Research Article

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Solitons and travelling waves structure for M-fractional Kairat-II equation using three explicit methods

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Abstract: Exact solutions of (1+1)-dimensional M-fractional Kairat-II equation are obtained *via* proposed three extended mathematical methods with the help of the computational software Mathematica. This model has many applications in optical fibers, which is used to describe the trajectory of optical pulses in optical fibers. The derived solutions are novel and newer existing in any kind of literature. The constructed solutions are in distinct form, such as trigonometric, hyperbolic, exponential, and rational functions. For the physical phenomena of concern fractional model, some obtained solutions are plotted in two-dimensional and three-dimensional by assigning the specific values to the parameters under the constrain conditions. Moreover, the proposed methods are enormously superbly mathematical tools to review wave solutions of several fractional models in nonlinear science.

Keywords: M-fractional Kairat-II equation, exact solutions, mathematical methods

1 Introduction

The whole world around us is fundamentally nonlinear. Most of the convoluted phenomena in real life such as fluid dynamics mass transfer, the propagation of waves, and evolution of gases in fluid dynamics, which are modelled by partial differential equations (PDEs). The understanding PDEs permit making a much better prediction and much broader applications on nature and life. These advanced models demonstrate to humanity why understanding solving PDEs are so imperative.

Fractional calculus was formulated in 1695, shortly after the enlargement of classical calculus. Fractional calculus is intensely related to the dynamics of intricated real-world problems. The subject of fractional calculus has seemed as influential and proficient mathematical tools during the past six decades, mainly due to its demonstrated applications in plentiful seemingly diverse and widespread fields of science and engineering. Naturally occurring phenomena are expressed in the form of fractional nonlinear PDEs, for example, fractional Bogoyavlensky–Konopelchenko equation [1], fractional Drinfeld–Sokolov equation [2], fractional Kuralay equation [3–7], fractional Zoomeron equation [8], fractional Kadomtsev–Petviashvili equation [9], fractional higher-order Sasa–Satsuma equation [10]. There have been settled sundry methods to solve the fractional nonlinear PDEs, such as Lie symmetry analysis [11], efficient (G/G')-expansion method [12], novel auxiliary equation method [13], novel direct extended algebraic method [14], modified Kudryashov method [15], modified (G/G')-expansion method [16], generalized algebra method [17], and many more methods [18–33].

Shallow water waves, plasma physics, differential geometry physics, and optical fibers are among the numerous physical phenomena that are governed by the nonlinear Kairat model, a prominent evolution equation. The Kairat model is a valuable instrument for describing the propagation of nonlinear waves in a variety of fields, precisely capturing the complex interplay between dispersion and nonlinearity.

Consider the (1+1)-dimensional M-fractional Kairat-II equation given in [34,35]

$$D_{M,\mathrm{xt}}^{2\alpha,\delta}H - 2D_{M,t}^{\alpha,\delta}H(D_{M,\mathrm{xx}}^{2\alpha,\delta}H) - 4D_{M,x}^{\alpha,\delta}H(D_{M,\mathrm{xt}}^{2\alpha,\delta}H) + D_{M,\mathrm{xxxt}}^{4\alpha,\delta}H = 0, \tag{1}$$

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where

$$D_{M,x}^{\alpha,\delta}H(x) = \underset{\sigma\to 0}{\text{Limit}}\left(\frac{H(x\,\mathrm{E}_{\delta}(\sigma x^{1-\alpha}) - H(x))}{\sigma}\right), \alpha \in (0,1], \quad \delta > 0, \tag{2}$$

where $E_\delta(\cdot)$ represents the truncated Mittag–Leffer (TML) function mentioned in previous studies [36,37].

The fractional Kairat-II equation is an integrable equation, and it is used to explain the deferential geometry of curves and equivalence aspects [38]. This equation is also helpful to study the behavior of optical solitons and pulses in nonlinear media, such as optical fibers [39]. This is a fact that very limited work had been done on Eq. (1) in the existing literature. For example, geometrical description of integrable Kairat equations has been discussed in Myrzakulova [34]. Furthermore, three mathematical methods called $\exp_a$ function method, modified simplest equation method, and generalized Kudryashov method have been used to study of Eq. (1) in the study of Awadalla et al. [35]. But here in our work, we have investigated novel exact results of Eq. (1) *via* application of three methods called extended simple equation method [40–46], extended G'/G-expansion method [47,48], and extended $\mathrm{Exp}(-\Psi(\phi))$-expansion [49,50], respectively. Some derived exact solutions were plotted in two-dimensional (2D) and three-dimensional (3D) with the assistance of 12.1 Mathematica software. The investigated solutions have fruitful applications in mathematical physics.

The arrangement of this work is as follows: in Section 2, proposed methods are explained. In Section 3, exact solutions of Eq. (1) are derived. In Section 4, Results and Discussion is explained, and finally, in Section 5, conclusion of the work has been explored.

2 Overview of the integration algorithms

In this section, we describe the algorithm of the three methods for finding the exact solutions to a nonlinear PDE. Consider the nonlinear PDE,

$$R(U, U_t, U_x, U_{xx}, U_{xt}, \ldots) = 0. \tag{3}$$

Let

$$U = U(\xi), \quad \xi = kx - \omega t. \tag{4}$$

Substituting Eq. (3) into Eq. (2),

$$S(U, U', U'', \ldots) = 0. \tag{5}$$

2.1 Extended simple equation method

Let Eq. (5) have solution

$$U(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi). \tag{6}$$

Let Ψ satisfy

$$\Psi' = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3. \tag{7}$$

Substitute Eq. (6) with Eq. (7) into Eq. (5) and solve for Eq. (3).

