

Research Article

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Nonclassical correlation dynamics of Heisenberg XYZ states with (x, y) -spin-orbit interaction, x -magnetic field, and intrinsic decoherence effects

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Abstract: In quantum information, it is important to recognize the effects of additional interactions, such as spin-orbit interactions, on the quantum information resources of two-qubit Heisenberg states. Therefore, we study the nonlocal correlation dynamics affected by the intrinsic decoherence of the spin-spin-Heisenberg-XYZ interaction, which is supported by spin-orbit interactions (Dzyaloshinsky-Moriya) of the x and y directions together. The two spins are coupled to an external inhomogeneous magnetic field (EIMF) in the x -direction. We investigate and compare the nonclassical correlation dynamics of local quantum Fisher information, local quantum uncertainty, and log-negativity. The results show that spin-spin and spin-orbit interactions have a high capability to enhance non-local correlations in the presence of an external magnetic field. The enhanced non-local correlation can be further improved by strengthening the spin-spin and spin-orbit interactions, as well as by increasing the EIMF's inhomogeneity and uniformity, which increases the amplitudes and fluctuations of the generated non-local correlation oscillations. The degradation of non-local correlations due to intrinsic decoherence can be controlled by spin-spin interactions. These degradation correlations can be enhanced by

increasing the intensities of spin-orbit interactions, as well as by increasing the EIMF's inhomogeneity and uniformity.

Keywords: local quantum Fisher information, magnetic field, spin-orbit interaction

1 Introduction

Among the various quantum systems proposed for implementing quantum information and computation [1,2], superconducting circuits, trapped ions, and semiconductor quantum dots are crucial techniques for realizing quantum bits (qubits). Based on electron spins trapped in quantum dots, a quantum computer protocol has been initially proposed [3–5]. The electron, having a spin of ($\frac{1}{2}$), is the simplest natural qubit. Recently, quantum computation with electron spins (as a single-spin-qubit geometric gate) has been realized in quantum dots [6,7]. Due to electron tunneling from one dot to another, spin-spin coupling and spin-orbit coupling interactions between two qubits can be realized by considering a two-qubit system represented by two electrons in coupled quantum dots. Therefore, Heisenberg XYZ models describing spin-spin interactions are among the important proposed qubit systems. Two-qubit Heisenberg XYZ models have been realized in various systems, including bosonic atoms inside an optical lattice [8], trapped ions [9], superconductor systems [10], and linear molecules [11]. These models have been updated to include the first order of spin-orbit coupling known as Dzyaloshinsky-Moriya interactions [12–14], realized through an antisymmetric superexchange interaction in La_2CuO_4 [15], and the second order of spin-orbit coupling known as the Kaplan-Shekhtman-Entin-Wohlman-Aharony interaction [16]. Additionally, Heisenberg XYZ models have been updated to include dipole-dipole interactions [17] and inhomogeneous external magnetic fields [18,19]. Recently, the spin-orbit interaction was experimentally realized in two magnetic cobalt layers [20] (with Co/Ag/Co system). Moreover, the spin-orbit interaction was

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experimentally implemented in the prototypical ferromagnet by polarized neutrons [21] and in ferroelectrics and antiferroelectrics [22] systems as well as in the epitaxial Ni/Cu(001) system [23]. The Heisenberg XYZ qubit models have shown several potential applications in teleportation, [24], quantum dense coding [25], thermodynamics [26], and quantum correlation generations [27].

Exploring two-qubit information dynamics in various proposed qubit systems, based on different types of non-local correlations (NLCs) (such as entanglement and quantum discord), is one of the most critical research fields in implementing quantum information and computation [28]. Quantum entanglement (QE), quantified by measures such as entropy [29], concurrence [30], negativity, and log-negativity [31], is a significant type of two-qubit NLCs [32,33]. It has a wide range of applications in quantum information fields, including quantum computation, teleportation [34,35], quantum optical memory [36], and quantum key distribution [37]. After implementing quantum discord as another type of qubits' NLCs beyond entanglement [38], several quantifiers have been introduced to address other NLCs [39] using Wigner–Yanase (WY) skew information [40] and quantum Fisher information (QFI) [41]. WY-skew-information minimization (local quantum uncertainty, LQU) [42] and WY-skew-information maximization (uncertainty-induced nonlocality) [43] have been introduced to quantify other NLCs beyond entanglement. Additionally, the minimization of QFI (local quantum Fisher information, LQFI) has been used to implement another two-qubit NLC [44,45]. LQU has a direct connection to LQFI [46,47], establishing more two-qubit NLCs in several proposed qubit systems [48], such as hybrid–spin systems (under random noise [49] and intrinsic decoherence [50]), two-coupled double quantum dots [51], the mixed-spin Heisenberg model [52], and the Heisenberg system [53].

The information dynamics of two-spin Heisenberg XYZ states have been investigated using the Milburn intrinsic decoherence model [54]. This includes studies on entanglement teleportation based on the Heisenberg XYZ chain [24,55], the LQFI of Heisenberg XXX states beyond IEMF effects [56], and quantum correlations of concurrence and LQU [57]. Previous works have focused on exploring the time evolution of the two-spin Heisenberg XYZ states' NLCs under limited conditions on spin–spin and spin–orbit interactions, as well as applied magnetic fields, to ensure residing quantum information resources of two-qubit X -states (having an X density matrix) [58–62].

Motivated by the aforementioned experimental evidence for realizations for the spin–orbit interaction having a high ability to support the generating NLCs, and the importance of general two-qubit Heisenberg states, this

study employs the Milburn intrinsic decoherence and Heisenberg XYZ models to explore the NLC dynamics of LQFI, LQU, and log-negativity for general two-qubit Heisenberg XYZ states with non- X density matrices, influenced by specific conditions on spin–spin and spin–orbit interactions, as well as applied magnetic fields.

