Research Article

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An analytical investigation to the (3+1)-dimensional Yu–Toda–Sassa–Fukuyama equation with dynamical analysis: Bifurcation

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Abstract: The (3 + 1)-dimensional Yu–Toda–Sasa–Fukuyama (YTSF) equation serves as a fundamental model for intricate nonlinear wave phenomena observed in various domains, including oceanography, coastal engineering, plasma physics, and high-speed fiber-optic communications. This study derives precise soliton solutions of the YTSF problem using a recently established $(G'/(G' + G + A))$-expansion approach, resulting in a comprehensive array of trigonometric, rational, and exponential waveforms. The resultant solutions include kink-type, antikink-type, periodic, and isolated solitary waves, each representing significant real-world phenomena such as rogue-wave creation, pulse propagation in optical fibers, and shallow-water wave dynamics. A thorough bifurcation analysis is performed, identifying important parameter "tipping points" where solution branches arise, disappear, or alter stability. This study reveals transitions from stable states to oscillatory or chaotic regimes, offering a prediction framework for the complex qualitative behavior of the equation. The two- and three-dimensional visualizations produced with Mathematica demonstrate the dynamic characteristics of the derived solutions for selected parameter sets. The results collectively underscore the practicality, adaptability, and effectiveness of the proposed strategy, while the bifurcation insights provide a robust framework for predicting and managing complicated wave patterns dictated by nonlinear partial differential equations.

Keywords: nonlinear equations, nonlinear evolution equation, Yu–Toda–Sasa–Fukuyama equation, $(G'/(G' + G + A))$-expansion approach, bifurcation, nonlinear partial differential equation

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1 Introduction

For the purpose of describing complex dynamic occurrences, nonlinear partial differential equations (NLPDEs) are indispensable in a wide variety of scientific and technical fields, such as fluid dynamics, plasma physics, optical fibers, and quantum mechanics. The nonlinear interactions, evolutions, and energy transfers of waves are described by these equations, which are fundamental to understanding the systems that exist in the real world. In contrast to linear equations, which simplify dynamics, NLPDEs provide a more accurate description of physical processes to the extent that they capture the complex behaviors that are the result of nonlinear interactions. One of the most important characteristics of NLPDEs is the presence of steady soliton solutions, which are waveforms that are self-sustaining and maintain their speed and form over time without losing energy. In both theoretical and applied science, NLPDEs are an essential tool because they enable researchers to gain a deeper understanding of the complex dynamics that are at play in a wide variety of physical systems through the study and the solution of these equations [1–9].

In this work, we focus on the following Yu–Toda–Sassa–Fukuyama (YTSF) equation [10,11]:

$$u_{xxxz} + 4u_xu_{xz} + 2u_zu_{xx} + 3u_{yy} - 4u_{xt} = 0. \tag{1}$$

This equation represents intricate wave dynamics in many physical systems by expanding its classical formulation to three spatial dimensions and one temporal dimension. This additional dimensionality facilitates a more authentic modeling of multidimensional wave propagation, integrating nonlinear, dispersive, and multidirectional phenomena. The investigation of soliton solutions for the YTSF equation is essential to understand the fundamental mechanisms of nonlinear wave propagation in complex dispersive systems. The YTSF equation serves as a mathematical framework for modeling various physical phenomena, including shallow water waves, plasma oscillations, and optical pulses. Solitons are stable, self-contained energy packets that can cross each other without altering their structure in these contexts. Researchers establish a crucial connection between the abstract structure of the equation and observable physical phenomena by deriving various explicit solutions, including kink, antikink, dark, and periodic solitons. This facilitates precise predictions of wave behavior, verifies the integrability of the equation, uncovers its hidden symmetries, and establishes critical benchmarks for numerical models. The stability and durability of soliton solutions position them as viable individuals for various technological applications, such as wave-based communication, energy transfer, and signal processing in nonlinear media. In oceanography, it elucidates the generation and interaction of rogue waves, unanticipated, substantial waves that provide considerable hazards to maritime activities, and helps to forecast shallow water dynamics crucial for coastal engineering. Nonlinear optics models ultrashort light pulses traversing optical fibers, which are essential for high-speed data transfer and contemporary telecommunication systems. The YTSF equation in plasma physics encapsulates nonlinear wave phenomena, including ion-acoustic waves, facilitating fusion research and space plasma diagnostics. It is pertinent to fluid dynamics and meteorology for examining turbulence and wave propagation in air systems. Some studies have investigated the behavior and solutions of the YTSF equation [12–16]. Among them, Du and Pang [10] investigated localized wave structures and demonstrated the emergence of rogue waves, breathers, and lump solitons, signifying the capacity of the equation to depict real-time wave evolution in actual scenarios. Tan and Li [11] examined the impact of varied coefficients on high-order solitons and hybrid wave phenomena, illustrating the adaptability of the YTSF equation to varying physical mediums, especially in nonlinear optical systems. In recent years, many researchers have developed a wide range of approaches to derive exact analytical solutions of the nonlinear evolution equation. Some of the prominent approaches include the Riccati–Bernoulli sub-ordinary differential equation (ODE) method [17], the Sardar subequation technique [18], Hirota bilinear transformation method [19], the modified Sardar subequation technique [20], the sine-Gordon method [21], the modified exp-function method [22], the Hirota bilinear method [23], the tanh–coth expansion method [24], the new Kudryashov approach [25], the modified Kudryshov method [26], the modified extended direct algebraic technique [27], Lie symmetry approach [28], ϕ^6 model expansion scheme [29], the $\left(\frac{G'}{G}\right)$-expansion method [30], $\left(\frac{1}{G'}\right)$-expansion technique [31], the $\left(\frac{G'}{G},\frac{1}{G}\right)$-expansion method [32], the Jacobi elliptic function expansion [33], and the modified simple equation method [34]. Among them, Elsherbeny *et al.* [35] recently studied the extended quadratic-cubic form of self-phase modulation and nonlinear chromatic dispersion to extract perturbed quiescent optical solitons for the Fokas–Lenells equation. To obtain the solutions, they have used the projective Riccati equation scheme and the improved Kudryashov's method. Akram *et al.* [36] have investigated the extraction of exact solutions of the nonlinear extended quantum Zakharov–Kuznetsov equation by utilizing well-established techniques such as the $\left(\frac{G'}{G},\frac{1}{G}\right)$-expansion method and the generalized exponential rational function method. For specific parameter values, solutions for singular, periodic, hyperbolic, and rational functions are obtained. Using the generalized $\left(\frac{G'}{G}\right)$-expansion approach, Hossain *et al.* [37] have examined both soliton and additional closed-form solutions of the Drinfeld–Sokolov–Wilson and Burgers equations. They have obtained numerous soliton solutions, such as periodic soliton, irregular periodic soliton solutions, kink-shaped soliton, bell-shaped soliton, singular soliton, and single soliton.