2.2 Extended (G'/G)-expansion method

Let (5) have solution

$$U = A_0 + \sum_{i=-N}^{N} A_i\left(\frac{G'}{G}\right). \tag{8}$$

Let

$$G'' = -\lambda G' - \mu G. \tag{9}$$

Substitute Eq. (8) with Eq. (9) into Eq. (5) and solve for Eq. (3).

2.3 Extended $\mathrm{Exp}(-\Psi(\phi))$-expansion method

Suppose Eq. (5) has solution

$$U = \sum_{i=-N}^{N} A_i(\mathrm{Exp}(-\Psi(\phi)))^i\,. \tag{10}$$

Let

$$\Psi' = \mathrm{Exp}(-\Psi(\phi)) + \mu\mathrm{Exp}(\Psi(\phi)) + \lambda. \tag{11}$$

Substitute (10) with (11) into (5) and solve for Eq. (3).

3 Applications

Consider

$$H(x,y) = U(x,y), \xi = \frac{(\Gamma(\delta+1))(\eta t^\alpha + \sigma x^\alpha)}{\alpha} + \theta. \tag{12}$$

Substituting Eq. (12) into Eq. (1),

$$\sigma^2 U^{(3)} - 3\sigma(U')^2 + U' = 0. \tag{13}$$

3.1 Application of extended simple equation method

We recover different forms of the solutions for the studied model. Let Eq. (13) have solution

$$U = A_1\Psi + \frac{A_{-1}}{\Psi} + A_0. \tag{14}$$

Substituting (14) with (7) into (13),

Case 1: $c_3 = 0$,

Family-I

$$A_0 = A_0, A_{-1} = 0, A_1 = \frac{c_1^2\sigma^2 + 1}{2c_0\sigma}, c_2 = \frac{c_1^2\sigma^2 + 1}{4c_0\sigma^2}. \tag{15}$$

Substituting Eq. (15) into Eq. (14),

$$U_1 = A_0 - \left(\frac{(c_1^2\sigma^2 + 1)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{4c_2c_0\sigma}\right), \tag{16}$$

$4c_0c_2 > c_1^2$.

Family-II

$$A_0 = A_0, A_{-1} = -2c_0\sigma, A_1 = 0, c_2 = \frac{c_1^2\sigma^2 + 1}{4c_0\sigma^2}. \tag{17}$$

Substituting Eq. (17) into Eq. (14),

$$U_2 = A_0 + \left(\frac{4c_0c_2\sigma}{c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)}\right), \tag{18}$$

$4c_0c_2 > c_1^2$.

Case 2: $c_0 = c_3 = 0$,

$$A_0 = A_0, A_{-1} = 0, A_1 = \frac{2ic_2}{c_1}, \sigma = \frac{i}{c_1}. \tag{19}$$

Substituting Eq. (19) into Eq. (14),

$$U_3 = A_0 + \left(\frac{(2ic_2)(c_1\exp(c_1(\xi + \xi_0)))}{c_1(1 - c_2\exp(c_1(\xi + \xi_0)))}\right), c_1 > 0, \tag{20}$$

$$U_4 = A_0 - \left(\frac{(2ic_2)(c_1\exp(c_1(\xi + \xi_0)))}{c_1(c_2\exp(c_1(\xi + \xi_0)) + 1)}\right), c_1 < 0. \tag{21}$$

Case 3: $c_1 = 0, c_3 = 0$,

Family-I

$$A_0 = A_0, A_{-1} = 0, A_1 = 2c_2\sigma, c_0 = \frac{1}{4c_2\sigma^2}. \tag{22}$$

Substituting Eq. (22) into Eq. (14),

$$U_5 = A_0 + 2\sigma(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0))), c_0c_2 > 0, \tag{23}$$

$$U_6 = A_0 + 2\sigma(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0))), c_0c_2 < 0. \tag{24}$$

Family-II

$$A_0 = A_0, A_{-1} = -\frac{1}{2c_2\sigma}, A_1 = 0, c_0 = \frac{1}{4c_2\sigma^2} \tag{25}$$

Substituting Eq. (27) into Eq. (16),

$$U_7 = A_0 - \left(\frac{1}{\frac{(2c_2\sigma)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0)))}{c_2}}\right), c_0c_2 > 0, \tag{26}$$

$$U_8 = A_0 - \left(\frac{1}{\frac{(2c_2\sigma)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0)))}{c_2}}\right), c_0c_2 < 0. \tag{27}$$

Family-III

$$A_0 = A_0, A_{-1} = -\frac{1}{8c_2\sigma}, A_1 = 2c_2\sigma, c_0 = \frac{1}{16c_2\sigma^2}. \tag{28}$$

