Research Article

Wen-Yuan Li, Nong-Sen Li, Rui-Gang Zhang, Yi-Lin Tian, and Ji-Feng Cui*

Interaction solutions of high-order breathers and lumps for a (3+1)-dimensional conformable fractional potential-YTSF-like model

https://doi.org/10.1515/phys-2025-0187
received November 08, 2024; accepted June 27, 2025

Abstract: In this study, the breather solutions and interaction solutions for the (3+1)-dimensional conformable fractional potential Yu-Toda-Sasa-Fukuyama-like model are conducted. The fractional derivative is described using the conformable derivative. By using certain commutative methods, we derived the breather solutions and the interaction solutions of breathers and lumps in terms of Gramian. Solutions of these forms have never been studied. The features of the solutions were examined. Each interval of the breather has one and only one peak as well as trough, and the interaction between the breather wave and the breather wave or the lump wave is elastic. In addition, the effect of a fractional order change on the shape of the breathers is discussed through 3-dimensional images, and this change is able to be reduced to integer order and different fractional order.

Keywords: potential-YTSF equation, conformable derivative, Hirota bilinear form, breathers, interaction solutions

1 Introduction

Fractional differential equations (FDEs) hold a significant place in the field of science, and their frequent use in research and industrial applications has piqued the interest and attention of numerous researchers. These equations have been used in a number of fields, such as telecommunications [1], transportation, optics [2], instruments instrumentation [3], geochemistry geophysics, fluid dynamics [4], and information processing [5]. FDEs represent an extension of integer-order differential equations, suitable for describing materials with historical memory and hereditary properties, and have advantages over integer-order differential models in many fields. Researchers have identified the nonlocality of fractional order differentiation as a key factor in its ability to describe a range of complex mechanical and physical behaviors [6]. Nonlinear fractional partial differential equations (FPDEs), which have been applied in number of fields of physics and engineering including thermodynamics [7], mathematical computational biology [8], optics [9], environmental sciences ecology, and geology [10], require the utilization of scientific fields to identify accurate or approximative solutions to the issues presented by these systems [11]. However, for FPDEs, it is typically impossible to obtain an exact solution. Li and Chen presented Galerkin finite element method, spectral method and the finite difference method as techniques to solve FPDEs [12]. Ammi *et al.* developed an entirely discrete scheme for solving the time fractional diffusion equation, utilizing time finite difference method and space finite element method [13]. Cen *et al.* employed the upwind finite difference method to FDEs, offering stability and a post-test error analysis of discrete schemes, in addition to adaptive methods [14]. Bayrak and Demir provided approximations to analytical solutions for arbitrary order space-time FDEs using a semi-analytical method and then obtained fractional power series solutions using residual power series method [15]. Furthermore, they provide stability and error estimates for this scheme. Even when dealing with applications that necessitate the use of algebra or other sciences, it remains challenging to identify exact solutions.

On the other hand, the Bogoyavlenskii-Schiff (BS) equations are widely used to study solitons and nonlinear waves in the region, including complex nonlinear problems in hydrodynamics, weakly dispersive media. Furthermore, the BS equation is deeply related to many of the classical equations in fluid dynamics [16].

$$-4\upsilon_t + \Phi\upsilon\upsilon_z = 0, \quad \Phi = \partial^2 + 4\upsilon + 2\upsilon_x\partial^{-1}. \tag{1}$$

* **Corresponding author: Ji-Feng Cui,** College of Science, Inner Mongolia University of Technology, Hohhot 010051, China, e-mail: cjf@imut.edu.cn
Wen-Yuan Li, Nong-Sen Li, Yi-Lin Tian: College of Science, Inner Mongolia University of Technology, Hohhot 010051, China
Rui-Gang Zhang: School of Mathematical Sciences, Inner Mongolia University, Hohhot 010021, China

Upon making $z = x$, Eq. (1) becomes the potential Kadomtsev–Petviashvili (KP) equation. Substituting $v_y = 0$, Eq. (1) becomes the potential Korteweg–de Vries (KdV) equation. Yu, Toda, Sasa, and Fukuyama in their study of BS equation proposed the following equation [17]:

$$[-4v_t + \Phi v v_z]_x + 3v_{yy} = 0, \quad \Phi = \partial^2 + 4v + 2v_x\partial^{-1}. \quad (2)$$

As a productive extension of KP equations and BS equations, Yu–Toda–Sasa–Fukuyama (YTSF) equations are commonly used to describe mixed reaction appearing in shallow water equations. They are also employed to study solitons and wave equations or weakly dispersive medium equations in nonlinear dynamics.

The solitary wave solutions of the established model equations are also a topic of great interest in the field of research. In soliton theory, the pursuit of analytical solutions to nonlinear evolution equations represents a pivotal area of inquiry. In the case of high-dimensional equations that describe complex physical phenomena, it is also of great importance to find their solutions. Since Korteweg proposed a solution to the KdV equation, which involved solitary wave solutions, numerous scholars have subsequently proposed various analytical methods with the objective of identifying the nonlinear equations' exact solutions, such as the Darboux transformation method proposed by Darboux [18], the bilinear transformation method proposed by Hirota [19,20], the simplest equation method [21], the semi-inverse method [22], the Jacobi elliptic function expansion method [23], and the homogeneous balance method [24]. However, the inherent complexity of physical problems, the presence of multiple interacting physical phenomena, and strong nonlinearity make it challenging to devise a method for obtaining unification of the analytical solutions of the resulting nonlinear partial differential equations.

In recent years, scholars have expanded and extended the Hirota bilinear form and its applications to obtain the soliton solutions such as breather solutions and lump solutions [25,26]. For instance, Masayoshi proposed a conjugate parameter method based on the Hirota method, which can transform the N-soliton solution into an N/2-breather solution [27]. Zheng analyzed the breathers and multi-soliton solutions of potential-YTSF equation based on the Hirota bilinear method [28]. Wang studied the soliton, breather and lump solutions of the fractional (2+1)-dimensional Boussinesq equation by the Hirota bilinear method and the variational approach [29]. Ma analyzed the solitons and confirmed the N-soliton condition of the Hirota bilinear form for the B-type KP equation, using the Hirota bilinear formulation [30]. Seadawy presented the application of the Hirota bilinear method, in conjunction with symbolic computation, which results in the following solutions: Y-shape, generalized breather, lump one strip, lump two strip, and lump periodic solution [31]. Jin-Bo *et al.* presented a bilinear form of the nonisospectral Ablowitz-Kaup-Newell-Segur equation and derived the N-soliton condition solutions by using the Hirota bilinear method [32]. Peng studied the stochastic Schrodinger-Hirota equation in refractive birefringent optical fibers with spatio-temporal chromatic dispersion and nonlinearity of the parametric law [33]. Kudryashov conducted a comprehensive analysis of the compatibility of the overdetermined system of equations, ultimately identifying the existence of optical solitons of the fourth order [34]. Kumar *et al.* utilized the generalized Kudryashov method and Lie symmetry to transform the specified partial differential equation to a system of ordinary differential equations [35].

The objective of this study is to propose new solutions and method for the resolution of (3+1)-dimensional fractional potential-YTSF equation. The study is structured as follows. Section 2 analyzes the properties of the conformable fractional order and reduces the (3+1)-dimensional conformable fractional equation to integer order. Section 3 obtains the Nth order breather solutions based on the KP hierarchy reduction. Section 4 investigates the interaction solutions taking the long-wave limit technique. Section 5 analyses the images of solutions in Sections 3 and 4. Section 6 provides the conclusion.

2 Fractional differential models and Hirota bilinear form

In this study, we focus on the following (3+1)-dimensional fractional potential YTSF-like model which reads [36]

$$-4D_{xt}^{2\beta}\varphi + D_{xxxz}^{4\beta}\varphi + 4D_x^{\beta}\varphi D_{xz}^{2\beta}\varphi + 2D_{xx}^{2\beta}\varphi D_z^{\beta}\varphi + 3D_{yy}^{2\beta}\varphi = 0, \quad (3)$$

where $D_x^{\beta}, D_y^{\beta}, D_z^{\beta}$, and D_t^{β}, order β is a fractional order derivative, $0 < \beta \le 1$. $\varphi(x, y, z, t)$ is a differentiable function on four independent variables. x, y, z, and t are independent variables under order β. This model exists in other areas of physics including dispersion relations, plasmas, fluid dynamics, and other areas of physics. A variety of effective methods exist for solving the (3+1)-dimensional YTSF equation, including the extended homoclinic test technique [37], the homoclinic assay, the G'/G-extension method [38], the variational methods based on two-scale fractal complex transforms and variational principles [39], the three-wave method [40], and the modulated phase shift method [41]. In 2014, Khalil and his team initially proposed a novel fractional derivative, designated the "Conformable Derivative" [42]. In Eq. (3), $D^{2\beta}_{xt}$ being a conformable fractional derivative of order 2β when independent variables x, t are order of 2β. The same reasoning leads to $D^{4\beta}_{xz}$ being a conformable fractional derivative of order 4β when x, z are order of 4β. Unlike classical integer derivative calculus, conformable fractional calculus and conformable fractional derivatives can simulate nonlinear phenomena on the nonsmooth boundaries as well as diffusion phenomenon or heat transmission phenomenon in porous media.

