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Research Article

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Bifurcation, chaotic behavior, and traveling wave solutions for the fractional (4+1)-dimensional Davey–Stewartson–Kadomtsev–Petviashvili model

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Abstract: This article investigates the traveling wave solution of the fractional (4+1)-dimensional Davey–Stewartson–Kadomtsev–Petviashvili model by using the complete discriminant system method. These solutions not only include rational function solutions, trigonometric function solutions, but also Jacobian function solutions. In order to illustrate the propagation of these solutions in the field of nonlinear optics and water wave models, some three-dimensional, two-dimensional, and contour maps are drawn. Meanwhile, the phase portrait of two-dimensional dynamical systems and its perturbation systems are studied using the planar dynamical system analysis method. By drawing phase diagrams, it is easy to observe the stability, periodicity, and chaotic behavior of two-dimensional dynamical systems through geometric visualization, which can also provide strong basis for researchers to design corresponding control systems.

Keywords: truncated M-fractional derivative, complete discriminant system, planar dynamical system, bifurcation, periodic disturbance

1 Introduction

The fractional (4+1)-dimensional Davey–Stewartson–Kadomtsev–Petviashvili (DSKP) model is described as follows [1]:

$$\begin{aligned}&4D_{M,xt}^{2\alpha,\Upsilon}H - D_{M,xxxy}^{4\alpha,\Upsilon}H + D_{M,xyyy}^{4\alpha,\Upsilon}H + 12D_{M,x}^{\alpha,\Upsilon}H \cdot D_{M,y}^{\alpha,\Upsilon}H\\&+ 12H \cdot D_{M,xy}^{2\alpha,\Upsilon}H - 6D_{M,zw}^{2\alpha,\Upsilon}H = 0,\end{aligned} \tag{1.1}$$

where $H = H(x, y, z, w, t)$ is a real-valued function. This equation is a new equation synthesized by the ancient Greek mathematician Fokas by combining the integrable Kadomtsev–Petviashvili (KP) equation and Davey–Stewartson (DS) equation. This equation is an important mathematical model in the fields of nonlinear optics and applied science, such as shallow water wave models, nonlinear phenomena in plasma physics, and ocean wave phenomena. Currently, research on Eq. (1.1) mainly focuses on its traveling wave solution. Alsharidi and Junjuan [1] obtained some soliton solutions of Eq. (1.1) using two different methods. The research in previous studies [2–6] mainly focuses on the study of traveling wave solutions for (4+1)-dimensional DSKP equation. For example, Ahmad *et al.* [2] considered the (4+1) DSKP equation using $(\frac{\mathfrak{g}'}{\mathfrak{g}'+\mathfrak{g}+\mathbb{A}})$-expansion method. El-Shorbagy *et al.* [3] obtained the solitary wave solutions of (4+1)-dimensional DSKP equation using the modified Sardar sub-equation method, the improved $\mathfrak{F}$-expansion method. Ahmad *et al.* [4] studied the exact solutions of the DSKP equation using the modified tanh method along associated with the Ricatti equation. Rehman *et al.* [5] obtained the solitary wave solution of the (4+1)-dimensional DSKP equation using Sardar sub-equation method, the Kudryashov's method, and the $(\frac{1}{\vartheta(\zeta)}, \frac{\vartheta'(\zeta)}{\vartheta(\zeta)})$ method, respectively. Rabie *et al.* [6] obtained the soliton solutions of the (4+1)-dimensional DSKP equation using the modified extended mapping method. In recent years, with the rapid development of fractional-order derivatives [7,8], various types of fractional-order derivatives have been proposed, and many experts and scholars are studying the traveling wave solutions [9–12] and dynamic properties [13–16] of fractional partial differential equations. Particularly, the study of fractional (4+1)-dimensional DSKP equation is still a hot topic in current

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research, and there are still many very important problems that need to be solved urgently. This article will focus on the study of traveling wave solution and dynamic behavior of fractional (4+1)-dimensional DSKP equation.

The remaining sections are described as follows: in Section 2, the phase portraits and chaotic behaviors of two-dimensional dynamical system and its perturbation system are discussed. In Section 3, the solitary wave solutions of the fractional (4+1)-dimensional DSKP equation are constructed. In Section 4, three-dimensional, two-dimensional, and contour maps of some solutions of Eq. (1.1) are drawn. In Section 5, the solution obtained in this article is discussed in relation to existing solutions. In Section 6, a brief conclusion is given.

