (2)$$

Among

$$\begin{bmatrix} F_{t,sl} \\ F_{t,pl} \\ F_{t,ch} \end{bmatrix} = \begin{bmatrix} F_{n,sl} \\ F_{n,pl} \\ F_{n,ch} \end{bmatrix} \begin{pmatrix} \mu_{sl} & \mu_{pl} & \mu_{ch} \end{pmatrix}, \quad (3)$$

where μ_{sl} , μ_{pl} , and μ_{ch} are the friction coefficients of the abrasive-workpiece contact surface at different stages, respectively.

2.1 Sliding force

In the sliding stage, the workpiece only undergoes elastic deformation. The grinding edge does not play a grinding

role, only sliding on the surface of the workpiece. Therefore, the sliding force between the workpiece and the abrasive particles can be solved by the friction relationship between the two.

In the sliding stage, the normal force between the abrasive particles and the workpiece changes with the contact area between them. Therefore, the normal sliding force between a single abrasive particle and the workpiece is

$$F'_{n,sl} = \bar{P} \cdot \bar{S}, \quad (4)$$

where $\bar{P}$ is the average contact pressure of a single abrasive particle-workpiece and $\bar{S}$ is the effective contact area of a single abrasive particle-workpiece.

In the geometric dynamics analysis of abrasive grains, the parabolic function is usually used to approximately replace the grinding path. The deviation between the radius of the grinding wheel and the curvature radius of the grinding path is [49]

$$\Delta = \pm \frac{4v_w}{v_s d_s}. \quad (5)$$

There is a certain linear relationship between $\bar{P}$ and Δ , which is given by

$$\bar{P} = P_0 \Delta = \pm \frac{4P_0 v_w}{v_s d_s}, \quad (6)$$

where P_0 is a constant, which can be obtained by experiments.

N'_{dt} is defined as the distribution density of dynamic effective abrasive particles on the surface of grinding wheel. Therefore, the total number of dynamic effective abrasive grains on the grinding wheel surface is:

$$N_{dt} = N'_{dt} \Gamma = N'_{dt} l_s b = N'_{dt} (a_p d_s)^{\frac{1}{2}} b, \quad (7)$$

where Γ is the area of the grinding area, l_s is the grinding arc length, b is the grinding width, a_p is the grinding depth, and d_s is the radius of the grinding wheel.

According to the research, at different stages of grinding wheel grinding, the proportion of dynamic effective abrasive grains involved in grinding is different. It is assumed that the proportions of dynamic effective grains in the sliding, plowing, and chip formation processes are ξ_{sl} , ξ_{pl} , and ξ_{ch} , respectively. Then, the number of abrasive grains corresponding to three stages is as follows:

$$\begin{bmatrix} N_{d,sl} \\ N_{d,pl} \\ N_{d,ch} \end{bmatrix} = N_{dt} \begin{bmatrix} \xi_{sl} \\ \xi_{pl} \\ \xi_{ch} \end{bmatrix}, \quad (8)$$

where $N_{d,sl}$, $N_{d,pl}$, and $N_{d,ch}$ are the dynamic effective abrasive grain counts for sliding, plowing, and chip formation, respectively.

Combining Eqs. (4), (6), and (7) yields

$$F_{n,sl} = N_{d,sl} \cdot F'_{n,sl} = N_{d,sl} \cdot \bar{P} \cdot \bar{S} = N_{d,sl} \frac{4P_0 v_w}{v_s d_s} \bar{S}. \quad (9)$$

Let $k_1 = 4P_0 \bar{S} N'_{dt} \xi_{sl}$, then

$$F_{n,sl} = k_1 \frac{N_{dt} \xi_{sl} v_w b}{v_s d_s} = k_1 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}}. \quad (10)$$

According to Li *et al.*, the dynamic friction coefficient between the abrasive grain workpiece during UAG is [50]

$$\mu_{sl} = \frac{2}{\pi} \left(\frac{C_1 \bar{S}}{F_{n,sl}} + C_2 \right) \frac{(\zeta + 1)\zeta}{\sqrt{\zeta^2 + 1}} k_2, \quad (11)$$

where k_2 is a complete elliptic integral of the first type and is constant.

Therefore, the tangential sliding force during UAG is

$$\begin{aligned} F_{t,sl} &= \mu_{sl} F_{n,sl} = k_1 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \frac{1}{\pi} \left(\frac{C_1 \bar{S}}{F_{n,sl}} + C_2 \right) \frac{2(\zeta + 1)\zeta}{\sqrt{\zeta^2 + 1}} k_2 \\ &= \frac{2k_1 k_2}{\pi} \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \left(\frac{C_1 \bar{S}}{F_{n,sl}} + C_2 \right) \frac{(\zeta + 1)\zeta}{\sqrt{\zeta^2 + 1}} \\ &= k_3 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \left(\frac{C_1 \bar{S}}{F_{n,sl}} + C_2 \right) \frac{v_s + 2\pi f A}{\sqrt{(2\pi f A)^2 + v_s^2}}. \end{aligned} \quad (12)$$

Among them, $k_3 = 2\pi k_1 k_2$.