Definition. Assume $g:(0,\infty)\to R$ is a function. Conformable fractional derivative is defined as follows when $t>0$ and $0<\alpha\le 1$:

$$D_t^{\alpha}x(t)=\lim_{\varepsilon\to 0}\frac{x(t+\varepsilon t^{1-\alpha})-x(t)}{\varepsilon}. \tag{4}$$

A few properties of conformable fractional derivatives are listed as follows [36].

Assume $\alpha\in(0,1]$ and x and y are α-order differentiable for any independent variable $t>0$. Then

(i) $D_t^{\alpha}(a_1x+a_2y)=a_1D_t^{\alpha}x+a_2D_t^{\alpha}y,\ a_1,a_2\in R$.
(ii) $D_t^{\alpha}t^r=rt^{r-\alpha}$ for any $r\in R$.
(iii) $D_t^{\alpha}\lambda=0$, for any constant value functions $x(t)=\lambda$.
(iv) $D_t^{\alpha}xy=xD_t^{\alpha}y+yD_t^{\alpha}x$.
(v) $D_t^{\alpha}\frac{x}{y}=\frac{xD_t^{\alpha}y-yD_t^{\alpha}x}{y^2}$.
(vi) If x in this case can be differential, then $D_t^{\alpha}x(t)=t^{1-\alpha}\frac{\mathrm{d}x}{\mathrm{d}t}$.

By using certain commutative methods $\varphi(x,y,z,t)=\psi(\frac{x^{\beta}}{\beta},\frac{y^{\beta}}{\beta},\frac{z^{\beta}}{\beta},\frac{t^{\beta}}{\beta})=\psi(X,Y,Z,\tau)$, and it is used directly to obtain

$$-4D_{X\tau}^2\psi+D_{XXXZ}^4\psi+4D_X\psi D_{XZ}^2\psi+2D_{XX}^2\psi D_Z\psi+3D_{YY}^2\psi=0. \tag{5}$$

Using the transformation $\xi=X+\varpi Z$ in Eq. (5), we obtain

$$-4\psi_{\xi\tau}+\varpi\psi_{\xi\xi\xi\xi}+6\psi_{\xi}\psi_{\xi\xi}+3\psi_{YY}=0. \tag{6}$$

Through the transformation

$$\psi=2(\ln f)_{\xi}+\psi_0, \tag{7}$$

substitute Eq. (7) in Eq. (6) to obtain its bilinear form

$$(3D_Y^2+\varpi D_{\xi}^4-4D_{\xi}D_{\tau})f\cdot f=0, \tag{8}$$

where f is a real function, D_{ξ}, D_Y, and D_{τ} are the differential bilinear Hirota operators defined by

$$D_{\xi}^{k_1}D_Y^{k_2}D_{\tau}^{k_3}(A\cdot B)=\left(\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi'}\right)^{k_1}\left(\frac{\partial}{\partial Y}-\frac{\partial}{\partial Y'}\right)^{k_2}\left(\frac{\partial}{\partial\tau}-\frac{\partial}{\partial\tau'}\right)^{k_3} A(\xi,Y,\tau)\cdot B(\xi',Y',\tau')|_{\xi'=\xi,Y'=Y,\tau'=\tau},$$

where A is a function of ξ, Y, and τ; B is a function of ξ', Y', and τ and k_1, k_2, and k_3 are non-negative integers.

3 Mth order breather solutions for Eq. (5)

We start with the KP hierarchy of deterministic solutions and use the Gramian equation to deduce a bilinear form of the breathing solution as listed below:

$$(D_{x_1}^4-4D_{x_1}D_{x_3}+3D_{x_2}^2)\phi\cdot\phi=0, \tag{9}$$

then the following solution can be obtained in terms of Gramian

$$\phi=\det_{1\le r,j\le N}(m_{r,j}^{(n)}) \tag{10}$$

and the elements of the matrix $m_{r,j}^{(n)}$ satisfy the following conditions [43]:

$$\partial_{x_1}m_{r,j}^{(n)}=\Psi_r^{(n)}\Phi_j^{(n)}, \tag{11}$$

$$\partial_{x_2}m_{r,j}^{(n)}=\Psi_r^{(n+1)}\Phi_j^{(n)}+\Psi_r^{(n)}\Phi_j^{(n-1)}, \tag{12}$$

$$\partial_{x_3}m_{r,j}^{(n)}=\Psi_r^{(n+2)}\Phi_j^{(n)}+\Psi_r^{(n+1)}\Phi_j^{(n-1)}+\Psi_r^{(n)}\Phi_j^{(n-2)}, \tag{13}$$

$$m_{r,j}^{(n+1)}=m_{r,j}^{(n)}+\Psi_r^{(n)}\Phi_j^{(n)}, \tag{14}$$

$$\partial_{x_i}\Psi_r^{(n)}=\Psi_r^{(n+i)},\ (i=1,2,3), \tag{15}$$

$$\partial_{x_i}\Phi_j^{(n)}=-\Phi_j^{(n-i)},\ (i=1,2,3), \tag{16}$$

where r and $j\in Z_+$, Z_+ is the set of positive integers. And $\Psi_r^{(n)}$ and $\Phi_j^{(n)}$ satisfy

$$m_{r,j}^{(n)}=\delta_{rj}+(p_r+q_j)^{-1}\Psi_r^{(n)}\Phi_j^{(n)}, \tag{17}$$

$$\Psi_r^{(n)}=p_r^n(p_r+q_j)e^{\xi_r}, \tag{18}$$

$$\Phi_j^{(n)}=(-q_j)^{-n}e^{\eta_j}, \tag{19}$$

$$\xi_r=p_rx_1+p_r^2x_2+p_r^3x_3+\xi_{r0}, \tag{20}$$

$$\eta_j=q_jx_1-q_j^2x_2+q_j^3x_3+\eta_{j0}, \tag{21}$$

where δ_{rj} is the Kronecker delta notation. Then, compare Eqs (8) and (9), and make $f=\phi$, it can be obtained immediately that

$$x_1=i\xi,\ x_2=i\sqrt{-\varpi}\,Y,\ x_3=-i\varpi\tau. \tag{22}$$

Theorem 1. *The breather solution which in bilinear form* (8) *in the Gramian forms is like*

$$f = \left| \delta_{rj} + \frac{p_r + q_r}{p_r + q_j} e^{\lambda_r + \eta_j} \right|_{1 \le r, j \le N}, \tag{23}$$

with

$$\lambda_r = ip_r\xi + ip_r^2\sqrt{-\varpi}\,Y - ip_r^3\varpi\tau + \lambda_r^0,$$
$$\eta_j = iq_j\xi - iq_j^2\sqrt{-\varpi}\,Y - iq_j^3\varpi\tau + \eta_j^0,$$
$$p_{2k-1} = -\Omega_k + \frac{\theta_k}{2},\; p_{2k} = -\Omega_k^* - \frac{\theta_k}{2},\; \lambda_{2k}^{0\,*} = \lambda_{2k-1}^0,$$
$$q_{2k-1} = \Omega_k + \frac{\theta_k}{2},\; q_{2k} = \Omega_k^* - \frac{\theta_k}{2},\; \eta_{2k}^{0\,*} = \eta_{2k-1}^0,$$

where $k, r,$ *and* $j \in Z_+$; N *is a positive even number*; $\Omega_k, p_r, q_j, \lambda_r^0,$ *and* η_j^0 *are plural and* θ_k *are real numbers.*

