

Research Article

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Novel exact solitons to the fractional modified mixed-Korteweg–de Vries model with a stability analysis

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Abstract: A novel class of exact soliton solutions has been derived for the truncated M proportional modified mixed-Korteweg–de Vries (KdV) model, a mathematical physics model that elucidates the flat-topped electron distribution characterized by strong nonlinearity, resulting in a wave with narrower width and higher velocity. For our purpose, first we will convert the concerned model into corresponding ordinary differential equation by applying the wave transformation. Then we obtained the solutions by using the unified and modified simplest equation techniques, yielding results that include periodic, dark, kink, and many others. The influence of the derivatives was also explored. The soliton solutions are presented in two-dimensional (2D), three-dimensional (3D), and contour plots. The results have important applications in fluid dynamics, nonlinear optics, ocean engineering, and related fields. In addition, a stability analysis of the concerned equation was done, to confirm the stability of the considered equation as well as obtained results. At the end, it is demonstrated that the used techniques are simple as well as more effective for solving other nonlinear fractional models.

Keywords: nonlinear modified mixed-KdV model, truncated M proportional derivative, stability analysis, exact soliton solutions

1 Introduction

Fractional calculus is important in many branches of science and engineering. Fractional partial differential equations (FPDEs) provide the best description for a wide range of naturally occurring phenomena. Several methods have been developed over time to extract the output from FPDEs: improved tanh-function method [1], Riccati equation expansion method [2], the generalized Riccati expansion scheme [3], Khater II technique [4,5], the $(G'/G, 1/G)$ -expansion scheme [6,7], and the modified sub-ordinary differential equation (ODE) method [8], among others.

In our study, we employed two straightforward yet highly effective techniques: the unified and the modified simplest equation techniques. These methods have demonstrated their versatility in various applications. For instance, the unified scheme has been successfully utilized to gain the various kinds of wave solutions of the Biswas–Arshed model, as discussed in previous studies [9,10]. Moreover, it has been applied to derive various exact solitons of nonlinear evolution models, as shown in the study by Wang *et al.* [11]. In addition, distinct analytical solitons for the Gilson–Pickering equation have been achieved using this technique [12]. Similarly, the modified simplest equation technique has been effectively used to obtain traveling wave solitons of the coupled Higgs model and Maccari's equation [13], as well as distinct analytical solitons of the Gardner model [14], and to gain the wave solitons of the nonlinear Schrödinger model [15,16].

One important mathematical physics model is the nonlinear (1+1)-dimensional modified mixed Korteweg–de Vries (mm-KdV) model, which holds substantial significance in nonlinear optics, fluid dynamics, and many more [17–20]. The concerned equation has been addressed using various techniques in the past, for example, the extended rational sinh–cosh scheme, the extended rational sine–cosine scheme, the polynomial function method [20], and the homogeneous balance method [21].

The primary aim of this research is to investigate the novel exact solitons for the space-time nonlinear mm-KdV

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equation in the concept of truncated M-fractional derivative (TMFD) by employing two distinct techniques: the unified and the modified simplest equation techniques, along with conducting a stability analysis of the governing model. This work introduces effective methodologies that may be utilized to a broad range of nonlinear mathematical physics problems. The fractional nature of the concerned model was explained using the TMFD.

Our article is structured as follows: Section 2 outlines the model under consideration and provides a detailed mathematical analysis. Section 3 presents the unified technique and the results obtained using this method. Section 4 discusses the modified simplest equation technique and the corresponding results. Section 5 graphically illustrates some of the obtained results. Section 6 is dedicated to the stability analysis. Finally, Section 7 concludes the article.

Definition. (TMFD) Let's $v(x) : [0, \infty) \rightarrow \mathfrak{R}$, so TMFD of v of order α [22]:

$$D_{M,x}^{\alpha,\gamma} v(x) = \lim_{\alpha \rightarrow 0} \frac{v(xE_\gamma(\alpha x^{1-\alpha})) - v(x)}{\alpha}, \quad \alpha \in (0, 1], \gamma > 0, \quad (1)$$

where $E_\gamma(\cdot)$ indicates a truncated Mittag-Leffler function [23]:

$$E_\gamma(z) = \sum_{j=0}^i \frac{z^j}{\Gamma(\gamma j + 1)}, \quad \gamma > 0 \quad \text{and} \quad z \in \mathbb{C}.$$

Properties: Suppose $a, b \in \mathfrak{R}$, and g, f are α -differentiable at a point $x > 0$, according to [22]:

- (a) $D_{M,x}^{\alpha,\gamma}(ag(x) + bf(x)) = aD_{M,x}^{\alpha,\gamma}g(x) + bD_{M,x}^{\alpha,\gamma}f(x)$
- (b) $D_{M,x}^{\alpha,\gamma}(g(x) \cdot f(x)) = g(x)D_{M,x}^{\alpha,\gamma}f(x) + f(x)D_{M,x}^{\alpha,\gamma}g(x)$
- (c) $D_{M,x}^{\alpha,\gamma}\left(\frac{g(x)}{f(x)}\right) = \frac{f(x)D_{M,x}^{\alpha,\gamma}g(x) - g(x)D_{M,x}^{\alpha,\gamma}f(x)}{(f(x))^2}$
- (d) $D_{M,x}^{\alpha,\gamma}(C) = 0$, where C is a constant.
- (e) $D_{M,x}^{\alpha,\gamma}g(x) = \frac{x^{1-\alpha}}{\Gamma(\gamma + 1)} \frac{dg(x)}{dx}$.

This novel fractional derivative is utilized for many equations including Westervelt model [24], Fokas equation [25], Konopelchenko–Dubrovsky model [26,27], and many more.

