Research Article

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Robust control and preservation of quantum steering, nonlocality, and coherence in open atomic systems

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Abstract: We investigate the robustness of Einstein–Podolsky–Rosen (EPR) steering, nonlocality, and quantum coherence in a bipartite atomic system coupled to cavity fields under the influence of decoherence. The system consists of two non-interacting atoms, where each atom is confined within a cavity that interacts with another cavity field, which plays a crucial role in governing the dynamical evolution of two atoms. Through a combination of analytical and numerical investigations, we demonstrate that quantum steering, Bell nonlocality, and coherence can be not only preserved but also enhanced by appropriately tuning the cavity–cavity interaction strength, effectively mitigating environmental decoherence and extending the coherence lifetime of the system. Our results reveal that, under optimal conditions, steering, nonlocality, and coherence remain resilient against decoherence over extended timescales. These findings offer valuable insights into the controlled manipulation of quantum resources in open quantum systems and have significant implications for quantum information processing and secure communication technologies.

Keywords: open quantum systems, cavity–cavity interaction, photon loss, quantumness measures

1 Introduction

The concept of quantum steering was first introduced by Schrödinger in 1936 as a response to the Einstein–Podolsky–Rosen (EPR) paradox [1,2]. Decades later, Wiseman et al. established the fundamental connection between EPR steering, quantum entanglement, and Bell nonlocality, positioning steering as an intermediate quantum correlation [3]. Unlike entanglement, EPR steering exhibits an intrinsic asymmetry, allowing one party (Alice) to remotely influence the quantum state of another party (Bob), even when Bob lacks trust in Alice's measurement apparatus [4–6]. Quantum steering is recognized as a pivotal quantum resource with diverse applications, including randomness certification [7], randomness generation [8], asymmetric quantum networks [9], subchannel discrimination [10], and quantum key distribution (QKD) [11]. Enhancing the robustness and practical utilization of EPR steering is crucial for quantum information transmission and foundational aspects of quantum communication protocols. Recent advances have focused on relaxing the no-signaling condition to optimize the efficiency of EPR steering resources [12]. Notably, it has been demonstrated that quantum steering can be sequentially distributed among multiple observers, either through standard projective measurements [13] or unsharp measurements [12]. This concept of steering sharing has been extensively studied in bipartite systems [14] and further extended to the realm of genuine multipartite steering reuse [15], shedding light on new possibilities for multi-user quantum networks and resource allocation in quantum technologies.

The phenomenon of quantum coherence, rooted in the superposition principle of quantum states, plays a fundamental role in quantum theory and technological applications. It serves as a key resource across various fields, including quantum information processing [16], quantum optics [17], solid-state physics [18], and even biological systems [19]. Over the years, substantial research efforts have been dedicated to developing a rigorous theoretical framework for quantum coherence as a physical resource [20,21], along with the establishment of quantitative measures for its characterization. A major breakthrough in this area was made by Baumgratz *et al.*, who introduced a formal resource-theoretic framework for quantifying coherence [22]. This framework defines coherence through

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well-established measures such as the l_1 norm of coherence and the relative entropy of coherence, each with distinct physical interpretations. In multipath quantum interference experiments, as demonstrated in previous studies [23,24], the l_1 norm effectively quantifies coherence by capturing the wave-like nature of a quanton, making it an experimentally observable metric. On the contrary, in the asymptotic regime where the number of quantum copies approaches infinity [25], the relative entropy of coherence provides an optimal measure for the distillation of maximally coherent states via incoherent operations. In recent years, the resource theory of coherence has gained significant attention, with its applications extending beyond fundamental studies to practical implementations in various quantum technologies, including quantum metrology, quantum thermodynamics, and quantum computing [26,27]. These advancements underscore the pivotal role of coherence as a versatile quantum resource, further motivating the exploration of strategies for its protection and manipulation in open quantum systems.

In realistic quantum systems, decoherence and noise induced by interactions with the external environment pose fundamental challenges to the preservation of quantum resources. To mitigate these effects, various strategies have been proposed to protect and enhance quantum correlations and coherence [28–32]. Notably, extensive studies have demonstrated that non-Markovian environments, characterized by memory effects, can effectively preserve quantum coherence and correlations by enabling partial information backflow into the system [33–35]. Despite substantial advancements in improving the efficiency of EPR steering and coherence, an open question remains: How can these quantum resources be simultaneously protected against decoherence while being dynamically controlled? Given their crucial role in quantum technologies, understanding the interplay between quantum steering, nonlocality, and coherence is essential for optimizing quantum information processing and communication protocols. In this work, we investigate the preservation and manipulation of EPR steering, Bell nonlocality, and quantum coherence in a bipartite atomic system, where each atom is confined within a cavity that interacts with another cavity field. By systematically analyzing the system's quantum dynamics, we demonstrate that these quantum resources can be sustained and even enhanced through strategic tuning of inter-cavity coupling strengths, enabling robust control over quantum correlations despite environmental decoherence. By comparing the time evolution of EPR steering, Bell nonlocality, and quantum coherence, we show that, in the ideal cavity limit, high levels of quantumness measures persist throughout the system's evolution. Furthermore, we establish that an optimal selection of quantum model parameters allows for the long-term protection of quantum steering, nonlocality, and coherence, effectively mitigating decoherence effects.