2 Bifurcation and chaotic behavior

Definition 2.1. [17] Let $H:[0,+\infty)\to(-\infty,+\infty)$. For $0<\alpha\leq 1$, a truncated M-fractional derivative is denoted as

$$D_{M,x}^{\alpha,\Upsilon}H(x)=\lim_{\tau\to 0}\frac{H(xE_{\Upsilon}(\tau x^{1-\alpha}))-H(x)}{\tau},\quad 0<\alpha\leq 1,$$
$$\Upsilon\in(0,\infty),$$

where $E_{\Upsilon}(\cdot)$ is a truncated Mittag–Leffler function. The constant α stands for a fractional-order derivative.

Remark 2.2. In definition 2.1, $E_{\Upsilon}(\cdot)$ is a truncated Mittag–Leffler function defined as [18]

$$E_{\Upsilon}(z)=\Sigma_j^i\frac{x^j}{\Gamma(\gamma j+1)},\quad x\in[0,+\infty).$$

First, the wave transformation is given as follows:

$$\begin{aligned}&H(x,y,z,w,t)=H(\xi),\\&\xi=\frac{\Gamma(1+\gamma)}{\alpha}(ax^{\alpha}+by^{\alpha}+cz^{\alpha}+\tau w^{\alpha}-\lambda t^{\alpha}),\end{aligned}\tag{2.1}$$

where a, b, c, τ, and λ represent the real constants.

Substituting Eq. (2.1) into Eq. (1.1) and integrating it twice, such that the first integration constant is zero and the second integration constant is non-zero, yields an ordinary differential equation

$$ab(b^2-a^2)H''+6abH^2-H(4a\lambda+6c\tau)=C_1,\tag{2.2}$$

where C_1 is an integral constant.

When $ab^3-a^3b\neq 0$, the two-dimensional dynamic system of Eq. (2.2) can be rewritten as

$$\begin{cases}\dfrac{\mathrm{d}H}{\mathrm{d}\xi}=z,\\ \dfrac{\mathrm{d}z}{\mathrm{d}\xi}=\mu_2H^2+\mu_1H+\mu_0,\end{cases}\tag{2.3}$$

with Hamiltonian function

$$F(H,z)=\frac{1}{2}z^2-\frac{1}{3}\mu_2H^3-\frac{1}{2}\mu_1H^2-\mu_0H=f_0,\tag{2.4}$$

where $\mu_2=-\frac{6}{b^2-a^2}$, $\mu_1=\frac{4a\lambda+6c\tau}{ab(b^2-a^2)}$, $\mu_0=\frac{C_1}{ab(b^2-a^2)}$. f_0 is an integral constant.

Here, we suppose that $G(H)=\mu_2H^2+\mu_1H+\mu_0$ and $G'(H)=2\mu_2H+\mu_1$. Furthermore, suppose that $\mathbf{M}(H_j,0)=\begin{pmatrix}0&1\\2\mu_2H_j+\mu_1&0\end{pmatrix}$ is the coefficient matrix of (2.3) at the equilibrium point, where H_j $(j=1,2,3)$ is the root of equation $G(H_j)=0$. Thus, we have

$$\mathbf{det}(\mathbf{M}(H_j,0))=-G'(H_j)=-(2\mu_2H_j+\mu_1).\tag{2.5}$$

If $\mu_1^2-4\mu_0\mu_2>0$ and $\mu_2\neq 0$, we can obtain that the equation $G(H)=0$ has two unequal real roots denoted as $H_1=\frac{-\mu_1-\sqrt{\mu_1^2-4\mu_0\mu_2}}{2\mu_2}$ and $H_2=\frac{-\mu_1+\sqrt{\mu_1^2-4\mu_0\mu_2}}{2\mu_2}$. If $\mu_1^2-4\mu_0\mu_2=0$ and $\mu_2\neq 0$, we can obtain that the equation $G(H)=0$ has a real root denoted as $H_3=-\frac{\mu_1}{2\mu_2}$. According to the theory of planar dynamical systems [19–21], we can draw the phase diagram of system (2.3) for different parameters. When $\mu_1^2-4\mu_0\mu_2>0$, we obtain that $G'(H_1)>0$, then we have $(H_1,0)$ is the saddle point. When $\mu_1^2-4\mu_0\mu_2>0$, we have $G'(H_2)<0$; thus, $(H_2,0)$ is the center point. When $\mu_1^2-4\mu_0\mu_2=0$, $(H_3,0)$ is the degenerate saddle point. Using Maple software, we have drawn the two-dimensional phase portraits of Eqs (2.3), as shown in Figure 1.