2 The model representation and its mathematical treatment

Suppose a (1+1)-D nonlinear mm-KdV equation in the sense of TMFD is shown as follows:

$$D_{M,t}^{\alpha,\gamma} v + (a\sqrt{v} + bv)D_{M,x}^{\alpha,\gamma} v + \tau D_{M,3x}^{3\alpha,\gamma} v = 0, \quad (2)$$

where

$$D_{M,x}^{\alpha,\gamma} v = \lim_{\tau \rightarrow 0} \frac{v(xE_\gamma(\tau x^{1-\alpha})) - v(x)}{\tau}, \quad 0 < \alpha \leq 1, \quad \gamma \in (0, \infty).$$

Here, $E_\gamma(\cdot)$ denotes a Mittag-Leffler function given in the previous studies [22,23].

Here, $v = v(x, t)$ denotes a wave profile and a, b , and τ are the parameters. Eq. (2) shows a mm-KdV equation, which describes the flat-topped electron distribution involving a stronger nonlinearity that related to a smaller-width and higher-velocity of waves [28–32]. Eq. (2) is a more prominent representation of the concerned model's mm-KdV equation because of the use of a TMFD. The mm-KdV equation depicts a flat-topped electron distribution with more nonlinearity, which correlates to a narrower and faster wave. To find the solutions of Eq. (2), we assume the following:

$$v = f^2; \quad f = f(x, t). \quad (3)$$

By substituting Eq. (3) into Eq. (2), we yield

$$fD_{M,t}^{\alpha,\gamma} f + (af^2 + bf^3)D_{M,x}^{\alpha,\gamma} f + \tau(fD_{M,3x}^{3\alpha,\gamma} f + 3D_{M,x}^{\alpha,\gamma} f D_{M,2x}^{2\alpha,\gamma} f) = 0. \quad (4)$$

Suppose the following transformation:

$$f = V(\xi); \quad \xi = \theta \frac{\Gamma(\gamma + 1)}{\alpha} (x^\alpha - \lambda t^\alpha). \quad (5)$$

Here, V indicates the amplitude of the wave, θ denotes a wave number, while λ shows the soliton speed.

By using Eq. (5) in Eq. (4), we gain

$$-6\lambda V^2 + 4aV^3 + 3bV^4 + 12\tau\theta^2(VV'' + (V')^2) = 0. \quad (6)$$

The value of natural number $m = 1$ is obtained by utilizing the Homogenous balance scheme into Eq. (6).

3 Unified technique and its application

3.1 Unified method

Consider a nonlinear fractional partial differential equation (PDE) given as follows:

$$F(v, v^2 D_{M,t}^{\alpha,\gamma} v, D_{M,x}^{\alpha,\gamma} v, D_{M,2t}^{2\alpha,\gamma} v, D_{M,2x}^{2\alpha,\gamma} v, D_{M,xt}^{2\alpha,\gamma} v, \dots) = 0. \quad (7)$$

where “ v ” represents the wave function.

By applying the following wave transformations;

$$v(x, t) = V(\eta), \eta = \frac{\Gamma(\gamma + 1)}{\alpha}(x^\alpha + \kappa t^\alpha), \quad (8)$$

where “ V ” represents the amplitude of wave and κ indicates the velocity of wave. Putting Eq. (8) in Eq. (7), yields:

$$G(V, V^2V', V', V'', \dots) = 0. \quad (9)$$

The result of Eq. (9) is represented as follows:

$$V(\eta) = \alpha_0 + \sum_{i=1}^m \alpha_i \phi^i(\eta) + \sum_{i=1}^m \beta_i \phi^{-i}(\eta), \quad (10)$$

where $\alpha_0, \alpha_i, \beta_i$, ($i = 1, 2, 3, \dots, m$) are undeterminate. Consider a new function given as follows:

$$\phi'(\eta) = \delta + \phi^2(\eta), \quad (11)$$

where δ is a parameter. The solutions of Eq. (11) are given in the study by Chou et al. [10]:

This technique gives the kink soliton, periodic, and rational soliton solutions.

3.2 Application

For $m = 1$, Eq. (10) takes form:

$$V(\eta) = \alpha_0 + \alpha_1 \phi(\eta) + \beta_1 \phi^{-1}(\eta). \quad (12)$$

By substituting Eq. (12) and its first and second derivatives in Eq. (6), and with the help of Mathematica tool, we gain the solutions:

Set 1:

$$\left\{ \alpha_0 = -\frac{2a}{5b}, \alpha_1 = \pm \frac{a}{5b\sqrt{-\delta}}, \beta_1 = \pm \frac{a\sqrt{-\delta}}{5b}, \lambda = -\frac{16a^2}{75b}, \tau = \frac{a^2}{300b\delta\theta^2} \right\}. \quad (13)$$

Case 1:

$$v(x, t) = \left[\frac{a}{5b} - 2 \pm \sqrt{-\delta} \frac{1}{\frac{\sqrt{-(p^2+q^2)\delta} - p\sqrt{-\delta} \cosh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)}{p \sinh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) + q}} \right] \pm \frac{1}{\sqrt{-\delta}} \left[\frac{\sqrt{-(p^2+q^2)\delta} - p\sqrt{-\delta} \cosh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)}{p \sinh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) + q} \right]^2. \quad (14)$$

$$v(x, t) = \left[\frac{a}{5b} - 2 \pm \sqrt{-\delta} \frac{1}{\frac{-\sqrt{-(p^2+q^2)p} - p\sqrt{-\delta} \cosh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)}{p \sinh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) + q}} \right] \pm \frac{1}{\sqrt{-\delta}} \left[\frac{-\sqrt{-(p^2+q^2)p} - p\sqrt{-\delta} \cosh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)}{p \sinh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(\gamma+1)}{a}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) + q} \right]^2. \quad (15)$$

$$v(x, t) = \left[\frac{a}{5b} - 2 \pm \left[\frac{1}{1 + \frac{2p}{p + \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}} \right] \right. \\ \left. \pm \left[1 + \frac{2p}{p + \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)} \right] \right] \right]^2. \quad (16)$$

$$v(x, t) = \left[\frac{a}{5b} - 2 \pm \left[\frac{1}{-1 + \frac{2p}{p + \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}} \right] \right. \\ \left. \pm \left[-1 + \frac{2p}{p + \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)} \right] \right] \right]^2. \quad (17)$$