Substituting Eq. (28) into Eq. (14),

$$\begin{aligned} U_9 = A_0 &+ \left(\frac{2c_2\sigma(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0)))}{c_2}\right) \\ &- \left(\frac{1}{\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0))}{c_2(8c_2\sigma)}}\right), c_0c_2 > 0, \end{aligned} \tag{29}$$

$$\begin{aligned} U_{10} = A_0 &+ \left(\frac{2c_2\sigma(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0)))}{c_2}\right) \\ &- \left(\frac{1}{\frac{(8c_2\sigma)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0)))}{c_2}}\right), \quad c_0c_2 < 0. \end{aligned} \tag{30}$$

3.2 Application of extended (G'/G)-expansion method

We recover different forms of the solutions for the studied model. Let (3) have solution

$$U = A_0 + A_1\left(\frac{G'}{G}\right) + A_{-1}\left(\frac{G'}{G}\right)^{-1}. \tag{31}$$

Substituting (31) with (9) into (3),

Family-I:

$$A_0 = A_0, A_1 = -2\sigma, A_{-1} = 0, \mu = \frac{\lambda^2\sigma^2 + 1}{4\sigma^2}. \tag{32}$$

Substituting Eq. (32) into Eq. (31).

When $\lambda^2 - 4\mu > 0$,

$$U_{11} = A_0 - 2\sigma\left(\frac{\sqrt{\lambda^2-4\mu}\left(P_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)+P_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)\right)}{2\left(P_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)+P_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)\right)} - \frac{\lambda}{2}\right). \tag{33}$$

When $\lambda^2 - 4\mu < 0$,

$$U_{12} = A_0 - 2\sigma\left(\frac{\sqrt{4\mu-\lambda^2}\left(P_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)-P_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\right)}{2\left(P_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)+P_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\right)} - \frac{\lambda}{2}\right). \tag{34}$$

When $\lambda^2 - 4\mu = 0$,

$$U_{13} = A_0 - 2\sigma\left(\frac{P_2}{\xi P_2 + P_1} - \frac{\lambda}{2}\right). \tag{35}$$