2.2 Plowing force

The grinding force per unit area is solved by the relationship between the force per unit area between the abrasive particles and the workpiece and the component force in any direction. The grinding force per unit area is integrated, and then the complete force between the abrasive grain and the workpiece in the plowing stage is obtained.

According to Zhang *et al.* [37], the plowing force is mainly generated by the plastic flow of the materials. Therefore, the grinding force per unit area is related to the grinding parameters (such as grinding wheel speed, workpiece feed speed, and cutting depth) and certain mechanical properties of the material (such as elastic modulus and yield strength). Figure 2 shows a schematic diagram of the plowing force.

Figure 2(a) shows the relationship between the grinding force F_p per unit area and its component force dF_x in any direction. Figure 2(b) shows a schematic diagram of the plowing force of a single abrasive grain perpendicular to the conical surface.

According to Figure 2(c), the expression of force dF_x can be obtained as follows:

$$dF_x = F_p \cos \theta \cos \psi dA, \quad (13)$$

$$dA = \frac{1}{2} r l_g d\psi = a_g^2 \frac{\tan \theta}{\cos \theta} d\psi. \quad (14)$$

Substituting formula (14) in formula (13), we can obtain

$$dF_x = F_p \cos \theta \cos \psi dA = \frac{1}{2} F_p a_g^2 \tan \theta \cos \psi d\psi, \quad (15)$$

where dA is the effective contact area of the workpiece-abrasive, r is the radius of the S region, l_g is the busbar of the contact zone between the grain and the workpiece, and ψ is the angle between any direction and the workpiece feed direction.

Therefore, the normal plowing force and tangential plowing force acting on a single abrasive grain are

$$\begin{cases} dF_{t,pg} = dF_x \cos \theta \cos \psi = \frac{1}{2} F_p a_g^2 \sin \theta \cos^2 \psi d\psi \\ dF_{n,pg} = dF_x \sin \theta = \frac{F_p a_g^2 \sin^2 \theta \cos \psi}{2 \cos \theta} d\psi. \end{cases} \quad (16)$$

The integral can be obtained as follows:

$$\begin{cases} F'_{t,pg} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} F_p a_g^2 \sin \theta \cos^2 \psi d\psi = \frac{\pi}{4} F_p a_g^2 \sin \theta \\ F'_{n,pg} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{F_p a_g^2 \sin^2 \theta \cos \psi}{2 \cos \theta} d\psi = \frac{F_p a_g^2 \sin^2 \theta}{\cos \theta}. \end{cases} \quad (17)$$

According to the experimental results, the relationship between $F_p a_g^2$ and v_w , v_s , a_p can be established [38].

$$F_p a_g^2 = C_3 \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma. \quad (18)$$

Therefore,

$$\begin{cases} F_{t,pg} = N_{d,pl} F'_{t,pg} = \frac{\pi}{4} N_{d,pl} C_3 \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma \sin \theta \\ = k_4 b \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma \sin \theta \\ F_{n,pg} = N_{d,pl} F'_{n,pg} = N_{d,pl} C_3 \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma \sin \theta \tan \theta \\ = k_5 b \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma \sin \theta \tan \theta, \end{cases} \quad (19)$$

where, α, β, γ are the constants, respectively, $k_4 = \frac{\pi C_3}{4} \xi_{pl}$, and $k_5 = C_3 \xi_{pl}$.