Case 2:

$$v(x, t) = \left[\frac{a}{5b} - 2 \pm \sqrt{-\delta} \left[\frac{1}{\frac{\sqrt{(p^2 + q^2)\delta} - p\sqrt{\delta} \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}{p \sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) + q}} \right] \right. \\ \left. \pm \frac{1}{\sqrt{-\delta}} \left[\frac{\sqrt{(p^2 + q^2)\delta} - p\sqrt{\delta} \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}{p \sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) + q} \right] \right] \right]^2. \quad (18)$$

$$v(x, t) = \left[\frac{a}{5b} - 2 \pm \sqrt{-\delta} \left[\frac{1}{\frac{-\sqrt{(p^2 + q^2)\delta} - p\sqrt{\delta} \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}{p \sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) + q}} \right] \right. \\ \left. \pm \frac{1}{\sqrt{-\delta}} \left[\frac{-\sqrt{(p^2 + q^2)\delta} - p\sqrt{\eta} \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}{p \sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) + q} \right] \right] \right]^2. \quad (19)$$

$$v(x, t) = \left[\frac{a}{5b} \left(-2 \pm \sqrt{-\delta} \left[\frac{1}{\iota\sqrt{\delta} + \frac{-2\iota p\sqrt{\delta}}{p + \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right) - \iota\sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)} \right] \right) \right. \\ \left. \pm \frac{1}{\sqrt{-\delta}} \left[\iota\sqrt{\delta} + \frac{-2\iota p\sqrt{\delta}}{p + \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right) - \iota\sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)} \right] \right] \right]^2. \quad (20)$$

$$v(x, t) = \left[\frac{a}{5b} \left(-2 \pm \sqrt{-\delta} \left[\frac{1}{-\iota\sqrt{\delta} + \frac{2\iota p\sqrt{\delta}}{p + \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right) - \iota\sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)} \right] \right) \right. \\ \left. \pm \frac{1}{\sqrt{-\delta}} \left[-\iota\sqrt{\delta} + \frac{2\iota p\sqrt{\delta}}{p + \cos\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right) - \iota\sin\left(2\sqrt{\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)} \right] \right] \right]^2. \quad (21)$$

Set 2:

$$\left\{ \alpha_0 = -\frac{2a}{5b}, \alpha_1 = \pm \frac{a}{5b\sqrt{-\delta}}, \beta_1 = 0, \lambda = -\frac{16a^2}{75b}, \tau = \frac{a^2}{75b\delta\theta^2} \right\}. \quad (22)$$

Case 1:

$$v(x, t) = \left[\frac{a}{5b} \left(-2 \pm \frac{1}{\sqrt{-\delta}} \left[\frac{\sqrt{-(p^2 + q^2)\delta} - p\sqrt{-\delta} \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)}{p \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) + q} \right] \right] \right]^2. \quad (23)$$

$$v(x, t) = \left[\frac{a}{5b} \left(-2 \pm \frac{1}{\sqrt{-\delta}} \left[\frac{-\sqrt{-(p^2 + q^2)p} - p\sqrt{-\delta} \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right)}{p \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) + q} \right] \right] \right]^2. \quad (24)$$

$v(x, t)$

$$= \left[\frac{a}{5b} \left(-2 \pm \left[1 + (2p) \left/ \left(p + \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) \right] \right] \right]^2. \quad (25)$$

$v(x, t)$

$$= \left[\frac{a}{5b} \left(-2 \pm \left[-1 + (2p) \left/ \left(p + \cosh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta^{\frac{\Gamma(\gamma+1)}{a}}\left(x^\alpha + \frac{16a^2}{75b}t^\alpha\right) + d\right)\right) \right] \right] \right]^2. \quad (26)$$

Case 2:

$$v(x, t) = \left[\frac{a}{5b} \left(-2 \pm \frac{1}{\sqrt{-\delta}} \left[\frac{\sqrt{(p^2 + q^2)\delta} - p\sqrt{\delta} \cos \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right)}{p \sin \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) + q} \right] \right) \right]^2. \quad (27)$$

$$v(x, t) = \left[\frac{a}{5b} \left(-2 \pm \frac{1}{\sqrt{-\delta}} \left[\frac{-\sqrt{(p^2 + q^2)\delta} - p\sqrt{\delta} \cos \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right)}{p \sin \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) + q} \right] \right) \right]^2. \quad (28)$$

 $v(x, t)$

$$= \left[\frac{a}{5b} \left(-2 \pm \left[1 + (-2p) \left(p + \cos \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) - \iota \sin \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) \right] \right) \right]^2. \quad (29)$$

 $v(x, t)$

$$= \left[\frac{a}{5b} \left(-2 \pm \left[-1 + (2p) \left(p + \cos \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) - \iota \sin \left(2\sqrt{\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) \right] \right) \right]^2. \quad (30)$$

Set 3:

$$\left[\alpha_0 = -\frac{2a}{5b}, \alpha_1 = 0, \beta_1 = \pm \frac{2a\sqrt{-\delta}}{5b}, \lambda = -\frac{16a^2}{75b}, \tau = \frac{a^2}{75b\delta\theta^2} \right]. \quad (31)$$

Case 1:

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{\frac{\sqrt{-(p^2 + q^2)\delta} - p\sqrt{-\delta} \cosh \left(2\sqrt{-\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right)}{p \sinh \left(2\sqrt{-\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) + q}} \right) \right]^2. \quad (32)$$

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{\frac{-\sqrt{-(p^2 + q^2)\delta} - p\sqrt{-\delta} \cosh \left(2\sqrt{-\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right)}{p \sinh \left(2\sqrt{-\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) + q}} \right) \right]^2. \quad (33)$$