The manuscript structure includes the Milburn intrinsic decoherence equation, the Heisenberg XYZ model, and its solution in Section 2. In Section 3, we introduce the definitions of the NLCs' quantifiers: LQFI, LQU, and LN. Section 4 presents the outcomes of the dependence of these quantifiers on the physical parameters. Our conclusions are provided in Section 5.

2 Heisenberg spin model

Here, the Milburn intrinsic decoherence and Heisenberg XYZ models are used to examine the capabilities embedded in spin–spin interaction supported by the spin–orbit (Dzyaloshinsky–Moriya) interactions in the x and y directions (described by the first-order of spin–orbit couplings D_x and D_y), to generate essential NLCs between the two spin qubits under the effects of the uniformity and the inhomogeneity of an applied external inhomogeneous magnetic field (EIMF). For two spins (each described by the upper $|1_k\rangle$ and lower $|1_k\rangle$ states, where $(k = A, B)$), the Hamiltonian of the Heisenberg XYZ model with a spin–orbit interaction and an applied EIMF (Figure 1) is written as

$$\hat{H} = \sum_{\alpha=x,y,z} J_{\alpha} \hat{\sigma}_A^{\alpha} \hat{\sigma}_B^{\alpha} + \sum_{k=A,B} \vec{B}_k \cdot \vec{\sigma}_k + \vec{D}_{AB} \cdot (\vec{\sigma}_A \times \vec{\sigma}_B), \quad (1)$$

where $\vec{\sigma}_k = (\hat{\sigma}_k^x, \hat{\sigma}_k^y, \hat{\sigma}_k^z)$ represents the vector of Pauli matrices of the k -spin, and $\vec{B}_k = (B_k^x, B_k^y, B_k^z)$ represents the vector of the external magnetic field applying on k -spin. In our work, we consider that the EIMF is applied

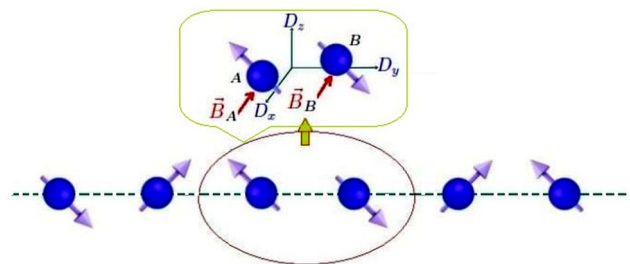


Figure 1: Diagram of a Heisenberg XYZ chain model, where two arbitrary spin–qubits (A and B) are selected with spin–orbit interaction vector $\vec{D}_{AB} = (D_x, D_y, D_z)$, and an EIMFs $\vec{B}_k$ in the x -direction.

only in the x -direction: $\vec{B}_k = (B_k^x, 0, 0)$, $B_A^x = B_m + b_m$, and $B_B^x = B_m - b_m$. Here, B_m and b_m represent the degree of the uniformity and the inhomogeneity of the applied EIMF, respectively. For the spin-orbit interaction vector $\vec{D}_{AB} = (D_x, D_y, D_z)$, we have $\vec{D}_{AB} \cdot (\vec{\sigma}_A \times \vec{\sigma}_B) = D_x \hat{C}_x + D_y \hat{C}_y + D_z \hat{C}_z$ with $\hat{C}_\alpha = \hat{\sigma}_A^{\alpha+1} \hat{\sigma}_B^{\alpha+2} - \hat{\sigma}_A^{\alpha+2} \hat{\sigma}_B^{\alpha+1}$ ($\alpha = x, y, z$). After considering only the spin-orbit interactions of the x and y directions $\vec{D}_{AB} = (D_x, D_y, 0)$ with an applied EIMF in the x direction, to support the spin-spin interaction in generating two-spin-qubit correlations, the considered Hamiltonian is written as

$$\begin{aligned} \hat{H} = & \sum_{i=x,y,z} J_i \hat{\sigma}_A^i \hat{\sigma}_B^i + D_x (\hat{\sigma}_A^y \hat{\sigma}_B^z - \hat{\sigma}_A^z \hat{\sigma}_B^y) \\ & + D_y (\hat{\sigma}_A^z \hat{\sigma}_B^x - \hat{\sigma}_A^x \hat{\sigma}_B^z) \\ & + (B_m + b_m) \hat{\sigma}_A^x + (B_m - b_m) \hat{\sigma}_B^x. \end{aligned} \quad (2)$$

In the two-spin-qubits basis: $\{|\psi_1\rangle = |1_A 1_B\rangle, |\psi_2\rangle = |1_A 0_B\rangle, |\psi_3\rangle = |0_A 1_B\rangle, |\psi_4\rangle = |0_A 0_B\rangle\}$, the two-spin system's Hamiltonian in (1) can read as a non- X matrix of

$$\hat{H} = \begin{pmatrix} J_z & \beta_- & \beta_+^* & J_x - J_y \\ \beta_-^* & -J_z & J_x + J_y & \beta_+ \\ \beta_+ & J_x + J_y & -J_z & \beta_-^* \\ J_x - J_y & \beta_+^* & \beta_- & J_z \end{pmatrix} \quad (3)$$

with $\beta_\pm = B_m \pm b_m + D_y + iD_x$. Generally, with a motion equation, this non- X Hamiltonian matrix generates two-qubit non- X states due to spin-spin interaction combined with x and y spin-orbit interactions. However, if we consider only the spin-orbit interaction in the z direction, $\vec{D} = (0, 0, D_z)$, as discussed in previous studies [58–62], the two-spin system's Hamiltonian (1) generates X -states characterized by a density X -matrix. Because deriving the analytical expressions for the eigenvalues and eigenvectors of the non- X Hamiltonian matrix (Eq. (3)) is very difficult, the eigenvalues are computed numerically.

The time evolution of the NLCs in the generated two spin-qubit states, represented by the density matrix $\hat{M}(t)$, will be explored using the Milburn intrinsic decoherence model [54], which is given by

$$\frac{d}{dt} \hat{M}(t) = -i[\hat{H}, \hat{M}] - \frac{\gamma}{2} [\hat{H}, [\hat{H}, \hat{M}]], \quad (4)$$

where γ is the intrinsic spin-spin decoherence (ISSD) coupling.