To date, very few studies have focused on the YTSF equation using modern analytical approaches, and to the best of our knowledge, no prior work has explored this equation *via* any form of expansion method. Therefore, the main purpose of this study is to derive new closed-form soliton solutions of the YTSF equation using a novel analytical technique, namely, the $\left(\frac{G'}{G'+G+A}\right)$-expansion method. This method offers a powerful framework for building a wide range of soliton solutions, including kink, antikink, periodic, and solitary waveforms, thus broadening the understanding of nonlinear wave dynamics [38–42]. This method has proven to be a highly effective technique, and several authors have made significant contributions to the field of nonlinear dynamics using it. For instance, the prior research has shown its success in yielding diverse novel exact solutions for various wave models. Mia and Paul [42] have applied it to the generalized shallow water wave equation to find periodic and singular solitary waves, while Tripathy and Sahoo [39] utilized it to discover a wide range of antikink, dark, bright, and singular soliton solutions for the ion sound Langmuir wave model. The method's utility extends to higher-dimensional systems as well, having been used by others [40,41] to derive exponential and trigonometric solutions for the (4+1)-dimensional Davey–Stewartson–Kadomtsev–Petviashvili equation and to find exact solutions for the (2+1)-dimensional Kadomtsev–Petviashvili–Benjamin–Bona–Mahony equation. Building on this established track record, our work extends the applicability of this powerful method to the YTSF equation, with the goal of constructing a variety of exact traveling wave solutions to reveal the rich nonlinear wave dynamics inherent in this high-dimensional model. The novelty of this current work lies in the ability to generate a wide range of closed-form soliton solutions, including trigonometric, rational, and exponential forms, using the proposed technique. Following the completion of a comprehensive bifurcation study, significant parameter "tipping points" are identified. These are the locations at which solution branches appear, disappear, or change stability. In addition, the study provides comprehensive 2D and 3D wave profiles of the derived solutions utilizing computational software like Mathematica, facilitating a thorough comprehension of their dynamic behavior. The results reported in this study demonstrate the flexibility, effectiveness, and practical relevance of the approach to finding novel soliton structures in complicated nonlinear evolution equations.

The article is organized as follows: Section 2 explains the main steps of the $\left(\frac{G'}{G'+G+A}\right)$-expansion method. In Section 3, the method is used to extract closed-form solutions of the governing YTSF model. Section 4 is devoted to the bifurcation analysis. Section 5 presents the results and discussion. Finally, Section 6 concludes the study.

2 Brief overview of the methodology

This is a modern analytical technique for deriving precise solutions of NLPDEs. It enhanced conventional methods by incorporating a more adaptable rational form that included an auxiliary function and its derivative. This method facilitated the derivation of various soliton solutions such as kink, antikink, periodic, rational, and exponential forms by converting the original partial differential problem into an ODE and employing a series solution ansatz. Owing to its universality and efficacy, it was extensively employed in the simulation of intricate physical phenomena, including optical solitons, rogue waves, and shallow water dynamics[38–40,42]. Considered a general nonlinear partial differential equation of the form:

$$P_1(u, u_x, u_y, u_t, u_{xx}, u_{yy}, u_{xy}, u_{xt}, u_{xxx} \dots) = 0, \tag{2}$$

where x, y, and t are independent variables and u is the unknown function. P_1 is a polynomial in u and its derivatives, incorporating nonlinear and higher order terms. The method involved the following key steps:

Step 1: For traveling wave solutions, assumed $\chi = m_1x + m_2y + m_3z - \omega t$, where m_1, m_2, m_3, and ω were constants with the transformation of the dependent variable as follows:

$$u(x, y, z, t) = U(\chi). \tag{3}$$

By applying this transformation to Eq. (2), we derived the following ODE:

$$P_2[U(\chi), U'(\chi), U''(\chi), U'''(\chi), \dots] = 0. \tag{4}$$

Step 2: Assumed a solution form of the transformed ODE (Eq. (4)) of the form:

$$U(\chi) = \sum_{j=0}^{r} d_j H^j, \tag{5}$$

where $H = \frac{G'}{G'+G+L}$ and $G = G(\chi)$ are the functions that needed to be determined that satisfied the following equation:

$$G'' + MG' + NG + LN = 0. \tag{6}$$

where L, M, N, and d_j $(j = 0, 1, 2, \dots, N)$ were constants.

Step 3: The highest-order nonlinear terms were balanced with the highest-order derivative terms in the transformed ODE to determine the degree r of the polynomial.

Step 4: Eq. (5) is substituted into Eq. (4) to obtain a polynomial in powers of H. By equating the coefficients of each power of H to zero, a system of algebraic equations for the coefficients d_j $(j = 0, 1, 2, \dots, r)$ was derived. The coefficients d_j were then obtained by solving the resulting system.

Step 5: The second-order ODE given in Eq. (6) is solved to obtain $G(\chi)$. Finally, the values of d_j and the evaluated form of $G(\chi)$ were substituted into Eq. (5) to obtain the closed-form solutions.

3 Extraction of solutions of the governing model YTSF

In this section, the aforementioned method was applied to the YTSF model given in Eq. (1) to extract the wave solutions. The following transformation is considered to implement the proposed technique:

$$u(x, y, z, t) = U(\chi), \quad \text{where } \chi = m_1 x + m_2 y + m_3 z - \omega t. \tag{7}$$

The YTSF model given in Eq. (1) is transformed into the following ODE by employing Eq. (7):

$$m_1^3 m_3 U^{iv} + 6m_1^2 m_3 U' U'' + \eta U'' = 0, \tag{8}$$

where $\eta = 4m_1\omega + 3m_2^2$. Upon integrating Eq. (8) and assuming the integration constant as zero, we obtained:

$$m_1^3 m_3 U''' + 3m_1^2 m_3 U'^2 + \eta U' = 0. \tag{9}$$

By applying the transformation $U' = \psi(\chi)$, Eq. (9) is transformed into:

$$m_1^3 m_3 \psi'' + 3m_1^2 m_3 \psi^2 + \eta\psi = 0. \tag{10}$$

The value $r = 2$ is obtained by applying the homogeneous balance principle to Eq. (10). Consequently, the solution is derived as follows:

$$\psi(\chi) = a_0 + a_1 H + a_2 H^2. \tag{11}$$

Eq. (11) is substituted into Eq. (10), and after collecting all terms in powers of H, the coefficients of the different powers of H were set to zero, yielding the following set of algebraic equations:

$$\begin{cases} d_0\eta + d_1 m_3 m_1^3 MN - 2d_1 m_3 m_1^3 N^2 + 2d_2 m_3 m_1^3 N^2 + 3d_0^2 m_3 m_1^2 = 0, \\ d_1\eta + d_1 m_3 m_1^3 M^2 - 6d_1 m_3 m_1^3 MN + 6d_2 m_3 m_1^3 MN + 6d_1 m_3 m_1^3 N^2 \\ \quad - 12d_2 m_3 m_1^3 N^2 + 2d_1 m_3 m_1^3 N + 6d_0 d_1 m_3 m_1^2 = 0, \\ d_2\eta - 3d_1 m_3 m_1^3 M^2 + 4d_2 m_3 m_1^3 M^2 + 9d_1 m_3 m_1^3 MN \\ \qquad - 24d_2 m_3 m_1^3 MN + 3d_1 m_3 m_1^3 M - 6d_1 m_3 m_1^3 N^2 + 24d_2 m_3 m_1^3 N^2 \\ \qquad - 6d_1 m_3 m_1^3 N + 8d_2 m_3 m_1^3 N + 3d_1^2 m_3 m_1^2 + 6d_0 d_2 m_3 m_1^2 = 0, \\ 2d_1 m_3 m_1^3 M^2 - 10d_2 m_3 m_1^3 M^2 - 4d_1 m_3 m_1^3 MN + 30d_2 m_3 m_1^3 MN \\ \quad - 4d_1 m_3 m_1^3 M + 10d_2 m_3 m_1^3 M + 2d_1 m_3 m_1^3 N^2 - 20d_2 m_3 m_1^3 N^2 \\ \quad + 4d_1 m_3 m_1^3 N - 20d_2 m_3 m_1^3 N + 2d_1 m_3 m_1^3 + 6d_1 d_2 m_3 m_1^2 = 0, \\ 6d_2 m_3 m_1^3 M^2 - 12d_2 m_3 m_1^3 MN - 12d_2 m_3 m_1^3 M + 6d_2 m_3 m_1^3 N^2 \\ \quad + 12d_2 m_3 m_1^3 N + 6d_2 m_3 m_1^3 + 3d_2^2 m_3 m_1^2 = 0. \end{cases} \tag{12}$$