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{\sqrt{-\delta} + \frac{2p\sqrt{-\delta}}{p + \cosh \left(2\sqrt{-\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right) - \sinh \left(2\sqrt{-\delta} \left(\theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) + d \right) \right)}} \right) \right]^2. \quad (34)$$

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{-\sqrt{-\delta} + \frac{2p\sqrt{-\delta}}{p + \cosh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \sinh\left(2\sqrt{-\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}} \right) \right]^2. \quad (35)$$

Case 2:

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{\frac{\sqrt{(p^2+q^2)\delta} - p\sqrt{\delta} \cos\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}{p \sin\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) + q}} \right) \right]^2. \quad (36)$$

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{\frac{-\sqrt{(p^2+q^2)\delta} - p\sqrt{\delta} \cos\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}{p \sin\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) + q}} \right) \right]^2. \quad (37)$$

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{\iota\sqrt{\delta} + \frac{-2\iota p\sqrt{\delta}}{p + \cos\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \iota \sin\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}} \right) \right]^2. \quad (38)$$

$$v(x, t) = \left[-\frac{2a}{5b} \left(1 \pm \frac{\sqrt{-\delta}}{-\iota\sqrt{\delta} + \frac{2\iota p\sqrt{\delta}}{p + \cos\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right) - \iota \sin\left(2\sqrt{\delta}\left(\theta\frac{\Gamma(y+1)}{a}\left(x^a + \frac{16a^2}{75b}t^a\right) + d\right)\right)}} \right) \right]^2. \quad (39)$$

4 The modified simplest equation scheme and its application

$$V(\Xi) = \sum_{s=1}^m b_s \phi^s(\Xi), \quad (43)$$

The fundamental steps of this scheme are as follows:

Step 1: Assuming a nonlinear PDE:

$$F(v, v^2, v^2 v_x, v_{xx}, v_{xt}, \dots) = 0. \quad (40)$$

Here, $v = v(x, t)$ indicates a profile. Let us apply a given relation:

$$v(x, t) = V(\Xi), \quad \Xi = x + \lambda t. \quad (41)$$

Substituting Eq. (41) into Eq. (40), a nonlinear ordinary differential equation is gained

$$Z(V, V^2 V', V'', \dots) = 0. \quad (42)$$

Step 2: Consider the result of Eq. (42) given as follows:

where $b_s (s = 1, 2, 3, \dots, m)$ represent undeterminate and $b_m \neq 0$. A new function $\phi(\Xi)$ fulfills the ODE:

$$\phi'(\Xi) = \phi^2(\Xi) + \overline{\phi}, \quad (44)$$

where $\overline{\phi}$ is a constant. Notice the solutions of Eq. (44) based on $\overline{\phi}$ are given in the study by Chou *et al.* [16].

Step 3: Using Eq. (43) with Eq. (44) in Eq. (42). Collecting the coefficients of each power of ϕ^s , putting equal to zero.

Step 4: putting Eq. (42), the values of b_s and λ in Eq. (11) will give results of Eq. (40).

This technique provides the dark soliton, singular soliton, bright soliton, dark-bright soliton, and many more solutions.

4.1 Application of modified simplest equation technique

For $m = 1$, Eq. (43) becomes into

$$V(\Xi) = b_0 + b_1 \psi(\Xi). \quad (45)$$

Substituting Eqs (45) and (44) into Eq. (6), one obtains the following set for discussion. Set:

$$\left\{ b_0 = -\frac{2a}{5b}, b_1 = \pm \frac{2a}{5b\sqrt{-\Theta}}, \lambda = -\frac{16a^2}{75b}, \tau = \frac{a^2}{75b\theta^2\Theta} \right\}. \quad (46)$$

Case 1: if $\Theta < 0$.

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \tanh(\sqrt{-\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right]^2. \quad (47)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(-\sqrt{-\Theta} \coth \left(\sqrt{-\Theta} \theta \frac{\Gamma(1+\gamma)}{a} \right. \right. \right. \right. \\ \left. \left. \left. \times \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right]^2. \quad (48)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(\sqrt{-\Theta} \left(\left(-\tanh \left(2\sqrt{-\Theta} \theta \frac{\Gamma(1+\gamma)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \pm \operatorname{isech} \left(2\sqrt{-\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right) \right]^2. \quad (49)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \left(-\coth \left(2\sqrt{-\Theta} \theta \frac{\Gamma(1+\gamma)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right. \right. \right. \\ \left. \left. \left. \pm \operatorname{csch} \left(2\sqrt{-\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right]^2. \quad (50)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{2} \left(\left(\tanh \left(\frac{\sqrt{-\Theta}}{2} \theta \frac{\Gamma(1+\gamma)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right. \right. \right. \right. \right. \\ \left. \left. \left. \coth \left(\frac{\sqrt{-\Theta}}{2} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right) \right]^2. \quad (51)$$

Case 2: if $\Theta > 0$.

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(\sqrt{\Theta} \tan \left(\sqrt{\Theta} \theta \frac{\Gamma(1+\gamma)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right)^2. \quad (52)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(-\sqrt{\Theta} \cot \left(\sqrt{\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right)^2. \quad (53)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(\sqrt{\Theta} \left(\left(\tan \left(2\sqrt{\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \pm \sec \left(2\sqrt{\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right) \right]^2. \quad (54)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(\sqrt{\Theta} \left(\left(-\cot \left(2\sqrt{\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \pm \csc \left(2\sqrt{\Theta} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right) \right]^2. \quad (55)$$

$$v(x, t) = \left[\frac{2a}{5b} \left(-1 \pm \frac{1}{\sqrt{-\Theta}} \left(\frac{\sqrt{\Theta}}{2} \left(\left(\tan \left(\frac{\sqrt{\Theta}}{2} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. - \cot \left(\frac{\sqrt{\Theta}}{2} \theta \frac{\Gamma(\gamma+1)}{a} \left(x^\alpha + \frac{16a^2}{75b} t^\alpha \right) \right) \right) \right) \right]^2. \quad (56)$$

5 Physically illustrations

Here, we will provide the solutions' physical behaviors using contour, 3D, and 2D graphs. In addition, 2D graphs are created for various α values to illustrate the impact of the derivative (Figures 1–4).