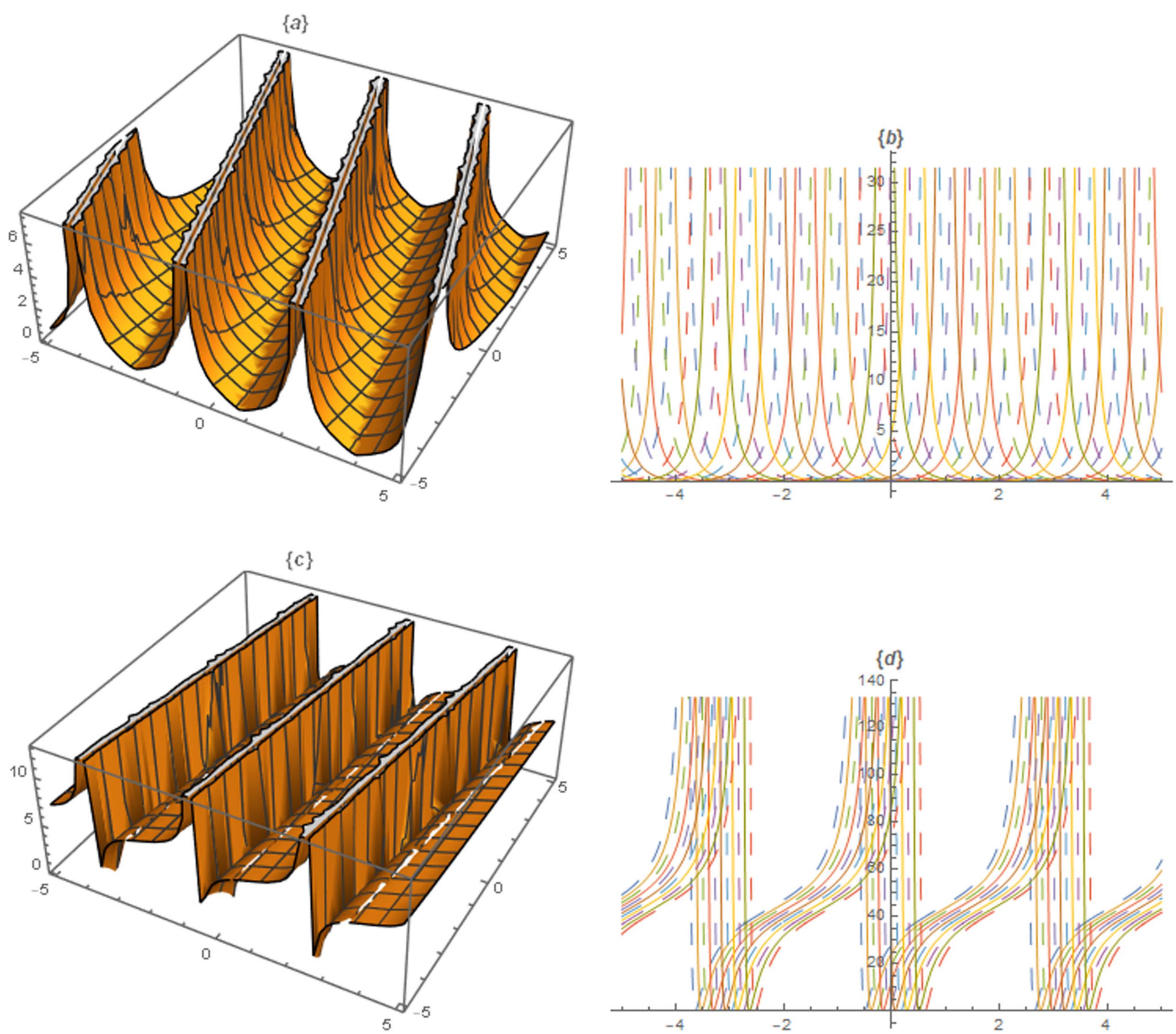

Figure 1: Profile of solutions U_1 (a, b) and U_2 (c, d) are plotted with $c_0 = 1$, $c_1 = 0.03$, $\xi_0 = 2.1$, $\alpha = 1$, $A_0 = -0.1$, $\Gamma = 1$, $\delta = 1$, $\eta = 1$, $\theta = 0.01$, $\sigma = 1.8$, and $\alpha = 1$, $A_0 = 6.1$, $c_0 = 1$, $c_1 = 0.3$, $\Gamma = 1$, $\delta = 1$, $\eta = -0.4$, $\theta = 0.01$, $\xi_0 = 2.1$, and $\sigma = 1.8$, respectively.

Family-II:

$$A_0 = A_0,\ A_1 = 0,\ A_{-1} = \frac{\lambda^2\sigma^2 + 1}{2\sigma},\ \mu = \frac{\lambda^2\sigma^2 + 1}{4\sigma^2}. \tag{36}$$

Substituting Eq. (36) into Eq. (31).

When $\lambda^2 - 4\mu > 0$,

$$U_{14} = A_0 + \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\left(\frac{\sqrt{\lambda^2 - 4\mu}\left(P_1 \sinh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right) + P_2 \cosh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)\right)}{2\left(P_2 \sinh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right) + P_1 \cosh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)\right)} - \frac{\lambda}{2}\right)}\right). \tag{37}$$

When $\lambda^2 - 4\mu < 0$,

$$U_{15} = A_0 - \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\left(\frac{\sqrt{4\mu - \lambda^2}\left(P_2 \cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right) - P_1 \sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\right)}{2\left(P_2 \sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right) + P_1 \cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\right)} - \frac{\lambda}{2}\right)}\right). \tag{38}$$

When $\lambda^2 - 4\mu = 0$,

$$U_{16} = A_0 - \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\left(\frac{P_2}{\xi P_2 + P_1} - \frac{\lambda}{2}\right)}\right). \tag{39}$$