The aforementioned algebraic equations are solved using Mathematica, which yielded the following two sets of solutions:

Set-I:

$$\begin{aligned} &\eta = -(M^2 - 4N)m_1^3 m_3,\ d_0 = -2N(-M + N + 1)m_1, \\ &d_1 = 2(M^2 - 3MN - M + 2N^2 + 2N)m_1, \\ &d_2 = -2(M - N - 1)^2 m_1. \end{aligned} \tag{13}$$

Set-II:

$$\begin{aligned} &\eta = (M^2 - 4N)m_1^3 m_3,\ d_0 = -\frac{1}{3}(M^2 - 6MN + 6N^2 + 2N)m_1, \\ &d_1 = 2(M^2 - 3MN - M + 2N^2 + 2N)m_1, \\ &d_2 = -2(M - N - 1)^2 m_1. \end{aligned} \tag{14}$$

In the case of ***Set-I***, the closed-form wave solutions were as follows:

Scenario I: When $\Omega = M^2 - 4N > 0$,

$$\begin{aligned} \psi_{11}(\chi) = &-2N(-M + N + 1)m_1 \\ &+ 2(M^2 - 3MN - M + 2N^2 + 2N)m_1 \\ &\times \left[\frac{C_1(M + \sqrt{\Omega}) + C_2(M - \sqrt{\Omega})e^{\chi\sqrt{\Omega}}}{C_1(M + \sqrt{\Omega} - 2) - C_2(-M + \sqrt{\Omega} + 2)e^{\chi\sqrt{\Omega}}}\right] \\ &- 2(M - N - 1)^2 m_1 \\ &\times \left[\frac{C_1(M + \sqrt{\Omega}) + C_2(M - \sqrt{\Omega})e^{\chi\sqrt{\Omega}}}{C_1(M + \sqrt{\Omega} - 2) - C_2(-M + \sqrt{\Omega} + 2)e^{\chi\sqrt{\Omega}}}\right]^2. \end{aligned} \tag{15}$$

Scenario II: When $\Omega = M^2 - 4N < 0$,

$$\begin{aligned} \psi_{12}(\chi) = &-2N(-M + N + 1)m_1 \\ &+ 2(M^2 - 3MN - M + 2N^2 + 2N)m_1 \\ &\times \left[\frac{A\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + B\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}{C\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + D\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}\right] \\ &- 2(M - N - 1)^2 m_1 \\ &\times \left[\frac{A\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + B\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}{C\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + D\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}\right]^2, \end{aligned} \tag{16}$$

where

$$\begin{aligned} &A = C_1 M - C_2\sqrt{-\Omega}, \quad B = C_1\sqrt{-\Omega} + C_2 M, \\ &C = C_1(M - 2) - C2\sqrt{-\Omega}, \quad D = C_1\sqrt{-\Omega} + C_2(M - 2). \end{aligned}$$

Similarly, for ***Set-II***:

Case-I: When $\Omega = M^2 - 4N > 0$

$$\begin{aligned} \psi_{21}(\chi) = &-\frac{1}{3}(M^2 - 6MN + 6N^2 + 2N)m_1 \\ &+ 2(M^2 - 3MN - M + 2N^2 + 2N)m_1 \\ &\times \left[\frac{C_1(M + \sqrt{\Omega}) + C_2(M - \sqrt{\Omega})e^{\chi\sqrt{\Omega}}}{C_1(M + \sqrt{\Omega} - 2) - C_2(-M + \sqrt{\Omega} + 2)e^{\chi\sqrt{\Omega}}}\right] \\ &- 2(M - N - 1)^2 m_1 \\ &\times \left[\frac{C_1(M + \sqrt{\Omega}) + C_2(M - \sqrt{\Omega})e^{\chi\sqrt{\Omega}}}{C_1(M + \sqrt{\Omega} - 2) - C_2(-M + \sqrt{\Omega} + 2)e^{\chi\sqrt{\Omega}}}\right]^2. \end{aligned} \tag{17}$$

Case-II: When $\Omega = M^2 - 4N < 0$

$$\psi_{22}(\chi) = -\frac{1}{3}(M^2 - 6MN + 6N^2 + 2N)m_1 + 2(M^2 - 3MN - M + 2N^2 + 2N)m_1\left[\frac{A\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + B\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}{C\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + D\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}\right] - 2(M - N - 1)^2 m_1\left[\frac{A\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + B\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}{C\cos\left(\frac{\chi\sqrt{-\Omega}}{2}\right) + D\sin\left(\frac{\chi\sqrt{-\Omega}}{2}\right)}\right]^2, \tag{18}$$

where

$$A = C_1M - C_2\sqrt{-\Omega}, B = C_1\sqrt{-\Omega} + C_2M,$$
$$C = C_1(M - 2) - \mathrm{C}2\sqrt{-\Omega}, D = C_1\sqrt{-\Omega} + C_2(M - 2).$$

4 Bifurcation analysis

Bifurcation analysis is essential to understand how the qualitative behavior of a dynamical system changes significantly as a parameter is incrementally modified. These alterations may encompass the emergence or elimination of equilibrium points, changes in their stability, or the formation of wholly novel patterns, such as periodic oscillations. Bifurcation analysis elucidates the values of essential parameters, providing significant information on the evolution of complex systems and their abrupt transitions in various scientific and technical fields [43–46]. For this analysis, we considered the dynamical system as follows.

