Research Article

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Study in the parameter influence on underwater acoustic radiation characteristics of cylindrical shells

Abstract: At present, the cylindrical shell was regarded as the dominating structure in underwater vehicles, which raised the crucial significance to research underwater vehicles' vibration and acoustic radiation features. In this study, the analytical expression of vibration–acoustic theory had been given according to the derivation with the vibration–acoustic theory of ribless cylindrical shell structure. Meanwhile, the effects of key parameters on vibration–acoustic characteristics are investigated including modulus, density, thickness, loss factor, etc. The research shows that the stiffness and damping of the shell directly affected the vibration of the structure. Furthermore, the performance of vibration attenuation and noise reduction of the shell had been enhanced by increasing the modulus, thickness, and loss factor in materials.

Keywords: cylindrical shell, vibration, acoustic radiation, influence parameters

1 Introduction

Research shows that when the underwater navigator sails, the mechanical equipment, propeller, water flow impact, and other external loads will cause a vibration response [1–3], which greatly affects the concealment of the underwater navigator. The cylindrical shell structure, which is the main form of the underwater navigation body, is crucial to study its underwater vibration and acoustic radiation properties [4].

In the earlier study, Junger [5] found that the presence of attached water reduces the natural frequency of the shell and significantly affects the structural low-frequency vibration, by analyzing the influence of fluid load on the vibration properties of the cylindrical shell. In the literature [6], Junger derives the systematic Lagrange equation and studies the acoustic radiation induced by shell vibration in an external acoustic medium, pointing out that the fluid loading on the shell reaction force is equivalent to the additional mass and damping forces. Bleich and Baron [7] introduced the modal superposition method into the investigation of underwater shell vibration to emphasize the necessity of solving the equation of shell and acoustic field simultaneously under the given object surface and far-field conditions, which promoted the research in coupled vibration problem. Williams et al. [8] expanded the velocity potential and boundary conditions into an infinite series of their respective characteristic functions with the semi-analytic method employed and converted those infinite series into terms of finite series with deducible coefficients based on the principle of minimum mean error. Then, the acoustic radiation in cylindrical shells with infinite and finite in length had been compared and analyzed by a semi-analytical method. Junger and Feit [9] analyzed the acoustic vibration problem of the beam, plate, spherical shell, and cylindrical shell and summarized the work and results of shell stimulated vibration and acoustic radiation.

Overall, early studies have established theoretical computational methods, and subsequent scholars carried out further studies on the problems of limited application scope and low computational accuracy. Ji et al. [10] and Zou et al. [11] presented an element group method and a mixed analytical-numerical substructure method separately to improve the computational efficiency of the hull-substructure coupled and fluid-structure interacted vibration and acoustic radiation of a submerged

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cylindrical shell-type vehicle. Ning [12] Researched the influence of ring rib arrangement on vibration and acoustic radiation of cylindrical shells and shows that the ring rib structure can suppress the transmission of medium and high-frequency vibration. Sun et al. [13] and Tong et al. [14,15] proved that the calculation precision for natural frequency and radiation efficiency of thick shells is inferior to thin shells by studying the modal of cylindrical shells and investigating the effects of layer angle on natural frequencies. Guo et al. [16] studied the effects of tensile and compressive stresses, stress direction, value size, and distribution on vibration and sound radiation of cylindrical shells for the problem of Vibration and sound radiation of submerged finite cylindrical shells with pre-stress.

In 2019, Wang et al. [17–20] carried out a vibro-acoustic behavior study of cylindrical shells in ice-covered water and analyzed the vibro-acoustic behavior of submerged double-walled cylindrical shells with general boundary conditions and the free vibration of stiffened cylindrical shells with variable thickness by a precise transfer matrix method (PTMM). Multiple experiments have shown that the PTMM is reliable and the result from PTMM is credible. Chai and Wang [21], Wang et al. [22], and Ye and Wang [23] established cylindrical shell and plate models for nonlinear vibration analysis by Donnel theory and discussed the influence of material, porosity distribution, and size on vibration characteristics. Li et al. [24–26] proposed a semi-analytical method to analyze the vibration response of cylindrical shells with arbitrary boundary restraints. Zhang et al. [27] adopted FEM/BEM algorithm and, via the virtual source chain model to calculate the modal source strengths and modal coordinate responses of cylindrical shell, analyzed the influence of the seabed parameters on the acoustic radiation of the cylindrical shell. Guo et al. [28] proposed a new method that has a wide application scope and good accuracy in the solution of the vibro-acoustic behaviors of an elastic cylindrical shell partially coupled with fluids theoretically. Tang et al. [29] proposed a new method to simplify the calculation process of radiated sound power by simplifying the cylindrical shell to a beam and adding mass to approximate fluid-structure coupling. Zhang et al. [30] proposed a sound radiation calculation method by using dominant modes to predict the sound radiation from a cylindrical shell that can reduce the number of displacement monitoring points as possible on the structure surface. Du et al. [31] and Du et al. [32] propose efficient calculation

methods for analyzing the vibration characteristics of the Spherical Cap and stiffened plate.

To effectively control the acoustic radiation of the cylindrical shell structure, scholars have launched research in materials [33,34] and structure [35–37]. The acoustic characteristics of cylindrical shells with additional materials and noise reduction measures are the focus of attention. Ding et al. [38] and Dai et al. [39] studied the radiation characteristic and active control of the cylindrical shell and provided theoretical support for the effective control of the acoustic vibration response of the submerged structure. Zou et al. [40] and Liu et al. [41] established an analytical formulation theoretically for the shell-coating-fluid system to calculate underwater acoustic radiation for the cylindrical shell structure covering the acoustic cover layer.

Based on the previous studies mentioned above, the vibration–acoustic theory of single-layer cylindrical shells had been deduced. Considering the case of underwater single-layer cylindrical shells, the influence of parameters, such as modulus, density, thickness, and loss on the vibration and acoustic performance with finite-length cylindrical shells, had been calculated, while the acoustic radiation characteristics of underwater cylindrical shells had been analyzed in this work. Results provided the guidelines for the acoustic stealth design for underwater vehicles.