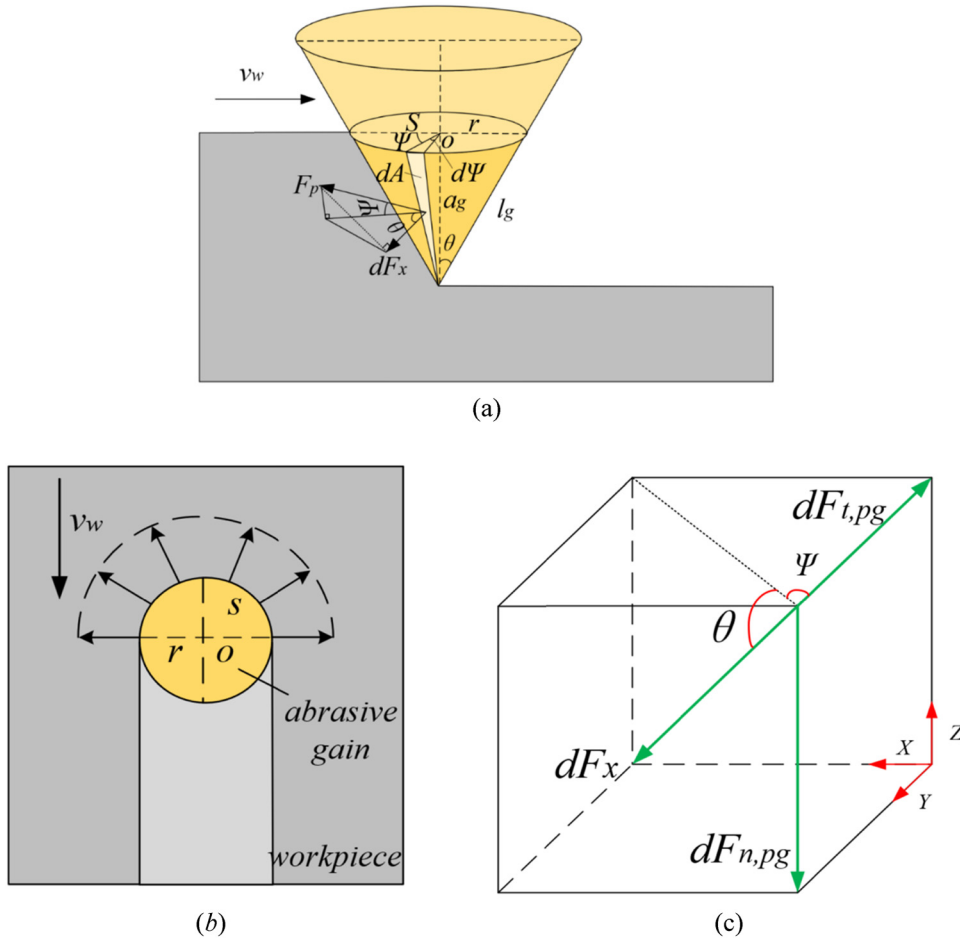


Figure 2: Diagram of plowing force. (a) The relationship between the grinding force per unit area and its component force dF_x in any direction is plotted, (b) detailed map of S-region plowing force, and (c) simplified diagram of tangential plowing force and normal plowing force.

At the same time, the friction coefficient between the workpiece-grinding wheel contact surface in the plowing stage can be obtained as follows:

$$\mu_{pl} = \frac{F_{t,pl}}{F_{n,pl}} = \frac{\pi}{4 \tan \theta}. \quad (20)$$

2.3 Chip formation force

At this stage, the workpiece material is sheared by the cutting edge of the tool to form chips. Therefore, the grinding force prediction model at this stage can be established by the characteristics of the material.

Typically, the energy consumed to remove the unit volume of material is recorded as the grinding specific energy [38], which is expressed by u :

$$u = \frac{v_s F_t}{bv_w a_p}. \quad (21)$$

Therefore, the grinding specific energy u_{ch} of a single abrasive grain in the chip formation stage can be regarded as

$$u_{ch} = \frac{v_s F'_{t,ch}}{bv_w a_p}. \quad (22)$$

In general, the grinding specific energy u_{ch} can be divided into two parts: static grinding specific energy $u_{ch,st}$ and dynamic grinding specific energy $u_{ch,dy}$ [47].

$$u_{ch} = u_{ch,st} + u_{ch,dy}. \quad (23)$$

The specific energy of static grinding is a constant, which is determined by the material properties of workpiece-grinding wheel. The dynamic specific grinding energy is determined by the workpiece and grinding wheel materials and grinding parameters.

2.3.1 Shear strain effect of material removal process

In the case of shear deformation, the general form of the dynamic-plasticity constitutive relation of the material is [49]

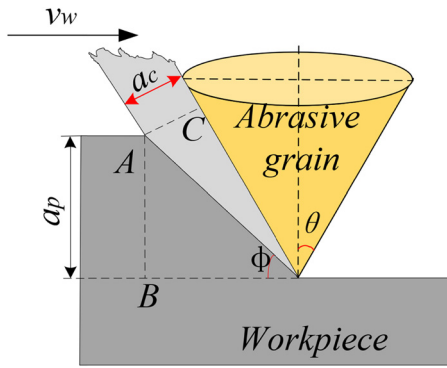


Figure 3: Single grain negative rake angle shear deformation diagram.

$$\tau = h(\gamma, \gamma', T), \quad (24)$$

where τ is the shear flow stress, γ is the shear strain, γ' is the shear strain rate, and T is the deformation temperature.

Figure 3 shows the negative rake angle shear deformation diagram of single grain grinding. In general, with the increase in the shear strain γ and the shear strain rate γ' , the shear flow stress τ of the material also increases. With the increase in the deformation temperature T , the shear flow stress τ of the material will decrease. The dynamic specific grinding energy $u_{ch,dy}$ is proportional to the change in the shear flow stress τ , which is the result of multiple effects.