Next, we directly add a periodic disturbance to the second equation of system (2.3)

$$\begin{cases}\dfrac{\mathrm{d}H}{\mathrm{d}\xi}=z,\\ \dfrac{\mathrm{d}z}{\mathrm{d}\xi}=\mu_2H^2+\mu_1H+\mu_0+A\sin(k\xi),\end{cases}\tag{2.6}$$

where A and k are the constants (Figure 2).

3 Traveling wave solutions of Eq. (1.1)

According to the polynomial complete discriminant system method proposed by Professor Liu [22], we can obtain the single traveling wave solution of Eq. (1.1) in this section. In recent years, many experts [23–25] have utilized Professor Liu's complete discriminant system method to construct single traveling wave solutions for nonlinear partial differential equations.

Multiplying both sides of Eq. (2.2) by H' integral simultaneously yields

$$(H')^2 = \frac{4}{a^2 - b_2}H^3 + \frac{2(2a\lambda + 3c\tau)}{ab(b^2 - a^2)}H^2 + \frac{2C_1}{ab(b^2 - a^2)}H + \frac{2C_2}{ab(b^2 - a^2)}. \tag{3.1}$$

Here, we suppose that

$$\Psi = \left(\frac{4}{a^2 - b^2}\right)^{\frac{1}{3}}H, \quad \chi_2 = \frac{2(2a\lambda + 3c\tau)}{ab(b^2 - a^2)}\left(\frac{4}{a^2 - b^2}\right)^{-\frac{2}{3}},$$
$$\chi_1 = \frac{2C_1}{ab(b^2 - a^2)}\left(\frac{4}{a^2 - b^2}\right)^{-\frac{1}{3}}, \quad \chi_0 = \frac{2C_2}{ab(b^2 - a^2)}. \tag{3.2}$$

Substituting Eq. (3.2) into Eq. (3.1), we have

$$(\Psi')^2 = \Psi^3 + \chi_2\Psi^2 + \chi_1\Psi + \chi_0, \tag{3.3}$$

here, let us assume that the third-order polynomial is $P(\Psi) = \Psi^3 + \chi_2\Psi^2 + \chi_1\Psi + \chi_0$, and the complete discriminative system of the polynomial is