2 Vibration–acoustic theory of cylindrical shells with isotropic materials

With the purpose to reveal the vibration mechanism of a finite-length cylindrical shell shown in Figure 1, the governing equations in cylindrical coordinates based on elasticity theory had been first established, which are presented as follows.

$$
\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} \beta^2 \ddot{u} \\ \beta^2 \ddot{v} \\ \beta^2 \ddot{w} \end{Bmatrix}, \tag{1}
$$

where $\beta^2 = 2\rho/E(1 + \mu)(1 - 2\mu)$, in which ρ, E, and μ are the density, Young's modulus, and Poisson's ratio of the elastomer, respectively. L_{mn} represent the partial differential operators.

According to Donnell's theory [42,43], the kinetics equation of a free-vibration cylindrical thin shell was written as
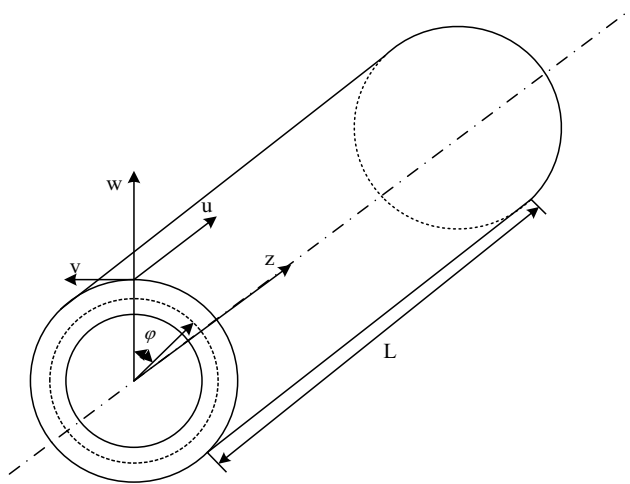
Figure 1: Schematic diagram of the cylindrical shell.

$$\begin{cases} \dfrac{\partial^2 u}{\partial z^2} + \dfrac{1-\mu}{2a^2}\dfrac{\partial^2 u}{\partial \phi^2} + \dfrac{1+\mu}{2a}\dfrac{\partial^2 v}{\partial z \partial \phi} + \dfrac{\mu}{a}\dfrac{\partial w}{\partial z} - \rho\dfrac{1-\mu^2}{E}\dfrac{\partial^2 u}{\partial t^2} = 0 \\[2mm] \dfrac{1+\mu}{2a}\dfrac{\partial^2 u}{\partial z \partial \phi} + \dfrac{1-\mu}{2}\dfrac{\partial^2 v}{\partial z^2} + \dfrac{1}{a^2}\dfrac{\partial^2 v}{\partial \phi^2} + \dfrac{1}{a^2}\dfrac{\partial w}{\partial \phi} - \rho\dfrac{1-\mu^2}{E}\dfrac{\partial^2 v}{\partial t^2} \\[2mm] \quad = 0 \\[2mm] \dfrac{\mu}{a}\dfrac{\partial u}{\partial z} + \dfrac{1}{a^2}\dfrac{\partial v}{\partial \phi} + \dfrac{w}{a^2} + \beta^2\left(a^2\dfrac{\partial^4 w}{\partial z^4} + 2\dfrac{\partial^4 w}{\partial z^2 \partial \phi^2} + \dfrac{1}{a^2}\dfrac{\partial^4 w}{\partial \phi^4}\right) \\[2mm] -\rho\dfrac{1-\mu^2}{E}\dfrac{\partial^2 w}{\partial t^2} = \dfrac{1-\mu^2}{Eh}(f(a,\phi,z) - p(a,\phi,z)), \end{cases} \quad (2)$$

which can be rewritten into the matrix as

$$\frac{Eh}{1-\mu^2}[L_D]\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} + \rho h \omega^2 \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$
$$= \begin{Bmatrix} 0 \\ 0 \\ f(a,\phi,z) - p(a,\phi,z) \end{Bmatrix}, \quad (3)$$

where a denote the radius of the cylinder, h denote the shell thickness, $\beta^2 = h^2/12a^2$, and $[L_D]$ denote the Donnell operator.

The harmonic field in a cylindrical shell resulting from a small perturbation within the ideal fluid medium can be described by the Helmholtz equation with Neumann boundary conditions and Somerfield radiation conditions [44–46]. The continuity condition of normal velocity on the coupling surface was

$$\left.\frac{\partial p(r,\varphi,z)}{\partial r}\right|_{r=a}$$
$$= \begin{cases} \rho_0 \omega^2 w(a,\varphi,z), & 0 \le z \le L, \\ 0, & z < 0 \text{ and } z > L, \end{cases} \quad (4)$$

where L denote the length of the cylindrical shell, ρ_0 denote media density, $k = \omega/c_0$, and ω and c_0 are a number of acoustic waves, angular frequency of acoustic waves, and the speed of sound in a certain medium.

The wave propagation within a cylindrical shell with finite length was reflected by the end faces, and the vibration of the shell was presented as the form of the stationary wave, which relied on the boundary conditions of the shell. As to simplify the theoretical analysis, the transverse bulkhead support of submarine pressure shell had been generally regarded as simply supported at both ends. Hence, the boundary conditions of simply supported at both ends of the cylindrical shell had to be satisfied [47–49].

$$v = w = N_z = M_z, \quad z = 0 \text{ and } z = L, \quad (5)$$

in which N_z is the force and M_z is the moment.