After calculating the eigenvalues V_k ($k = 1, 2, 3, 4$) and the eigenstates $|V_k\rangle$ of the Hamiltonian in Eq. (3), the two-spin density matrix $\hat{M}(t)$ of Eq. (4) can be obtained numerically using the following solution, given by

$$\hat{M}(t) = \sum_{m,n=1}^4 U_{mn}(t) S_{mn}(t) |V_m\rangle \langle V_n| \hat{M}(0) |V_n\rangle \langle V_m|. \quad (5)$$

This solution depends on the unitary interaction $U_{mn}(t)$ and the ISSD coupling $S_{mn}(t)$, given by the following terms:

$$\begin{aligned} U_{mn}(t) &= e^{-i(V_m - V_n)t}, \\ S_{mn}(t) &= e^{-\frac{\gamma}{2}(V_m - V_n)^2 t}. \end{aligned} \quad (6)$$

Eq. (5) is used to numerically calculate and explore the dynamics of the NLCs within the two-spin-qubit states' Heisenberg XYZ model under the effects of spin-orbit interactions along the x and y directions and an applied external magnetic field in the x direction.

3 NLC quantifiers

Here, the two-spin NLCs will be measured by the following quantifiers: LQFI, LQU, and LN.

• LQFI

Here, we use LQFI to quantify another type of two-spin Heisenberg-XYZ correlation beyond entanglement. After calculating the two-spin eigenvalues π_k ($k = 1, 2, 3, 4$) and the eigenstates $|\Pi_k\rangle$ of the density matrix of Eq. (5), which has the representation matrix $M(t) = \sum_m \pi_m |\Pi_m\rangle \langle \Pi_m|$ with $\pi_m \geq 0$ and $\sum_m \pi_m = 1$, the LQFI is calculated using the closed expression given by [41,44,45]

$$F(t) = 1 - \pi_R^{\max},$$

which depends on the highest eigenvalue $\pi_R^{\max}$ of the symmetric matrix $R = [r_{ij}]$. Based on the Pauli spin- $\frac{1}{2}$ matrices σ^i ($i = 1, 2, 3$) and the elements $\xi_{mn}^i = \langle \Pi_m | I \otimes \sigma^i | \Pi_n \rangle$, the symmetric matrix elements r_{ij} are given by

$$r_{ij} = \sum_{\pi_m + \pi_n \neq 0} \frac{2\pi_m \pi_n}{\pi_m + \pi_n} \xi_{mn}^i (\xi_{nm}^j)^{\dagger}.$$

For a maximally correlated two-spin-qubit state, the LQFI function converges to $F(t) = 1$. Otherwise, the LQFI function oscillates and is bounded by the inequality $0 < F(t) < 1$, indicating that the states have partial LQFI NLC.

• LQU

Also, we use LQU of WY skew information [40] to realize another type of two-spin-qubits' NLC [40,42,43]. For the two-spin density matrix $M(t)$ of Eq. (5), the LQU can be calculated by the following closed expression [42]:

$$U(t) = 1 - \lambda_{\max}(\Lambda_{AB}), \quad (7)$$

which depends on the largest eigenvalue $\lambda_{\max}$ of the 3×3 -matrix $\Lambda = [a_{ij}]$, which have the following elements:

$$a_{ij} = \text{Tr} \{ \sqrt{M(t)} (\sigma_i \otimes I) \sqrt{M(t)} (\sigma_j \otimes I) \}.$$

The LQU function oscillates and is bounded by the inequality $0 \leq U(t) \leq 1$. It converges to $U(t) = 1$, otherwise indicating that the states have partial LQU NLC.

- **Logarithmic negativity (LN)**

We employ LN [31] to measure the generated two-spin-qubit entanglement. The LN expression is based on the negativity's definition μ_t , which is defined as the absolute sum of the negative eigenvalues of the partial transposition matrix $(M(t))^T$ of the two-spin-qubit density matrix $M(t)$ of Eq. (5). The LN can be expressed as

$$N(t) = \log_2[1 + 2\mu_t]. \quad (8)$$

The LN function vanishes, $N(t) = 0$, for a disentangled two-spin state. It converges to its maximum value, $N(t) = 1$, for a maximally entangled two-spin state. Otherwise, LN oscillates and is bounded by the inequality $0 \leq N(t) \leq 1$, indicating that the two-spin states have partial entanglement.

In the following, we work in a system of units where $\hbar = 1$, and employ the nondimensionalized parameter method as described in previous studies [24,63,64]. We also consider the case of spin-spin interactions with anti-ferromagnetic couplings satisfying $J_a > 0$. Meanwhile, the other physical parameters, including the spin-orbit couplings and the degree of uniformity and inhomogeneity of the magnetic field, satisfy $D_x, D_y, B_m, b_m \geq 0$. Small values of these parameters indicate weak spin-orbit interaction and a weak applied magnetic field.

4 Two-spin qubit dynamics

To explore the generation of non-local correlations between two spin qubits, we consider that the two spins are initially in their uncorrelated upper states $|1_A\rangle \otimes |1_B\rangle$. In this state, the density matrix has no non-local correlations according to the considered quantifiers. Our focus is on the effects of J_a spin-spin interactions (D_x and D_y) and inhomogeneous x -direction magnetic field parameters (B_m and b_m) in the presence of ISSD coupling.