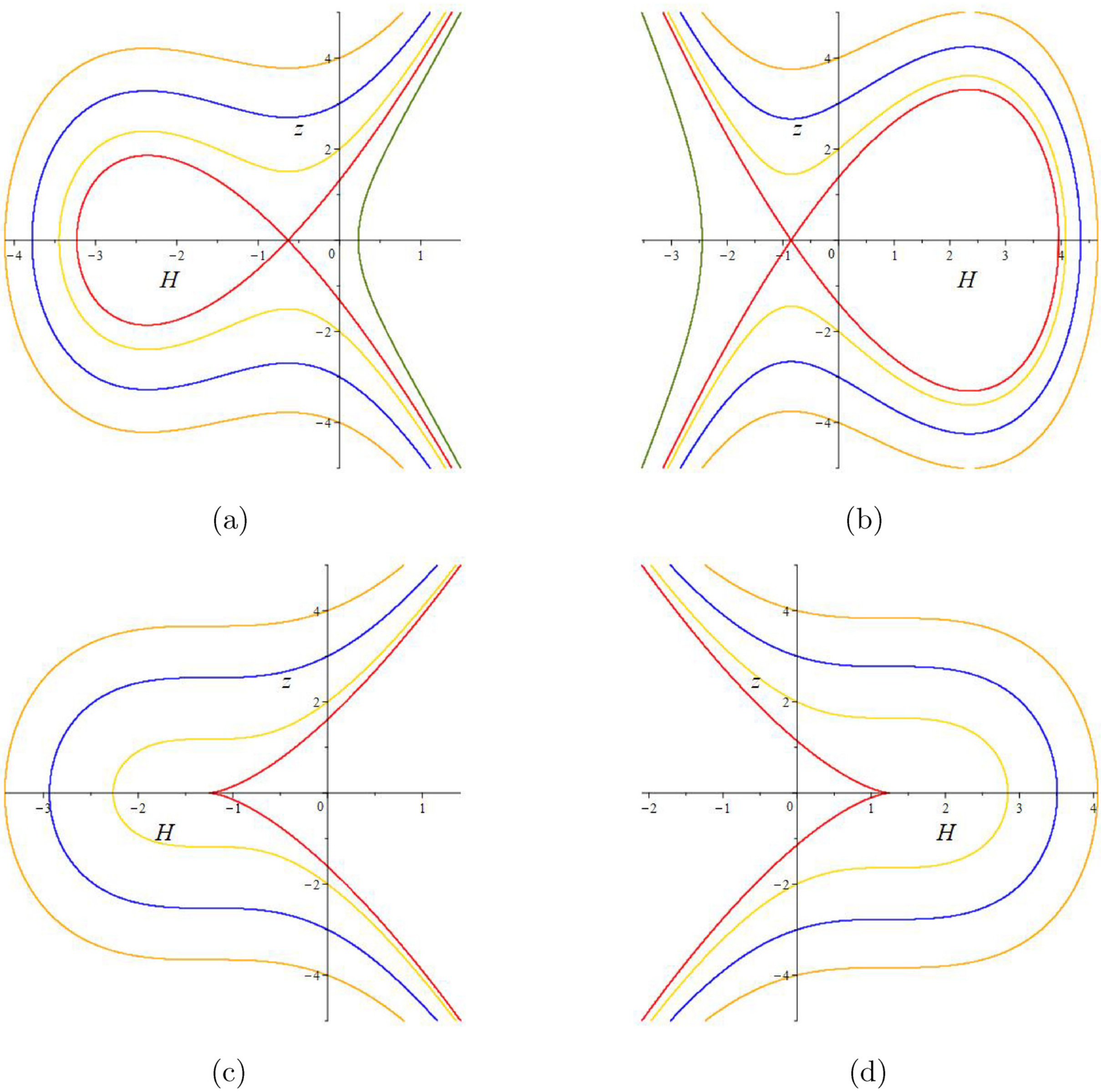


Figure 1: 2D phase portraits of Eq. (2.3). (a) $\mu_0 = 3, \mu_1 = 6, \mu_2 = 3$, (b) $\mu_0 = 2, \mu_1 = \frac{3}{2}, \mu_2 = -1$, (c) $\mu_0 = \frac{25}{8}, \mu_1 = 5, \mu_2 = 2$, (d) $\mu_0 - \frac{25}{16}, \mu_1 = \frac{5}{2}, \mu_2 = -1$.

$$\triangle = -27\left(\frac{2\chi_2^3}{27} + \chi_0 - \frac{\chi_1\chi_2}{3}\right)^2 - 4\left(\chi_1 - \frac{\chi_2^2}{3}\right)^3,$$
$$D_1 = \chi_1 - \frac{\chi_2^2}{3}. \tag{3.4}$$

Therefore, the integral expression of Eq. (3.3) is

$$\int \frac{d\Psi}{\sqrt{\Psi^3 + \chi_2\Psi^2 + \chi_1\Psi + \chi_0}} = \pm\left(\frac{4}{a^2 - b_2}\right)^{\frac{1}{3}}(\xi - \xi_0), \tag{3.5}$$

where ξ_0 is an integral constant.

Case 1. $\triangle = 0$, $D_1 < 0$, namely, $-27(\frac{2\chi_2^3}{27} + \chi_0 - \frac{\chi_1\chi_2}{3})^2 - 4(\chi_1 - \frac{\chi_2^2}{3})^3 = 0$, $\chi_1 - \frac{\chi_2^2}{3} < 0$. At this point, the equation $P(\Psi) = 0$ has a double real and a single real root, i.e., $P(\Psi) = (\Psi - \beta_1)^2(\Psi - \beta_2)$, where $\beta_1 \neq \beta_2$.