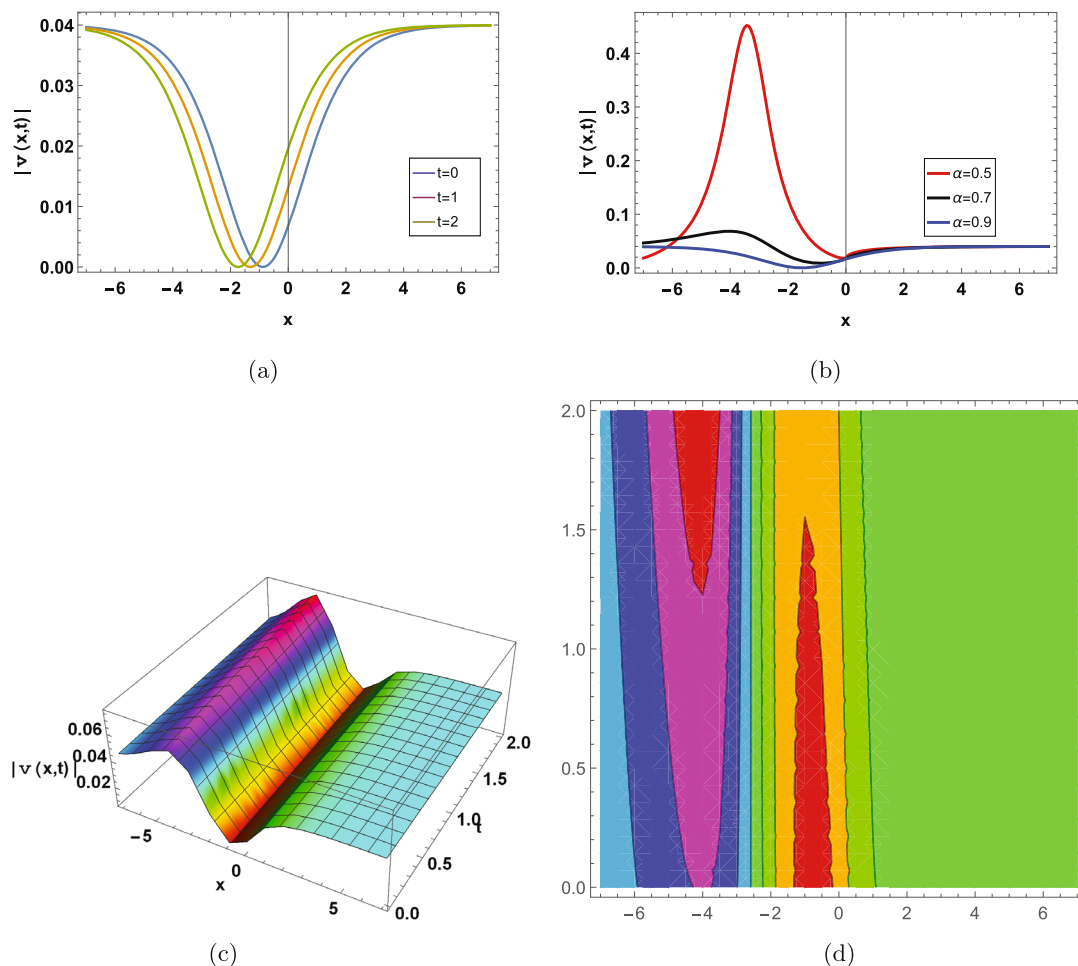


Figure 1: (Kink soliton solution) graph of $|v(x, t)|$ is given in Eq. (26) for : $a = 1, b = 1, p = 0.5, \theta = 0.5, d = 1, \eta = -1$, and $\gamma = 1$. (a) A two-dimensional graph when $x \in (-7, 7)$ at α is 1, blue curve if value of t is 0, orange line if t is 1, while green line if t is 2. (b) A two-dimensional plot if $x \in (-7, 7)$ and $0 < t < 2$, while red line when $\alpha = 0.5$, black line when $\alpha = 0.7$, and blue line when $\alpha = 0.9$. (c) A three-dimensional plot if $\alpha = 0.8$ for $t \in (0, 2)$. (d) A contour graph when $\alpha = 0.8$ for $0 < t < 2$. (c) $\alpha = 0.8$, (d) $\alpha = 0.8$.

6 A stability analysis

It serves to describes how the model reacts over time and how it behaves in response to outside disturbances. This analysis of various equations are explained in the literature, including [33,34].

Now, we will discuss the stability analysis of Eq. (2). For this purpose, we consider the given condition;

$$\mathcal{U} = \frac{1}{2} \int_{-\infty}^{\infty} v^2 dx, \quad (57)$$

where $\mathcal{U}$ indicates the momentum and the power of possibility is represented through $v(x, t)$. There is a mandatory criterion for stability in the following;

$$\frac{\partial \mathcal{U}}{\partial \lambda} > 0, \quad (58)$$

where λ represents a speed of wave, substituting Eq. (26) in Eq. (57) yields:

$$\mathcal{U} = \frac{1}{2} \int_{-10}^{10} \left[\frac{2a}{5b} \left(-1 + \frac{1}{\sqrt{-\theta}} \left(-\sqrt{\theta} \cot \left(\sqrt{\theta} \theta \left(x + \frac{16a^2}{75b} t \right) \right) \right) \right) \right]^4 dx, \quad (59)$$