2.3.2 Shear strain effect

The negative rake angle shear deformation diagram of a single abrasive grain acting on the workpiece is shown in Figure 3. Under two-dimensional shear conditions, γ and γ' in the shear band can be expressed as follows:

$$\gamma = \frac{\cos \theta}{\sin \phi \cos(\phi + \theta)}, \quad (25)$$

$$\gamma' = \frac{v_s \cos \theta}{\Delta s \cos(\phi + \theta)}, \quad (26)$$

where ϕ is the shear angle, and Δs is the thickness of shear band.

The relationship between the shear band thickness Δs and the average undeformed chip thickness a_c is [49]

$$\frac{a_c}{\Delta s \sin \phi} \approx 10. \quad (27)$$

The relationship between the average undeformed chip thickness a_c of a single abrasive grain and the grinding depth a_p is

$$\kappa = \frac{a_c}{a_p} = \frac{\sin \phi}{\cos(\phi + \theta)}. \quad (28)$$

Therefore,

$$\gamma' = \frac{v_s \cos \theta}{\Delta s \cos(\phi + \theta)} = \frac{10v_s \cos \theta}{a_c \cos(\phi + \theta)}. \quad (29)$$

Therefore, the shear strain rate γ' is proportional to the wheel speed v_s and inversely proportional to the average undeformed chip thickness a_c .

It is noted that ϕ and a_c are physical quantities related to the grinding parameters, and based on empirical equations

$$\gamma' = k_6 d_s^e v_s^f a_p^g v_w^{1-f}, \quad (30)$$

where k_6 is a constant.

2.4 Calculation of force in chip formation stage

Studies have shown that there is the following relationship between dynamic specific grinding specific energy $u_{ch,dy}$ and shear strain rate γ' [38].

$$\begin{aligned} u_{ch,dy} &= k_7 \ln \left(\frac{\gamma'}{\gamma'_0} \right) = k_7 \ln \left(\frac{10v_s \cos \theta}{\gamma'_0 a_c \cos(\phi + \theta)} \right) \\ &= k_7 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right). \end{aligned} \quad (31)$$

where k_7 and γ'_0 are constants.

Thus,

$$\begin{aligned} u_{ch} &= \lambda u_{ch,dy} = \lambda k_7 \ln \left(\frac{k_6 d_s^{0.25} v_s^{1.5} a_p^{-0.25} v_w^{-0.5}}{\gamma'_0} \right) \\ &= k_8 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right). \end{aligned} \quad (32)$$

According to Eqs. (24) and (32) can be obtained.

$$F'_{t,ch} = \frac{u_{ch} b v_w a_p}{v_s} = k_8 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right) \frac{b v_w a_p}{v_s}. \quad (33)$$

where $k_8 = \lambda k_7$.

Therefore,

$$F_{t,ch} = F'_{t,ch} \cdot N_{d,ch} = k_8 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right) \frac{b v_w a_p^{1.5} d_s^{0.5}}{v_s}. \quad (34)$$

It was shown that the coefficient of friction between the abrasive grains-workpiece during the grinding stage is

$$\mu_{ch} = \frac{\pi}{4 \tan 60^\circ}. \quad (35)$$

Therefore,

$$F_{n,ch} = \frac{F_{t,ch}}{\mu_{ch}} = k_8 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right) \frac{b v_w a_p^{1.5} d_s^{0.5}}{v_s} \cdot \frac{4 \tan 60^\circ}{\pi} \quad (36)$$

$$= k_9 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right) \frac{b v_w a_p^{1.5} d_s^{0.5}}{v_s},$$

$$\text{where, } k_9 = k_8 \frac{4 \tan 60^\circ}{\pi}.$$

2.5 Summary

This can be obtained from Eqs. (10), (12), (19), (34), and (36).

$$\left\{ \begin{aligned} F_n &= F_{n,sl} + F_{t,pg} + F_{t,ch} = k_1 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \\ &\quad + k_5 b \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma \sin \theta \tan \theta \\ &\quad + k_9 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right) \frac{b v_w a_p^{1.5} d_s^{0.5}}{v_s} \\ F_t &= F_{t,sl} + F_{t,pg} + F_{t,ch} = k_3 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \\ &\quad \times \left(\frac{C_1 \bar{S}}{F_{n,sl}} + C_2 \right) \frac{v_s + 2\pi f A}{\sqrt{(2\pi f A)^2 + v_s^2}} \\ &\quad + k_4 b \left(\frac{v_w}{v_s} \right)^\alpha a_p^\beta d_s^\gamma \sin \theta \\ &\quad + k_8 \ln \left(\frac{k_6 d_s^e v_s^f a_p^g v_w^{1-f}}{\gamma'_0} \right) \frac{b v_w a_p^{1.5} d_s^{0.5}}{v_s}. \end{aligned} \right. \quad (37)$$

3 Experimental verification platform

Through the above theoretical analysis, the predictive force model of UAG of γ -TiAl intermetallic compounds has been established. In order to solve the unknowns in Eq. (37) and verify the predictive force model, a corresponding experimental verification platform is needed. This section introduces the experimental verification platform.