Our first analysis, starting from Figure 2, illustrates the dynamics of non-local correlations (LQFI, LQU, and LN) between two spin qubits. These correlations are generated by the couplings $(J_x, J_y, J_z) = (0.8, 0.8, 0.8)$, supported by varying intensities of x and y spin-orbit interactions. This is done in the presence of an inhomogeneous x -direction magnetic field with small uniformity and inhomogeneity $(B_m, b_m) = (0.3, 0.5)$, and in the absence of intrinsic spin-spin decoherence ($\gamma = 0$). Figure 2(a) with $(D_x, D_y) =$

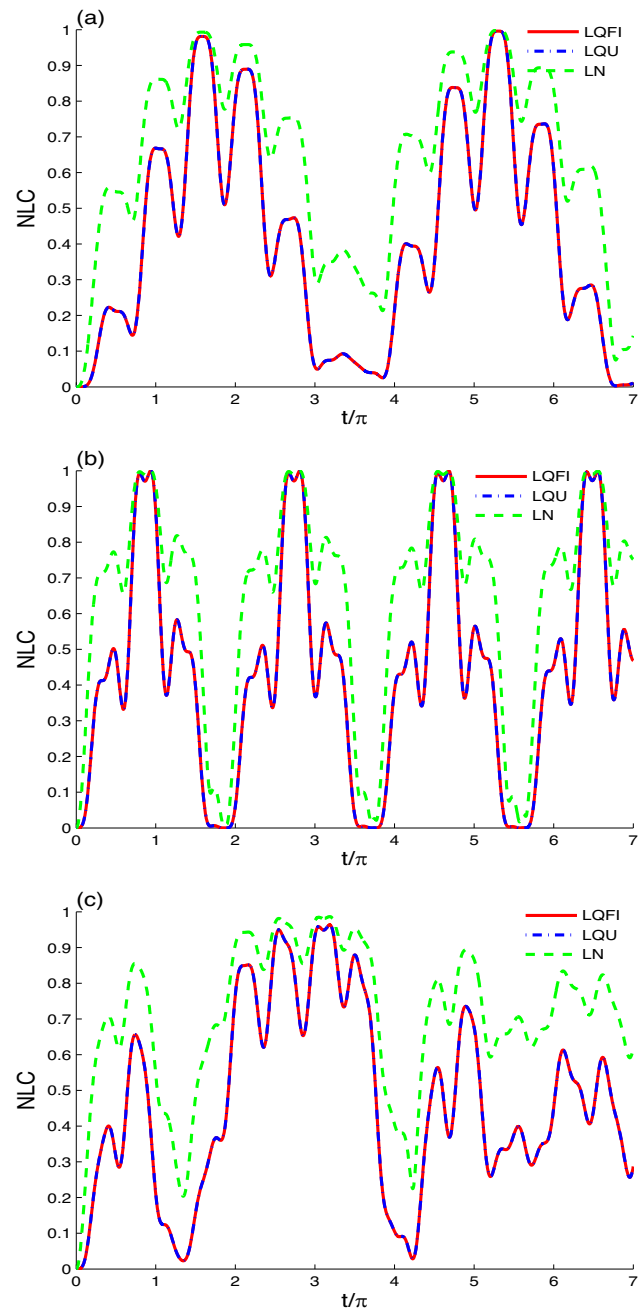


Figure 2: Time evolution of the LQFI, LQU, and LN are shown with the two-spin couplings $(J_x, J_y, J_z) = (0.8, 0.8, 0.8)$ and the applied magnetic field parameters $(B_m, b_m) = (0.3, 0.5)$ for different x, y spin-orbit interactions: $(D_x, D_y) = (0.0, 0.0)$ in (a), $(D_x, D_y) = (0.5, 0.0)$ in (b), and $(D_x, D_y) = (0.5, 0.5)$ in (c).

$(0.0, 0.0)$ shows that the LQFI, LQU, and log-negativity grow and reach their maximum values. They are subject to slow quasi-regular oscillations with the same frequencies and different amplitudes. LQFI and LQU have the same behavior, i.e., the two spin qubit correlation is called “LQFI–LQU correlation.” The amplitude of the LN is always

larger than that of the LQFI and LQU. Under these circumstances of a weak coupling regime ($J_a = 0.8$) and the applied inhomogeneous x -direction magnetic field (with weak uniformity and inhomogeneity), the initial pure uncorrelated two-spin state evolves into various time-dependent partially correlated states. At specific times, it transforms into maximally correlated states. The two-spin states exhibit maximal LQFI–LQU correlation ($F(t) = U(t) = 1$) and log-negativity ($N(t) = 1$) simultaneously. At particular times, we observe that partially entangled two-spin states have neither LQFI nor LQU correlation.

The D_x -spin–orbit interaction (D_x, D_y) = (0.5, 0) dramatically improves the appearance of the intervals of the maximal LQFI–LQU correlation and log-negativity entanglement, as well as the intervals in which two-spin entangled states have no LQFI or LQU correlation. The effects of weak spin–orbit interactions in the x direction only are shown in Figure 2(b). As illustrated, the regularity and fluctuations of the generated LQFI–LQU correlation and log-negativity entanglement are significantly greater than previously observed in the absence of x and y spin–orbit interactions. The weak D_x spin–orbit interaction dramatically enhances the intervals of maximal LQFI–LQU correlation and log-negativity entanglement, as well as the intervals where two-spin entangled states have neither LQFI nor LQU correlation. In Figure 2(c), we combined the D_x and D_y spin–orbit interactions (D_x, D_y) = (0.5, 0.5). As shown, the fluctuations of the two-spin NLCs between their partial and maximal values are significantly fewer than in Figure 2(a) and (b). Additionally, the NLC frequencies have been reduced, and their lower bounds have shifted upward. This indicates that the combined D_x and D_y spin–orbit interactions enhance the generated partial two-spin–qubit LQFI–LQU correlation and log-negativity entanglement.