When $\Psi > \beta_2$, the integral Eq. (3.5) can be rewritten as

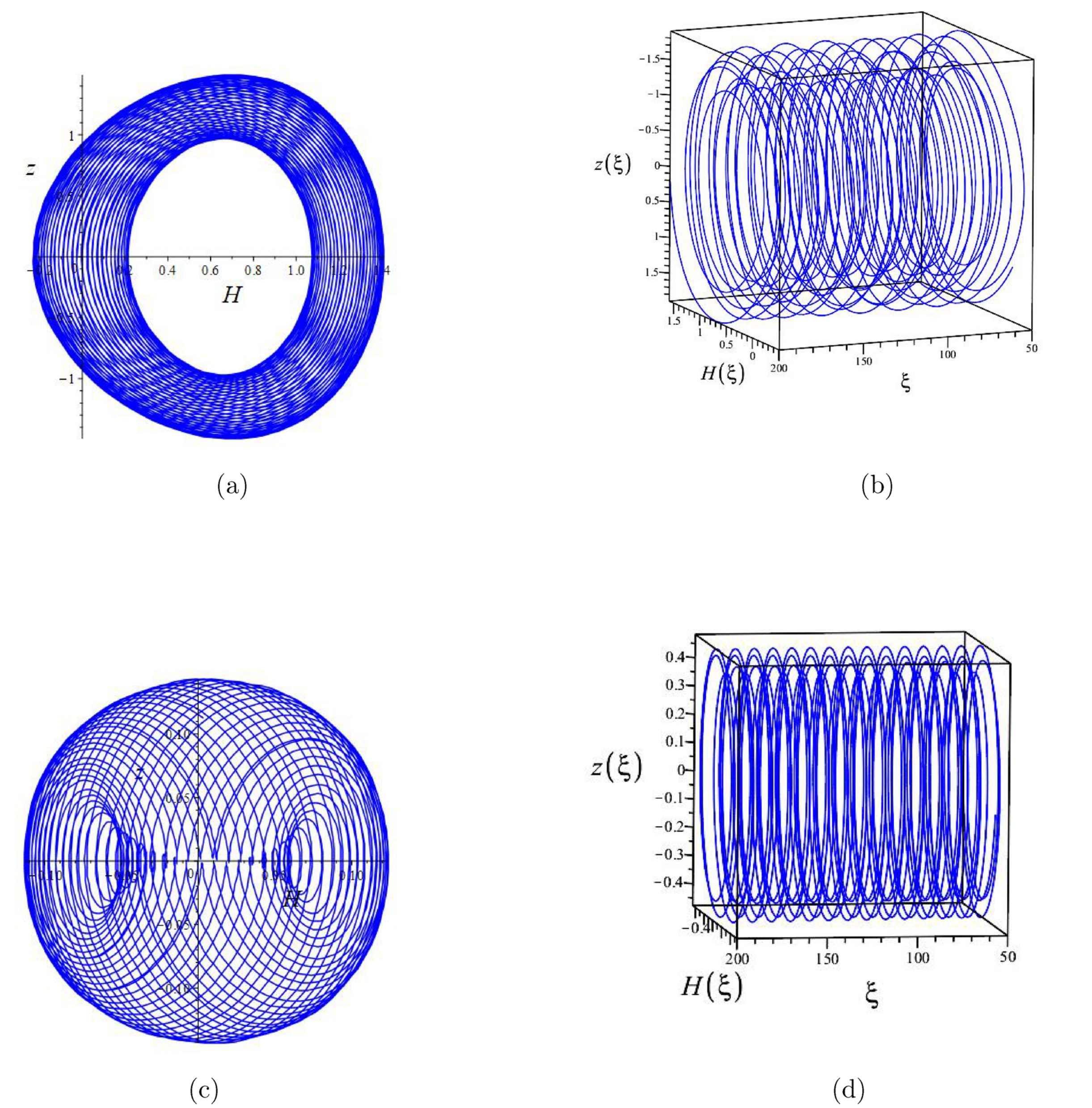


Figure 2: 2D and 3D phase portraits of Eq. (2.3). (a) $\mu_0 = 2$, $\mu_1 = -\frac{3}{2}$, $\mu_2 = -2$, $A = 0.38$, $k = 1.35$, (b) $\mu_0 = 2$, $\mu_1 = -\frac{3}{2}$, $\mu_2 = -2$, $A = 0.38$, $k = 1.35$, (c) $\mu_0 = -4$, $\mu_1 = -5$, $\mu_2 = 2$, $A = 0.38$, $k = 0.6$, (d) $\mu_0 = -4$, $\mu_1 = -5$, $\mu_2 = 2$, $A = 0.38$, $k = 0.6$.