The VMC-850 vertical machining center is selected to build a platform for the main body of the grinding test. A one-dimensional longitudinal vibration ultrasonic system is installed on the spindle to realize the longitudinal ultrasonic vibration of the grinding wheel. At the same time,

Kistler 9257 B dynamometer, 5070 multi-channel charge amplifier, 5697 data acquisition card, and DynoWare software are used to collect normal grinding force F_n and tangential grinding force F_t . One-dimensional longitudinal vibration ultrasonic grinding test platform is shown in Figure 4.

The test used an ordered diamond grinding wheel with an average particle size of 115 μm . The width and diameter of the grinding wheel are 10 mm and 20 mm, respectively. The workpiece is γ -TiAl intermetallic compounds (Ti-45Al-2Mn-2Nb), Table 1 shows the material parameters of γ -TiAl [3]. The length of the workpiece along the grinding direction is 30 mm, and the radial width along the grinding wheel is 15 mm. The workpiece is clamped on the worktable by a fixture. The ultrasonic frequency is 35 kHz.

4 Model calculation and verification

4.1 Calculation of unknown coefficients of the model

Since Eq. (37) contains some unknown vectors, the model is not complete and cannot predict the grinding force of UAG of γ -TiAl intermetallic compounds. Therefore, it is necessary to obtain the grinding force value through experiments to solve the unknown coefficients.

12 groups of experimental results are randomly selected, as shown in Table 2, and the data in the table are substituted in formula (37) to solve the unknown coefficient. The results are shown in Table 3.

Substitute the unknown coefficient in Eq. (37).

$$\left\{ \begin{aligned} F_n &= F_{n,sl} + F_{t,pg} + F_{t,ch} = 10.75 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \\ &\quad + 0.74 \left(\frac{v_w}{v_s} \right)^{0.047} a_p^{0.33} d_s^{1.56} b \sin \theta \tan \theta \\ &\quad + 1.01 \ln(4.29 d_s^{0.64} v_s^{-0.22} a_p^{-0.48} v_w^{1.22}) \frac{v_w a_p^{1.5} d_s^{0.5} b}{v_s} \\ F_t &= F_{t,sl} + F_{t,pg} + F_{t,ch} = -2.18 \frac{a_p^{0.5} v_w b}{v_s d_s^{0.5}} \\ &\quad \times \left(\frac{288.17}{F_{n,sl}} - 25.27 \right) \frac{v_s + 2\pi f A}{\sqrt{(2\pi f A)^2 + v_s^2}} \\ &\quad + 0.22 \left(\frac{v_w}{v_s} \right)^{0.047} a_p^{0.33} d_s^{1.56} b \sin \theta \\ &\quad + 11.23 \ln(4.29 d_s^{0.64} v_s^{-0.22} a_p^{-0.48} v_w^{1.22}) \frac{v_w a_p^{1.5} d_s^{0.5} b}{v_s}. \end{aligned} \right. \quad (38)$$

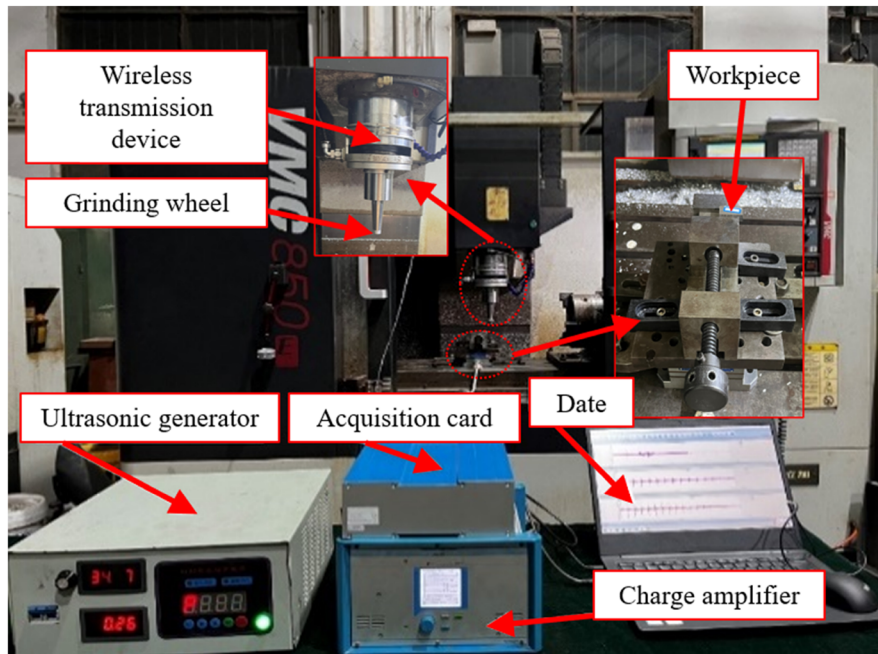


Figure 4: One-dimensional longitudinal vibration ultrasonic grinding test platform.