The structure of this article is as follows. Section 2 presents the Hamiltonian formulation of the quantum system and describes its dynamical evolution, along with a concise review of quantum steering and coherence. Additionally, it provides a detailed numerical analysis, offering a comprehensive discussion of the obtained results. Finally, Section 3 summarizes the key findings and outlines potential directions for future investigations.

2 Model and quantum resources

In the quantum regime, physical systems must be treated as open systems due to their inevitable interactions with the surrounding environment. In this work, we consider a quantum model consisting of a single atom confined in a cavity, which is coupled to another cavity. The total Hamiltonian governing the atom–cavity system is given by

$$H_T = H_A + H_F + H_{A-F} + H_{C-C}. \tag{1}$$

The atomic Hamiltonian is expressed as

$$H_A = \frac{\hbar\omega_0}{2}\sigma_z, \tag{2}$$

where ω_0 denotes the atomic transition frequency, and σ_z is the Pauli matrix. The Hamiltonian describing the cavity fields is

$$H_F = \hbar \sum_{i=1,2} \omega_i c_i^\dagger c_i, \tag{3}$$

where $\omega = \omega_1 = \omega_2$ represents the frequency of the cavity modes, and c_i ($c_i^\dagger$) are the annihilation (creation) operators. The interaction Hamiltonians governing the atom-field coupling (H_{A-F}) and the inter-cavity coupling (H_{C-C}) are given by

$$H_{A-F} = \hbar\alpha(c_1\sigma_+ + c_1^\dagger\sigma_-), \tag{4}$$

$$H_{C-C} = \hbar\beta(c_1 c_2^\dagger + c_1^\dagger c_2), \tag{5}$$

where α and β represent the coupling strengths of the atom-field and cavity–cavity interactions, respectively. The operators σ_- and σ_+ denote the atomic lowering and raising operators. The Hamiltonian (1) considered in this work describes an atom trapped within a cavity that is linked to a second cavity, a configuration that holds significant importance in the field of cavity quantum electrodynamics (QED). This model effectively encapsulates the fundamental dynamics of open quantum systems, where interactions with the environment play a crucial role.

By introducing coupling between the two cavities, this Hamiltonian goes beyond the traditional Jaynes–Cummings model, enabling the exploration of quantum correlations across multiple parties, specifically EPR steering and Bell nonlocality, within a system of two atoms. The model features adjustable parameters, including the strength of the atom's interaction with the field (α) and the strength of the connection between the cavities (β). These parameters provide a means to manipulate the quantum properties of the system, which is particularly useful for examining how these properties can be maintained in the presence of decoherence [28,36]. One of the strengths of this model is its ability to realistically represent physical systems while remaining mathematically controllable.

Considering the dissipative effects in both cavities, the time evolution of the density operator ρ, which describes the combined system of the atom and the cavities, is governed by the following master equation:

$$\frac{\partial\rho}{\partial t} = \frac{i}{\hbar}[\rho, H_T] - \sum_{i=1}^{2}\frac{\mu_i}{2}[c_i^\dagger c_i\rho + \rho c_i^\dagger c_i - 2c_i\rho c_i^\dagger], \tag{6}$$

where μ_1 and μ_2 denote the photon decay rates of the respective cavities. The distinction between non-Markovian and Markovian dynamics is determined by conditions $\alpha > \mu_1/4$ and $\alpha \le \mu_1/4$ [28,37,38], respectively.

The atomic density operator evolves as

$$\rho_A(t) = \begin{pmatrix} \rho_A^{00}(0)|\kappa(t)|^2 & \rho_A^{01}(0)\kappa(t) \\ \rho_A^{10}(0)\kappa^*(t) & 1 - \rho_A^{11}(0)|\kappa(t)|^2 \end{pmatrix}. \tag{7}$$

The function $\kappa(t)$, which encapsulates the system's temporal evolution, is given by [28]