by using the criterion given in Eq. (58), we obtain

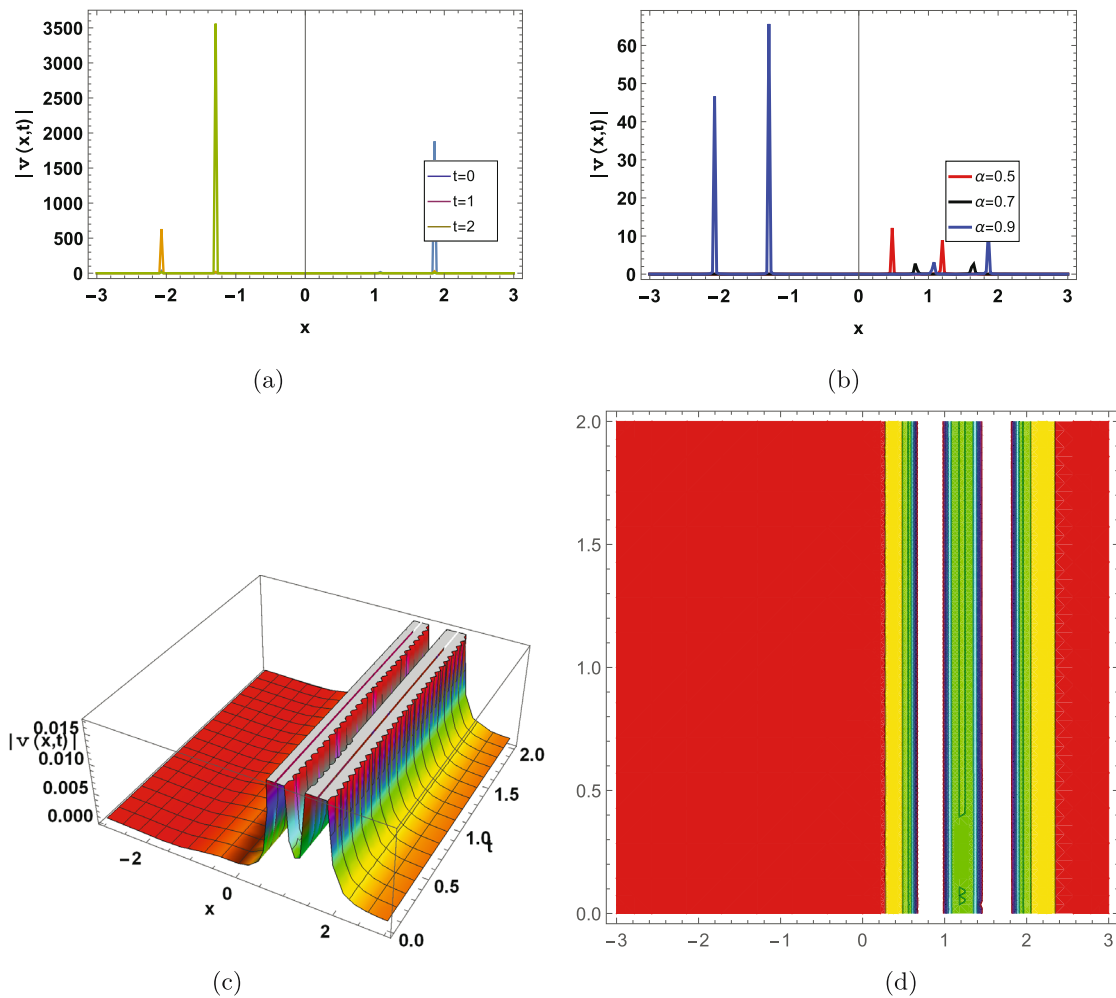


Figure 2: (Periodic wave soliton) graph of $|v(x, t)|$ is given in Eq. (30) with the following parameter values: $a = 0.1, b = 1, p = 1, \theta = 1, d = 0.5, \eta = 1$, and $\gamma = 1$. (a) A two-dimensional graph when $x \in (-3, 3)$ at a is 1, blue curve if value of t is 0, orange line if t is 1, while green line if t is 2. (b) A two-dimensional plot if $x \in (-3, 3)$ and $0 < t < 2$, while red line when $\alpha = 0.5$, black line when $\alpha = 0.7$, and blue line when $\alpha = 0.9$. (c) A three-dimensional plot if $a = 0.8$ for $t \in (0, 2)$, and (d) represents a contour graph when $a = 0.8$ for $0 < t < 2$. (c) $a = 0.8$, and (d) $a = 0.8$.

$$\begin{aligned}
 & \left(8a^4 \left[\theta t \sqrt{\overline{c}} \left[\csc^2 \left(\theta \sqrt{\overline{c}} \left(-\frac{16a^2}{75b} t + 5 \right) \right) + 14 \right] \csc^2 \left(\theta \sqrt{\overline{c}} \left(-\frac{16a^2}{75b} t + 5 \right) \right) - \theta t \sqrt{\overline{c}} \right. \right. \\
 & \quad \times \csc^2 \left(\theta \sqrt{\overline{c}} \left(5 + \frac{16a^2}{75b} t \right) \right) \left[\csc^2 \left(\theta \sqrt{\overline{c}} \left(5 + \frac{16a^2}{75b} t \right) \right) + 14 \right] 2\theta t \sqrt{\overline{c}} \cot^2 \left(\theta \sqrt{\overline{c}} \left(-\frac{16a^2}{75b} t + 5 \right) \right) \\
 & \quad \times \csc^2 \left(\theta \sqrt{\overline{c}} \left(-\frac{16a^2}{75b} t + 5 \right) \right) - 2\theta t \sqrt{\overline{c}} \cot^2 \left(\theta \sqrt{\overline{c}} \left(5 + \frac{16a^2}{75b} t \right) \right) \csc^2 \left(\theta \sqrt{\overline{c}} \left(5 + \frac{16a^2}{75b} t \right) \right) \\
 & \quad - 6 \left[2\theta t \sqrt{\overline{c}} \cot \left(\theta \sqrt{\overline{c}} \left(-\frac{16a^2}{75b} t + 5 \right) \right) \csc^2 \left(\theta \sqrt{\overline{c}} \left(-\frac{16a^2}{75b} t + 5 \right) \right) \right. \\
 & \quad \left. \left. + 2\theta t \sqrt{\overline{c}} \cot \left(\theta \sqrt{\overline{c}} \left(5 + \frac{16a^2}{75b} t \right) \right) \csc^2 \left(\theta \sqrt{\overline{c}} \left(5 + \frac{16a^2}{75b} t \right) \right) \right] \right) / (1875b^4 \theta \sqrt{\overline{c}}) > 0.
 \end{aligned} \tag{60}$$

Hence, the given condition is fulfilled. So, Eq. (2) is a stable equation.