Whereafter, the kinetics equation of the shell was solved through the modal expansion method. In the investigation of the acoustic and vibration under the symmetric excitation, the displacement solution in the following form had been applied[32,50].

$$\begin{cases} u(\varphi,z) = \sum\limits_{m=1}^{+\infty}\sum\limits_{n=0}^{+\infty} U_{mn}\cos(n\varphi)\cos(k_m z), \\[2mm] v(\varphi,z) = \sum\limits_{m=1}^{+\infty}\sum\limits_{n=0}^{+\infty} V_{mn}\sin(n\varphi)\sin(k_m z), \\[2mm] w(\varphi,z) = \sum\limits_{m=1}^{+\infty}\sum\limits_{n=0}^{+\infty} W_{mn}\cos(n\varphi)\sin(k_m z). \end{cases} \quad (6)$$

Thereafter, the radial excitation force and surface sound pressure were expanded into the series of the (m,n) order modal namely $\{\cos(n\varphi)\sin(k_m z)\}$ of radial displacement; thus,

$$\begin{Bmatrix} f(a,\varphi,z) \\ p(a,\varphi,z) \end{Bmatrix} = \sum_m \sum_n \begin{Bmatrix} f_{mn} \\ p_{mn} \end{Bmatrix} \cos(n\varphi)\sin(k_m z), \quad (7)$$

the expansion coefficient was determined by the following equation:

$$\begin{Bmatrix} f_{mn} \\ p_{mn} \end{Bmatrix} = \frac{\varepsilon_n}{\pi L} \int\limits_{-\pi}^{\pi}\int\limits_{0}^{L} \begin{Bmatrix} f(a,\varphi,z) \\ p(a,\varphi,z) \end{Bmatrix} \cos(n\varphi)\sin(k_m z)\mathrm{d}\varphi\mathrm{d}z, \quad (8)$$

where ε_n was the Neumann factor. $\varepsilon_n = 1$ while $n = 0$ and $\varepsilon_n = 2$ when $n \ge 1$. The modal equation was obtained by substitution of equations (6) and (7) into the kinetics equation of the cylindrical thin shell,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{bmatrix} = \frac{a^2}{\rho_p c_p^2 h} \begin{bmatrix} 0 \\ 0 \\ f_{mn} - p_{mn} \end{bmatrix}. \quad (9)$$

In which each element a_{ij} in the eigenmatrix depended on the governing equation forms corresponding to different thin shell theories. Considering that the frequency band was lower than the critical frequency of the shell, the

kinetics equation based on Donnell's shell theory mentioned in the above section had been selected; thus,

$$a_{11} = -\Omega^2 + k_m^2 a^2 + \frac{1}{2}(1-\mu)n^2, \quad a_{22} = -\Omega^2 + \frac{1}{2}(1-\mu)k_m^2 a + n^2,$$

$$a_{12} = a_{21} = \frac{1}{2}(1+\mu)nk_m a, \qquad a_{23} = a_{32} = n, \tag{10}$$

$$a_{13} = a_{31} = \mu k_m a, \qquad\qquad a_{33} = 1 - \Omega^2 + \beta^2(k_m^2 a + n^2)^2,$$

where $c_p = \sqrt{E/\rho_p(1-\mu^2)}$, which represented the longitudinal wave velocity while the shell was extended into a flat plate. $\Omega = \omega a/c_p$, which represented the longitudinal wave velocity while the shell was extended into a flat plate. $\Omega = \omega a/c_p$, n, and $k_m a$ were marked as dimensionless quantities.

As revealed from equation (3), the normal mode velocity satisfied the following equation:

$$\dot{W}_{mn} Z_{mn}^M = f_{mn} - p_{mn}. \tag{11}$$

Assuming $p_{mn} = 0$, the mechanical impedance in the (m,n) order normal mode of the cylindrical shell surface with finite length had to be obtained as

$$Z_{mn}^M = i\rho_p h\omega|a_{ij}|/\Omega^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}. \tag{12}$$

Moreover, according to the Green's function G, a fundamental solution of the wave equation, sound pressure at the arbitrary point of $A(r, \phi, z)$ in the acoustics field was expressed as

$$p(A) = \iint\limits_{s} G(A|A_0)\dot{w}(A_0)\mathrm{d}s, \tag{13}$$

where $A_0(a, \varphi_0, z_0)$ represented the arbitrary point on the surface of the shell while $\dot{w}(A_0)$ was the radial velocity of shell surface. Hence, Green's function satisfied with

$$G(A|A_0) = \frac{\rho_0\omega}{4\pi^2} \cdot \sum_{n=0}^{\infty} \varepsilon_n \, \cos(n(\varphi - \varphi_0))$$
$$\times \int_{-\infty}^{\infty} \frac{iH_n^{(1)}(\sqrt{k^2 - k_z^2}\, r)}{k_r a H_n^{(1)\prime}(\sqrt{k^2 - k_z^2}\, a)} e^{ik_z|z-z_0|}\mathrm{d}k_z, \tag{14}$$

where $k^2 = k_r^2 + k_z^2$, k_r and k_z denote the radial and axial components of a wave vector.

By substituting the radial modal velocity into the above equation, the sound pressure at the arbitrary point within acoustic field was able to be deduced as

$$p(r, \varphi, z) = -\frac{i\rho_0\omega^2}{4\pi^2} \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} W_{mn} \cos n\varphi \cdot$$
$$\times \int_{-\infty}^{\infty} \frac{iH_n^{(1)}(\sqrt{k^2 - k_z^2}\, r)}{\sqrt{k^2 - k_z^2}\, H_n^{(1)\prime}(\sqrt{k^2 - k_z^2}\, a)} \tag{15}$$
$$\times \frac{k_m[1 - (-1)^m e^{-ik_z L}]}{k_m^2 - k_z^2} e^{ik_z z}\mathrm{d}k_z,$$

Which expressed in the following form:

$$p_{mn} = \sum_q Z_{nmq}\dot{W}_{qn}. \tag{16}$$

As $\beta = k_z/k$, $\alpha = ka\sqrt{1-\beta^2}$, $Z_n(\alpha) = iH_n^{(1)}(\alpha)/H_n^{(1)\prime}(\alpha)$, and the dimensionless radiation impedance Z_{nmq} was written as

$$z_{nmq} = \frac{Z_{nmq}}{\rho_0 c_0 S} = R_{nmq} - i\chi_{nmq}$$
$$= \frac{8mq\pi}{\varepsilon_n(kL)^3} \int_0^{\infty} \frac{Z_n(\alpha)N_{mq}(\beta)\mathrm{d}\beta}{(1-\beta^2)^{1/2}[\beta^2 - (k_m/k)^2][\beta^2 - (k_q/k)^2]}, \tag{17}$$

where

$$N_{mq}(\beta) = \begin{cases} \cos^2(kL\beta/2), & m, q = 1, 3, 5 \ldots \\ \mp i\sin(kL\beta/2)\cos(kL\beta/2), & m(q) = 1, 3, 5 \ldots; \\ & q(m) = 2, 4, 6 \ldots \\ \sin^2(kL\beta/2), & m, q = 2, 4, 6 \ldots. \end{cases} \tag{18}$$

The acoustic radiation impedance reflected the interaction between the medium and the sound source, which was considered the fundamental of structural-acoustic radiation analysis. As introduced in the equation above, non-zero axial half-wave numbers (m and q) are exhibited only when both of them are odd or even numbers.