$$\kappa(t) = L^{-1}\left[\frac{F(s)}{G(s)}\right], \tag{8}$$

where

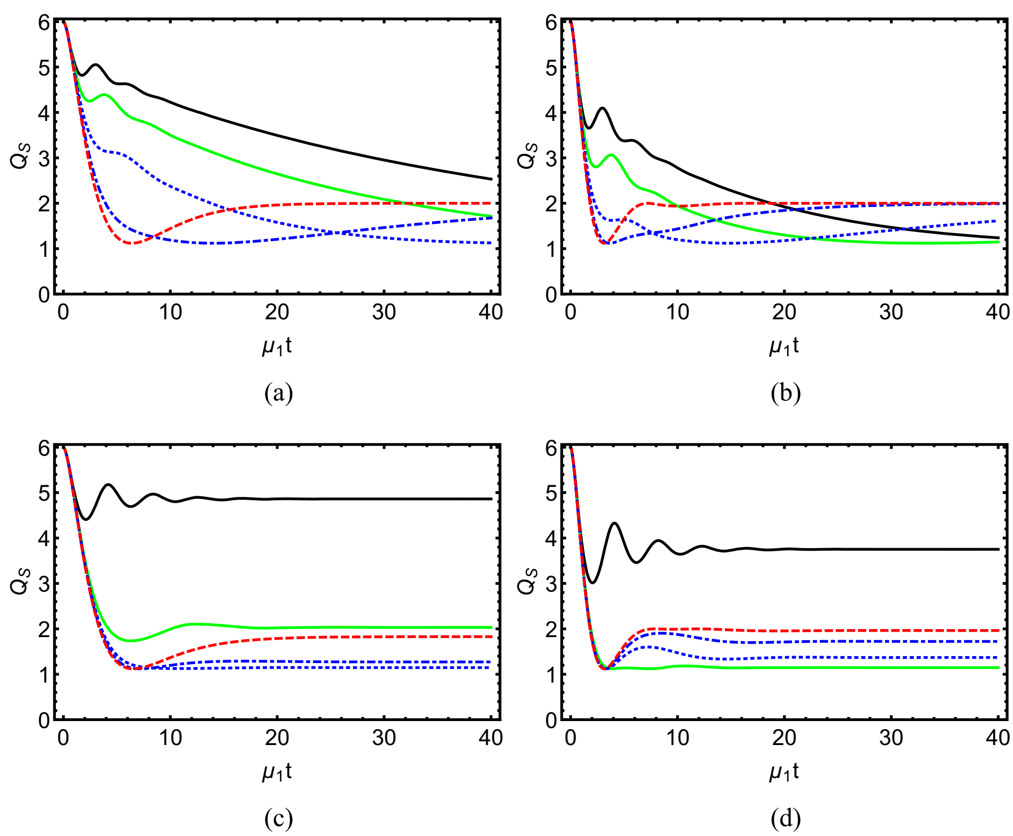

Figure 1: Dynamics of the Q_S (EPR steering measure) for the two atoms, governed by the EPR steering inequality defined in equation (12), plotted as a function of time $\mu_1 t$ for various β values with $\delta = 0$ when the atoms start from a Bell state $\rho_{A-A}(0) = 1/2(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 01|)$. Labels (a), (b), (c), and (d) are for $\alpha = 0.24/0.5\mu_2 = 0.24\mu_1$, $\alpha = 0.4/0.5\mu_2 = 0.4\mu_1$, $\alpha = 0.24\mu_1$ with $\mu_2 = 0$, and $\alpha = 0.4\mu_1$ with $\mu_2 = 0$, respectively. For the case of (a) and (b): black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\beta = 2\mu_1$, $\beta = 1.5\mu_1$, $\beta = 1\mu_1$, $\beta = 0.5\mu_1$, and $\beta = 0$, respectively. For the case of (c) and (d): black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\beta = 1.5\mu_1$, $\beta = 0.5\mu_1$, $\beta = 0.3\mu_1$, $\beta = 0.2\mu_1$, and $\beta = 0.1\mu_1$, respectively.

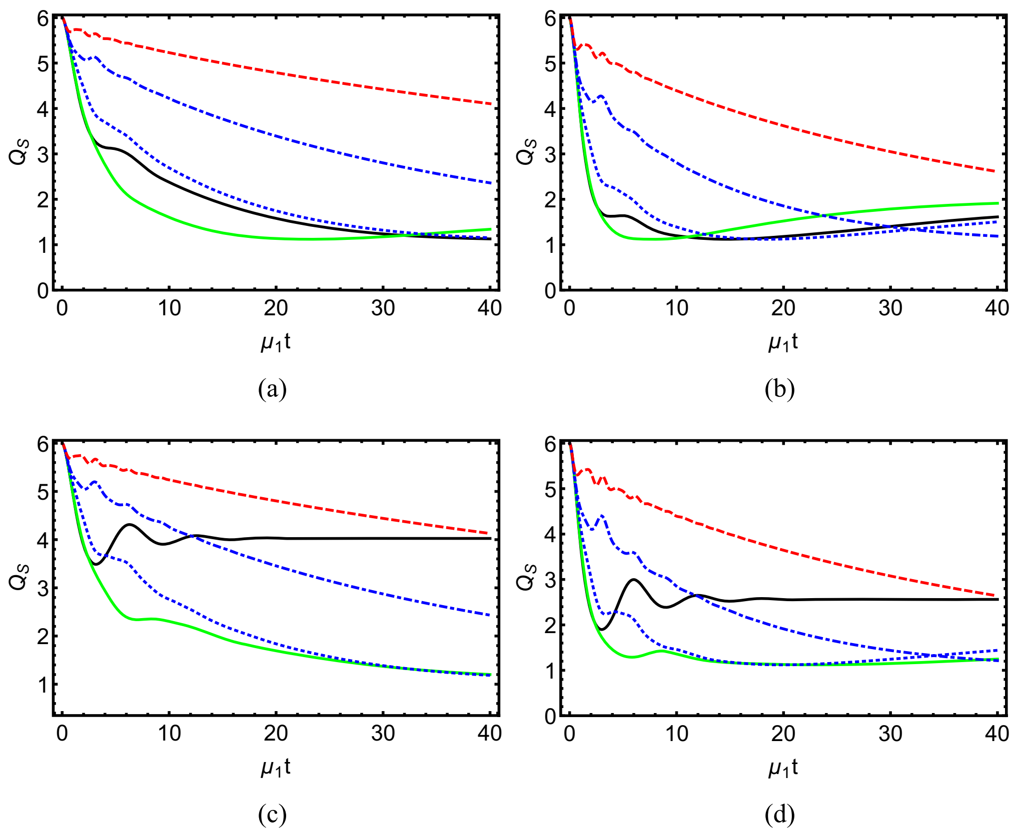

Figure 2: Dynamics of the Q_S (EPR steering measure) for the two atoms, governed by the EPR steering inequality defined in equation (12), plotted as a function of time $\mu_1 t$ for various δ values with $\beta = \mu_1$ when the atoms start from a Bell state $\rho_{A-A}(0) = 1/2(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 01|)$. Labels (a), (b), (c), and (d) are for $\alpha = 0.24/0.5\mu_2 = 0.24\mu_1$, $\alpha = 0.4/0.5\mu_2 = 0.4\mu_1$, $\alpha = 0.24\mu_1$ with $\mu_2 = 0$, and $\alpha = 0.4\mu_1$ with $\mu_2 = 0$, respectively. Black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\delta = 0$, $\delta = 0.5\mu_1$, $\delta = 2\mu_1$, $\delta = 3\mu_1$, and $\delta = 5\mu_1$, respectively.