$$\pm\left(\frac{4}{a^2-b_2}\right)^{\frac{1}{3}}(\xi-\xi_0) = \int\frac{d\Psi}{(\Psi-\beta_1)\sqrt{\Psi-\beta_2}} = \begin{cases}\frac{1}{\sqrt{\beta_1-\beta_2}}\ln\left|\frac{\sqrt{\Psi-\beta_2}-\sqrt{\beta_1-\beta_2}}{\sqrt{\Psi-\beta_2}+\sqrt{\beta_1-\beta_2}}\right|, & (\beta_1>\beta_2),\\ \frac{2}{\sqrt{\beta_2-\beta_1}}\arctan\sqrt{\frac{\Psi-\beta_2}{\beta_2-\beta_1}}, & (\beta_1<\beta_2).\end{cases} \tag{3.6}$$

From Eqs (2.1) and (3.6), we obtain

$$H_1(x,y,z,w,t) = \left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}}\left\{(\beta_1-\beta_2)\tanh^2\left[\frac{\sqrt{\beta_1-\beta_2}}{2}\left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}} \times\left(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha-\lambda t^\alpha)-\xi_0\right)\right] + \beta_2\right\},\ \beta_1>\beta_2, \tag{3.7}$$

$$H_2(x,y,z,w,t) = \left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}}\left\{(\beta_1-\beta_2)\cdot\coth^2\left[\frac{\sqrt{\beta_1-\beta_2}}{2}\left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}} \times\left(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha-\lambda t^\alpha) - \xi_0\right)\right] + \beta_2\right\}, \beta_1>\beta_2, \tag{3.8}$$

$$H_3(x,y,z,w,t) = \left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}}\{(-\beta_1+\beta_2) \cdot\tan^2\left[\frac{\sqrt{-\beta_1+\beta_2}}{2}\left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}} \times\left(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha-\lambda t^\alpha) - \xi_0\right)\right] + \beta_2\right\}, \quad \beta_1<\beta_2. \tag{3.9}$$

Case 2. $\triangle = 0$, $D_1 = 0$, namely, $-27(\frac{2\chi_2^3}{27}+\chi_0-\frac{\chi_1\chi_2}{3})^2 - 4(\chi_1-\frac{\chi_2^2}{3})^3 = 0$, $\chi_1-\frac{\chi_2^2}{3}=0$. At this point, the equation $P(\Psi)=0$ has a triple real root, i.e., $P(\Psi)=(\Psi-\beta_3)^3$; and then, we obtain traveling wave solution of Eq. (1.1)

$$H_4(x,y,z,w,t) = 4\left(\frac{4}{a^2-b^2}\right)^{-\frac{2}{3}}\left[\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha + \tau w^\alpha-\lambda t^\alpha)-\xi_0\right]^{-2}+\beta_3. \tag{3.10}$$

Case 3. $\triangle > 0$, $D_1 < 0$, namely, $-27(\frac{2\chi_2^3}{27}+\chi_0-\frac{\chi_1\chi_2}{3})^2 - 4(\chi_1-\frac{\chi_2^2}{3})^3 > 0$, $\chi_1-\frac{\chi_2^2}{3}<0$. At this point, the equation $P(\Psi)=0$ has three different real roots, i.e., $P(\Psi) = (\Psi-\beta_4)(\Psi-\beta_5)(\Psi-\beta_6)$, where $\beta_4<\beta_5<\beta_6$.

When $\beta_4<\Psi<\beta_6$, we make the variable replacement $\Psi=\beta_4+(\beta_5-\beta_4)\sin^2\vartheta$. From Eq. (3.6), we have