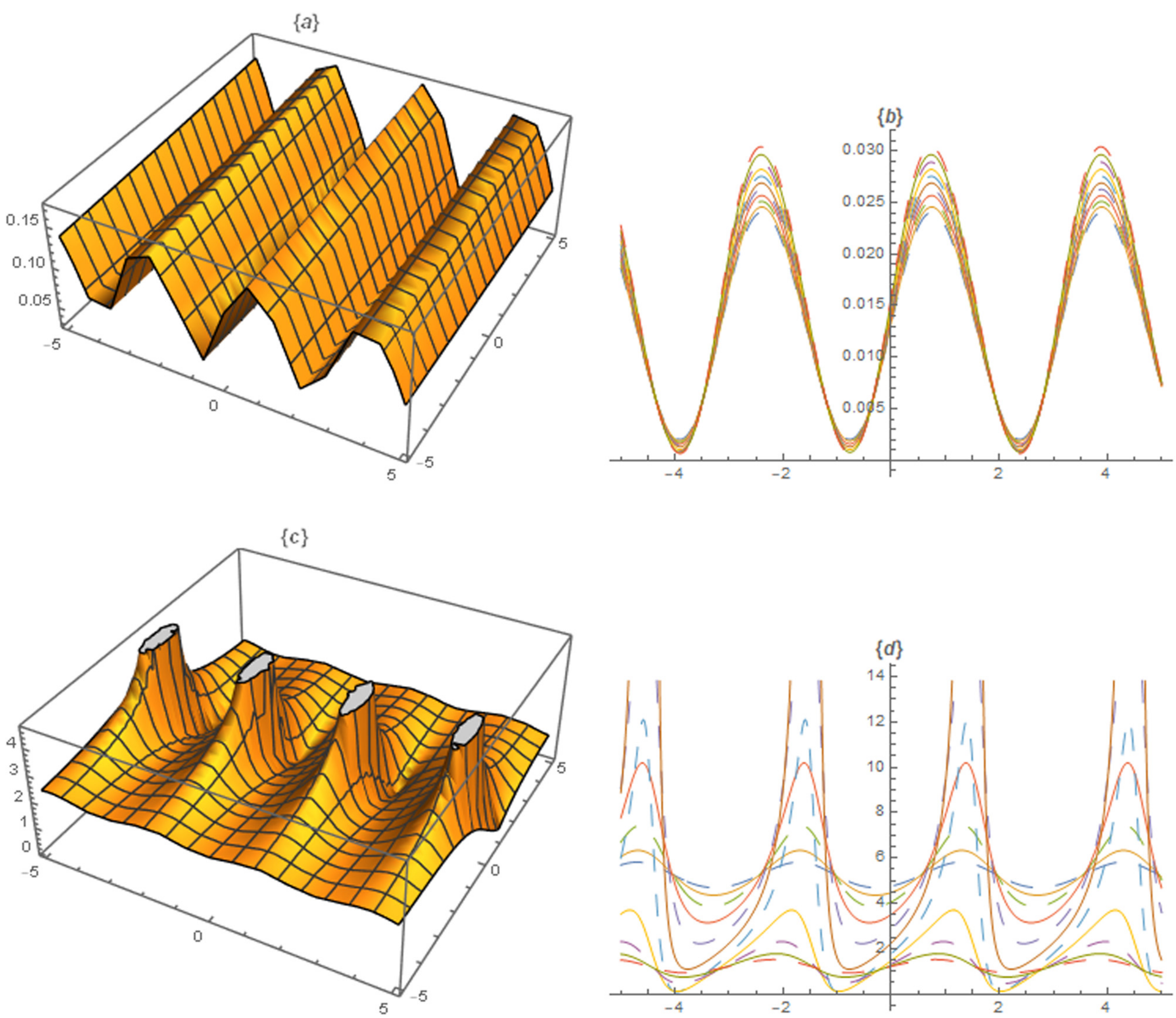

Figure 2: Profiles of solutions U_3 (a, b) and U_4 (c, d) are plotted with $c_1 = 0.03$, $c_2 = 0.03$, $L = 2.1$, $\alpha = 1$, $A_0 = -0.1$, $\Gamma = 1$, $\delta = 1$, $\eta = 1$, $\theta = 0.01$ and $\alpha = 1$, $A_0 = 1.1$, $c_1 = -0.5$, $c_2 = 3$, $\Gamma = 1$, $\delta = 1.1$, $\eta = 1.1$, $\theta = 0.01$, $\xi_0 = 2.1$, respectively.

3.3 Application of extended $\mathrm{Exp}(-\Psi(\phi))$-expansion method

We recover different forms of the solutions for the studied model. Let (3) have solution as

$$U = A_0 + A_1 \exp(-\Psi(\phi)) + A_{-1} \exp(-\Psi(\phi))^{-1}. \qquad (40)$$

Substituting (40) with (11) into (3),

Family-I:

$$A_0 = A_0, A_1 = -2\sigma, A_{-1} = 0, \mu = \frac{\lambda^2\sigma^2 + 1}{4\sigma^2}. \qquad (41)$$

When $\lambda^2 - 4\mu > 0$, $\mu \neq 0$,

$$U_{17} = A_0 - 2\sigma \log \times \left[\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}(\xi + \xi_0)\sqrt{\lambda^2 - 4\mu}\right) - \lambda}{2\mu}\right]. \qquad (42)$$

When $\lambda^2 - 4\mu > 0$, $\mu = 0$,

$$U_{18} = A_0 - 2\sigma\left[-\log\left(\frac{\lambda}{\exp(\lambda(\xi + \xi_0)) - 1}\right)\right]. \qquad (43)$$