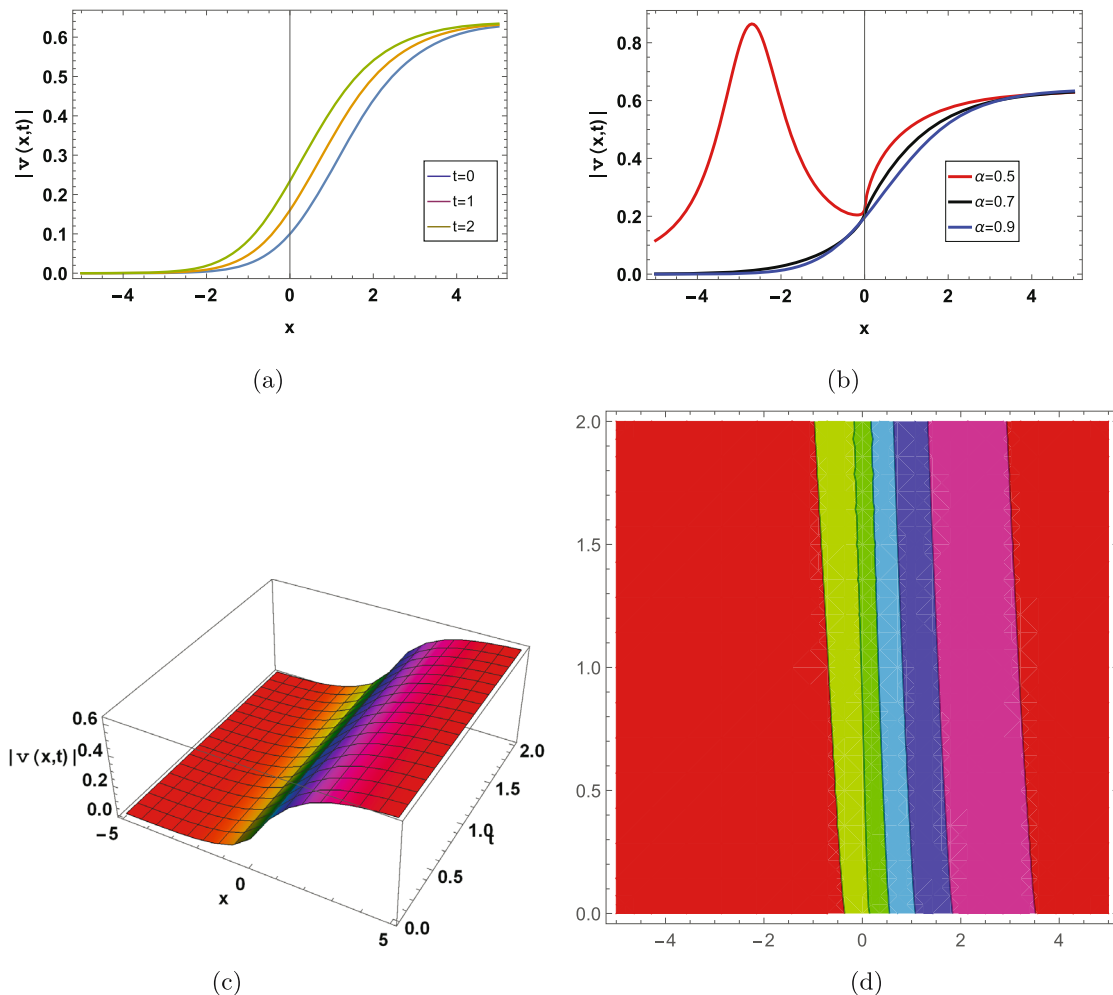


Figure 3: (Dark soliton solution) graph for $|v(x, t)|$ is given in Eq. (48) for: $a = 1$, $b = 1$, $\theta = 0.5$, $\bar{U} = -1$, and $\gamma = 1$. (a) A two-dimensional graph when $x \in (-5, 5)$ at $\alpha = 1$, blue curve if value of t is 0, orange line if t is 1, while green line if t is 2. (b) A two-dimensional plot if $x \in (-5, 5)$ and $0 < t < 2$, while red line when $\alpha = 0.5$, black line when $\alpha = 0.7$, and blue line when $\alpha = 0.9$. (c) A three-dimensional plot if $\alpha = 0.8$ for $t \in (0, 2)$. (d) A contour graph when $\alpha = 0.8$ for $0 < t < 2$. (c) $\alpha = 0.8$, (d) $\alpha = 0.8$.

7 Results and discussion

Here, we give a comparative analysis of our findings alongside present solutions in the literature. Alquran *et al.* [20] demonstrates the acquisition of various topological and nontopological wave solutions through the application of the extended rational sinh-cosh method, the extended rational sine-cosine method, and a polynomial function method. Similarly, Butt *et al.* [21] successfully derived dark, bright, and periodic solitons by employing the homogeneous balance scheme.

In contrast, our research utilizes the concerned equation in conjunction along TMFD, which has enabled us to gain a diverse range of wave solitons. These include dark, periodic, and kink soliton solutions, among others, through

the implementation of the unified technique and the modified simplest equation method. Kink soliton solutions have many applications, including nonlinear fibers, signal processing, electromagnetism, and complex media, *etc.* Periodic wave solutions are useful in plasma physics, nonlinear optics, energy harvesting, *etc.* Dark soliton has many applications in different fields, including ultrafast optics, nonlinear dynamics, quantum computing, *etc.* Furthermore, our work contributes to the field by incorporating a stability analysis of the solutions, a topic that has not been addressed in prior studies. The results we have obtained hold significant potential for future research into the model and related areas, such as nonlinear optics, telecommunications, and ocean engineering.