Figure 3(a) and (b) illustrates that higher spin–spin interaction couplings ($(J_x, J_y, J_z) = (1, 0.5, 1.5)$ in (a) and $(J_x, J_y, J_z) = (5, 1, 1.5)$ in (b)) significantly enhance the two-spin LQFI–LQU correlation and log-negativity entanglement. By comparing the generated spin–spin NLCs shown in Figure 2(c) and 3(a), we find that relatively strong couplings of J_a -spin–spin interactions ($(J_x, J_y, J_z) = (1, 0.5, 1.5)$), supported by weak $D_{x,y}$ -spin–orbit interactions ($(D_x, D_y) = (0.5, 0.5)$), increase the amplitudes and frequencies of the LQFI–LQU correlation and log-negativity entanglement oscillations. Figure 3(a) and (b) shows that higher J_a -couplings lead to that the spin–spin NLCs' oscillations have more fluctuations. The time positions of the maximal LQFI–LQU correlation and log-negativity entanglement are enhanced. Figure 3(c) is plotted to demonstrate the capability of spin–spin interactions ($J_a = 0.8$), supported

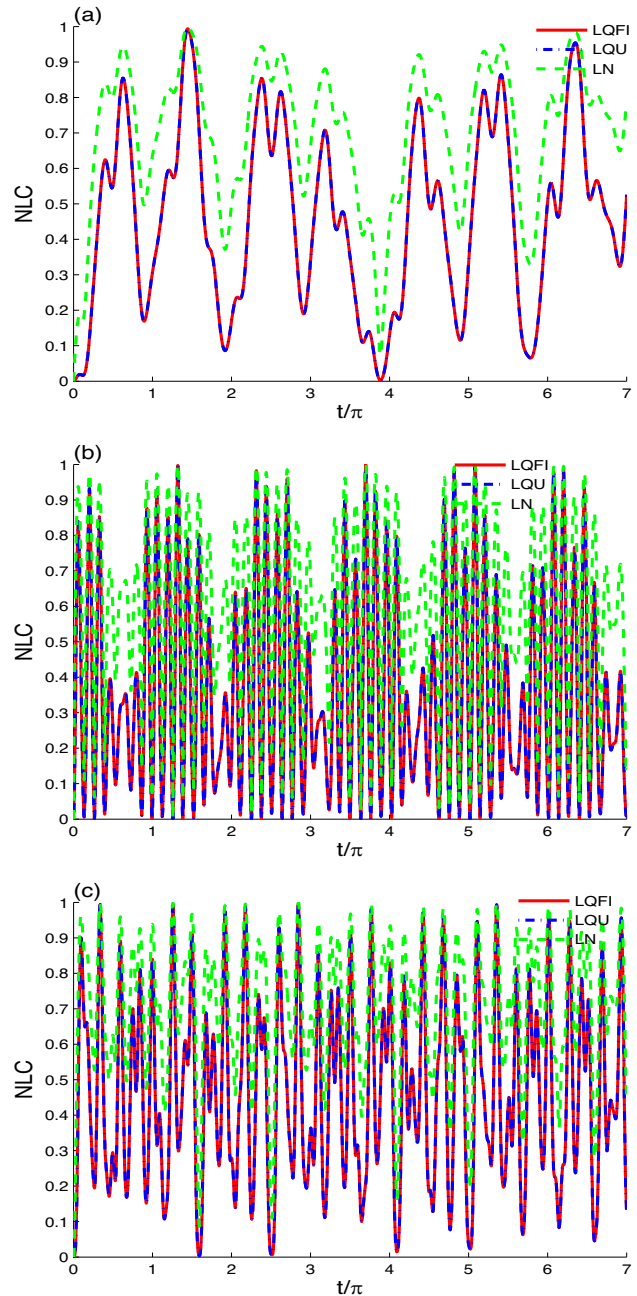


Figure 3: Time evolution of the LQFI, LQU, and LN of Figure 2(c) are plotted for different two-spin couplings: $(J_x, J_y, J_z) = (1, 0.5, 1.5)$ in (a) and $(J_x, J_y, J_z) = (5, 1, 1.5)$ in (b). In (c), they are plotted for x, y spin–orbit couplings $D_x = D_y = 2$ in (c).

by x, y -spin–orbit interactions ($D_x = D_y = 2$), to enhance the generated spin–spin NLCs when an external magnetic field with weak determinants $((B_m, b_m) = (0.3, 0.5))$ is applied. By comparing the qualitative dynamics of the generated LQFI–LQU correlation and log-negativity entanglement shown in Figure 2(c) ($D_x = D_y = 0.5$) with those in Figure 3(c) ($D_x = D_y = 2$), we can deduce that

$D_{x,y}$ -spin-orbit interactions play a significant role in enhancing the generated LQFI-LQU correlation and log-negativity entanglement. Their amplitudes are increased, and their oscillations exhibit more fluctuations between extreme values. Additionally, strong x, y -spin-orbit interactions potentially strengthen and accelerate the generation of LQFI-LQU correlation and log-negativity entanglement due to J_a -spin-spin interactions.

Figure 4 illustrates the LQFI-LQU correlation and log-negativity entanglement dynamics of Figure 3(a) (where $(J_x, J_y, J_z) = (1, 0.5, 1.5)$, $b_m = 0.5$, and $D_x = D_y = 0.5$) for different uniformities of the applied EIMF. Figure 4(a) illustrates that with a large uniformity ($B_f = 2$), increasing the EIMF uniformity delays the growth of LQFI, LQU, and log-negativity. It also increases the fluctuations of the two-spin state between different partially and maximally correlated states. The generations of the LQFI-LQU correlation and log-negativity entanglement are shown in Figure 4(b) (with $B_m = 10$), confirming that increasing the EIMF uniformity enhances the ability of strong J_a -spin-spin interactions, supported by weak x, y -spin-orbit interactions, to create partially and maximally correlated states with greater

stability. However, the generated spin-spin NLCs become more sensitive to the EIMF uniformity.