When $\lambda^2 - 4\mu = 0 \quad \lambda \neq 0, \mu \neq 0$,

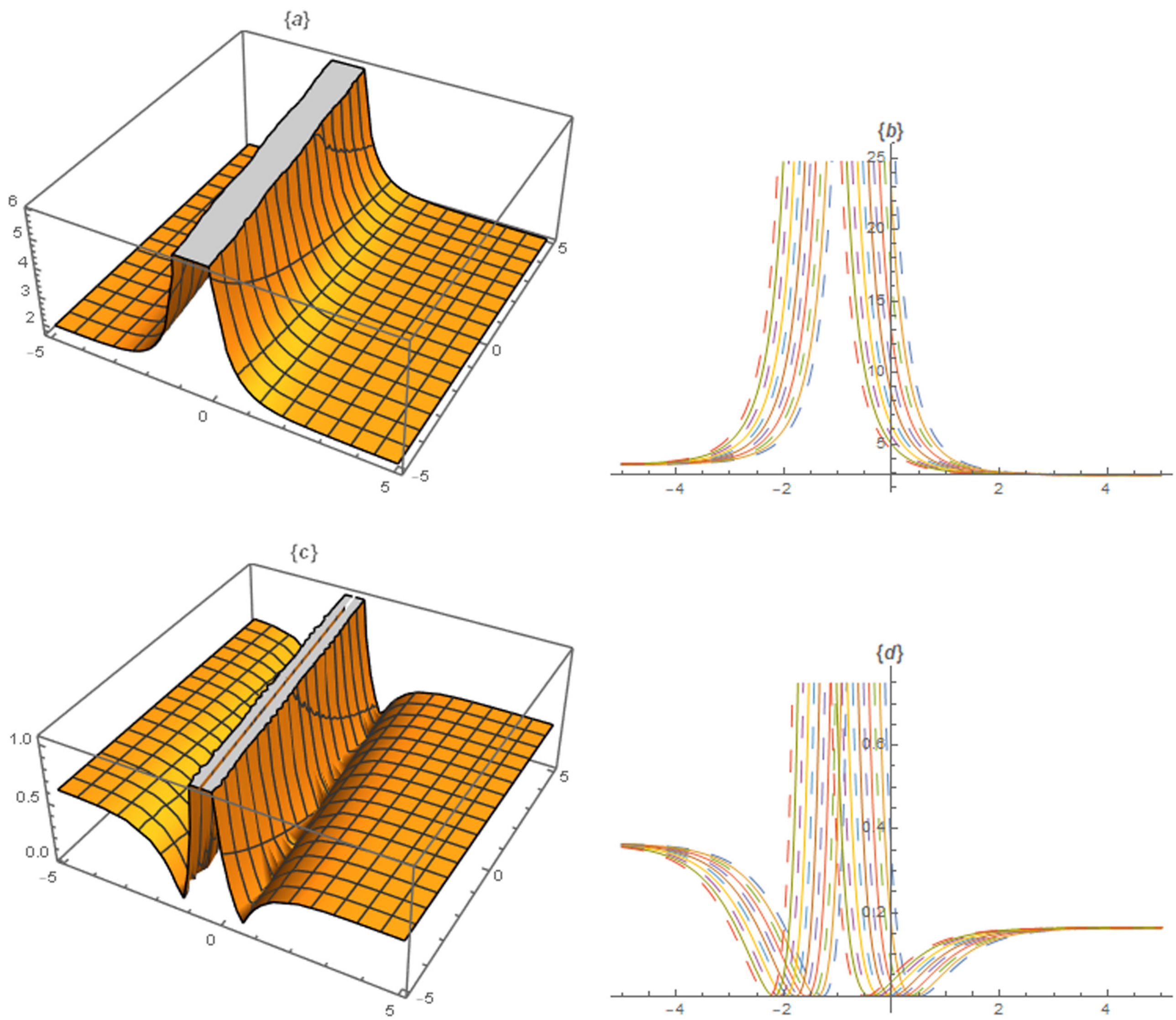

Figure 3: Profile of solutions U_9 (a, b) and U_{10} (c, d) are plotted with $\alpha = 1$, $A_0 = -0.1$, $c_0 = -1.1$, $c_2 = 0.8$, $\Gamma = 1$, $\delta = 1$, $\eta = 0.1$, $\theta = 1$, $\xi_0 = 0.01$, $\sigma = 0.5$ and $\alpha = 1$, $A_0 = -0.1$, $c_0 = -1.1$, $c_2 = 0.6$, $\Gamma = 1$, $\delta = 1$, $\eta = 0.1$, $\theta = 1$, $\xi_0 = 0.01$, $\sigma = 0.5$, respectively.

$$U_{19} = A_0 - 2\sigma \log\left(-\frac{2(\lambda(\xi + \xi_0) + 2)}{\lambda^2(\xi + \xi_0)}\right). \tag{44}$$

When $\lambda^2 - 4\mu < 0$,

$$U_{20} = A_0 - 2\sigma \log \times \left(\frac{-\sqrt{4\mu - \lambda^2}\tan\left(\frac{1}{2}(\xi + \xi_0)\sqrt{4\mu - \lambda^2}\right) - \lambda}{2\mu}\right). \tag{45}$$

Family-II:

$$A_0 = A_0, A_1 = 0, A_{-1} = \frac{\lambda^2\sigma^2 + 1}{2\sigma}, \mu = \frac{\lambda^2\sigma^2 + 1}{4\sigma^2}. \tag{46}$$

When $\lambda^2 - 4\mu > 0$, $\mu \neq 0$,

$$U_{21} = A_0 + \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\log\left(\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}(\xi + \xi_0)\sqrt{\lambda^2 - 4\mu}\right) - \lambda}{2\mu}\right)}\right). \tag{47}$$

