Erratum

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Erratum to "On Soliton structures in optical fiber communications with Kundu–Mukherjee–Naskar model (Open Physics 2021;19:679–682)"

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Abstract: In a recent paper (Open Physics 2021;19:679–682), Khalil Salim Al-Ghafri investigated the soliton structures of the integrable Kundu–Mukherjee–Naskar (KMN) equation in optical fiber communication. Some exact solutions such as the W-shaped soliton solution, the bright soliton solutions (type-I and type-II) and dark soliton solution are derived using the ansatz approach. In this erratum article, we show that all these four solutions presented in Open Physics 2021;19:679–682 do not satisfy the original KMN equation; hence they are not the exact solutions of the integrable KMN equation.

1 Introduction

Recently, Khalil Salim Al-Ghafri investigated the integrable Kundu–Mukherjee–Naskar (KMN) equation [1] in optical fiber communication and derived few exact solutions [2]. Different research groups have carried out various innovation activities to evaluate different properties of the KMN equation such as exact and approximate solutions, integrable structures, symmetries, conserved quantities, etc. The KMN equation is given as

$$i\Psi_t + a\Psi_{xy} + ib\Psi(\Psi\Psi_x^* - \Psi^*\Psi_x) = 0, \tag{1}$$

where $\Psi(x, y, t)$ represents the optical soliton profile. To investigate the various optical soliton solutions of KMN Eq. (1), Al-Ghafri [2] used the traveling wave transformation as:

$$\Psi(x, y, t) = \psi[\xi(x, y, t)]e^{i\phi(x,y,t)}, \tag{2}$$

where $\psi[\xi(x, y, t)]$ and $\phi(x, y, t)$ denote the amplitude and phase of the soliton, respectively. The variable $\xi(x, y, t)$ and phase component $\phi(x, y, t)$ are defined as:

$$\xi(x, y, t) = \kappa x + \lambda y + \nu t, \tag{3}$$

$$\phi(x, y, t) = \alpha x + \beta y + \omega t + \theta[\xi(x, y, t)], \tag{4}$$

where, κ, λ, ν, α, β, ω represent constant parameters and $\theta[\xi(x, y, t)]$ is another real function. Applying the transformation given by Eq. (2) to Eq. (1) and collecting the real and imaginary parts lead to the following pair of ordinary differential equations:

$$(a\alpha\lambda + a\beta\kappa + \nu)\psi' + 2a\kappa\lambda\psi'\theta' + a\kappa\lambda\psi\theta'' = 0, \tag{5}$$

$$\begin{aligned} &a\kappa\lambda\psi'' - (a\alpha\beta + \omega) + 2b\alpha\psi^3 - (a\beta\kappa + a\alpha\lambda + \nu)\psi\theta' \\ &\quad - a\kappa\lambda\psi\theta'^2 + 2b\kappa\psi^3\theta' = 0, \end{aligned} \tag{6}$$

where prime in the superscript of the dependent variables ψ and θ denotes the derivative with respect to the variable ξ. On solving Eq. (5) for θ', we arrive at:

$$\theta' = -\frac{a\alpha\lambda + a\beta\kappa + \nu}{2a\kappa\lambda} + \frac{C}{a\kappa\lambda\psi^2}, \tag{7}$$

where C is the integration constant. Using Eq. (7) in Eq. (6), Al-Ghafri [2] obtained an ordinary nonlinear differential equation as:

$$\psi'' + A_1 + A_2\psi^3 - A_3\psi^{-3} = 0, \tag{8}$$

where

$$A_1 = \frac{(a\alpha\lambda + a\beta\kappa + \nu)^2 - 4a\kappa\lambda(\omega + a\alpha\beta) + 8b\kappa C}{4a^2\kappa^2\lambda^2}, \tag{9}$$

$$A_2 = \frac{-b\kappa(a\alpha\lambda + a\beta\kappa + \nu) + 2ab\alpha\kappa\lambda}{a^2\kappa^2\lambda^2}, \tag{10}$$

$$A_3 = \frac{C^2}{a^2\kappa^2\lambda^2}. \tag{11}$$

On multiplying Eq. (8) by ψ' and further integrating with respect to variable ξ, Al-Ghafri [2] obtained:

$$\psi'^2 + A_1\psi^2 + \frac{A_2}{2}\psi^4 + A_3\psi^{-2} + 2A_0 = 0, \tag{12}$$