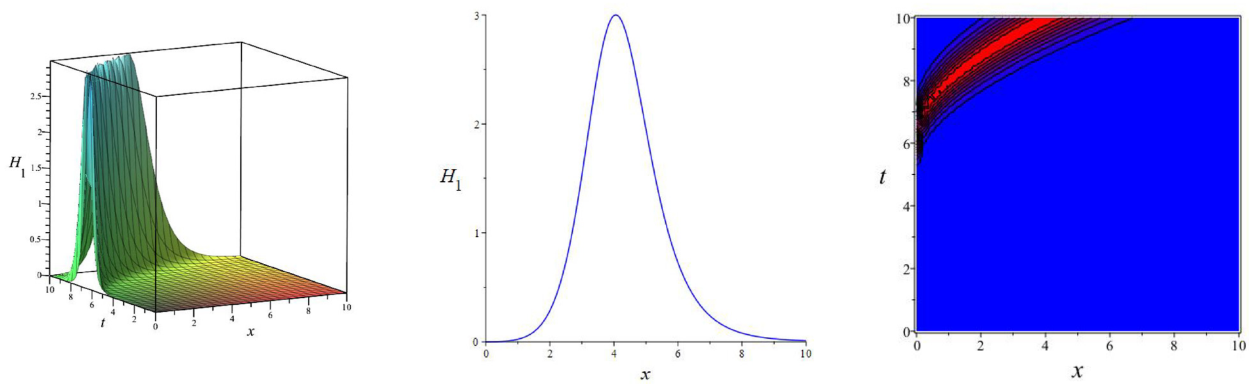
$$\pm\left(\frac{4}{a^2-b_2}\right)^{\frac{1}{3}}(\xi-\xi_0) = \int\frac{d\Psi}{\sqrt{P(\Psi)}} = \int\frac{2(\beta_5-\beta_4)\sin\vartheta\cos\vartheta d\vartheta}{\sqrt{\beta_6-\beta_4}(\beta_5-\beta_4)\sin\vartheta\cos\vartheta\sqrt{1-m^2\sin^2\vartheta}} = \frac{2}{\sqrt{\beta_6-\beta_4}}\int\frac{d\vartheta}{\sqrt{1-m^2\sin^2\vartheta}}, \tag{3.11}$$

where $m^2=\frac{\beta_5-\beta_4}{\beta_6-\beta_4}$.

$$H_5(x,y,z,w,t) = \left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}}\left\{\beta_4+(\beta_5-\beta_4)\mathbf{sn}^2\left[\frac{\sqrt{\beta_6-\beta_4}}{2} \times\left(\frac{4}{a^2-b^2}\right)^{\frac{1}{3}}\left(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha - \lambda t^\alpha)-\xi_0\right), m\right]\right\}. \tag{3.12}$$

When $\Psi>\beta_6$, we make the variable replacement $\Psi=\frac{-\beta_5\sin^2\vartheta+\beta_6}{\cos^2\vartheta}$. From Eq. (3.6), we also obtain the solution of Eq. (1.1)

$$H_6(x,y,z,w,t) = \left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}} \times\left\{\frac{\beta_6-\beta_5\mathbf{sn}^2\left[\frac{\sqrt{\beta_6-\beta_4}}{2}\left(\frac{4}{a^2-b^2}\right)^{\frac{1}{3}}(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha-\lambda t^\alpha)-\xi_0), m\right]}{\mathbf{cn}^2\left[\frac{\sqrt{\beta_6-\beta_4}}{2}\left(\frac{4}{a^2-b^2}\right)^{\frac{1}{3}}(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha-\lambda t^\alpha)-\xi_0), m\right]}\right\}. \tag{3.13}$$



(a) (b) (c)

Figure 3: Graphics of H_1 at $a=2, b=\sqrt{5}, c=4, \chi_2=3, \chi_1=\chi_0=0, \lambda=3, \tau=\sqrt{5}-1, \gamma=\frac{1}{2}, \alpha=\frac{1}{2}, y=z=w=1$: (a) 3D surface, (b) 2D surface at $t=10$, and (c) contour plot.

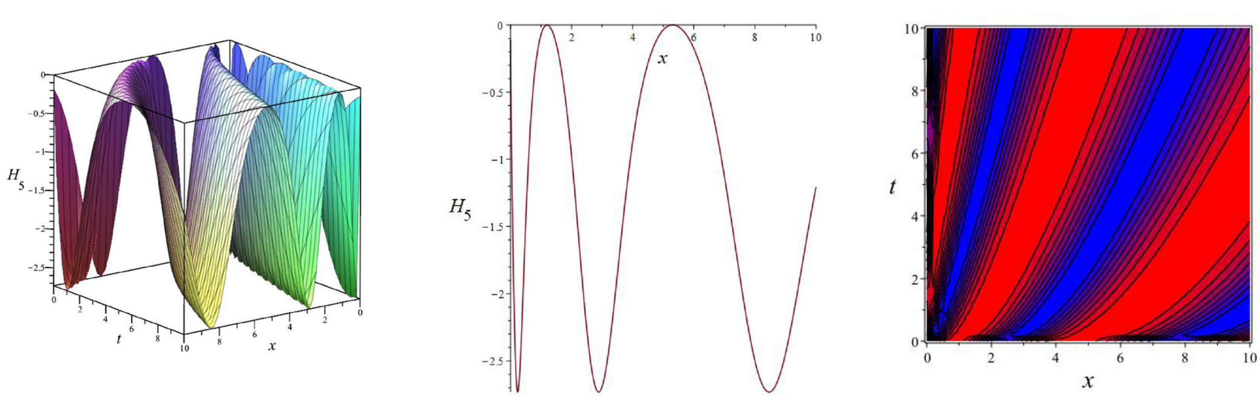
Case 4. $\triangle<0$, namely, $-27(\frac{2\chi_2^3}{27}+\chi_0-\frac{\chi_1\chi_2}{3})^2-4(\chi_1-\frac{\chi_2^2}{3})^3<0$. At this point, the equation $P(\Psi)=0$ has one real root, i.e., $P(\Psi)=(\Psi-\beta_7)(\Psi^2+p\Psi+q)$, where $p^2-4q<0$.