In the upcoming analysis of Figure 5, we maintain the same parameter values as in Figure 3a (with $(J_x, J_y, J_z) = (1, 0.5, 1.5)$, $B_m = 0.3$, and $D_x = D_y = 0.5$) and examine different magnetic field inhomogeneities: $b_m = 2$ in (a) and $b_m = 10$ in (b). In the case of Figure 5(a), we observe that greater EIMF uniformities improve the efficiency of generating LQFI-LQU correlations and log-negativity entanglement. The uniformity of the EIMF increases the fluctuations of the two-spin state between different partially and maximally correlated states. The timing of the maxima ($F(t) = U(t) = N(t) \approx 1$) and minima (zero-value) ($F(t) = U(t) = N(t) \approx 0$) of the generated LQFI-LQU correlations and log-negativity entanglement is enhanced. Figure 5(b) demonstrates that an increase in EIMF inhomogeneity significantly enhances the generated two-spin-qubits' NLCs, leading to greater amplitudes and fluctuations in the NLC oscillations.

The next illustrations in Figures 6–8 depict the time evolutions of NLCs of LQFI, LQU, and log-negativity in the presence of non-zero ISSD coupling. By comparing the results of Figure 2(a) ($\gamma = 0.0$) with those of Figure 6(a)

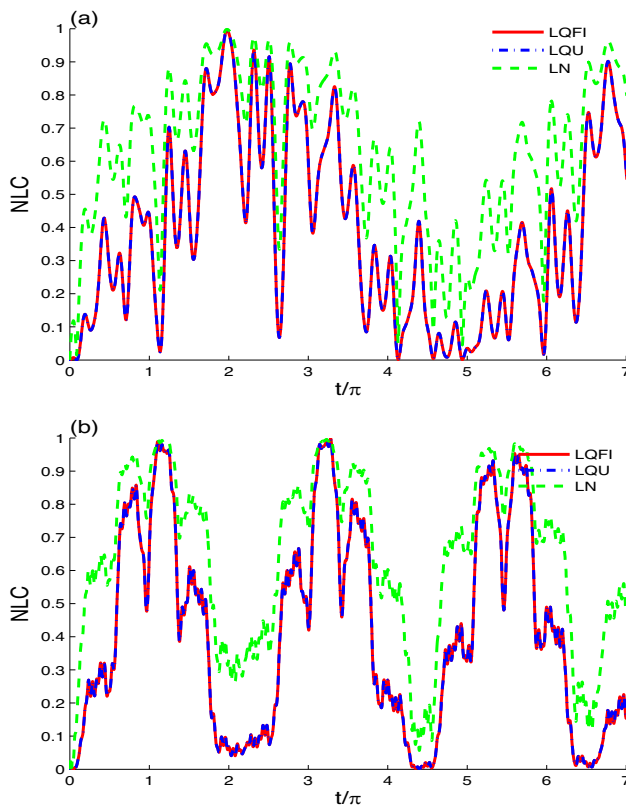


Figure 4: Time evolutions of LQFI, LQU, and LN of Figure 3(a) (for $(J_x, J_y, J_z) = (1, 0.5, 1.5)$, $b_m = 0.5$, and $D_x = D_y = 0.5$) are plotted for different EIMF uniformities: $B_m = 2$ in (a) and $B_m = 10$ in (b).

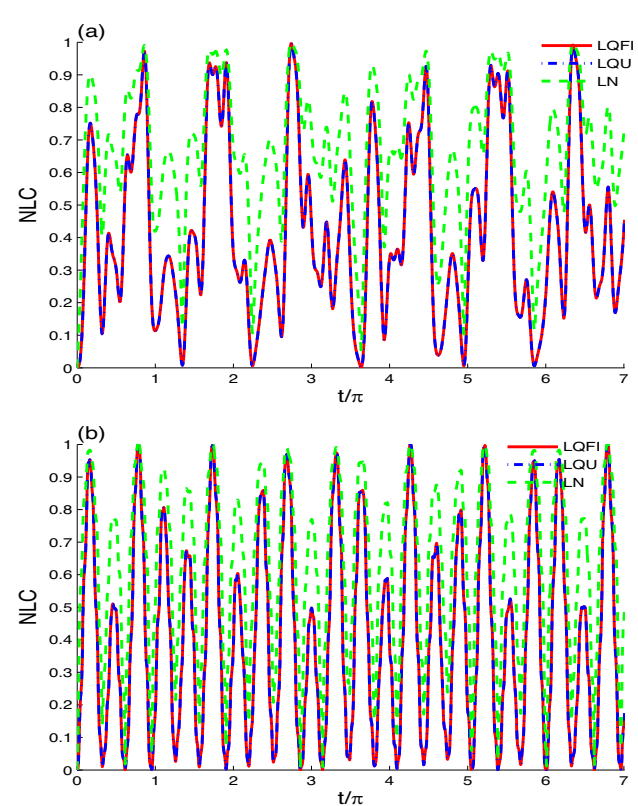


Figure 5: Time evolutions of LQFI, LQU, and LN of Figure 3(a) (for $(J_x, J_y, J_z) = (1, 0.5, 1.5)$, $B_m = 0.3$, and $D_x = D_y = 0.5$) are plotted for different EIMF inhomogeneities: $b_m = 2$ in (a) and $b_m = 10$ in (b).

($\gamma = 0.05$), we observe that LQFI, LQU, and log-negativity exhibit different decaying oscillatory dynamical evolutions. The generations of Heisenberg-XYZ states' NLCs, due to $J_a = 0.8$ spin-spin couplings and the applied magnetic field $(B_m, b_m) = (0.3, 0.5)$ without spin-orbit interaction, are weakened and exhibit different amplitudes, which decrease with increasing ISSD coupling. After a certain time interval, with non-zero ISSD coupling, LQFI and LQU show different NLCs with varying amplitudes but

similar behaviors. Moreover, the robustness of LQFI and log-negativity against the ISSD effect is greater for LQU.