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where A_0 is an arbitrary integration constant. Al-Ghafri [2] used the transformation $\psi^2 = V$ in Eq. (12) and arrived at:

$$V'^2 + 4A_1V^2 + 2A_2V^3 + 4A_3 + 8A_0V = 0. \tag{13}$$

Al-Ghafri [2] further differentiated the above equation [Eq. (13)] to obtain:

$$V'' + 4A_0 + 4A_1V + 3A_2V^2 = 0. \tag{14}$$

2 Erratum to the study of Al-Ghafri [2]

Al-Ghafri [2] obtained different exact solutions of Eq. (14). But, it must be noted that the solutions obtained from Eq. (14) are the square of amplitude of exact solutions of KMN equation [Eq. (1)] only if it satisfies Eq. (13) at the same time. Hence, the solutions obtained from Eq. (14) must also satisfy Eq. (13) to be the exact solutions of Eq. (1).

If we carefully look at Eq. (13), it has an extra parameter A_3 which is not present in Eq. (14). When we use the ansatz of different kinds of solutions that were used in the study of Al-Ghafri [2] in Eq. (13), we would obtain an extra constraint condition for parameter A_3 for each solution. These constraint conditions give the relationship of A_3 with other parameters present in the system. Hence, one must not arbitrarily choose all the parameters present in the system.

But, all the four solutions presented in the study of Al-Ghafri [2] *i.e.*, W shaped soliton, Bright soliton (type-I), Bright soliton (type-II) and the Dark soliton do not satisfy Eq. (14) and Eq. (13) simultaneously. Hence, it can be clearly stated that these four solutions (Eqs (19)–(22) presented in the study of Al-Ghafri [2]) are not the exact solutions of the KMN Eq. (1). These facts are explained in detail in the following subsections.

2.1 Erratum to the W-shaped soliton solution given in Eq. (19) of the study of Al-Ghafri [2]

Al-Ghafri [2] used the following ansatz for W-shaped soliton solution as:

$$V(\xi) = l + \frac{m\,\mathrm{sech}^2 p\xi}{4 - [1 - \tanh p\xi]^2} + \frac{n\,\mathrm{sech}^4 p\xi}{(4 - [1 - \tanh p\xi]^2)^2}, \tag{15}$$

where the parameters l, m, n are related to A_1, A_2 as $l = -\frac{4A_1}{3A_2}$, $m = \frac{16A_1}{A_2}$, $n = -\frac{32A_1}{A_2}$ with $p = \sqrt{A_1}$. The constants A_1, A_2 follow the conditions $A_1 > 0$, $A_2 < 0$ and A_0 is taken to be zero as it is an arbitrary constant.

As we have discussed before, the solution (15) must satisfy Eq. (14) and Eq. (13) simultaneously to be an exact solution of the KMN equation. On substituting Eq. (15) in Eq. (13), we obtain a constraint:

$$A_3 = -\frac{16A_1^3}{27A_2^2}. \tag{16}$$

As $A_1 > 0$ and $A_2 < 0$, A_3 is going to be negative according to Eq. (16). However, Eq. (11) suggests that A_3 is always positive. As a result, we state that the W-shaped soliton Eq. (15) presented in the study of Al-Ghafri [2] is not an exact solution of KMN Eq. (1).

2.2 Erratum to the bright soliton solution of type-I given in Eq. (20) of the study of Al-Ghafri [2]

Al-Ghafri [2] used the following ansatz for bright solitons of type I:

$$V(\xi) = l + \frac{m\,\mathrm{sech}^2 p\xi}{4 - [1 - \tanh p\xi]^2} + \frac{n\,\mathrm{sech}^4 p\xi}{(4 - [1 - \tanh p\xi]^2)^2}, \tag{17}$$

where the parameters l, m, n are related to A_1, A_2 as $l = 0$, $m = -\frac{16A_1}{A_2}$, $n = \frac{32A_1}{A_2}$, with $p = \sqrt{-A_1}$. The constants A_1, A_2 follow the conditions $A_1 < 0$ and $A_2 > 0$ and A_0 is taken to be zero as it is an arbitrary constant.

As we have discussed before, the solution (17) must satisfy Eq. (14) and Eq. (13) simultaneously to be an exact solution of KMN equation. On substituting Eq. (17) in Eq. (13), we obtain:

$$A_3 = 0. \tag{18}$$

On comparing Eqs (18) and (11), we find that $C = 0$. However, Fig. 1 in the study of Al-Ghafri [2] is plotted for $C = 1$. Hence, the solution (20) presented in the study of Al-Ghafri [2] is not correct.