When $\lambda^2 - 4\mu > 0$, $\mu = 0$,

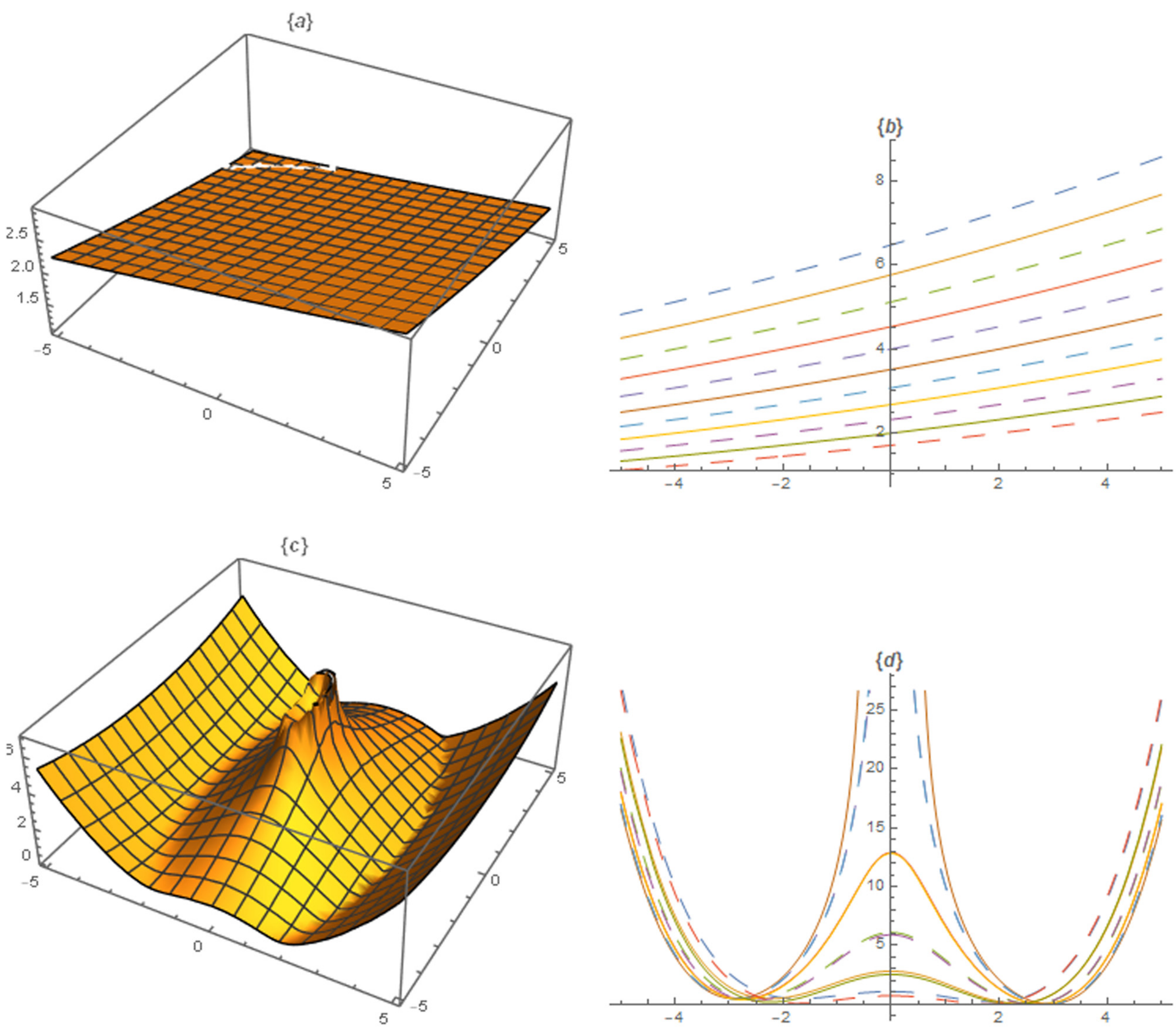

Figure 4: Profiles of solutions U_{16} (a, b) and U_{18} (c, d) are plotted with $\alpha = 1$, $A_0 = 1.2$, $\Gamma = 1$, $\delta = -1.1$, $\eta = -0.1$, $\theta = 0.5$, $\lambda = 0.3$, $P_1 = 0.04$, $P_2 = 0.4$, $\sigma = -0.5$ and $\alpha = 2.1$, $A_0 = 2.5$, $\Gamma = 1$, $\delta = 1$, $\eta = 0.1$, $\theta = -0.1$, $\lambda = 0.4$, $\xi_0 = -0.01$, and $\sigma = 0.5$, respectively.