Therefore, in combination with equations (11) and (17), the coupling kinetics equation of excitation source-cylindrical shell-external flow field was yielded,

$$Z_{nm}^M \dot{W}_{mn} + Z_{nmm}\dot{W}_{mn} = f_{mn} - \sum_{q=1, q\neq m}^{\infty} Z_{nmq}\dot{W}_{qn}, \tag{19}$$

Once the modal vibration velocity was obtained, the sound pressure, intensity, radiated power, radiation efficiency, and other associated acoustic quantities were able to be deduced as consequence.

Wherein, the acoustic radiation power of the shell surface was

$$W(\omega) = \frac{1}{2}\mathrm{Re}\iint\limits_{s} p(a, \varphi, z)\dot{w}(\varphi, z)\mathrm{d}s$$
$$= \frac{S}{4}\mathrm{Re}\left\{ \sum_q\sum_m\sum_n \frac{1}{\varepsilon_n}\dot{W}_{qn}Z_{nmq}\dot{W}_{qn}^{\star} \right\}. \tag{20}$$

The radial mean square velocity on the shell surface was

$$\langle v_r^2 \rangle = \frac{1}{2S}\iint\limits_{s} \dot{w}(\varphi, z)\dot{w}(\varphi, z)^{\star}\mathrm{d}s$$
$$= \frac{1}{4}\sum_m\sum_n \frac{1}{\varepsilon_n}\dot{W}_{mn}\dot{W}_{mn}^{\star}. \tag{21}$$

And the radiation efficiency was

$$\sigma(\omega) = \frac{W(\omega)}{\rho_0 c_0 S < v_r^2 >} = \frac{\text{Re}\left\{\sum_q \sum_m \sum_n \frac{1}{\varepsilon_n} \dot{W}_{qn} Z_{nmq} \dot{W}_{qn}^{\star}\right\}}{\rho_0 c_0 \sum_m \sum_n \frac{1}{\varepsilon_n} \dot{W}_{mn} \dot{W}_{mn}^{\star}}. \quad (22)$$

3 Theoretical model of calculation conditions

In this paper, a cylindrical shell with a radius of 0.5 m and a length of 1.2 m is taken as an example to study the parameter influence on underwater acoustic radiation characteristics of cylindrical shells. The density ρ, modulus E, thickness h, loss factor η and Poisson's ratio v were valued of the cylindrical shell as the controlling variables , specific inputs are shown in Tables 1–5.

According to the above theory, the vibration–acoustic calculation theory of the cylindrical shell had been programmed, and the vibration and acoustic radiation response curves of the ideal homogeneous shell plate under the influence of different parameters were calculated by Matlab to understand the influence introduced by parameters setting. For cylindrical shells, the radial exciting force acted at $(L/2, 0)$ of the shell with the amplitude of 1 N. Fluid density

Table 1: Total level of vibration and radiation power of shell under different density materials

Groups	Density (kg·m⁻³)	Cylindrical shell	
		Overall mean square vibration velocity level (dB)	Overall acoustic radiation power level (dB)
1-1	0.1	121.74	101.23
1-2	0.05	125.31	101.41
1-3	0.01	130.86	103.86
1-4	0.001	137.60	103.96

Table 2: The overall vibration level and the overall sound power level of the shell with different elastic moduli of materials

Groups	Elastic modulus (GPa)	Cylindrical shell	
		Overall mean square vibration velocity level (dB)	Overall acoustic radiation power level (dB)
2-1	200	121.15	99.68
2-2	100	124.05	99.04
2-3	50	130.86	102.09
2-4	25	138.22	106.63

Table 3: The overall vibration level and the overall sound power level of the shell with different shell thicknesses of materials

Groups	Shell thickness (mm)	Cylindrical shell	
		Overall mean square vibration velocity level (dB)	Overall acoustic radiation power level (dB)
3-1	20	126.25	88.73
3-2	15	129.17	91.16
3-3	10	132.84	88.29
3-4	5	140.75	88.61

$\rho_f = 1.025 \times 10^3\,\text{kg·m}^{-3}$, sound velocity $cf = 1{,}500\,\text{m·s}^{-1}$, reference velocity $v_{\text{ref}} = 10^{-9}\,\text{m·s}^{-1}$, and reference sound power $W_{\text{ref}} = 10^{-12}\,\text{W}$ were set. Respected to the value presented in Tables 1–5, step size was 2 Hz per step, a total of 750 steps.

4 Analysis of underwater acoustic radiation parameters of cylindrical shell

4.1 Influence of material density on vibro-acoustic characteristics of cylindrical shell

The vibration and acoustic performance of the theoretical model of the cylindrical shell had been calculated according to the structural model described in Section 1. The differences in vibration and acoustic performance of structures under different densities had been compared. Results are revealed in Figure 2. In addition, overall vibration levels and the overall acoustic power levels within the 0–1,500 Hz frequency band are shown in Table 1.

Figure 2 and Table 1 reveal that with the increase in material density, the mean square velocity of structure

Table 4: The overall vibration level and the overall sound power level of the shell with different material losses

Groups	Material loss factor	Cylindrical shell	
		Overall mean square vibration velocity level (dB)	Overall acoustic radiation power level (dB)
4-1	0.1	121.74	101.23
4-2	0.05	125.31	101.41
4-3	0.01	130.86	103.86
4-4	0.001	137.60	103.96

Table 5: The overall vibration level and the overall sound power level of the shell with different Poisson's ratios of materials

Groups	Poisson's ratio	Cylindrical shell	
		Overall mean square vibration velocity level (dB)	Overall acoustic radiation power (dB)
5–1	0.4	128.51	101.37
5–2	0.3	128.80	101.30
5–3	0.2	130.86	102.09
5–4	0.1	130.59	102.75

and the acoustics radiation power in the vibro-acoustic performance of the simply supported cylindrical shell deceased. However, within the designed variation range of density in the calculation conditions, theoretical calculation models exhibited their characteristics. With the decreasing of density, the peak of vibration of the structure shifted toward a higher frequency with a lower modal density in the same frequency band.