Table 1: Mechanical properties of γ -TiAl intermetallic compounds

Density ($\text{g}\cdot\text{cm}^{-3}$)	Elastic modulus (GPa)	Yield strength (MPa)	Ductility at room temperature (%)	Creep limit ($^{\circ}\text{C}$)	Antioxidation limit ($^{\circ}\text{C}$)
3.7–3.9	160–176	450–630	1–4	1,000	900–1,000

Table 2: Twelve groups of experimental results for calculating unknown coefficients

a_p (mm)	v_w ($\text{mm}\cdot\text{min}^{-1}$)	v_s (rpm)	A (mm)	F_n	$F_t(N)$
0.002	150	3,000	0.002	8.41	4.15
0.002	200	4,000	0.004	6.68	3.53
0.002	250	5,000	0.006	3.22	3.08
0.004	150	2,000	0.004	13.79	7.36
0.004	100	3,000	0.006	8.39	4.72
0.004	250	4,000	0	15.41	7.93
0.004	200	5,000	0.002	15.98	6.79
0.006	200	2,000	0.006	7.02	4.72
0.006	250	3,000	0.004	16.99	8.66
0.008	200	3,000	0	17.88	9.16
0.008	150	4,000	0.006	10.5	4.75
0.008	100	5,000	0.004	9.14	3.37

4.2 Model validation

By solving the unknown quantities in the model, the complete prediction formula of the grinding force of UAG

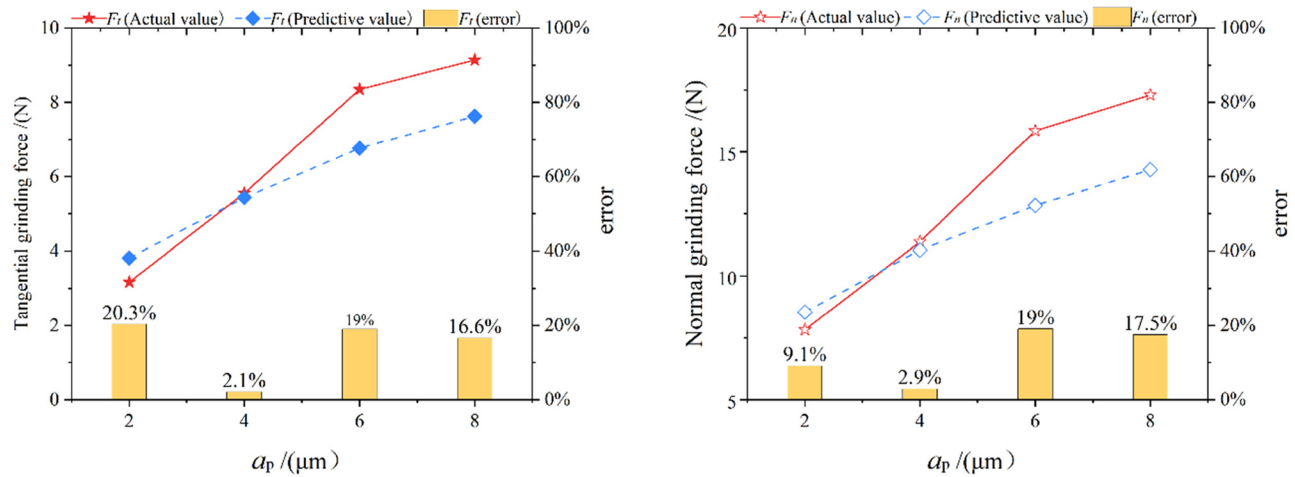
γ -TiAl intermetallic compounds was obtained. However, the accuracy of the prediction model still has some unknowns. Therefore, it is necessary to compare the predicted value of grinding force with the experimental value under different processing parameters to verify the rationality of the model.

Figure 5 shows the comparison between the predicted value and the actual value of the model at different grinding depths a_p under $v_w = 150 \text{ mm} \cdot \text{min}^{-1}$, $v_s = 4,000 \text{ rpm}$, $A = 4 \mu\text{m}$. It can be seen from Figure 5 that the maximum deviation between the predicted value and the actual value of F_t under different grinding depth conditions is 20%, and the maximum deviation between the predicted value and the actual value of F_n is 19%. And with the increase in a_p , the grinding force also showed an upward trend, which is in line with the basic law.