$$\begin{aligned} F(s) &= -4\beta^2 - (2s + 2i\omega + \mu_1)(2s + 2i\omega + \mu_2), \\ G(s) &= 2\alpha^2(2s + 2i\omega + \mu_2) + (s + i(\omega + \delta))[4(\beta^2 \\ &\quad + (s + i\omega)^2) + 2\mu_2(s + i\omega) \\ &\quad + \mu_1(2s + 2i\omega + \mu_2)]. \end{aligned} \tag{9}$$

Here, L^{-1} represents the inverse Laplace transform and $\delta = \omega_0 - \omega$ represents the atom-field detuning.

It is well-established that for non-interacting subsystems, the full dynamics of a two-qubit system can be determined from the evolution of each individual qubit coupled to its respective environment [37]. Using the dynamics of a single qubit, we can derive the time-evolved density matrix for the two-atom system, with its elements expressed as follows:

$$\begin{aligned} \rho^{11}_{A-A}(t) &= \rho^{11}_{A-A}(0)|\kappa(t)|^4, \\ \rho^{22}_{A-A}(t) &= \rho^{11}_{A-A}(0)|\kappa(t)|^2(1 - |\kappa(t)|^2) + \rho^{22}_{A-A}(0)|\kappa(t)|^2, \\ \rho^{33}_{A-A}(t) &= \rho^{11}_{A-A}(0)|\kappa(t)|^2(1 - |\kappa(t)|^2) + \rho^{33}_{A-A}(0)|\kappa(t)|^2, \\ \rho^{44}_{A-A}(t) &= 1 - (\rho^{11}_{A-A}(t) + \rho^{22}_{A-A}(t) + \rho^{33}_{A-A}(t)). \end{aligned} \tag{10}$$

The non-diagonal density matrix elements evolve as

$$\begin{aligned} &\rho^{12}_{A-A}(t) = \rho^{12}_{A-A}(0)\kappa(t)|\kappa(t)|^2, \\ &\rho^{13}_{A-A}(t) = \rho^{13}_{A-A}(0)\kappa(t)|\kappa(t)|^2, \\ &\rho^{14}_{A-A}(t) = \rho^{14}_{A-A}(0)\kappa(t)^2, \quad \rho^{23}_{A-A}(t) = \rho^{23}_{A-A}(0)|\kappa(t)|^2, \\ &\rho^{24}_{A-A}(t) = \rho^{13}_{A-A}(0)\kappa(t)(1 - |\kappa(t)|^2) + \rho^{24}_{A-A}(0)\kappa(t), \\ &\rho^{34}_{A-A}(t) = \rho^{12}_{A-A}(0)\kappa(t)(1 - |\kappa(t)|^2) + \rho^{34}_{A-A}(0)\kappa(t). \end{aligned} \tag{11}$$

The condition $\rho^{ij}_{A-A}(t) = \rho^{ji}_{A-A}(t)$ ensures the Hermiticity of the density matrix. Using equations (10) and (11), the two-atom dynamics for any given initial state can be fully determined. This allows for the investigation of quantum resources, such as quantum steering, nonlocality, and coherence, as a function of the model parameters.

We take into consideration the EPR steering inequality [39–41] in order to study the dynamics of quantum steering for the two-atom state in the present model. If the steering inequality is violated, the atom state is steerable. For a quantum state in X-form with the Bloch decomposition in terms of vectors $\vec{r} = (0, 0, r)$ and $\vec{s} = (0, 0, s)$, the quantum steering inequality for the two atoms via performing the Pauli measurements is formulated as

$$\begin{aligned}\sum_{l=1,2}&[(1-b_l)\log(1-b_l)+(1+b_l)\log(1+b_l)]\\&-[(1+r)\log(1+r)+(1-r)\log(1-r)]\\&+\frac{1}{2}(1+b_3-r-s)\log(1+b_3-r-s)\\&+\frac{1}{2}(1+b_3+r+s)\log(1+b_3+r+s)\\&+\frac{1}{2}(1-b_3+r-s)\log(1-b_3+r-s)\\&+\frac{1}{2}(1-b_3-r+s)\log(1-b_3-r+s)\le 2,\end{aligned} \tag{12}$$

where

$$\begin{aligned}b_1&=2(\rho^{23}_{A-A}\rho^{14}_{A-A})\\b_2&=2(\rho^{23}_{A-A}-\rho^{14}_{A-A})\\b_3&=\rho^{11}_{A-A}-\rho^{22}_{A-A}-\rho^{33}_{A-A}+\rho^{44}_{A-A}\\r&=\rho^{11}_{A-A}+\rho^{22}_{A-A}-\rho^{33}_{A-A}-\rho^{44}_{A-A}\\s&=\rho^{11}_{A-A}-\rho^{22}_{A-A}+\rho^{33}_{A-A}-\rho^{44}_{A-A}.\end{aligned} \tag{13}$$

When the inequality is violated, the two-atom steering is obtained.