As shown in Figure 6(b) and (c), increasing the intensities of x, y -spin-orbit interactions reduces the robustness of NLCs against the ISSD effect. The amplitudes of NLCs significantly decrease as the x, y -spin-orbit interactions increase. Moreover, LQFI and LQU exhibit sudden changes at different times. The phenomenon of sudden changes has been studied both theoretically [65] and experimentally [66]. For very strong x, y -spin-orbit interactions ($D_k = 2$) Figure 6(c), we observe that the log-negativity of the two-spin qubit drops instantly to zero at a specific time and remains zero for an extended period (i.e., the sudden-death LN-entanglement phenomenon occurs). After this, the disentangled two-spin states exhibit only different stable NLCs of LQFI and LQU. We can conclude that the decay of NLCs due to ISSD can be intensified by increasing the intensities of x, y -spin-orbit interactions.

Figure 7 illustrates the time evolutions of LQFI, LQU, and LN of Figure 6(b) and (c), but for strong spin-spin couplings with $(J_x, J_y, J_z) = (1, 0.5, 1.5)$. By comparing Figures 6(b), (c) and 7(a), (b), we find that strong spin-spin couplings $(J_x, J_y, J_z) = (1, 0.5, 1.5)$ reduce the ISSD effect and

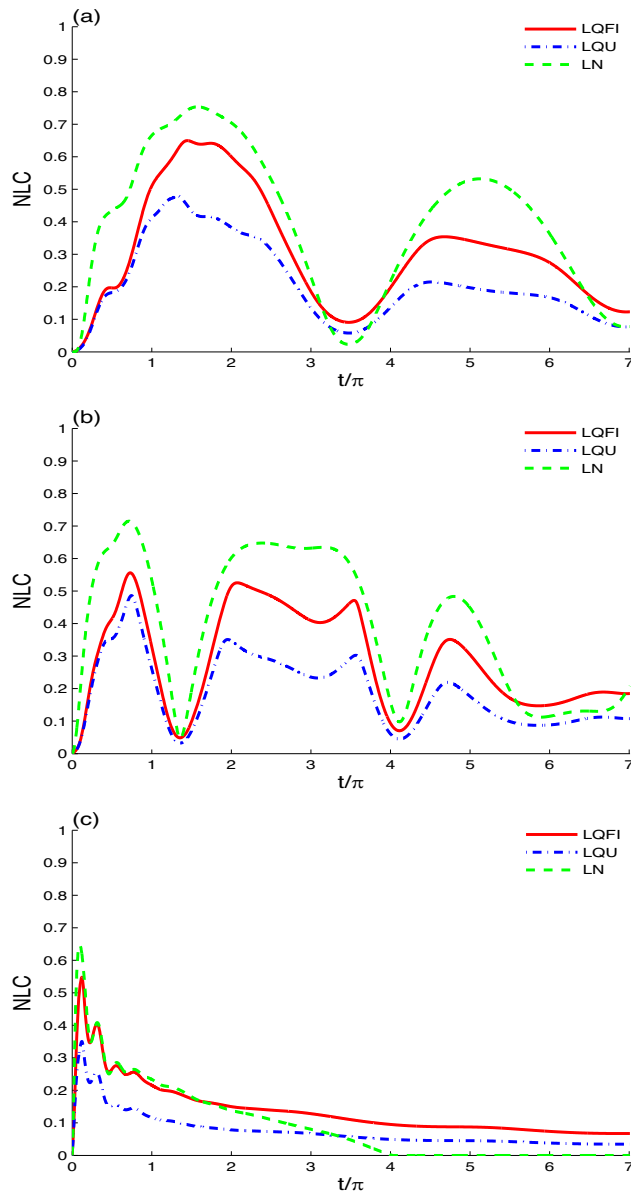


Figure 6: Time evolutions of LQFI, LQU, and LN of Figure 2(a) are shown in the presence of the ISSD effect ($\gamma = 0.05$) with EIMF uniformity and inhomogeneity $(B_m, b_m) = (0.3, 0.5)$, and two-spin couplings $J_a = 0.8$ for different couplings: $D_k = 0$ ($k = x, y$) in (a), $D_k = 0.5$ in (b), and $D_k = 2$ in (c).

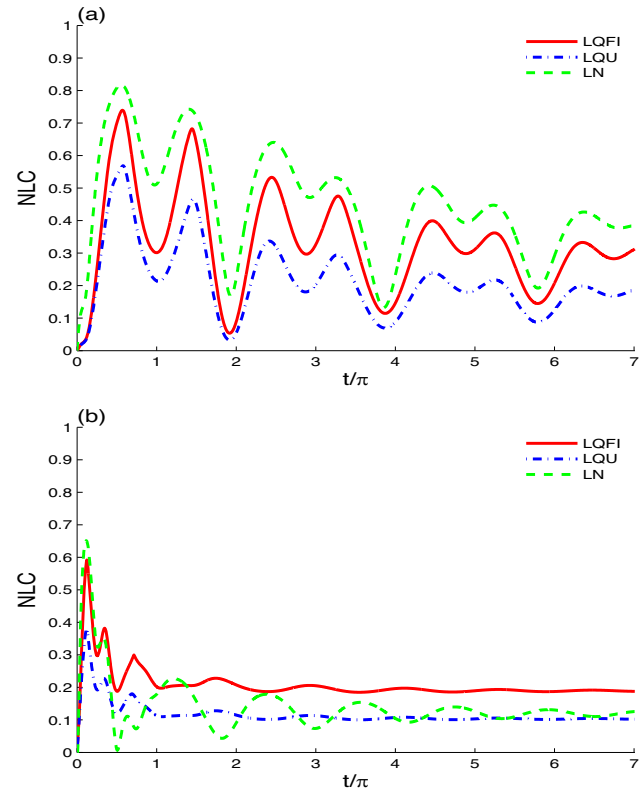


Figure 7: Time evolutions of LQFI, LQU, and LN of Figure 6(b) and (c) are shown but for strong spin-spin couplings with $(J_x, J_y, J_z) = (1, 0.5, 1.5)$.

improve the robustness of the NLCs (against the ISSD effect) of LQFI, LQU, and LN. For very strong spin-orbit interactions $D_k = 2$ (Figure 7(b)), the sudden-death LN-entanglement phenomenon does not occur, except instantaneously at $t \approx 0.5\pi$. The generated two-spin states have different stable partial NLCs of LQFI, LQU, and LN. In this case, the decay of NLCs due to ISSD can be mitigated by enhancing the spin-spin interactions.