For the cylindrical shell, two independent domains existed in and outside of the shell, which lead to a certain analogousness to the case with the baffle in radiation performance. The mean vibration velocity level and acoustics radiation power level were barely affected by the density. In comparison under the same radiation mode, the peak value of acoustic vibration response increased slightly with the decrease in density. Nevertheless, limited to the extremely dense radiation modes and the calculation accuracy, position, and amplitude of peaks were failed to detective in some frequency bands, which occurred with errors in certain levels. However, in general, the acoustics power level of the underwater cylindrical shell was hardly disturbed by the density. The variation in density mainly affected the modal distribution, and the magnitude of the radiation energy was similar. Whereas, the surface velocity increased with the decrease in the density, which was significantly related. Compared with the 1–4 working condition, the overall mean square vibration velocity and the overall acoustic radiation power in the frequency band of 0–1,500 Hz decreased by 4.57 and −0.6 dB, respectively, under the 1–1 working condition.

4.2 Influence of elasticity modulus in materials on vibro-acoustic characteristics of cylindrical shell

The vibration and acoustic performance of the theoretical model of the cylindrical shell had been calculated according to the structural model described in Section 1. The

(a)

(b)

(c)
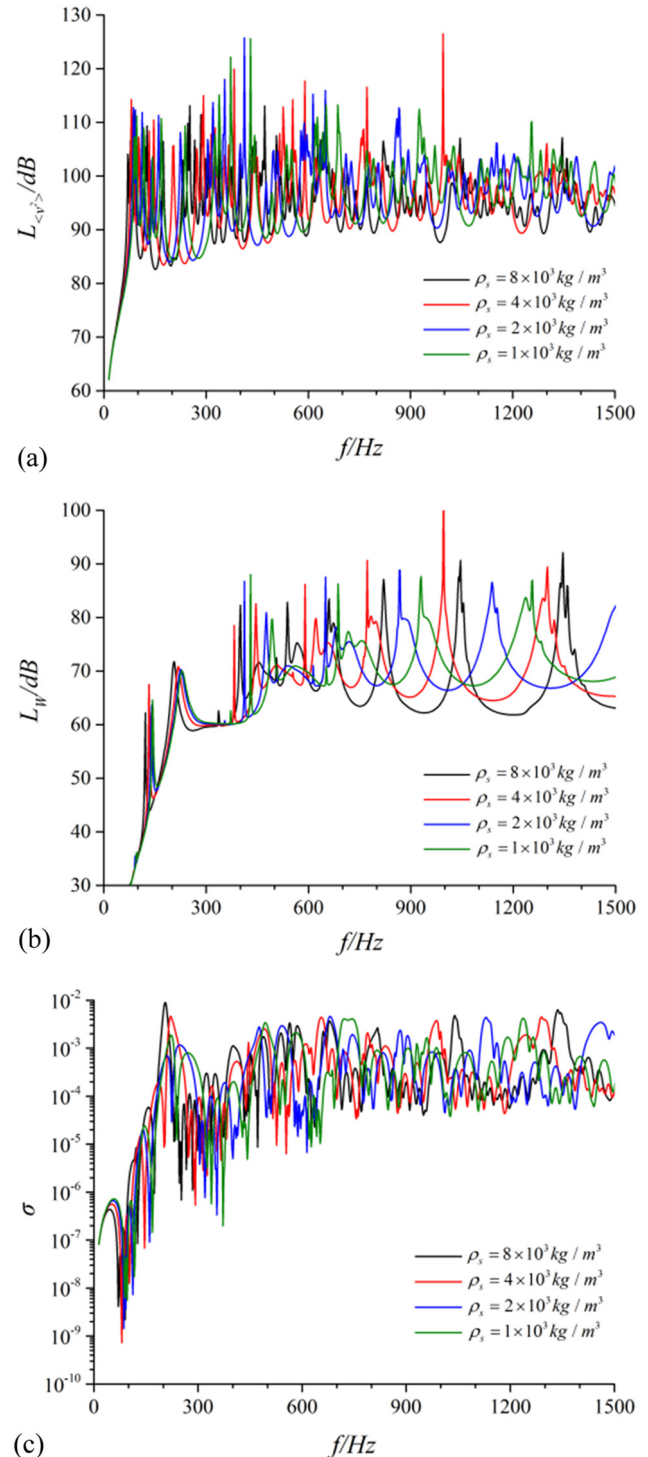
Figure 2: Contrast of vibration and noise performance for the cylindrical shell under different densities. (a) Radial quadratic velocity level, (b) sound radiation power level, and (c) sound radiation efficiency.

differences in performance of the structure with the influence of different elasticity modulus in materials had been compared. Results are revealed in Figure 3. In addition, the overall vibration levels and the overall acoustics power

(a)

(b)