Letting

$$A = m_1^3 m_3, \quad B = 3m_1^2 m_3, \quad \omega = \eta.$$

Then the normalized equation (Eq. (10) converted into):

$$A\psi'' + B\psi^2 + \omega\psi = 0. \tag{19}$$

Introduced:

$$\psi' = z, \quad \Rightarrow \quad \psi'' = z'.$$

Accordingly, the dynamical system was formulated as follows:

$$\begin{cases}\psi' = z,\\ z' = -\frac{1}{A}(B\psi^2 + \omega\psi).\end{cases} \tag{20}$$

Equilibrium points satisfied:

$$z = 0, \quad -\frac{1}{A}(B\psi^2 + \omega\psi) = 0.$$

That yielded:

$$\psi(B\psi + \omega) = 0 \quad \Rightarrow \quad \psi = 0, \quad \psi = -\frac{\omega}{B}. \tag{21}$$

Jacobian at equilibrium:

$$J = \begin{bmatrix} 0 & 1 \\ -\frac{1}{A}(2B\psi + \omega) & 0\end{bmatrix}. \tag{22}$$

The eigenvalues are as follows:

$$\lambda = \pm i\sqrt{\frac{1}{A}(2B\psi + \omega)}\,. \tag{23}$$

The bifurcation behavior depended on ω, if $\omega > 0$: Two equilibrium points, potentially one center and one saddle. if $\omega < 0$: One saddle and one center. if $\omega = 0$: At the bifurcation point, the equilibria coalesce (saddle-node or pitchfork bifurcation) (Figure 1).

The phase portraits of the YTSF equation offered an extensive qualitative examination of the variations in a nonlinear wave system in relation to a control parameter, demonstrating a dominant pitchfork bifurcation with immediate practical implications. In the scenario where $\omega < 0$, the system exhibited a singular, stable fixed point at the origin, which, in a physical setting, signified a stable and predictable state, such as a solitary wave (soliton) propagating undistorted in an optical fiber or a stable, simple pattern in a fluid or plasma. As the parameter approached the critical value of $\omega = 0$, this stable condition became unstable, indicating the emergence of a physical instability where the simple wave solution was no longer applicable. For $\omega > 0$, the system experienced a significant transition: the original fixed point at the origin became unstable and two new stable fixed points symmetrically positioned arose. This transition signified a fundamental alteration in physical systems, as an individual stable state evolved into a multistable regime. This manifested itself as symmetry breaking, wherein a uniform state became unstable, compelling the system to select between two new, stable patterns; the instability of a traveling wave that resulted in the emergence of two new stable waveforms; or as the basic process underlying all-optical switching, whereby a system was effectively transitioned between two distinct stable states through slight perturbation.

5 Graphs and discussion

This section presented the physical interpretation of various soliton solutions of the YTSF equation, which were obtained by the proposed analytical method. For selected parameter values, these solutions were illustrated using 3D