Bell nonlocality is a fundamental manifestation of quantum mechanics, providing a means to test quantum correlations that cannot be reconciled with classical explanations. The Bell–Clauser–Horne–Shimony–Holt (BCHSH) inequality serves as a standard criterion for quantifying nonlocality in quantum systems. A violation of this inequality signifies the presence of nonlocal correlations and can be formulated as follows:

$$B_l = 2\max\{B_{l1}, B_{l2}\}, \tag{14}$$

where the quantities B_{l1} and B_{l2} are given by

$$B_{l1}=\sqrt{\mathfrak{b}_1+\mathfrak{b}_2},\quad B_{l2}=\sqrt{\mathfrak{b}_1+\mathfrak{b}_3}. \tag{15}$$

Here, $\mathfrak{b}_1$, $\mathfrak{b}_2$, and $\mathfrak{b}_3$ are parameters derived from the elements of the system's density matrix, defined as

$$\begin{aligned}\mathfrak{b}_1&=4(|\rho^{14}_{A-A}|+|\rho^{23}_{A-A}|)^2,\\\mathfrak{b}_2&=4(|\rho^{14}_{A-A}|-|\rho^{23}_{A-A}|)^2,\\\mathfrak{b}_3&=(\rho^{11}_{A-A}-\rho^{22}_{A-A}-\rho^{33}_{A-A}+\rho^{44}_{A-A})^2.\end{aligned} \tag{16}$$

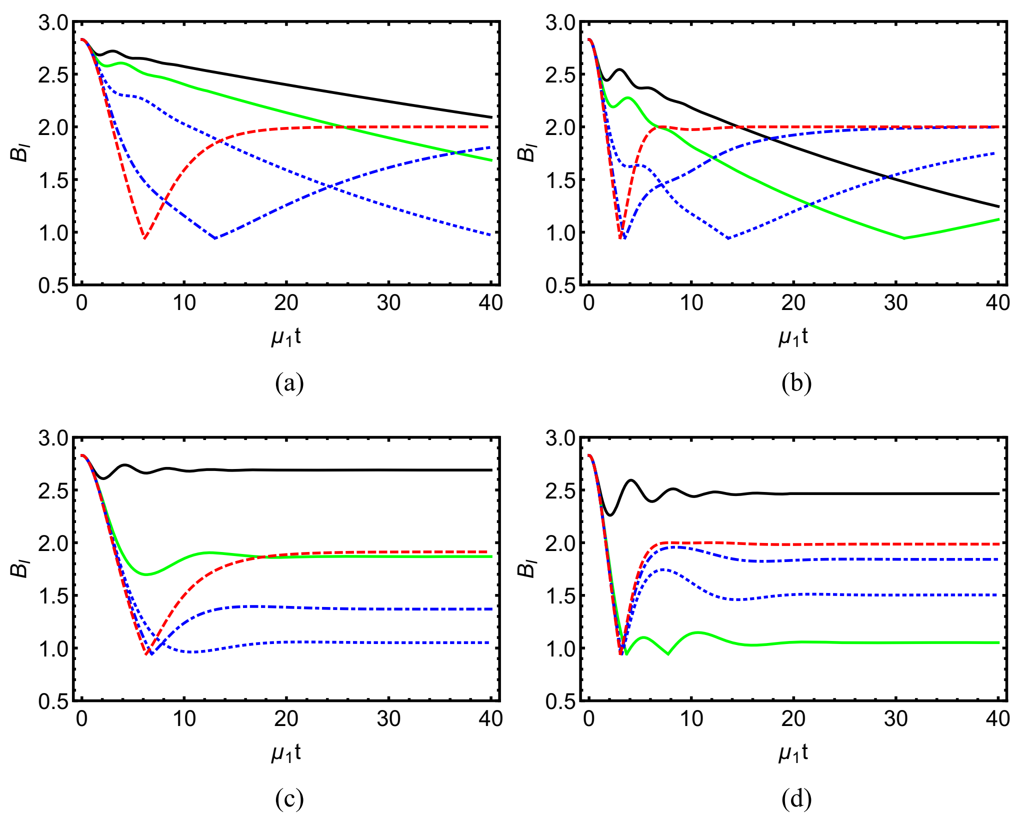

Figure 3: Dynamics of Bell nonlocality for the two atoms, quantified by the Bell-CHSH measure B_l from equation (14), plotted as a function of time $\mu_1 t$ for various β values with $\delta = 0$ when the atoms start from a Bell state $\rho_{A-A}(0) = 1/2(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 01|)$. Labels (a), (b), (c), and (d) are for $\alpha = 0.24/0.5\mu_2 = 0.24\mu_1$, $\alpha = 0.4/0.5\mu_2 = 0.4\mu_1$, $\alpha = 0.24\mu_1$ with $\mu_2 = 0$, and $\alpha = 0.4\mu_1$ with $\mu_2 = 0$, respectively. For the case of (a) and (b): black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\beta = 2\mu_1$, $\beta = 1.5\mu_1$, $\beta = 1\mu_1$, $\beta = 0.5\mu_1$, and $\beta = 0$, respectively. For the case of (c) and (d): black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\beta = 1.5\mu_1$, $\beta = 0.5\mu_1$, $\beta = 0.3\mu_1$, $\beta = 0.2\mu_1$, and $\beta = 0.1\mu_1$, respectively.