Theorem 2. *Equation* (7) *has a Mth order breather solutions by Theorem 1 of the form*

$$\psi = 2(\ln f)_\xi + \psi_0, \tag{24}$$

$$f = \Xi \begin{vmatrix} \Lambda_{11} & \Lambda_{12} & \dots & \Lambda_{1,M} \\ \Lambda_{21} & \Lambda_{22} & \dots & \Lambda_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{M,1} & \Lambda_{M,2} & \dots & \Lambda_{M,M} \end{vmatrix}, \tag{25}$$

with

$$\Lambda_{k,k} = \begin{pmatrix} \frac{1}{\theta_k e^{\varsigma_k}} + \frac{1}{\theta_k} & \frac{1}{\Omega_k^* - \Omega_k} \\ \frac{1}{\Omega_k^* - \Omega_k} & \frac{1}{\theta_k e^{\varsigma_k^*}} + \frac{1}{\theta_k} \end{pmatrix},$$

$$\Lambda_{k,l} = \begin{pmatrix} \frac{1}{\Omega_l - \Omega_k + \frac{\theta_k + \theta_l}{2}} & \frac{1}{\Omega_l^* - \Omega_k + \frac{\theta_k - \theta_l}{2}} \\ \frac{1}{\Omega_k^* - \Omega_l + \frac{\theta_k - \theta_l}{2}} & \frac{1}{\Omega_k^* - \Omega_l^* + \frac{\theta_k + \theta_l}{2}} \end{pmatrix} (k \ne l),$$

$$\varsigma_k = i\theta_k\xi - 2i\theta_k\Omega_k\sqrt{-\varpi}\,Y - i\theta_k\left(3\Omega_k^2 + \frac{\theta_k^2}{4}\right)\varpi\tau + \varsigma_k^0,$$

$$\Xi = e^{\sum_{k=1}^{M}\varsigma_k + \varsigma_k^*}\Pi_{k=1}^{M}\theta_k^2,$$

where $M = N/2, k,$ *and* $l \in Z_+$, *and* $\varsigma_k^0 = \lambda_k^0 + \eta_k^0$. *Because the denominator should not be zero,* Ω_k *is not real. Since* $2(\ln f)_\xi = (2f_\xi/f)$ *and* Ξ *is a function with no dependence on x, so* Ξ *can be about to be dropped.*

Let $M = 1$ in Eq. (25), to obtain the first-order breather solution

$$f = 1 + e^{\varsigma_1} + e^{\varsigma_1^*} + \left[1 - \left(\frac{\theta_1}{\Omega_1^* - \Omega_1}\right)^2\right] e^{\varsigma_1 + \varsigma_1^*}, \tag{26}$$

where $\varsigma_1 = i\theta_1\xi - 2i\theta_1\Omega_1\sqrt{-\varpi}\,Y - i\theta_1(3\Omega_1^2 + \theta_1^2/4)\varpi\tau + \varsigma_1^0$.

Let $M = 2$ in Eq. (25), to obtain the second-order breather solution

$$f = \begin{vmatrix} \frac{1}{\theta_1 e^{\varsigma_1}} + \frac{1}{\theta_1} & \frac{1}{\Omega_1^* - \Omega_1} & \frac{1}{\frac{\theta_1+\theta_2}{2} + \Omega_2 - \Omega_1} & \frac{1}{\frac{\theta_1-\theta_2}{2} + \Omega_2^* - \Omega_1} \\ \frac{1}{\Omega_1^* - \Omega_1} & \frac{1}{\theta_1 e^{\varsigma_1^*}} + \frac{1}{\theta_1} & \frac{1}{\frac{\theta_1-\theta_2}{2} + \Omega_1^* - \Omega_2} & \frac{1}{\frac{\theta_1+\theta_2}{2} + \Omega_1^* - \Omega_2^*} \\ \frac{1}{\frac{\theta_1+\theta_2}{2} + \Omega_1 - \Omega_2} & \frac{1}{\frac{\theta_2-\theta_1}{2} + \Omega_1^* - \Omega_2} & \frac{1}{\theta_2 e^{\varsigma_2}} + \frac{1}{\theta_2} & \frac{1}{\Omega_2^* - \Omega_2} \\ \frac{1}{\frac{\theta_2-\theta_1}{2} + \Omega_2^* - \Omega_1} & \frac{1}{\frac{\theta_1+\theta_2}{2} + \Omega_2^* - \Omega_1^*} & \frac{1}{\Omega_2^* - \Omega_2} & \frac{1}{\theta_2 e^{\varsigma_2^*}} + \frac{1}{\theta_2} \end{vmatrix}, \tag{27}$$

where

$$\varsigma_1 = i\theta_1\xi - 2i\theta_1\Omega_1\sqrt{-\varpi}\,Y - i\theta_1\left(3\Omega_1^2 + \frac{\theta_1^2}{4}\right)\varpi\tau + \varsigma_1^0,$$

$$\varsigma_2 = i\theta_2\xi - 2i\theta_2\Omega_2\sqrt{-\varpi}\,Y - i\theta_2\left(3\Omega_2^2 + \frac{\theta_2^2}{4}\right)\varpi\tau + \varsigma_2^0.$$

4 Interaction solutions

Theorem 3. *Let* $N = \tilde{M} + \tilde{M}'$ *and making* $\theta_k \to 0$, *which keeps* $e^{\varsigma_k^0} = -1$ *by the way. Then, obtain the* $\tilde{M}$*-order lumps and* $\tilde{M}'$*-order breathers solutions of the form*

$$f = \begin{vmatrix} A_{\tilde{M}\times\tilde{M}} & B_{\tilde{M}\times\tilde{M}} \\ C_{\tilde{M}\times\tilde{M}} & D_{\tilde{M}\times\tilde{M}} \end{vmatrix}, \tag{28}$$

where $\tilde{M}$ and $\tilde{M}' \in Z_+$.

$$A_{k,k} = \begin{pmatrix} \mu_k & \frac{1}{\Omega_k^* - \Omega_k} \\ \frac{1}{\Omega_k^* - \Omega_k} & \mu_k^* \end{pmatrix},$$

$$A_{k,l}=\begin{pmatrix}\dfrac{1}{\Omega_l-\Omega_k} & \dfrac{1}{\Omega_l^*-\Omega_k}\\ \dfrac{1}{\Omega_k^*-\Omega_l} & \dfrac{1}{\Omega_k^*-\Omega_l^*}\end{pmatrix},$$

$$B_{k,l}=\begin{pmatrix}\dfrac{1}{\Omega_l-\Omega_k+\frac{\theta_l}{2}} & \dfrac{1}{\Omega_l^*-\Omega_k-\frac{\theta_l}{2}}\\ \dfrac{1}{\Omega_k^*-\Omega_l-\frac{\theta_l}{2}} & \dfrac{1}{\Omega_k^*-\Omega_l^*+\frac{\theta_l}{2}}\end{pmatrix},$$

$$C_{k,l}=\begin{pmatrix}\dfrac{1}{\Omega_l-\Omega_k+\frac{\theta_k}{2}} & \dfrac{1}{\Omega_l^*-\Omega_k+\frac{\theta_k}{2}}\\ \dfrac{1}{\Omega_k^*-\Omega_l+\frac{\theta_k}{2}} & \dfrac{1}{\Omega_k^*-\Omega_l^*+\frac{\theta_k}{2}}\end{pmatrix},$$

$$D_{k,k}=\begin{pmatrix}\dfrac{1}{\theta_k+e^{\varsigma_k}}+\dfrac{1}{\theta_k} & \dfrac{1}{\Omega_k^*-\Omega_k}\\ \dfrac{1}{\Omega_k^*-\Omega_k} & \dfrac{1}{\theta_k+e^{\varsigma_k^*}}+\dfrac{1}{\theta_k}\end{pmatrix},$$

$$D_{k,l}=\begin{pmatrix}\dfrac{1}{\Omega_l-\Omega_k+\frac{\theta_k+\theta_l}{2}} & \dfrac{1}{\Omega_l^*-\Omega_k+\frac{\theta_k-\theta_l}{2}}\\ \dfrac{1}{\Omega_k^*-\Omega_l-\frac{\theta_k+\theta_l}{2}} & \dfrac{1}{\Omega_k^*-\Omega_l^*+\frac{\theta_k+\theta_l}{2}}\end{pmatrix},$$

where

$$\mu_k = i\xi - 2i\Omega_k\sqrt{-\varpi}\,Y - 3i\Omega_k^2\varpi\tau,$$

$$\varsigma_k = i\theta_k\xi - 2i\theta_k\Omega_k\sqrt{-\varpi}\,Y - i\theta_k\left(3\Omega_k^2+\frac{\theta_k^2}{4}\right)\varpi\tau + \varsigma_k^0.$$

Let $\tilde{M} = \tilde{M}' = 1$, then the solution of Eq. (5) in the form of a first-order breather and one lump will be obtained