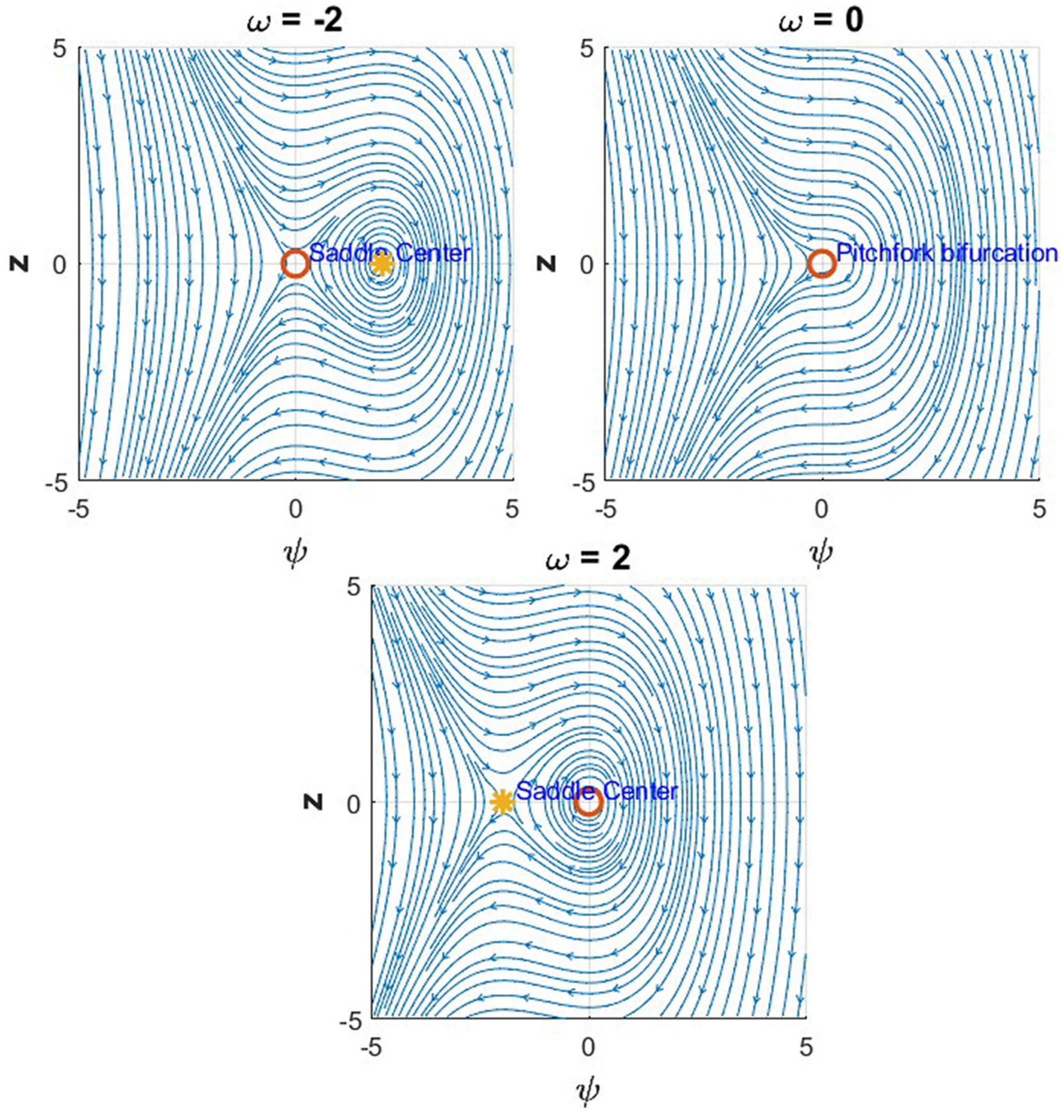

Figure 1: Bifurcation analysis of the given dynamical system.

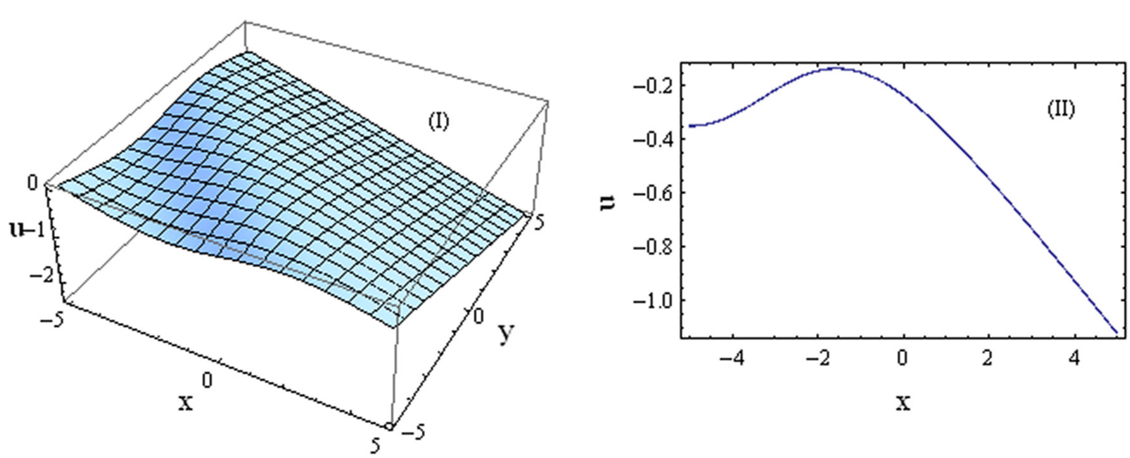

Figure 2: 3D and 2D plots of the solution based on ψ_{11} for t = 1, M = 1, N = 0.1, C_1 = 1, C_2 = 1, and z = 1.5.

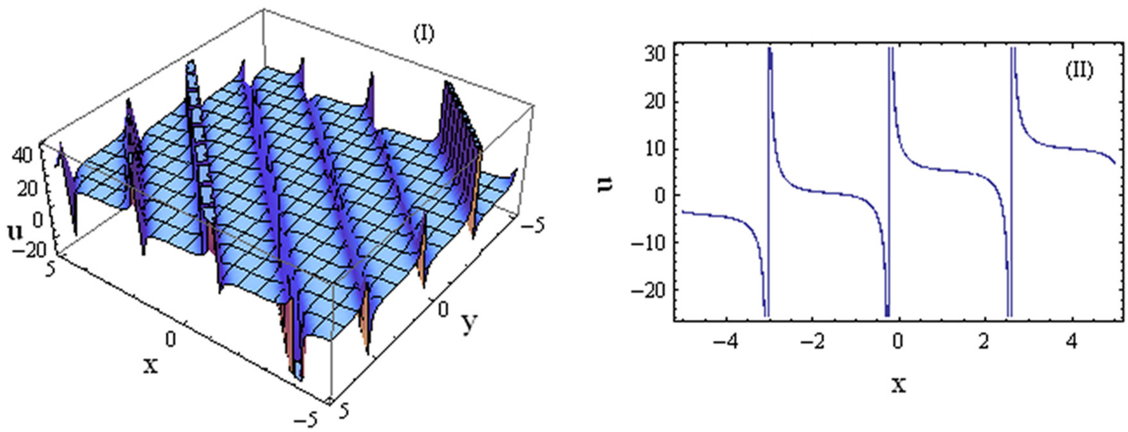

Figure 3: 3D and 2D plots of the solution based on ψ_{12} for $t = 1$, $M = 1$, $N = 1.5$, $C_1 = 0$, $C_2 = 1$, and $z = 1.5$.