When $\Psi>\beta_7$, we make the variable replacement $\Psi=\sqrt{\beta_7^2+p\beta_7+q}\tan^2\frac{\vartheta}{2}$. From Eq. (3.6) and the definition of Jacobian elliptic cosine function, we have

$$
\begin{aligned}
&H_7(x,y,z,w,t)\\
&=\left(\frac{4}{a^2-b^2}\right)^{-\frac{1}{3}}\left\{\beta_7+\frac{2\sqrt{\beta_7^2+p\beta_7+q}}{1+\mathbf{cn}^2\left[(\beta_7^2+p\beta_7+q)^{\frac{1}{4}}\left(\frac{4}{a^2-b^2}\right)^{\frac{1}{3}}\left(\frac{\Gamma(1+\gamma)}{\alpha}(ax^\alpha+by^\alpha+cz^\alpha+\tau w^\alpha-\lambda t^\alpha)-\xi_0\right),m\right]}\right.\\
&\quad\left.-\sqrt{\beta_7^2+p\beta_7+q}\right\}.
\end{aligned}
\tag{3.14}
$$

4 Numerical simulation

In this section, we plotted the three-dimensional, two-dimensional, and contour plots of the two solutions H_1 and H_5 of Eq. (1.1) by controlling different parameters, as shown in Figures 3 and 4. From Figure 3, it can be seen that the solution of Eq. (1.1) is a hyperbolic function solution H_1 and a bright soliton solution. From Figure 4, it can be concluded that the solution H_5 of Eq. (1.1) is a periodic wave



(a) (b) (c)

Figure 4: Graphics of H_5 at $a=\sqrt{5}, b=2, c=-\frac{7}{8}, \chi_2=3, \chi_1=0, \chi_0=-2, \lambda=1, \tau=\sqrt{5}, \gamma=\frac{1}{2}, \alpha=\frac{1}{2}, y=z=w=1$: (a) 3D surface, (b) 2D surface at $t=10$, and (c) contour plot.

solution. Figures 3(c) and 4(c) are contour maps indicating the basic contour of the understanding.

5 Discussion of results

Alsharidi *et al.* [1] obtained the exact solution of the DSKP equation using both the unified technique and modified extended tanh-expansion function technique, respectively. Compared with Alsharidi *et al.* [1], this article not only constructs the traveling wave solution of the DSKP equation using the fully discriminative system method, but also obtains the dynamic behavior of the DSKP equation using the planar dynamical system method. The solutions obtained in this article not only include common rational function solutions and trigonometric function solutions, but also more general Jacobian function solutions. In order to obtain the dynamic behavior of the equation more easily, this article also draws three-dimensional and two-dimensional phase diagrams of the two-dimensional dynamical system and its perturbation system. By analyzing the phase diagrams, it is easy to obtain the dynamic behavior of the two-dimensional dynamical system.

6 Conclusion

In this article, we obtained the traveling wave solution of the fractional (4+1)-dimensional DSKP equation. We not only discussed the planar phase portraits of the two-dimensional dynamical system of Eq. (1.1). Moreover, we have added periodic perturbations to the two-dimensional dynamical system and plotted the two-dimensional and three-dimensional phase prrtraits of the perturbed two-dimensional dynamical system. Based on Professor Liu's complete discriminant system method, we obtained the Jacobian function solution, trigonometric function solution, and hyperbolic function solution of Eq. (1.1). We also simulated the obtained solutions by drawing three-dimensional, two-dimensional, and contour maps. In future work, we will continue to study soliton solutions of fractional-order partial differential equations, such as multi-soliton solutions or experimental validations.

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Author contributions: Zhao Li: writing – orginal draft; Yueyong Jiang: supervision. All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

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