(c)
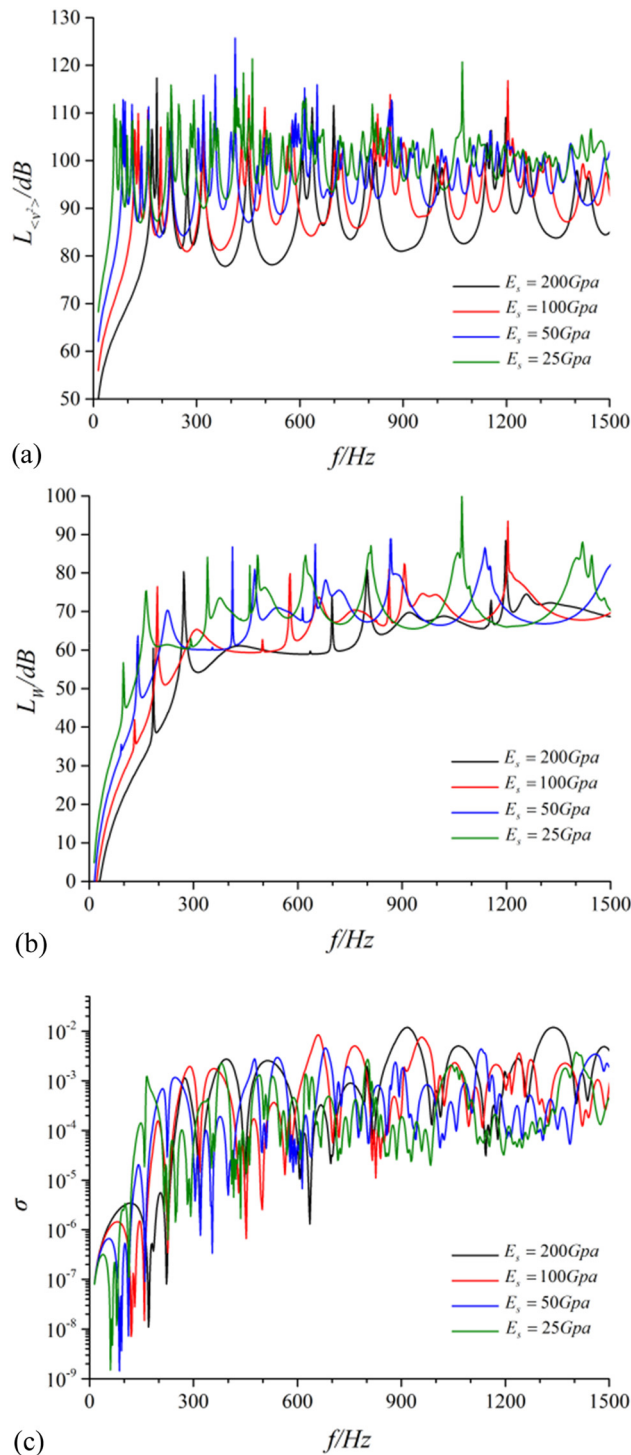
Figure 3: Contrast of vibration and noise performance for the cylindrical shell under different elasticity moduli. (a) Radial quadratic velocity level, (b) sound radiation power level, and (c) sound radiation efficiency.

levels within the 0–1,500 Hz frequency band are shown in Table 2.

Variation in the elasticity modulus of materials would significantly influence the vibration and acoustic performance

of cylindrical shell structures. Concretely, with the consideration of simply supported cylindrical shells, the vibration level and the acoustic radiation level of the cylindrical shells remarkably increased with the decrease in the elastic modulus. On the contrary, as the increment of elastic modulus, the acoustic radiation efficiency of the theoretical model increased consequently, which offsets the negative effect of reducing the acoustic radiation power resulting from the decrement of vibration level.
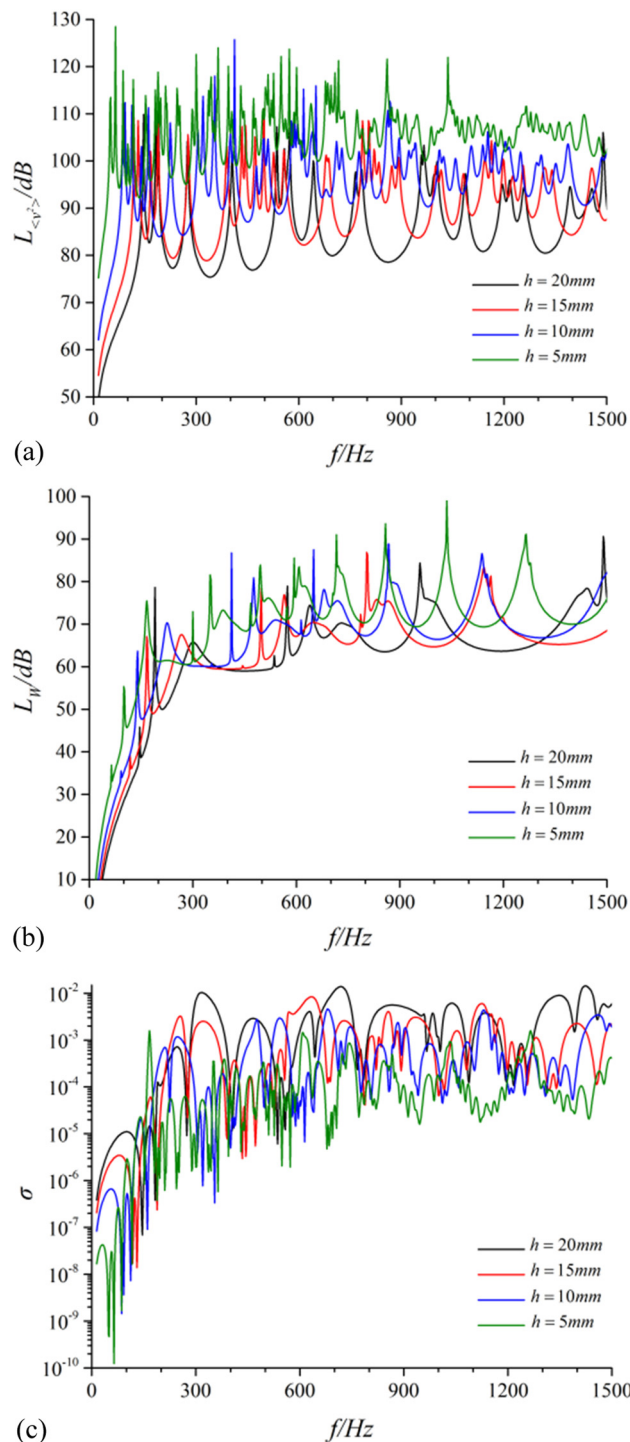
For the cylindrical shell, the level of mean vibration velocity and acoustics radiation power were greatly impacted by the elastic modulus of the material in evidence by the comparison of spectral peak size in the same order. The level of mean square velocity and acoustics radiation power increased with the decrease in the modulus. Compared with the 2–4 working condition, the overall mean square vibration velocity and the overall acoustic radiation power in the frequency band of 0–1,500 Hz decreased by 8.19 and 8.31 dB, respectively, under the 2–1 working condition, which is revealed in Table 2. Therefore, the performance of vibration and acoustic radiation of cylindrical shells was significantly affected by the elastic modulus of materials. And the composite materials with relatively large elastic modulus were a better choice for non-pressure shell design.