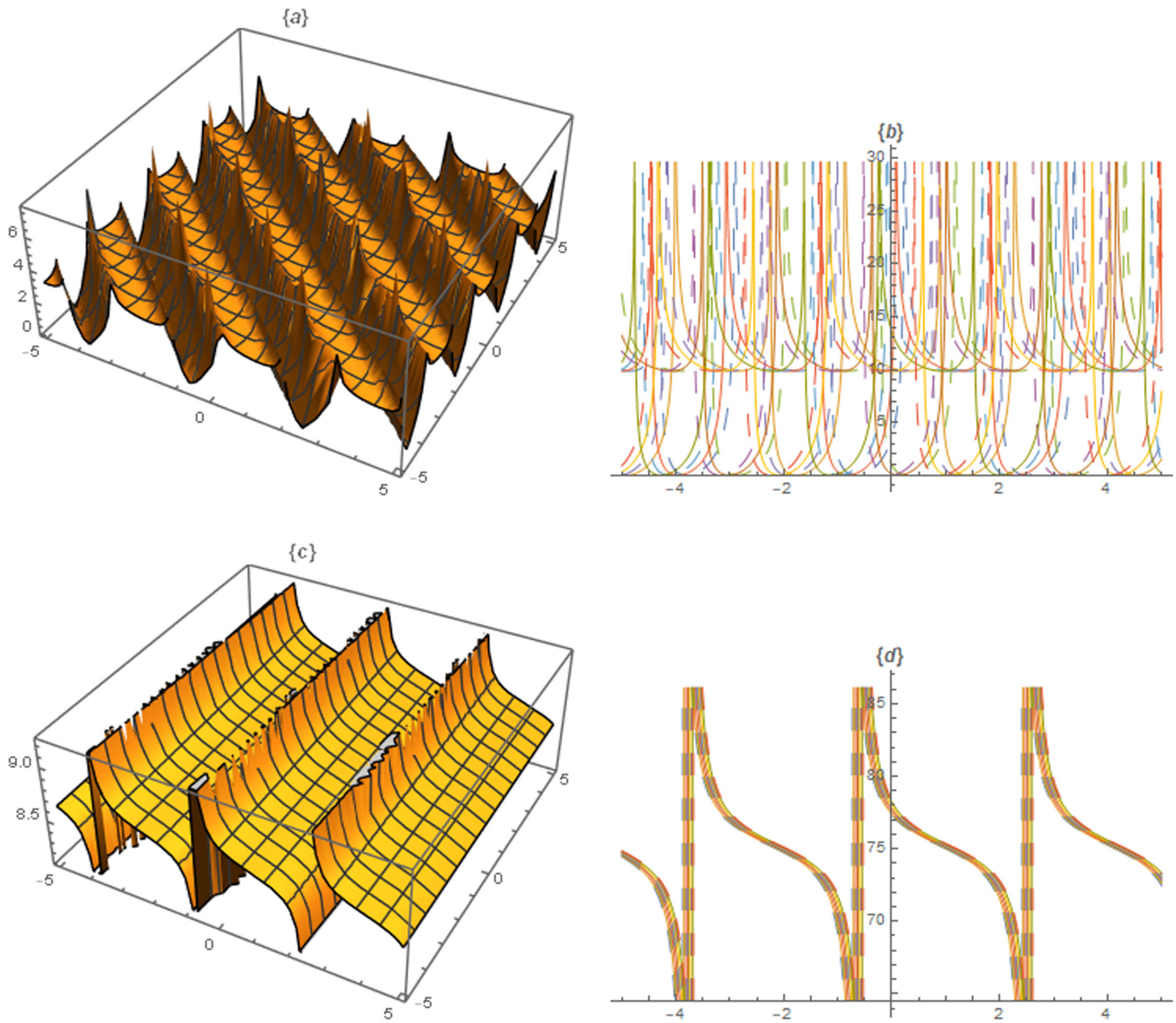

Figure 5: Profiles of solutions U_{20} (a, b) and U_{24} (c, d) are plotted with $\alpha = 1$, $A_0 = 0.5$, $\Gamma = 1$, $\delta = 1$, $\eta = 1.1$, $\theta = -1.2$, $\lambda = 0.4$, $\xi_0 = -0.01$, $\sigma = 0.5$ and $\alpha = 1$, $A_0 = -8.5$, $\Gamma = 1$, $\delta = 1$, $\eta = -0.1$, $\theta = 0.1$, $\lambda = 2.01$, $\xi_0 = -4.4$, $\sigma = 2.5$, respectively.

$$U_{22} = A_0 + \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\left(-\log\left(\frac{\lambda}{\exp(\lambda(\xi + \xi_0)) - 1}\right)\right)}\right). \tag{48}$$

When $\lambda^2 - 4\mu = 0 \quad \lambda \neq 0, \mu \neq 0$,

$$U_{23} = A_0 + \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\log\left(-\frac{2(\lambda(\xi + \xi_0) + 2)}{\lambda^2(\xi + \xi_0)}\right)}\right). \tag{49}$$

When $\lambda^2 - 4\mu < 0$,

$$U_{24} = A_0 + \left(\frac{\lambda^2\sigma^2 + 1}{(2\sigma)\log\left(\frac{-\sqrt{4\mu - \lambda^2}\tan\left(\frac{1}{2}(\xi + \xi_0)\sqrt{4\mu - \lambda^2}\right) - \lambda}{2\mu}\right)}\right). \tag{50}$$

The different graphs are sketched by the assistance of parametric values, and it is observed that the fractional operator has deep impact on physical behaviour of the solutions. Figures 1–5 illustrate the nonlinear dynamic nature of the M-fractional Kairat-II equation. The inferred graphical renderings illustrate several forms of travelling waves and solitons.

4 Results and discussion

We have discussed our derived results of Eq. (1) *via* application of three methods with the others results in the literature. Our obtained exact solutions have different forms such as trigonometric function, hyperbolic function, exponential function, and rational function after obtaining the values of A_1 and A_{-1} in Eq. (14), Eq. (31), and Eq. (40), respectively. This is fact that Eq. (1) is study analytically in the current literature only by two authors in [34,35] *via* using the different methods and derived some exact solutions as compared to our investigated many solutions. Moreover, our three solutions are approximately similar to others as:

- Our solution U_1 in Eq. (16) and solution $h(x,t)$ in Eq. (61) in [35] are resemble with each other.
- Our solution U_2 in Eq. (18) and solution Φ_{14} in Eq. (22) in [51] are similar appearance.
- There is some similarity between our solution U_{16} in Eq. (16) and solution $\mu_3(x,y,t)$ mentioned in Eq. (25) in the study of Ali *et al.* [52].

Furthermore, our all remaining results are new and have not be reputed in any existing literature. Hence, it has been concluded that our suggested methods offer a good groundwork to solve many fractional NPDEs problems in different fields.

5 Conclusion

In this article, we have constructed several type exact solutions of M-fractional Kairat-II model *via* successfully implementation of three extended mathematical methods with the support of computational software Mathematica 13.0. The concern model has many applications in optical fibers, which is used to describe the trajectory of optical pulses in optical fibers. The work formally furnishes algorithms for studying newly constructed systems that examine plasma physics, optical communications, oceans and seas, and the differential geometry of curves, among others. The derived exact solutions are in the form of trigonometric function, hyperbolic function, rational function, and exponential function. Moreover, many of the solutions obtained are new and have not been found before. For the physically description, few investigated solutions are plotted 2D and 3D by assigning the particular values to the parameters. This is fact that in current existing literature only two authors [34,35] derived few results of the concern model but here we have established many type exact solutions of Eq. (1). The constructed results show that the applied techniques are trustworthy, competent, and dominant in the analysis of various nonlinear fractional differential equations in diverse field of nonlinear science.

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Conflict of interest: The authors state no conflict of interest.

Data availability statement: The datasets used and/or analyzed during the current study available from the corresponding author on reasonable request.

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