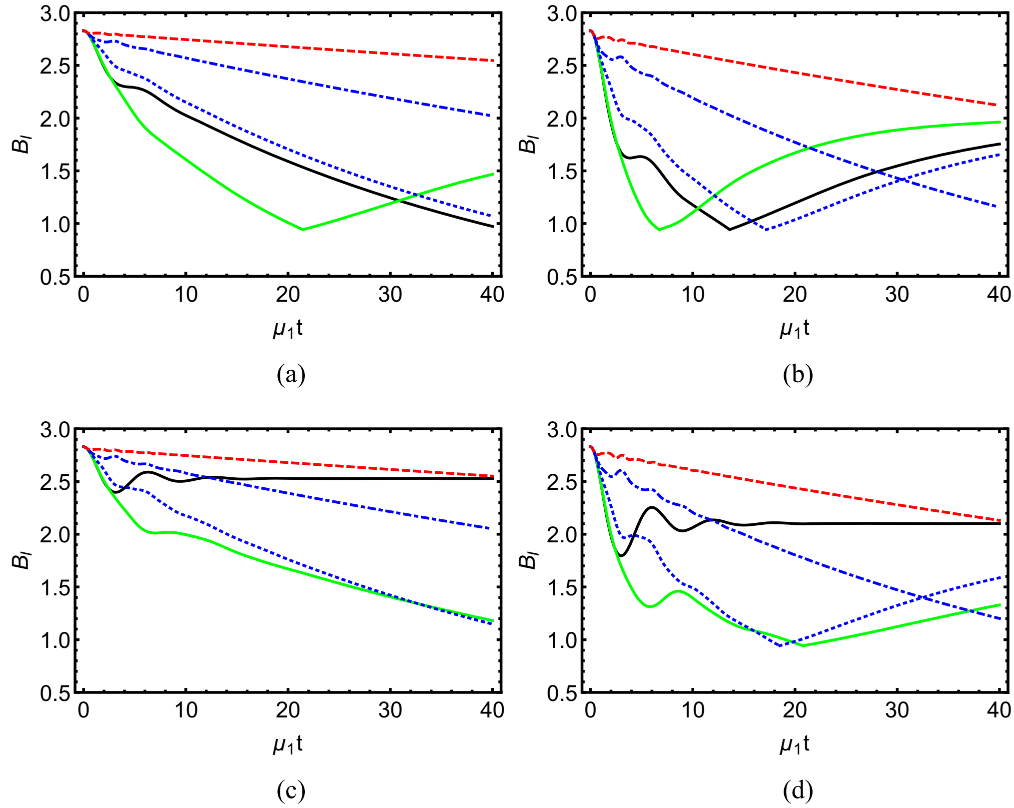

Figure 4: Dynamics of Bell nonlocality for the two atoms, quantified by the Bell-CHSH measure B_l from equation (14), plotted as a function of time $\mu_1 t$ for various δ values with $\beta = \mu_1$ when the atoms start from a Bell state $\rho_{A-A}(0) = 1/2(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 01|)$. Labels (a), (b), (c), and (d) are for $\alpha = 0.24/0.5\mu_2 = 0.24\mu_1$, $\alpha = 0.4/0.5\mu_2 = 0.4\mu_1$, $\alpha = 0.24\mu_1$ with $\mu_2 = 0$, and $\alpha = 0.4\mu_1$ with $\mu_2 = 0$, respectively. Black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\delta = 0$, $\delta = 0.5\mu_1$, $\delta = 2\mu_1$, $\delta = 3\mu_1$, and $\delta = 5\mu_1$, respectively.

These expressions highlight the pivotal role of off-diagonal density matrix elements in governing the degree of nonlocality exhibited by the system. Notably, the relationship $\mathfrak{b}_1 \geq \mathfrak{b}_2$ ensures that B_l accurately captures the maximal violation of the BCHSH inequality. This formalism is particularly relevant to two-qubit X states, where the density matrix structure simplifies the analysis, offering deeper insights into the emergence and characterization of nonlocal quantum correlations.

The dependence of two measures on the parameters β and δ during the dynamics is shown in Figures 1–4. For a generic case, the time evolution of quantum resources is affected by the coupling strength and the detuning parameter. As shown in the figures, we have displayed the measures of steering and Bell nonlocality as a function of the time $\mu_1 t$ for various β values. We find that the quantum measures decreases, from a maximally steerable state with Bell nonlocality with Tsirelson's bound, as the time increases. In the presence cavity loss $\mu_2 \neq 0$, the increase in the parameter β leads to delay the measures loss during the dynamics and the atoms state will be steerable with Bell nonlocality for long periods of time. This indicates that the increase in the interaction between the cavities will enhance the quantum steerability and Bell nonlocality. In the absence of cavity loss with $\mu_2 \neq 0$ (perfect cavity), we can observe the trapping phenomenon of quantum measures and the atoms state is always steerable with Bell nonlocality only in the large values of parameter β. In this context, the physical parameters act in a similar way on the quantum measures and when the Bell nonlocality's is satisfied, the quantum seteering inequality is violated for the atoms state. On the contrary, the presence of detuning effects can enhance the quantum measures and the atoms state can be steerable with Bell nonlocality more periods of time. From the obtained results, the EPR steering and nonlocality for the atoms state can be controlled during the time evolution through a proper choice of β and δ values.