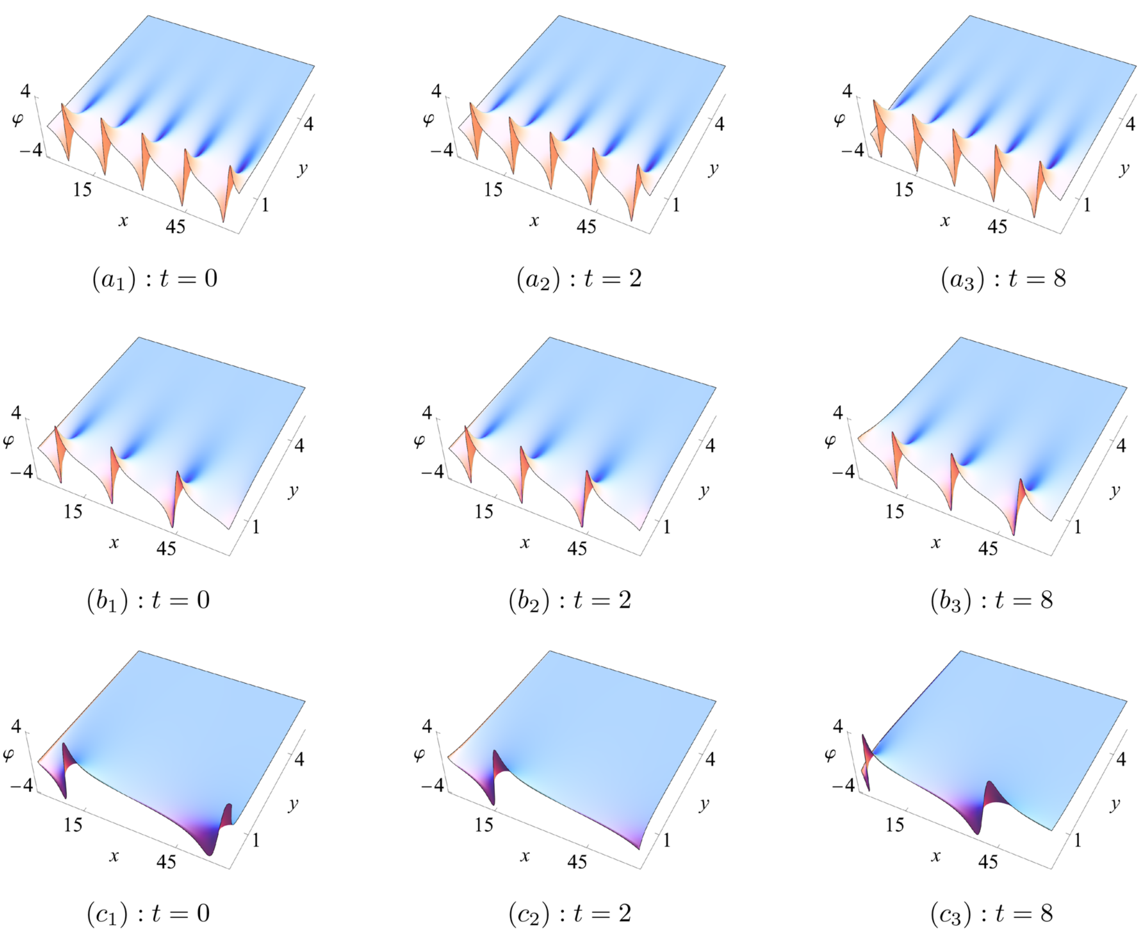

Figure 1: The impact of different values of β on the results of the first-order breather with $\varsigma_1^0 = 0, \theta = \frac{1}{2}, \varpi = -2, \Omega_1 = i$, and $z = 0$. (a) $\beta = 1$, (b) $\beta = \frac{7}{8}$, (c) $\beta = \frac{3}{5}$.

$$f = \begin{vmatrix} \mu_1 & \dfrac{1}{\Omega_1^* - \Omega_1} & \dfrac{1}{\Omega_2 - \Omega_1 + \frac{\theta_2}{2}} & \dfrac{1}{\Omega_2^* - \Omega_1 - \frac{\theta_2}{2}} \\ \dfrac{1}{\Omega_1^* - \Omega_1} & \mu_1^* & \dfrac{1}{\Omega_1^* - \Omega_2 - \frac{\theta_2}{2}} & \dfrac{1}{\Omega_1^* - \Omega_2^* + \frac{\theta_2}{2}} \\ \dfrac{1}{\Omega_1 - \Omega_2 + \frac{\theta_2}{2}} & \dfrac{1}{\Omega_1^* - \Omega_2 + \frac{\theta_2}{2}} & \dfrac{1}{\theta_2 e^{\varsigma_2}} + \dfrac{1}{\theta_2} & \dfrac{1}{\Omega_2^* - \Omega_2} \\ \dfrac{1}{\Omega_2^* - \Omega_1 + \frac{\theta_2}{2}} & \dfrac{1}{\Omega_2^* - \Omega_1^* + \frac{\theta_2}{2}} & \dfrac{1}{\Omega_2^* - \Omega_2} & \dfrac{1}{\theta_2 e^{\varsigma_2^*}} + \dfrac{1}{\theta_2} \end{vmatrix}, \tag{29}$$

where

$$\mu_1 = i\xi - 2i\Omega_1\sqrt{-\varpi}\,Y - 3i\Omega_1^2\varpi\tau,$$
$$\varsigma_2 = i\theta_2\xi - 2i\theta_2\Omega_2\sqrt{-\varpi}\,Y - i\theta_2\left(3\Omega_2^2 + \frac{\theta_2^2}{4}\right)\varpi\tau + \varsigma_2^0.$$

5 Discussion

Figure 1 presents the dissemination of the first-order breather within the $x - y$ plane under the influence of different fractional-order derivative. In each interval, the breathers only have a peak and a trough. Figure 2 shows

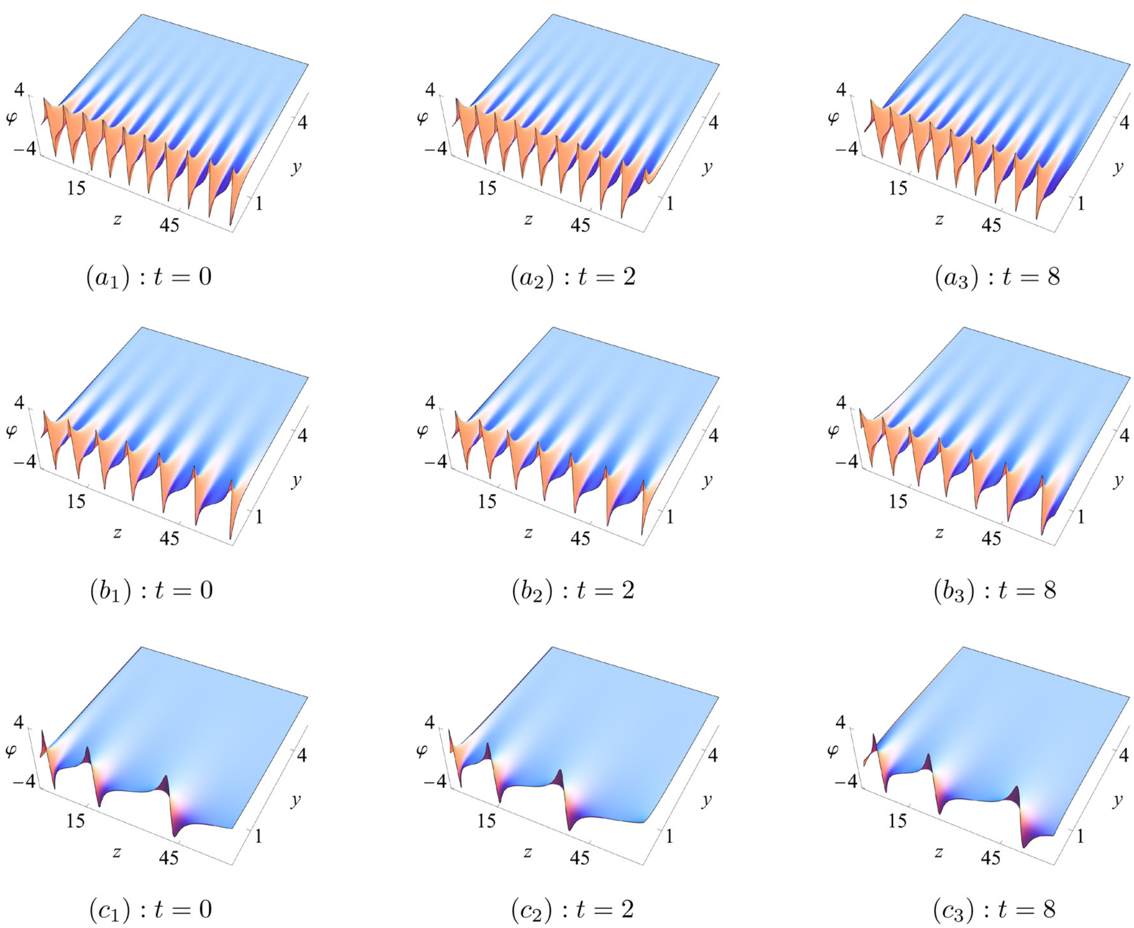

Figure 2: The impact of different values of β on the results of the first-order breather with $\varsigma_1^0 = 0$, $\theta = \frac{1}{2}$, $\varpi = -2$, $\Omega_1 = i$, and $x = 0$. (a) $\beta = 1$, (b) $\beta = \frac{7}{8}$, (c) $\beta = \frac{3}{5}$.