and 2D graphical representations, highlighting their spatial and temporal structures. The final forms of the YTSF solutions were derived by integrating the analytical expressions, providing a comprehensive view of the solitary wave profiles and their dynamical characteristics ψ_{11}, ψ_{12}, ψ_{21}, and ψ_{22} derived in Eqs. (15)–(18) with respect to χ. The corresponding 3D and 2D graphical representations of the solutions are shown in Figures 2–5. For all plots, we have chosen $m_1 = m_2 = m_3 = 1$. Solutions for Set I and $\Omega > 0$ using Eq. (15) with parameters values $t = 1$, $M = 1$, $N = 0.1$, $C_1 = 1$, $C_2 = 1$, and $z = 1.5$ are displayed in Figure 2. In Figure 2, part (I) shows the 3D plot and part (II) presents the 2D plot of the solution corresponding to ψ_{11}. It is clear from the figure that this solution varies gradually in both spatial directions, and it is smooth, continuous, and non-oscillatory. Also, the 2D plot shows a nonperiodic structure. From this solution structure, we can say that the solution is a kink-type solution. In Figure 3, we have shown the solution based on ψ_{12} for *Set-I* with $\Omega < 0$ at the parameter values $t = 1$, $M = 1$, $N = 1.5$, $C_1 = 0$, $C_2 = 1$, and $z = 1.5$. It is found from the 3D and 2D plots of the solution that the wave solution is periodic and singular. The solution corresponding to ψ_{21} for *Set-II* with $\Omega > 0$ is depicted in Figure 4 with the particular values of the parameters $t = 1$, $M = 1$, $N = 0.1$, $C_1 = 1$, $C_2 = 1$, and $z = 1.5$. From this figure, it is found that the solution can be described as an anti-kink-type solution. For *Set-II* and $\Omega < 0$, the final plot corresponding to the solution ψ_{22} is shown in Figure 5 with parameters $t = 1$, $M = 1$, $N = 1.5$, $C_1 = 0$, $C_2 = 1$, and $z = 1.5$. From Figure 5, it is clear that the solution is periodic and of singular type.

The YTSF equation yields a rich array of solutions that model various coherent wave structures in nonlinear media. Among these are kink and antikink solitons, which describe abrupt, front-like interfaces between two separate equilibrium states, analogous to phenomena like internal tidal bores in oceanography, switching fronts in fiber optics, or transition layers in plasmas. The equation also

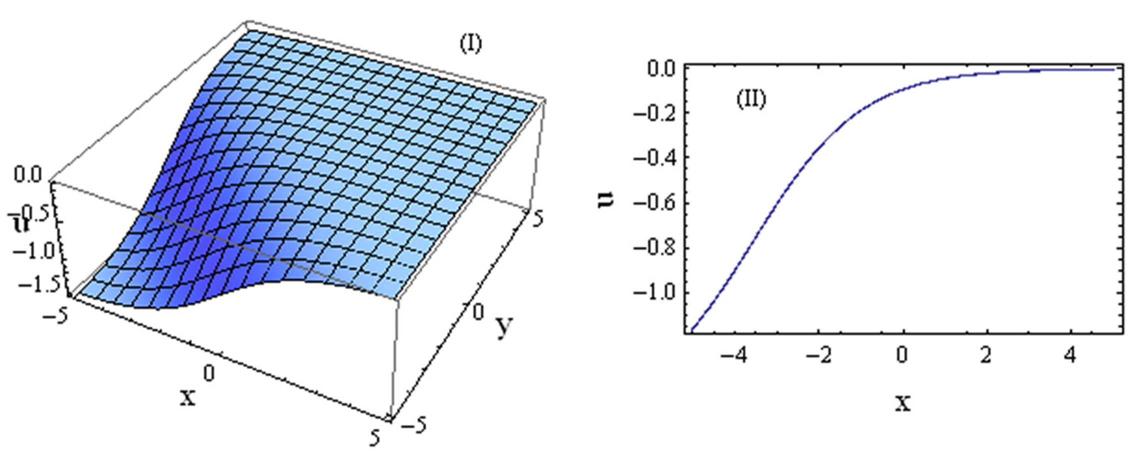

Figure 4: 3D and 2D plots of the solution based on ψ_{21} for $t = 1$, $M = 1$, $N = 0.1$, $C_1 = 1$, $C_2 = 1$, and $z = 1.5$.