4.3 Influence of shell thickness on vibro-acoustic characteristics of cylindrical shell

The vibration and acoustic performance of the theoretical model of the cylindrical shell had been calculated according to the structural model described in Section 1. The differences in performance of the structure with the influence of different thicknesses in materials had been compared. Results are revealed in Figure 4. In addition, the overall vibration levels and the overall acoustics power levels within the 0–1,500 Hz frequency band are shown in Table 3.

Analogous to the elastic modulus of materials, the thickness of shell h was also one of the crucial parameters that affected the structural stiffness, which was proportion to h^3. In addition, in comparison of different thicknesses, the underwater mean vibration velocity level of the cylindrical shell calculation model decreased overtly with the increase in shell thickness; however, the acoustic radiation efficiency increased and the acoustic power level decreased consecutively.

For the cylindrical shell, the mean vibration velocity level and the acoustic radiated power level were significantly affected by the thickness and the mean square vibration velocity level and the acoustic radiated power level of

(a)

(b)

(c)

Figure 4: Contrast of vibration and noise performance for the cylindrical shell under different shell thicknesses. (a) Radial quadratic velocity level, (b) sound radiation power level, and (c) sound radiation efficiency.

the cylindrical shell decreased dramatically with the increasing of the shell thickness. Compared with the 3–4 working conditions, the overall mean square vibration velocity and the overall acoustic radiation power in the frequency band of

0–1,500 Hz decreased by 17.07 and 6.95 dB, respectively, under the 3–1 working condition. Hence, the vibration and acoustic radiation properties of cylindrical shells were greatly affected by the shell thickness. The employment of composite materials in the shell was capable to maximize the thickness (stiffness) of the shell while satisfying the index of weight.

4.4 Influence of loss factors in materials on vibro-acoustic characteristics of cylindrical shell

The vibration and acoustic performance of the theoretical model of the cylindrical shell had been calculated according to the structural model described in Section 1. Results are illustrated in Figure 5. The differences in the performance of the structure with the influence of different loss factors in materials had been compared. The overall vibration levels and the overall acoustic power levels within the 0–1,500 Hz frequency band are shown in Table 4.

With different losses in materials, the underwater mean vibration velocity level and the acoustic radiation power level of the cylindrical shell calculation model with simply supported boundary decreased obviously with the increasing of loss factor. Despite the acoustic radiation efficiency increasing simultaneously, one was not enough to introduce higher acoustic radiation in the structure with large damping than that of the structure with small damping.

For the cylindrical shell, the increase in the loss leads to a significant increase in the radiation efficiency. The variation in the acoustic radiation within the frequency band was faint except for a conspicuous drop in the peak value. Compared with the 4–4 working condition, the overall mean square vibration velocity and the overall acoustic radiation power in the frequency band of 0–1,500 Hz decreased by 15.86 and 2.73 dB, respectively, under the 4–1 working condition. Therefore, the vibration and acoustic radiation performance of the cylindrical shell were remarkably affected by the loss in materials, which suggested that the damping performance should be improved as possible while maintaining the stiffness and strength of the shell to reduce the radiation spectrum significantly.
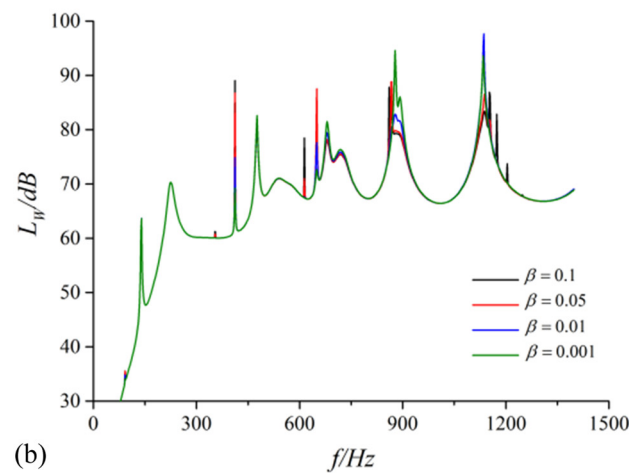
4.5 Influence of Poisson's ratio in materials on vibro-acoustic characteristics of cylindrical shell