Based on the study of Baumgratz *et al.* [22], the concept of L_1 norm is introduced for detecting the amount of coherence with respect to the off-diagonal elements of the density operator ρ^{12}. Mathematically, this coherence measure is defined as

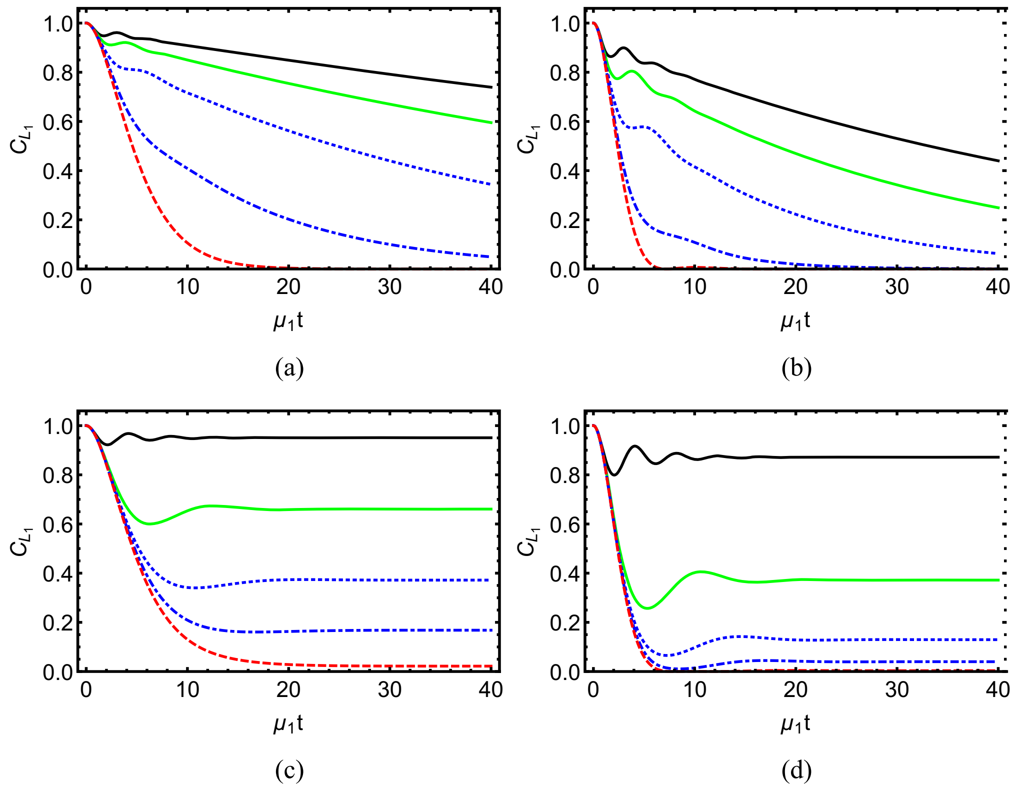

Figure 5: Time evolution of the quantum coherence C_{L_1} (equation (17)) for the two atoms as a function of time $\mu_1 t$ for various β values with $\delta = 0$ when the atoms start from a Bell state $\rho_{A-A}(0) = 1/2(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 01|)$. Labels (a), (b), (c), and (d) are for $\alpha = 0.24/0.5\mu_2 = 0.24\mu_1$, $\alpha = 0.4/0.5\mu_2 = 0.4\mu_1$, $\alpha = 0.24\mu_1$ with $\mu_2 = 0$, and $\alpha = 0.4\mu_1$ with $\mu_2 = 0$, respectively. For the case of (a) and (b): black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\beta = 2\mu_1$, $\beta = 1.5\mu_1$, $\beta = 1\mu_1$, $\beta = 0.5\mu_1$, and $\beta = 0$, respectively. For the case of (c) and (d): black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\beta = 1.5\mu_1$, $\beta = 0.5\mu_1$, $\beta = 0.3\mu_1$, $\beta = 0.2\mu_1$, and $\beta = 0.1\mu_1$, respectively.

$$C_{L_1} = \sum_{i,j} |\rho^{ij}_{A-A}| \quad \text{with } i \neq j. \tag{17}$$

The measure of coherence corresponding to the atom state is displayed in Figures 5 and 6 versus the time $\mu_1 t$ for various β and δ values in the presence and absence of the decay rate effect. Generally, the L_1 norm of quantum coherence is first decreased from its maximal value as the time $\mu_1 t$ increases. In the existence of cavity loss, the increase in the interaction between the cavities results in an enhancement in the amount of coherence. Additionally, we obtain that as the parameter β increases, coherence decays more slowly. In the absence of cavity loss, we can observe that coherence saturates at different maximum values for various β values. On the contrary, the increase of the parameter δ accompanied by an enhancement in the amount of coherence. As a result, the possibility for the control and preservation of L_1 norm of quantum coherence may occur through a proper choice of parameters β and δ with respect to the decay rate effect.