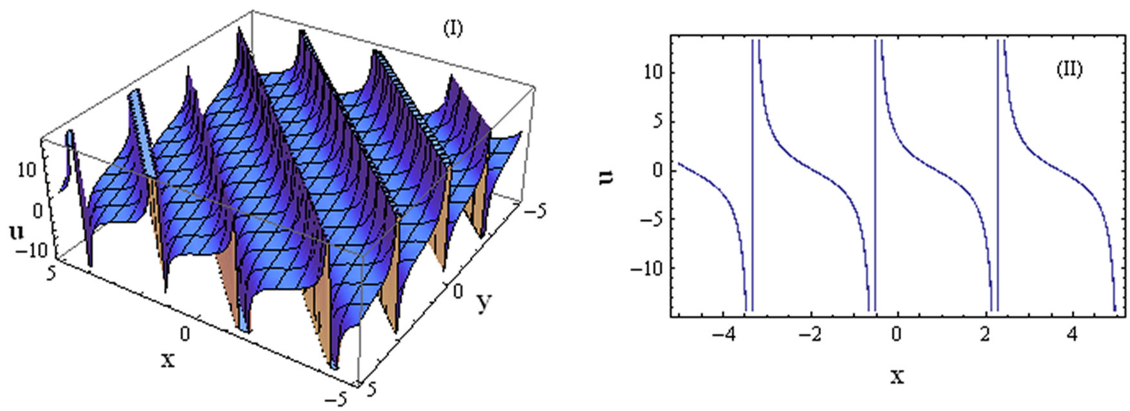

Figure 5: 3D and 2D plots of the solution based on ψ_{22} for $t = 1$, $M = 1$, $N = 1.5$, $C_1 = 0$, $C_2 = 1$, and $z = 1.5$.

generates periodic solutions, which represent stable, oscillatory wave trains resulting from the balance of dispersion and nonlinearity. These solutions are comparable to familiar wave packets in internal waves, oscillations in plasma, and pulse trains in optical systems. This diversity of solutions demonstrates the YTSF equation's capacity to accurately represent a wide spectrum of nonlinear wave behaviors in different dispersive media.

6 Comparative analysis

Recent research has explored exact solutions to higher-dimensional YTSF type equation, focusing on the dynamics and interactions of solitons. A shared methodological foundation, Hirota's bilinear method, is prevalent across these studies; however, the specific types and scope of solutions investigated show considerable diversity. Alternatively, this research employs a powerful method to derive exact soliton solutions for the YTSF equation, encompassing kink, antikink, periodic, and isolated types. A thorough bifurcation analysis is conducted to identify critical parameter thresholds and characterize transitions into oscillatory or chaotic regimes. The study's use of 2D and 3D visualizations provides a more comprehensive and predictive characterization of solution dynamics compared to prior work. Table 1 presents a comparative overview of these findings.

7 Conclusions

This study employed the powerful $\left(\frac{G'}{G'+G+A}\right)$-expansion method to derive a broad class of closed-form soliton solutions for the (3+1)-dimensional YTSF equation. The analytical solutions – which included kink-type, antikink-type, periodic, and solitary wave structures – underscored the equation's ability to describe complex real-world phenomena, from rogue waves and optical pulse propagation to wave dynamics in shallow water and plasma environments. In addition, a comprehensive bifurcation analysis

Table 1: Comparative summary of recent studies on YTSF equation

References	Solution types	Method(s)	Main contribution
Ma *et al.* [15]	N-solitons, localized waves	Hirota bilinear method	Explicit N-soliton solutions and multi-wave interaction structures
Hu *et al.* [16]	Kink multisolitons	Hirota method, symbolic computation	First report of kink-type soliton structures in YTSF framework
Guo *et al.* [13] generalized	Abundant solution families	Extended bilinear transformation	Dynamics of generalized YTSF with broad solution structures
Huang *et al.* [12]	Lump, soliton, N-soliton, lightest supersymmetric particle (LSP)	Bilinear method, spectral analysis	Combination of lump–soliton interactions with integrability verified *via* LSP
Present study	kink, antikink, periodic soliton and bifurcation analysis	$\left(\frac{G'}{G'+G+A}\right)$-expansion method	Novel exact soliton solutions and bifurcation-based nonlinear wave dynamics

was performed, which revealed critical transitional behaviors in the system. This analysis highlighted how minor changes in system parameters could lead to significant shifts in wave behavior, from stable solutions to oscillatory or chaotic regimes. The findings provided crucial insights for understanding and predicting the qualitative dynamics of these nonlinear systems. Visual representations of the solutions reinforced the theoretical findings, offering an intuitive understanding of the waveforms' evolution through space and time. In general, this research confirmed the effectiveness of the applied method for solving higher dimensional nonlinear evolution equations. The diversity of the obtained solutions, which included kink and antikink solitons analogous to internal tidal bores or switching fronts, as well as periodic solutions representing stable wave trains, demonstrated the YTSF equation's ability to accurately model a wide spectrum of nonlinear wave behaviors in various dispersive media. In conclusion, this work provided a robust framework for future investigations in nonlinear science and applied physics. The effectiveness of the method, along with the detailed analysis of the rich solution set of the YTSF equation, paves the way for further exploration of similar higher dimensional nonlinear models and their applications.

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