the dissemination of the first-order breather within the $z-y$ plane under the influence of different fractional-order derivative. Similar to the integer-order case, the breather propagates along the x-axis with no movement along the y-axis. The situation within the $z-y$ plane is similar to the situation within the $x-y$ plane. The further from the origin, the smaller the value of β, the longer the interval between reappearances of the breather. Since the image of the wave is continuous and does not change traits so the part where $t<0$ can be easily inferred. Figure 3 presents the dissemination of the first-order breather within the $x-z$ plane under the influence of different fractional-order derivative. The breather within the $x-z$ plane appear periodically like soliton. Similar to Figures 1 and 2, the value of β and the distance to the origin influence the interval between reappearances of the breather. Since the image of the wave is continuous and does not change traits, so the part where $t<0$ can be easily inferred. Figure 4 presents the first-order breather within the $x-y$ and $z-y$ planes. It can be seen as the case of integer order, but the case of fractional order like $x<0$ or $z<0$ or $t<0$ needs to be discarded. Comparing Figure 4 with Figures 1–3, it can be observed that the movement of the breathers is very similar, with only significant differences in the intervals.

Figure 5 presents the two breathers interaction within the $x-y$ plane. Although only half of the breather propagating around the x-axis can be observed, starting from $\tau=0$, the breather propagating around the y-axis can be gradually observed. For each interval, the breathers all have a peak and a trough during the interaction process, and the effect of β is similar to that in Figures 1 and 2. It also shows that two breathers holds their shapes, which indicates that interaction is elastic, allowing either to expand to the other side of the image when $t<0$ through these images, or to compare it directly with the integer order part of $\tau<0$. Figure 6 presents the case of integer order, how the two breathers interact with each other for τ from −3 to 8. Upon comparing Figure 6 with Figure 5, it can

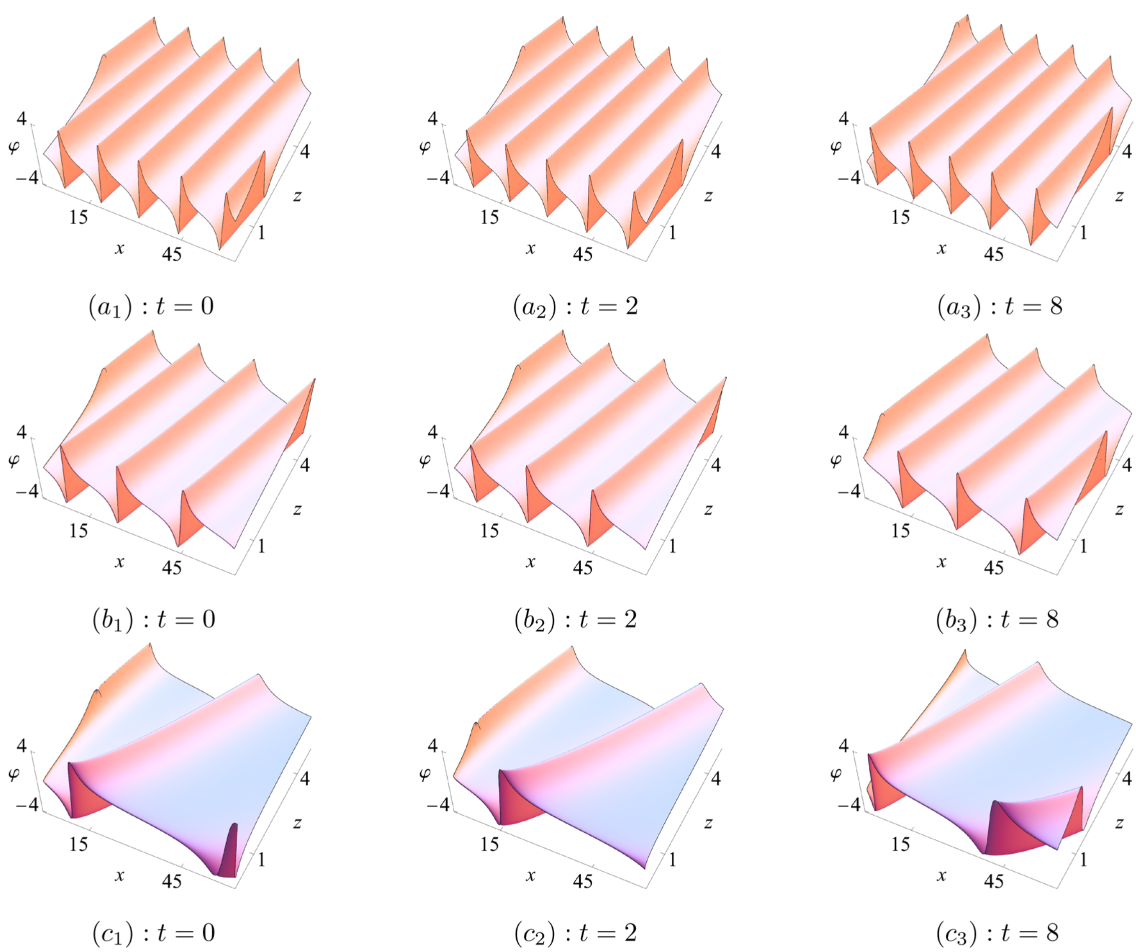

Figure 3: The impact of different values of β on the results of the first-order breather with $\varsigma_1^0=0$, $\theta=\frac{1}{2}$, $\varpi=-2$, $\Omega_1=i$, and $y=0$. (a) $\beta=1$, (b) $\beta=\frac{7}{8}$, (c) $\beta=\frac{3}{5}$.

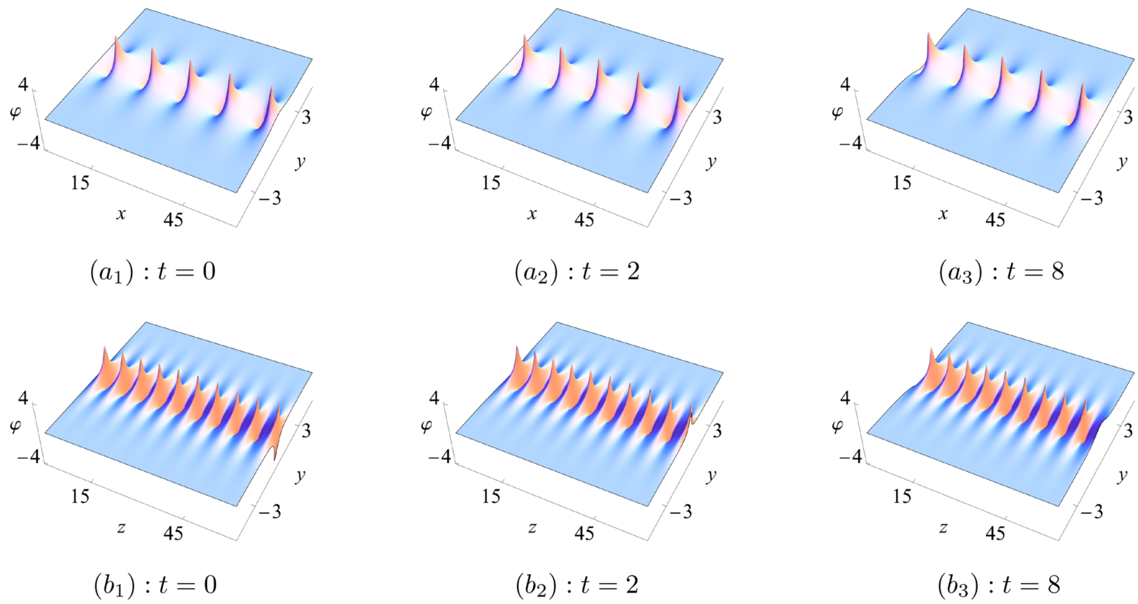

Figure 4: The first-order breather on the results of the first-order breather with $\varsigma_1^0 = 0, \theta = \frac{1}{2}, \beta = 1, \varpi = -2$, and $\Omega_1 = i$. (a) $z = 0$, (b) $x = 0$.