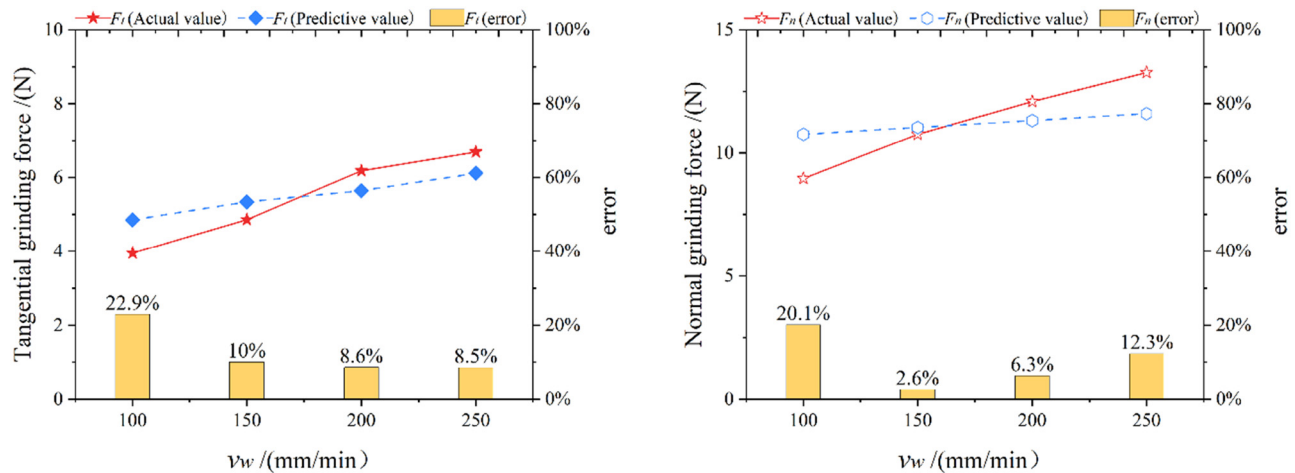
Figure 6 shows the comparison between the predicted value and the actual value of the model at $a_p = 4 \mu\text{m}$, $v_s = 4,000 \text{ rpm}$, $A = 4 \mu\text{m}$, for different feed rates v_w . It can be seen from Figure 6 that the maximum deviation of the predicted value of F_t from the actual value is 23%

Table 3: UAG of γ -TiAl intermetallic compounds unknown coefficient values

k_1	k_3	k_4	k_5	$\frac{k_6}{v_0}$	k_8	k_9	α	β	γ
10.75	-2.18	0.22	0.74	4.29	11.23	1.01	0.047	0.33	1.56
$C_1 S$	C_2	e	f	g					
288.17	-25.27	0.64	-0.22	-0.48					

**Figure 5:** Comparison between the predicted value and the actual value of the model at different grinding depths

a_p : $v_w = 150 \text{ mm} \cdot \text{min}^{-1}$, $v_s = 4,000 \text{ rpm}$, $A = 4 \mu\text{m}$.

**Figure 6:** Comparison between the predicted value and the actual value of the model at different feed speeds

v_w : $a_p = 4 \mu\text{m}$, $v_s = 4,000 \text{ rpm}$, $A = 4 \mu\text{m}$.

and the maximum deviation of the predicted value of F_n from the actual value is 20.1% at different feed rates. And as v_w increases, the grinding force tends to increase, which is in accordance with the law.

Figure 7 shows the comparison between the predicted value and the actual value of the model at $a_p = 4 \mu\text{m}$,

$v_w = 150 \text{ mm} \cdot \text{min}^{-1}$, $A = 4 \mu\text{m}$ and different grinding wheel speeds v_s . It can be seen from Figure 7 that at different speeds, the maximum deviation between the predicted value and the actual value of F_t is 16%, and the maximum deviation between the predicted value and the actual value of F_n is 20.3%. Moreover, with the increase in