The vibration and acoustic performance of the theoretical model of the cylindrical shell had been calculated according
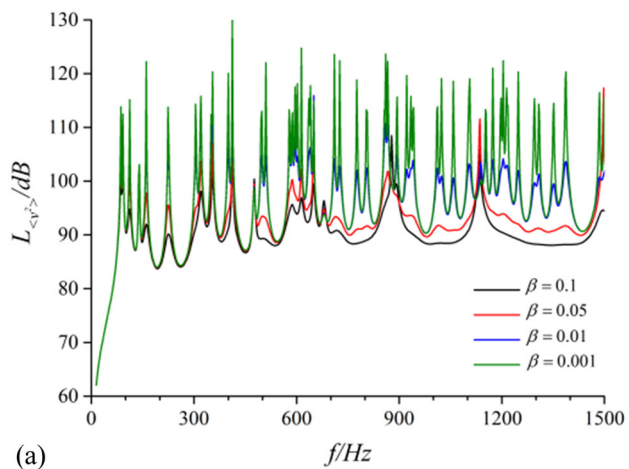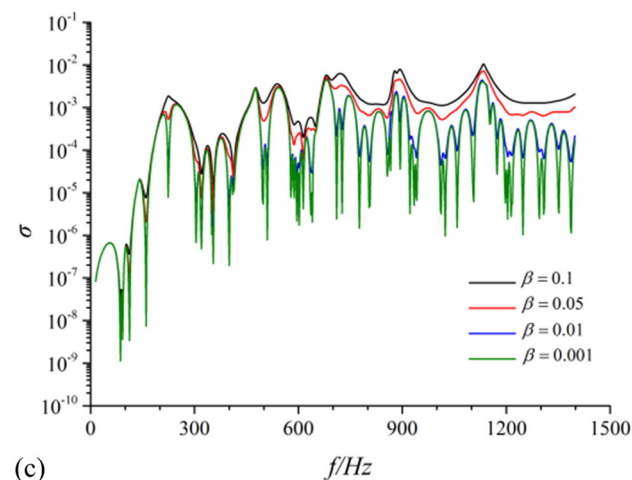
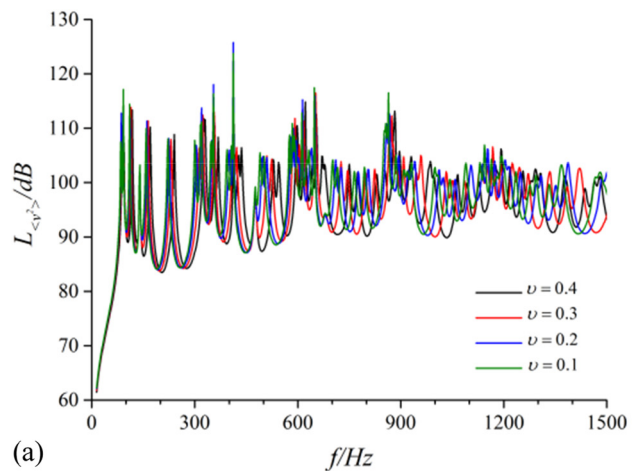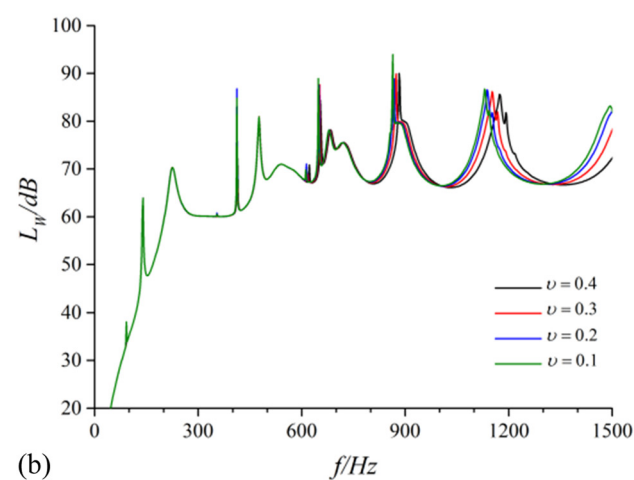**Figure 5:** Contrast of vibration and noise performance for the cylindrical shell under different dissipation factors. (a) Radial quadratic velocity level, (b) sound radiation power level, and (c) sound radiation efficiency.

Figure 6: Contrast of vibration and noise performance for the cylindrical shell under different Poisson ratios. (a) Radial quadratic velocity level, (b) sound radiation power level, and (c) sound radiation efficiency.
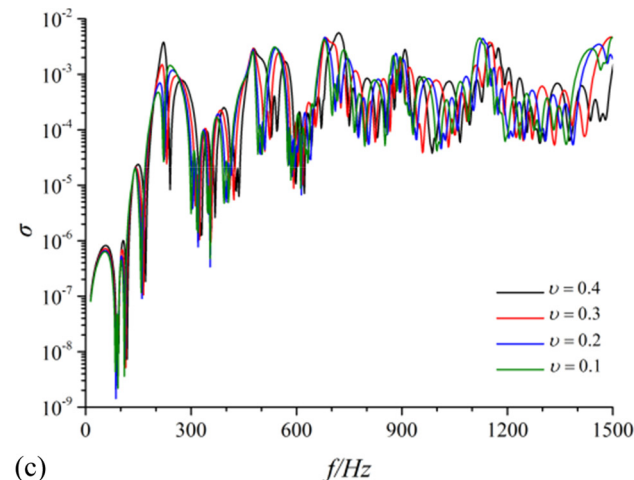
to the structural model described in Section 1. Results are illustrated in Figure 6. The differences in performance of the structure with the influence of different Poisson ratios in

materials had been compared. The overall vibration levels and the overall acoustic power levels within the 0–1,500 Hz frequency band are shown in Table 5.

With different Poisson's ratio parameters, the vibration and acoustic performance of the cylindrical shell with simply supported boundary are analogical. The mean square vibration velocity level and the acoustic radiation power level slightly decreased with the increase in Poisson's ratio in materials. For the cylindrical shell, compared with the 5-4 working conditions, the overall mean square vibration velocity and the overall acoustic radiation power in the frequency band of 0–1,500 Hz decreased by 2.08 and 1.38 dB, respectively, under the 5–1 working condition.

5 Conclusion

Based on the Donnel shell theory and modal expansion method, the article constructs the parameterized by the theoretical model of free vibrating cylindrical thin shells. This theory model has the advantages of fast convergence, easy parametric analysis, no repeated modeling, and high computational efficiency. In this work, the acoustic radiation characteristics of cylindrical shells had been analyzed. With consideration of the underwater simply supported cylindrical shells, the structural vibration and acoustic performance under different structural parameters had been calculated and compared. The influence of the parameters, such as modulus, density, thickness, and loss on the vibration and acoustic performance, had been revealed. The main conclusions were summarized as follows:

(1) With the increase in the cylindrical shell density, the mean square vibration velocity of the structure, and the acoustic radiation power were decreased.
(2) As the elastic modulus of materials decreased, the vibration and the acoustic radiation level of the cylindrical shell increased remarkably. With the higher the modulus, the acoustic radiation efficiency of the model was getting higher.
(3) The underwater mean square vibration velocity level and the sound power level of the cylindrical shell calculation model decreased significantly with the increase in the shell thickness, while the acoustic radiation efficiency increased.
(4) The underwater mean square vibration velocity level and the acoustic radiation power level of the cylindrical shell decreased markedly with the increase in the loss factor in the material. In spite of the increase in acoustic radiation efficiency, one was not enough to introduce a higher acoustic radiation in the structure with large damping than that of the structure with small damping.

(5) The mean square vibration velocity level and the acoustic radiation power level slightly decreased with the increase in Poisson's ratio in materials.

With the increase in the above parameters, the mean square vibration velocity level and the acoustic radiation power level of the cylindrical shell gradually decrease. Therefore, in structural design and material selection, it is necessary to ensure the maximum value of these parameters, which is expected to achieve the reduction of the surface vibration velocity and acoustic radiation power of the cylindrical shell. The research provided the theoretical guidelines in the acoustic stealth design for underwater vehicles, and the specific engineering application needs further study.

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