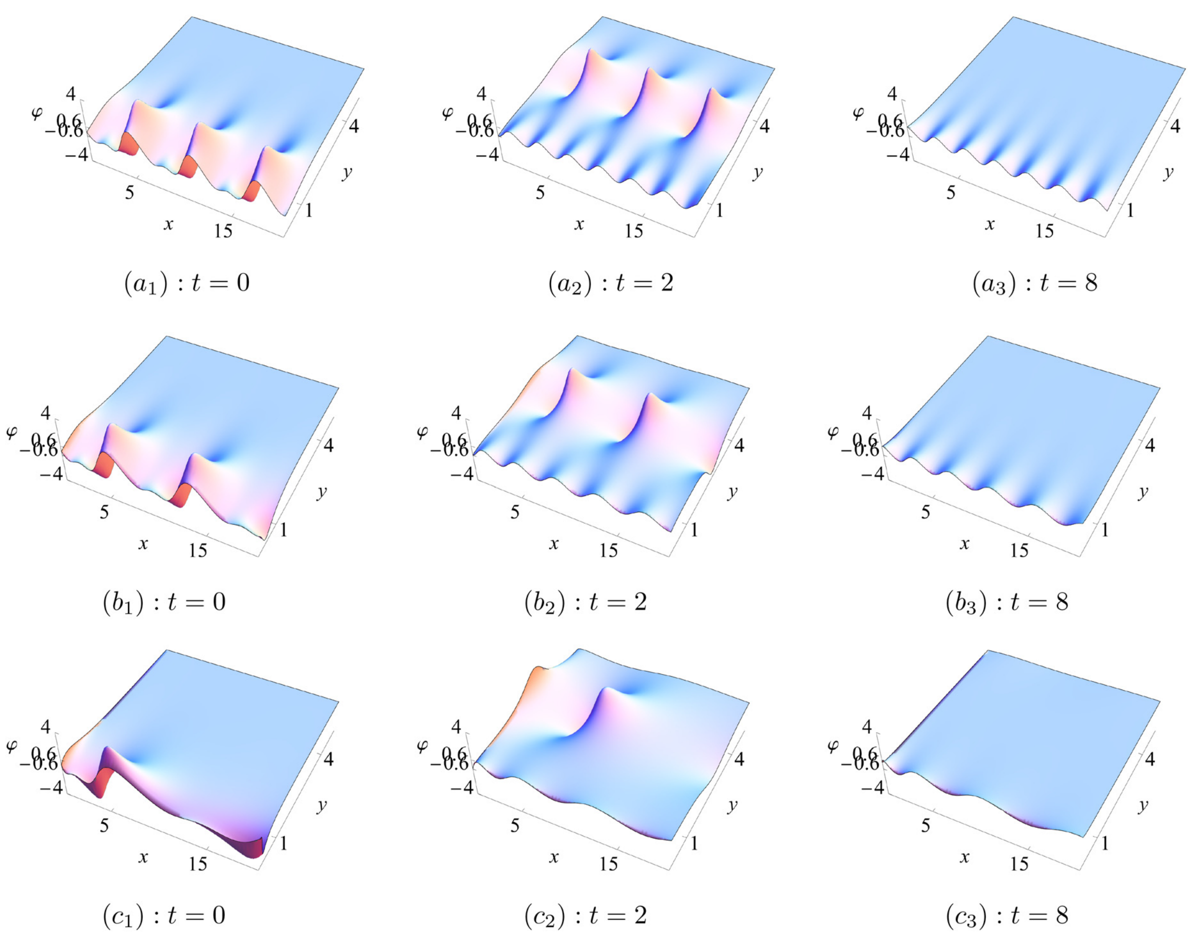

Figure 5: The impact of different values of β on the results of two breathers with $\varsigma_1^0 = 0, \varsigma_2^0 = 0, \theta_1 = 2, \theta_2 = 1, \varpi = -1, \Omega_1 = -\frac{i}{2}, \Omega_2 = \frac{1}{3} + i$, and $z = 0$. (a) $\beta = 1$, (b) $\beta = \frac{7}{8}$, (c) $\beta = \frac{3}{5}$.

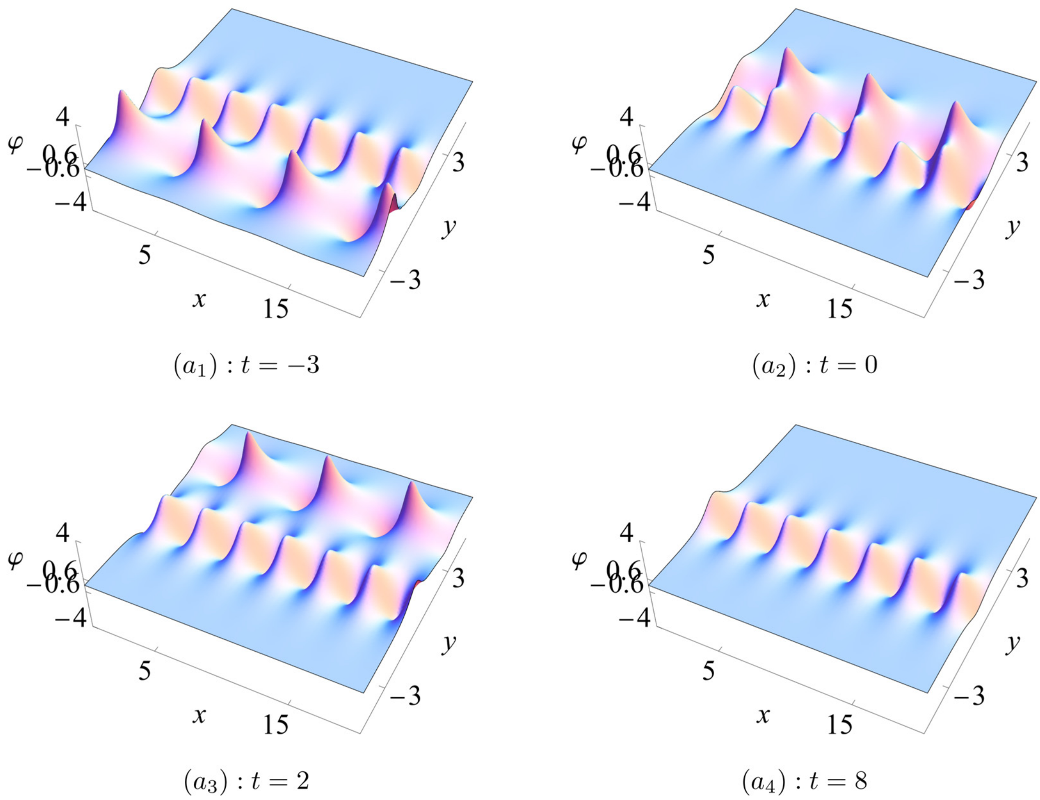

Figure 6: The first-order breather on the results of two breathers with $\varsigma_1^0 = 0$, $\varsigma_2^0 = 0$, $\theta_1 = 2$, $\theta_2 = 1$, $\varpi = -1$, $\Omega_1 = -\frac{i}{2}$, $\Omega_2 = \frac{1}{3} + i$, $\beta = 1$, and $z = 0$.

be noted that if we ignore the case when $\tau < 0$, the situation is still similar to Figures 1–4, but harder to recognize when $\tau = 0$.

Figure 7 presents the interactions solutions of a first-order breather and one lump within the $x - y$ plane, while the breather propagates in y directions and the lumps propagate at different speeds. In lump solutions, β has a similar effect as β in the breather solutions. Such a interaction is obviously elastic, so allowing either to compare it directly with the case of integer orders, or to expand the other side of the image when $t < 0$ from the $\tau > 0$ part; however, the second method is somewhat unintuitive. Figure 8 presents the interactions in the case of integer order, for τ from −3 to 8, the interaction of the lumps and the breathers can be observed. The shapes of the lumps and the breathers do not change before contact and after separation.

6 Conclusions

In this work, the (3+1)-dimensional fractional potential-YTSF-like model is studied. The transformation of Eq. (5) was utilized to effect the transformation of the (3+1)-dimensional fractional potential-YTSF-like equation into the (3+1)-dimensional potential-YTSF Eq. (6), thus resulting in the subsequent acquisition of its Hirota bilinear form. By using the KP hierarchy reduction, we have obtained the breather solutions in terms of Gramian and the N-order interaction solutions. From Figures 1–4, it can be observed that a first-order breather exhibits a single peak and trough. Figures 5 and 6 illustrate the interaction between two breathers that are propagated, respectively, along the x-axis and the y-axis. The long-wave limit technique is used to solve solution Eq. (24), which introduces the interaction solution Eq. (28). Figures 7 and 8 illustrate the interaction of the lumps and breathers, which undergo a shape change before contact and after separation. This indicates that the interaction is elastic. It is our hope that these results will contribute to the further exploration of phenomena in physical systems. The propagation behavior is found to be nonlinear and complex for different conformable fractional order parameters and amplitudes.