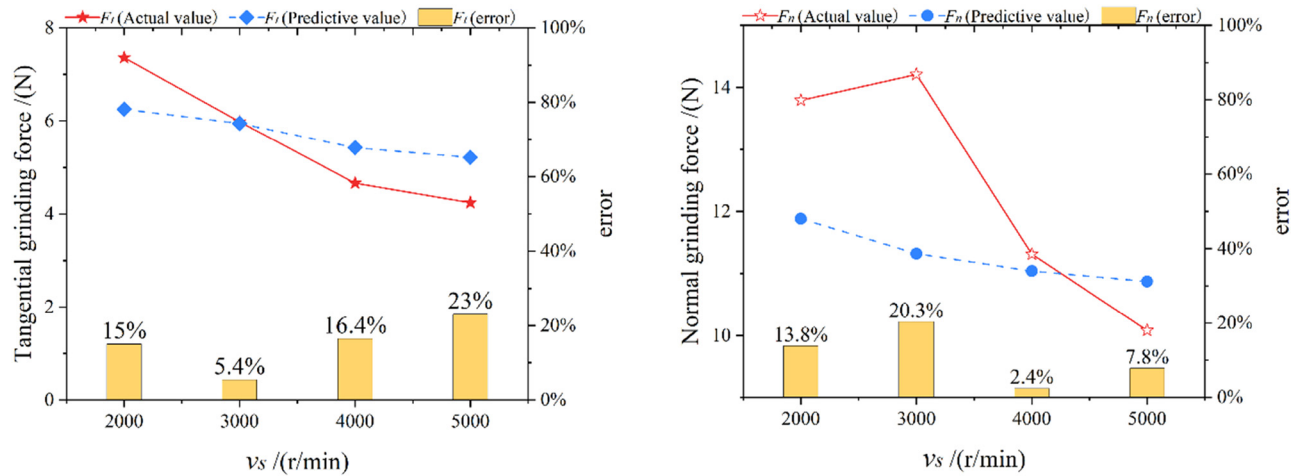


Figure 7: Comparison between the predicted value and the actual value of the model at different grinding wheel speeds
 $v_s: a_p = 4 \mu\text{m}$, $v_w = 150 \text{ mm} \cdot \text{min}^{-1}$, $A = 4 \mu\text{m}$.

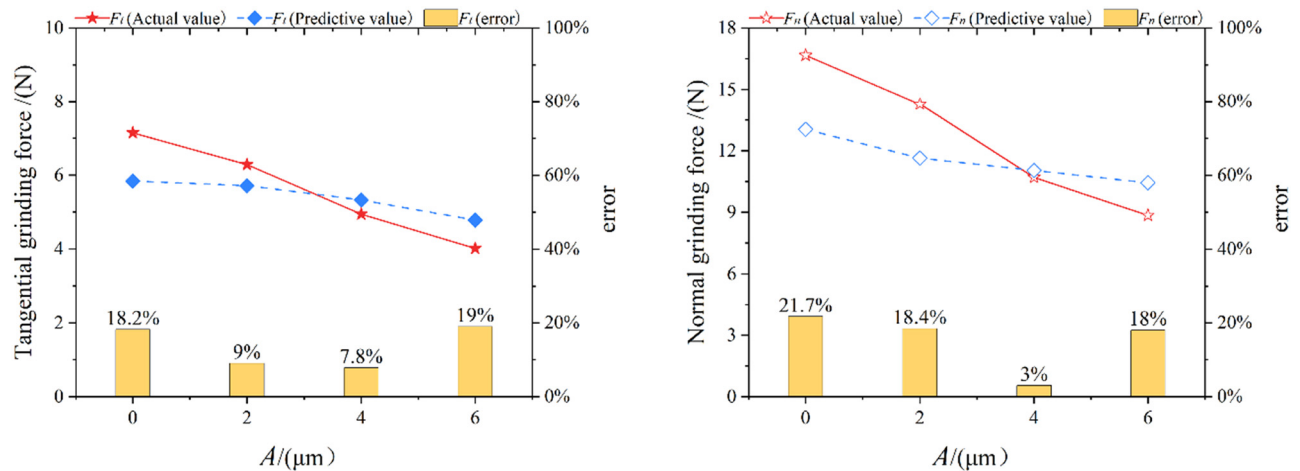


Figure 8: Comparison between the predicted value and the actual value of the model at different ultrasonic amplitudes
 $A: a_p = 4 \mu\text{m}$, $v_w = 150 \text{ mm} \cdot \text{min}^{-1}$, $v_s = 4,000 \text{ rpm}$.

v_s , the grinding force also shows a decreasing trend, which is in accordance with the law.

Figure 8 shows the comparison between the predicted value and the actual value of the model at different ultrasonic amplitudes of A , $a_p = 4 \mu\text{m}$, $v_w = 150 \text{ mm} \cdot \text{min}^{-1}$, $v_s = 4,000 \text{ rpm}$. It can be seen from Figure 8 that under different ultrasonic amplitudes, the maximum deviation between the predicted value of F_t and the actual value is 23%, and the maximum deviation between the predicted value and the actual value of F_n is 21.7%. With the increase in A , the grinding force also shows an upward trend, which conforms to the law.

Therefore, the maximum deviation between the predicted value and the actual value of F_t is 23%, and the maximum deviation between the predicted value and the

actual value of F_n is 21.7%. The results show that the predicted results are in good agreement with the experimental results.

5 Conclusion

In this study, a new UAG force prediction model is established. The model considers three key factors: the friction between the abrasive and the material, the plastic deformation of the material during the abrasive plowing process, and the shear strain effect of the material during the chip formation process. Based on this model, the UAG experiment of γ -TiAl intermetallic compounds was carried

out, and the unknown coefficients in the model were calculated.

The model was verified under different parameters. It was found that the maximum deviation between the predicted value and the actual value of F_t is 23%, and the maximum deviation between the predicted value and the actual value of F_n is 21.7%. The results show that the predicted results are in good agreement with the experimental results.

In addition, by modifying the correlation coefficient, the model can be used to predict the grinding force of different metal materials under different processing parameters.

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