2.3 Erratum to the bright soliton solution of type-II given in Eq. (21) of the study of Al-Ghafri [2]

Al-Ghafri [2] used the following ansatz for bright solitons of type II:

$$V(\xi) = l + \frac{m\,\text{sech}^2 p\xi}{4 - [1 - \tanh p\xi]^2} + \frac{n\,\text{sech}^4 p\xi}{(4 - [1 - \tanh p\xi]^2)^2}, \tag{19}$$

where the parameters l, m, n are related to A_1, A_2 as $l = -\frac{6A_1}{11A_2}$, $m = -\frac{32A_1}{11A_2}$, $n = \frac{64A_1}{11A_2}$, with $p = \sqrt{-\frac{2A_1}{11}}$. The constants A_1, A_2 follow the condition $A_1 < 0$, $A_2 > 0$ along with $39A_1^2 = 121A_0A_2$.

On substituting Eq. (19) in Eq. (13), we obtain an extra condition given by:

$$A_3 = \frac{180A_1^3}{1331A_2^2}, \tag{20}$$

which is missing in the original paper [2]. The value of C chosen for bright soliton of type II should be such that A_3 satisfies the above equation [Eq. (20)].

These facts are completely missing in the study of Al-Ghafri [2] and the solution (21) presented in the study of Al-Ghafri [2] is not correct.

2.4 Erratum to the dark soliton solution given in Eq. (22) of the study of Al-Ghafri [2]

Al-Ghafri [2] used the following ansatz for dark soliton:

$$V(\xi) = l + \frac{m\,\text{sech}^2 p\xi}{4 - [1 - \tanh p\xi]^2} + \frac{n\,\text{sech}^4 p\xi}{(4 - [1 - \tanh p\xi]^2)^2}, \tag{21}$$

where the parameters l, m, n are related to A_1, A_2 as $l = -\frac{26A_1}{33A_2}$, $m = \frac{32A_1}{11A_2}$, $n = -\frac{64A_1}{11A_2}$ with $p = \sqrt{\frac{2A_1}{11}}$. The constants A_1, A_2 follow the condition as $A_1 > 0$, $A_2 < 0$ along with $39A_1^2 = 121A_0A_2$. On substituting Eq. (21) in Eq. (13), we obtain an extra condition given by:

$$A_3 = \frac{4732A_1^3}{35937A_2^2}, \tag{22}$$

which is missing in the original paper [2]. The value of C for dark soliton should be chosen such that A_3 satisfies the above equation [Eq. (22)].

These facts are completely missing in the study of Al-Ghafri [2] and the solution (22) presented in the study of Al-Ghafri [2] is not correct.

Specifically, if such incorrect soliton profiles are assumed to be realizable in optical fiber systems, they could misrepresent pulse stability, lead to incorrect predictions of optical signal propagation [3]. Bright solitons are localized pulses of light that maintain their shape during propagation due to a balance between dispersion and nonlinearity. They are widely used in long-distance optical communication to minimize pulse broadening and signal distortion caused by group velocity dispersion [3]. Dark solitons are intensity dips (or voids) in a continuous wave background. They are less affected by perturbations such as amplifier noise compared to bright solitons, making them suitable for noise-resilient communication channels [3]. W-shaped solitons are localized waveforms with a double-dip or oscillatory profile, often arising in higher-order nonlinear systems. They are explored for error correction and regeneration of distorted signals, where the W-profile can reshape or stabilize degraded optical pulses. They may provide robustness in systems prone to higher-order dispersion and nonlinear instabilities. Including this perspective underscores the importance of verifying mathematical solutions before their application in physical models.

3 Conclusions

In the present article, we have shown that the exact soliton solutions of the integrable KMN equation that was presented in the study of Al-Ghafri [2] are not correct. All the four solutions, *i.e.*, the W-shaped soliton, the bright soliton (type-I and type-II) and dark soliton solutions, which are presented in the study of Al-Ghafri [2] must also satisfy Eqs (14) and (13) simultaneously to be exact solutions of the KMN equation. But, we have shown that those solutions (Eqs (19)–(22) presented in the study of Al-Ghafri [2]) do not satisfy Eq. (14) and Eq. (13) simultaneously; hence, they are not the exact solutions of the KMN Eq. (1). This discrepancy highlights the necessity for rigorous verification of solutions in mathematical physics, particularly in the context of optical fiber communication. Future research should focus on identifying valid solutions that truly adhere to the integrable nature of the KMN equation, ensuring that they can be reliably applied in practical scenarios.

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