The wave solution of fractional order can be seen as a modified result of the potential-YTSF equation. The Gramian solution is distinguished from the conventional

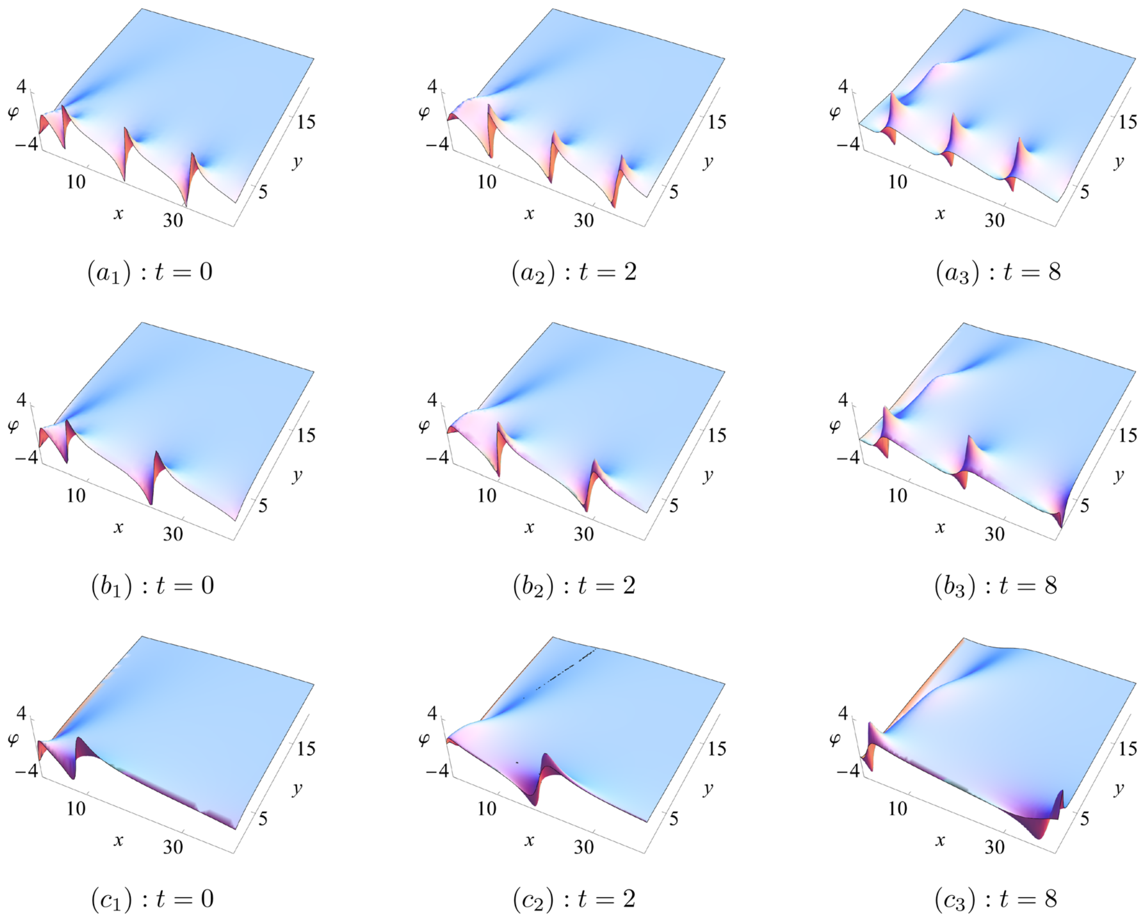

Figure 7: The impact of various values of β on the results of a first-order breather and one lump with $\varsigma_2^0 = 0, \theta = \frac{1}{2}, \varpi = -\frac{1}{2}, \Omega_1 = \frac{1}{2} + \frac{i}{2}, \Omega_2 = \frac{1}{10} - i$, and $z = 0$. (a) $\beta = 1$, (b) $\beta = \frac{7}{8}$, (c) $\beta = \frac{3}{5}$.

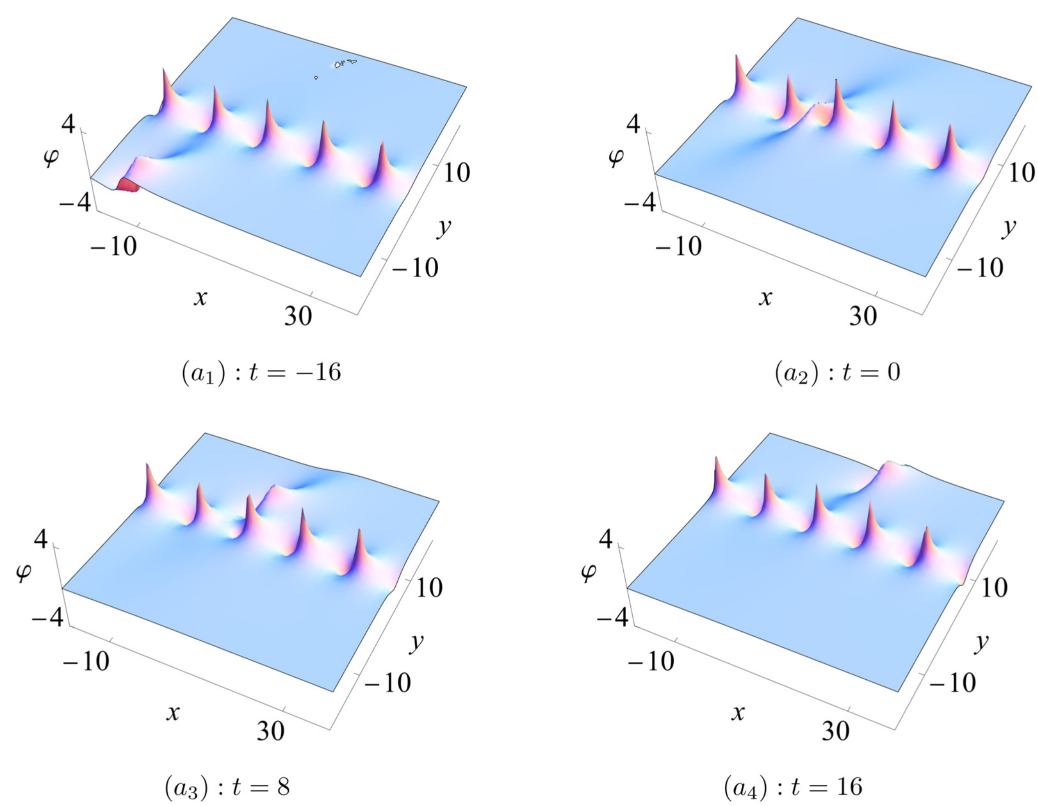

Figure 8: The first-order breather on the results of a first-order breather and one lump with $\varsigma_2^0 = 0, \theta = \frac{1}{2}, \varpi = -\frac{1}{2}, \Omega_1 = \frac{1}{2} + \frac{i}{2}, \Omega_2 = \frac{1}{10} - i, \beta = 1$, and $z = 0$.

travelling wave solution in that the x, y, and t obtained in terms of Gramian are independent of each other. The study of the Gramian solution can facilitate the design process in related industries. Complex propagation behavior which is nonlinear is also able to be observed for different fractional dimensions and amplitudes. The effects of these variables on the structure of the solution are different, and they are better at simulating water waves in real situations. The objective of this study is to contribute to the advancement of research in the field of physical systems. However, some of the other soliton type solution methods for this type of fractional order equations where other soliton-type solution methods are applicable do not allow the use of Gramian methods. We will be looking at the additional new solutions to the fractional order-YTSF equation as well as applying the variational method to other equations with different fractional order definitions.

Funding information: This work was supported by the National Natural Science Foundation of China (Approval Nos. 12062018, 12172333), Program for Young Talents of Science and Technology in Universities of Inner Mongolia Autonomous Region (Approval No. NJYT22075).

Author contributions: Wenyuan Li: writing – original draft preparation, formal analysis and investigation, writing – review and editing. Nongsen Li: methodology, software. Ruigang Zhang: writing – review and editing, funding acquisition, methodology. Yilin Tian: methodology, software. Jifeng Cui: supervision, writing – review and editing, funding acquisition. All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: Data sharing is not applicable to this article as no new data were created or analyzed in this study.