Figure 8(a) shows the time evolutions of LQFI, LQU, and LN from Figure 4(a) for $(J_x, J_y, J_z) = (1, 0.5, 1.5)$ and $D_x = D_y = 0.5$, considering the ISSD effect ($\gamma = 0.05$) after strengthening the EIMF uniformity with $(B_m, b_m) = (2, 0.5)$. In this case, we observe that the large EIMF uniformity $B_m = 2$ increases the NLCs' decay resulting from ISSD. Time intervals appear in which the disentangled two-spin states have only different stable NLCs of LQFI and LQU. Moreover, the robustness of LQFI and LN NLCs, against the ISSD effect, is reduced by increasing EIMF uniformity. The results shown in Figure 8(b) demonstrate that increasing the inhomogeneity of the EIMF to $b_m = 2$ enhances the degradation of the NLC functions. Under the parameters $b_m = 2$, $(J_x, J_y, J_z) = (1, 0.5, 1.5)$, and $D_k = 0.5$, we observe

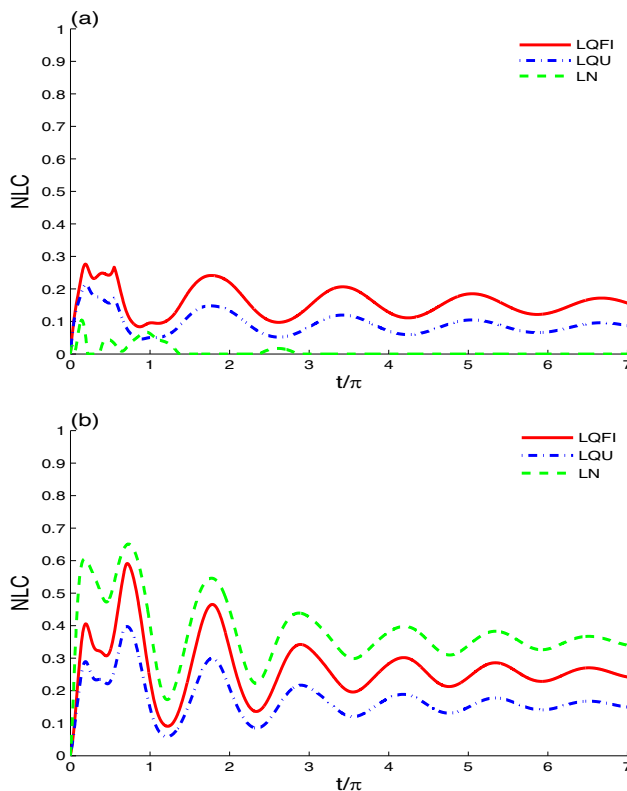


Figure 8: Time evolutions of LQFI, LQU, and LN in Figure 4(a), for $(J_x, J_y, J_z) = (1, 0.5, 1.5)$ and $D_x = D_y = 0.5$, are presented considering the ISSD effect ($\gamma = 0.05$) for EIMF uniformity with $(B_m, b_m) = (2, 0.5)$ in (a) and EIMF inhomogeneity with $(B_m, b_m) = (0.3, 2)$ in (b).

that the generated NLCs (LQFI, LQU, and entanglement) in Figure 4(a) degrade (due to the ISSD effect) and quickly reach their partially stable oscillatory behaviors, compared to the case with a small value of $b_m = 0.5$ in Figure 7(a). We find that the EIMF's inhomogeneity has a lesser ability to enhance the ISSD effect compared to its uniformity.

5 Conclusion

In this study, we use the Milburn intrinsic decoherence and the Heisenberg XYZ models to explore the capabilities of spin-spin and spin-orbit interactions (in the x and y directions) to generate NLCs (measured by LQFI, LQU, and LN) under the influence of the uniformity and inhomogeneity of the EIMF in the x direction. The generated NLCs are examined in the absence of the ISSD, as the parameters of spin-spin and spin-orbit interactions, as well as the EIMF's uniformity and inhomogeneity, are increased. It is found that the spin-spin Heisenberg XYZ and x, y -spin-orbit interactions have a strong ability to enhance non-local correlations with small external magnetic field parameters. The spin-orbit interactions significantly contribute to the enhancement of the generated two-spin-qubits LQFI-LQU correlation and log-negativity entanglement, increasing their oscillation amplitudes and fluctuations. In the presence of the ISSD, the generation of NLCs is weakened and exhibits varying amplitudes, decreasing as the ISSD coupling increases. The robustness of LQFI and log-negativity against the ISSD effect is greater than that of LQU. Sudden changes occur during the dynamics of LQU and LQFI, while sudden death occurs during the dynamics of log-negativity entanglement. The decay of NLCs due to ISSD can be enhanced by increasing the intensities of x, y -spin-orbit interactions. Strengthening the spin-spin interactions, however, weakens the NLCs' decay resulting from ISSD. The generated NLCs degrade quickly due to the ISSD effect with a large IMF's inhomogeneity, reaching their partially stable oscillatory behaviors. The ability of the IMF's inhomogeneity to increase the ISSD effect is small compared to that of the EIMF's uniformity. Our findings on the generated two-spin qubit NLCs (measured by LQFI, LQU, and LN) resulting from spin-spin interactions, supported by spin-orbit interactions in the x and y directions and an applied EIMF in the x direction, pave the way for exploring other quantum effects with the considered interaction directions or with different interaction directions. Additionally, these quantum effects can be explored using other differential motion equations with the considered interaction directions.

Furthermore, fractional quantum calculus and Riemann–Liouville integration [67,68] can be employed to investigate fractional quantum phenomena in the considered Heisenberg XYZ model using fractional differential motion equations [69,70].

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