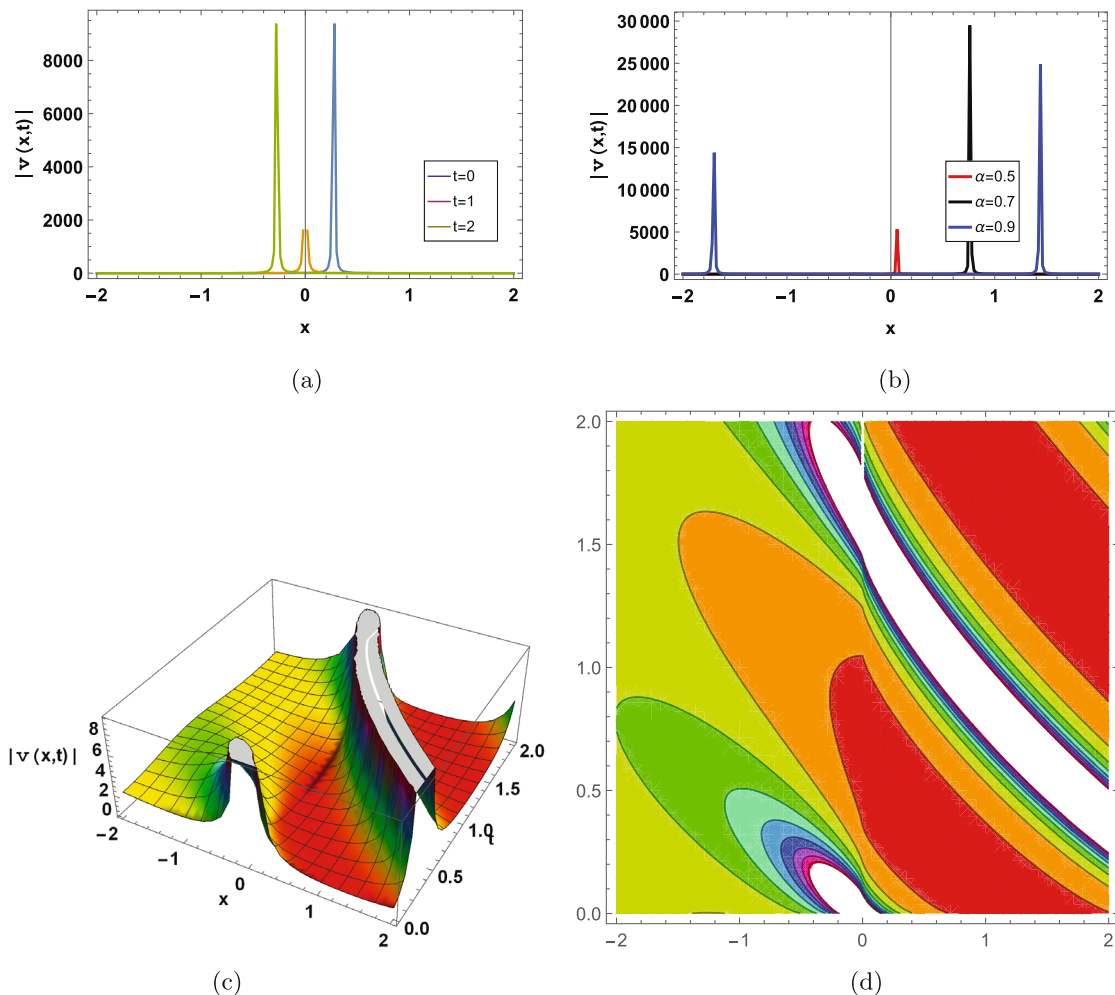


Figure 4: (Periodic wave solutions) graph for $|v(x, t)|$ is given in Eq. (54) for the following values: $\alpha = 4$, $b = 2$, $\theta = -1$, and $\bar{U} = \gamma = 1$. (a) A two-dimensional graph when $x \in (-2, 2)$ at α is 1, blue curve if value of t is 0, orange line if t is 1, while green line if t is 2. (b) A two-dimensional plot if $x \in (-2, 2)$ and $0 < t < 2$, while red line when $\alpha = 0.5$, black line when $\alpha = 0.7$, and blue line when $\alpha = 0.9$. (c) A three-dimensional plot if $\alpha = 0.8$ for $t \in (0, 2)$. (d) A contour graph when $\alpha = 0.8$ for $0 < t < 2$. (c) $\alpha = 0.8$, and (d) $\alpha = 0.8$.

8 Conclusion

In this article, we successfully attained the new exact solitons of the nonlinear (1+1)-D mm-KdV equation along with a novel definition of derivative. The modified-mixed Korteweg–de Vries equation is an equation that describes a flat-topped electron distribution with more nonlinearity, which correlates to a narrower and faster wave. The solutions has been achieved through the application of unified and modified simplest equation techniques. The resulting solutions encompass a diverse array of soliton types, including dark, periodic, kink, and other soliton types. The influence of derivations on these results has also been explored, pressing the distinctness of these results in comparison to being bones. In the future, the obtained

solutions can be observed experimentally like the solutions of other models [35,36].

To insure the accuracy as well as delicacy of the obtained results, Mathematica tool was used for calculation as well as verification. Also, we have shown the deduced results by two-, three-dimensional, and Contour graphs, as shown in numbers 1–4. The results are useful in different fields including, ocean engineering, fluid dynamics, etc.

Also, a stability analysis of the concerned equation has been performed to corroborate the stability and perfection of the attained results. The styles utilized are not only simple but also exceptionally effective in working nonlinear FPDEs. Likewise, these ways prove to be precious for addressing advanced-order nonlinear FPDEs. The

findings presented then offer substantial perceptivity and implicit operations across different fields of science as well as engineering.

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