3 Conclusion

In this work, we have investigated the robustness and control of quantum steering, Bell nonlocality, and coherence in a bipartite atomic system where each atom is confined within a cavity that interacts with another cavity field. By systematically analyzing the quantum dynamics under decoherence effects, we demonstrated how key system parameters – particularly inter-cavity coupling strength and detuning – govern the resilience and evolution of quantumness measures. Our results reveal that quantum steering, nonlocality, and coherence can be preserved and even enhanced through optimal tuning of system parameters, allowing significant levels of quantumness measures to persist over extended timescales. Specifically, increasing the inter-cavity interaction strength was found to delay the degradation of quantum resources, while an appropriate choice of detuning parameters further improved coherence, nonlocality, and steering, effectively

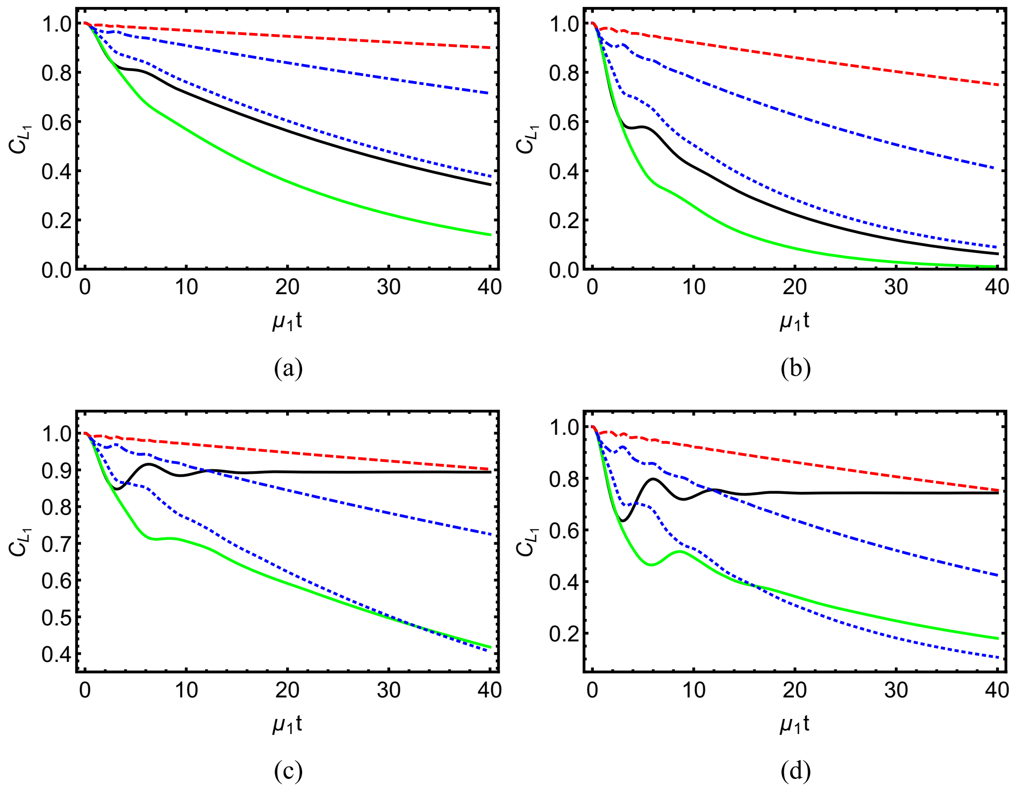

Figure 6: Time evolution of the quantum coherence C_{L_1} (equation (17)) for the two atoms as a function of time $\mu_1 t$ for various δ values with $\beta = \mu_1$ when the atoms start from a Bell state $\rho_{A-A}(0) = 1/2(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 01|)$. Labels (a), (b), (c), and (d) are for $\alpha = 0.24/0.5\mu_2 = 0.24\mu_1$, $\alpha = 0.4/0.5\mu_2 = 0.4\mu_1$, $\alpha = 0.24\mu_1$ with $\mu_2 = 0$, and $\alpha = 0.4\mu_1$ with $\mu_2 = 0$, respectively. Black line, green line, blue dotted line, blue dot-dashed line, and red dashed line are for $\delta = 0$, $\delta = 0.5\mu_1$, $\delta = 2\mu_1$, $\delta = 3\mu_1$, and $\delta = 5\mu_1$, respectively.

mitigating the impact of decoherence. A significant outcome of our study is that, in the ideal cavity limit, where cavity losses are minimized, the system retains robust quantum correlations, reinforcing the feasibility of coherence protection during the quantum dynamics. Furthermore, our findings demonstrate that EPR steering and Bell nonlocality exhibit similar dynamical behaviors under certain parameter conditions, indicating their interdependence in non-Markovian environments. Additionally, we showed that quantum coherence plays a crucial role in sustaining nonlocal correlations, and its controlled enhancement via cavity coupling provides a means for long-term coherence preservation. These findings offer valuable insights into the fundamental mechanisms governing quantum resource dynamics in open quantum systems, with direct implications for quantum information processing and secure quantum communication technologies. The ability to suppress decoherence and sustain quantum correlations is essential for the implementation of high-fidelity quantum operations, quantum state transfer, and distributed quantum networks. Future research should focus on extending these results to multipartite and high-dimensional quantum systems, exploring experimental realizations in cavity QED, superconducting circuits, or trapped-ion platforms, and developing advanced control techniques, including feedback optimization and adaptive tuning strategies, to further enhance quantum coherence, nonlocality and steering in realistic environments. These directions will contribute to the development of scalable, noise-resilient quantum technologies for future applications in quantum computing and quantum communication networks.

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Data availability statement: All data generated or analyzed during this study are included in this published article.

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