References

[1] Wang Y, Han X, Guo D, Lu L, Chen Y, Ouyang M. Physics-informed recurrent neural networks with fractional-order constraints for the state estimation of lithium-ion batteries. Batteries. 2022;8(10):148.
[2] Seadawy AR, Alsaedi BA. Dynamical stricture of optical soliton solutions and variational principle of nonlinear Schrödinger equation with Kerr law nonlinearity. Modern Phys Lett B. 2024;38(28):2450254.
[3] Saadatmandi A, Dehghan M. A new operational matrix for solving fractional-order differential equations. Comput Math Appl. 2010;59(3):1326–36.
[4] Seadawy AR. Stability analysis solutions for nonlinear three-dimensional modified Korteweg-de Vries-Zakharov-Kuznetsov equation in a magnetized electron-positron plasma. Phys A Stat Mech Appl. 2016;455:44–51.
[5] Malik S, Mir AH. Synchronization of fractional order neurons in presence of noise. IEEE/ACM Trans Comput Biol Bioinform. 2020;19(3):1887–96.
[6] Zeid SS. Approximation methods for solving fractional equations. Chaos Solitons Fractals. 2019;125:171–93.
[7] Bagyalakshmi M, Saratha S, Sai Sundara Krishnan G, Yıldırım A. Nonlinear analysis of irregular temperature distribution in a heat exchanger using fractional derivative. J Therm Anal Calorim. 2022;147(23):13769–79.
[8] Mabrouk SM, Wazwaz AM, Rashed AS. Monitoring dynamical behavior and optical solutions of space-time fractional order double-chain deoxyribonucleic acid model considering the Atangana's conformable derivative. J Appl Comput Mech. 2024;10(2):383–91.
[9] Mabrouk S, Rezazadeh H, Ahmad H, Rashed A, Demirbilek U, Gepreel KA. Implementation of optical soliton behavior of the space-time conformable fractional Vakhnenko-Parkes equation and its modified model. Opt Quant Electr. 2024;56(2):222.
[10] Abdou M, Ouahid L, Kumar S. Plenteous specific analytical solutions for new extended deoxyribonucleic acid (DNA) model arising in mathematical biology. Modern Phys Lett B. 2023;37(34):2350173.
[11] Halim A, Kumar BR, Niranjan A, Nigam A, Schneider W, Ahuja CK, et al. A colour image segmentation method and its application to medical images. Signal Image Video Process. 2024;18(2):1635–48.
[12] Li C, Chen A. Numerical methods for fractional partial differential equations. Int J Comput Math. 2018;95(6–7):1048–99.
[13] Ammi MRS, Jamiai I, Torres DF. A finite element approximation for a class of Caputo time-fractional diffusion equations. Comput Math Appl. 2019;78(5):1334–44.
[14] Cen Z, Le A, Xu A. A posteriori error analysis for a fractional differential equation. Int J Comput Math. 2017;94(6):1185–95.
[15] Bayrak MA, Demir A. A new approach for space-time fractional partial differential equations by residual power series method. Appl Math Comput. 2018;336:215–30.
[16] Bruzón M, Gandarias M, Muriel S, Ramíres Kh SS, Romero F. The Calogero-Bogoyavlenskii-Schiff equation in dimension 2+1. Teoret Mat Fiz. 2003;137:14–26.
[17] Yu SJ, Toda K, Sasa N, Fukuyama T. N soliton solutions to the Bogoyavlenskii-Schiff equation and a quest for the soliton solution in (3+1) dimensions. J Phys A Math General. 1998;31(14):3337.
[18] Li N, Liu QP. Smooth multisoliton solutions of a 2-component peakon system with cubic nonlinearity. SIGMA Symmetry Integrability Geom Meth Appl. 2022;18:066.
[19] Zhou HJ, Chen Y. High-order soliton solutions and their dynamics in the inhomogeneous variable coefficients Hirota equation. Commun Nonlinear Sci Numer Simulat. 2023;120:107149.
[20] Zuo DW, Zhang GF. Exact solutions of the nonlocal Hirota equations. Appl Math Lett. 2019;93:66–71.
[21] Kuo CK, Chen YC, Wu CW, Chao WN. Novel solitary and resonant multi-soliton solutions to the (3+1)-dimensional potential-YTSF equation. Modern Phys Lett B. 2021;35(19):2150326.

[22] Manafian J, Ilhan OA, Ali KK, Abid S. Cross-kink wave solutions and semi-inverse variational method for (3+1)-dimensional potential-YTSF equation. East Asian J Appl Math. 2020;10(3):549–65.
[23] Gui-Qiong X, Zhi-Bin L. Applications of Jacobi elliptic function expansion method for nonlinear differential-difference equations. Comm Theoret Phys. 2005;43(3):385.
[24] Butt AR, Huma Z, Jannat N. A novel investigation of dark, bright, and periodic soliton solutions for the Kadomtsev-Petviashvili equation via different techniques. Opt Quantum Electron. 2023;55(2):168.
[25] Tian Y, Cui J, Zhang R. Exact traveling wave solutions of the strain wave and (1+1)-dimensional Benjamin-Bona-Mahony equations via the simplest equation method. Modern Phys Lett B. 2022;36(23):2250103.
[26] Sun Y. Breather and interaction solutions for a (3+1)-dimensional generalized shallow water wave equation. Qualitative Theory Dynam Syst. 2023;22(3):91.
[27] Tajiri M. Some Remarks on Similarity and Soliton Solutions of Nonlinear Klein-Gordon Equation. J Phys Soc Japan. 1984;53(11):3759–64.
[28] Zheng M, Li M. High-order breathers and semi-rational solutions of the (2+1)-dimensional Yu-Toda-Sasa-Fukuyama equation. Modern Phys Lett B. 2021;35(26):2150422.
[29] Wang KJ, Liu JH, Si J, Shi F, Wang GD. N-soliton, breather, lump solutions and diverse traveling wave solutions of the fractional (2+1)-dimensional Boussinesq equation. Fractals. 2023;31(3):2350023.
[30] Ma WX, Yong X, Lü X. Soliton solutions to the B-type Kadomtsev-Petviashvili equation under general dispersion relations. Wave Motion. 2021;103:102719.
[31] Seadawy AR, Ahmad A, Rizvi ST, Ahmed S. Bifurcation solitons, Y-type, distinct lumps and generalized breather in the thermophoretic motion equation via graphene sheets. Alex Eng J. 2024;87:374–88.
[32] Jin-Bo B, Ye-Peng S, Deng-Yuan C. Soliton Solutions for Nonisospectral AKNS Equation by Hirota's Method. Commun Theoret Phys. 2006 Mar;45(3):398. doi: 10.1088/0253-6102/45/3/004.
[33] Peng C, Tang L, Li Z, Chen D. Qualitative analysis of stochastic Schrödinger-Hirota equation in birefringent fibers with spatiotemporal dispersion and parabolic law nonlinearity. Results Phys. 2023;51:106729.
[34] Kudryashov NA. Optical solitons of the Schrödinger-Hirota equation of the fourth order. Optik. 2023;274:170587.
[35] Kumar S, Nisar KS, Niwas M. On the dynamics of exact solutions to a (3+1)-dimensional YTSF equation emerging in shallow sea waves: Lie symmetry analysis and generalized Kudryashov method. Results Phys. 2023;48:106432.
[36] Badshah F, Tariq KU, Wazwaz AM, Mehboob F. Lumps, solitons and stability analysis for the (3+1)-dimensional fractional potential-YTSF-like model in weakly dispersive medium. Phys Scr. 2023;98(12):125263.
[37] Guo Y, Dai Z, Li D. New exact periodic solitary-wave solution of MKdV equation. Commun Nonlinear Sci Numer Simulat. 2009;14(11):3821–4.
[38] Sahoo S, Ray SS, Abdou M. New exact solutions for time-fractional Kaup-Kupershmidt equation using improved (G'/G)-expansion and extended (G'/G)-expansion methods. Alex Eng J. 2020;59(5):3105–10.
[39] Lu J. Application of variational principle and fractal complex transformation to (3+1)-dimensional fractal potential-YTSF equation. Fractals. 2024;32(1):2450027.
[40] Guo Y, Li D, Wang J. The new exact solutions of the Fifth-Order Sawada-Kotera equation using three wave method. Appl Math Lett. 2019;94:232–7.
[41] Chen M, Wang Z. Nonlinear interactions of two-kink-breather solution in Yu-Toda-Sasa-Fukuyama equation by modulated phase shift. Phys Scr. 2023;98(9):095241.
[42] Atangana A, Alqahtani RT. Modelling the spread of river blindness disease via the caputo fractional derivative and the beta-derivative. Entropy. 2016;18(2):40.
[43] Sun Y, Wu XY. Studies on the breather solutions for the (2+1)-dimensional variable-coefficient Kadomtsev-Petviashvili equation in fluids and plasmas. Nonlinear Dynam. 2021;106(3):2485–95.