Research Article

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Interpreting optical effects with relativistic transformations adopting one-way synchronization to conserve simultaneity and space–time continuity

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Abstract: We revise the optical effects of the Sagnac type where the moving closed contour is traversed by a photon in the observable invariant time interval T. Light propagation is described using relativistic transformations adopting an internal one-way synchronization procedure, not equivalent to the standard two-way Einstein synchronization. We show that for the reciprocal linear Sagnac effect, where the emitter–receiver C^* is stationary and the contour is in motion, T is no longer invariant for the standard Lorentz transforms, reflecting a weak form of the relativity principle. Instead, the relativity principle is fully preserved and T is invariant for transforms based on conservation of simultaneity. We prove that in the standard linear Sagnac effect, if the local one-way speed along the optical fiber is assumed to be c, the photon cannot cover the whole closed contour in the interval T. The uncovered "missing" section reflects a breach in space-time continuity related to the "time gap" of the transforms based on relative simultaneity. Our revision confirms the well-known result that the Lorentz transforms fail in interpreting these effects. Together with other examples, the results of the reciprocal linear effect invalidate the conventionalist claim that relative and absolute simultaneity are equivalent. The reciprocal effect can then be used for testing Lorentz and light speed invariance.

Keywords: one-way speed of light, Sagnac effect, Lorentz invariance, conservation of simultaneity, relativity principle, foundations of relativity theory

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1 Introduction

The Michelson–Morley optical experiment of 1887 had the aim to detect the famous "ether wind," the ether being the medium where light propagates according to Maxwell's laws of electromagnetism. No ether wind was detected, and the experiment provided a surprising null result that gave support to Einstein's theory of special relativity of 1905, where light is assumed to propagate in empty space at the same speed c relative to any inertial observer in motion (light speed invariance), as described by the Lorentz transformations (LTs).

After decades of controversy and the criticisms of epistemologists and physicists [1–8] to Einstein synchronization, recent advances in optical experiments, such as the linear Sagnac effect verified in 2003 by Wang *et al.* [9,10], the reciprocal linear Sagnac effect proposed in 2023 by Spavieri and Haug [11,12], and their interpretations [13–31], justify the early criticisms of Lorentz and light speed invariance, suggesting that a paradigm shift is taking place in relativity theory, practically unnoticed to most physicists.

To measure the one-way speed of light c traveling from point A to point B, with fixed distance AB = L, Einstein adopted a procedure for synchronizing two spatially separated clocks, one at A and the other at B, assuming that the one-way light speed coincides with the average round-trip light speed $c = 2L/T$, where T is the time interval measured by clock A in the light round-trip from A to B and back to A. Then, with Einstein synchronization, the clock at B is set at $t = L/c$ when light reaches it.

Epistemologists [1–5] and physicists soon criticized Einstein synchronization procedure, pointing out that, since the one-way speed from A to B can be different from the return speed from B to A, Einstein synchronization leaves undetermined and arbitrary (conventional) the one-way speed. At this point in the evolution of special relativity, in 1977, the physicists Mansouri and Sexl [6] introduced a set of coordinate transformations in agreement with the requirement of Einstein synchronization, but with a speed from A to B that can be different from the return speed from B to A, depending on the synchronization parameter ε:

$$\begin{aligned} t' &= \frac{t}{\gamma} - \frac{\varepsilon x'}{c^2} = \gamma\left[t\left(1 + \frac{\varepsilon v}{c^2} - \frac{v^2}{c^2}\right) - \frac{\varepsilon x}{c^2}\right] \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \text{ LT} \quad t' = t/\gamma \text{ LTA} \\ x' &= \gamma(x - vt) \quad y' = y \quad z' = z \\ c' &= c'(\varepsilon) = \frac{dx'}{dt'} = \frac{c}{1 + v/c - \varepsilon/c} \end{aligned} \tag{1}$$

In the generalized transformations (1) from frame S to S' in relative motion with velocity v, the factor $\gamma = (1 - v^2/c^2)^{-1/2}$ and the parameter ε can assume any arbitrary value from $\varepsilon = 0$ to $\varepsilon = v$. With $\varepsilon = v$, we obtain the standard LT. With $\varepsilon = 0$, we have the so-called Lorentz transformations based on absolute simultaneity (LTA). The time transform of the LT and LTA differs by the value of ε only. The one-way speed of light is assumed to be c in frame S, while in frame S', the local (differential) light speed is $c' = c'(\varepsilon) = dx'/dt'$. Light speed invariance, $c' = c$, holds for the LT only ($\varepsilon = v$).

Hypothetically, on account of the arbitrariness of synchronization and adhering to the conventionality of the light speed, all these transformations are physically equivalent to the LTs. Hence, as pointed out in mainstream journals, the original postulate that the one-way light speed is c, as originally introduced by Einstein, has changed and, presently, the universal constant c no longer represents the local one-way light speed, but the observable average round-trip light speed $c = 2L/T$ [6–8]. Then, different one-way speeds correspond different, but physically equivalent types of LTs. Thus, special relativity can be described either with the standard LT, based on relative simultaneity, or any other transformations with different synchronization, *e.g.*, the LTA, which have been used by many physicists, although under different names (*e.g.*, Tangherlini transforms [32], Selleri transforms [13–16], ALT [17,18]). Hence, for physicists [6,8,33–37] adhering to the "conventionalist" view of Mansouri and Sexl, the LT and LTA are physically equivalent and interchangeable and foresee the same relativistic effects, even though they adopt different, conventional values for the one-way light speed.

The theme of how to measure the one-way speed of light is stimulating and has been widely discussed in the literature because it touches on fundamental aspects of relativity theory. However, in most textbooks of special relativity, the related fundamental subject of clock synchronization is not discussed. Hence, many physicists, unaware of the issue of synchronization, use the most common relativistic transformations available that are the LT based on the standard Einstein synchronization with the measurable invariant two-way average round-trip light speed c. Articles dealing with the theme of synchronization, as those of refs [6–40], may help improve and complement the standard of available textbooks and taught physics in universities worldwide.

In many cases, there are no problems with the use of the LT with the average speed c. Hence, when studying, for example, the solutions of Maxwell's equations in vacuum, in practically all textbooks, it is presented that electromagnetic waves propagate at speed c independent of the direction. However, it is worth emphasizing that, in this case, the reference frame where light propagation is described corresponds to the preferred frame S of the transformations (1) where space is assumed to be isotropic, and the one-way speed of light coincides with the average speed c for, *e.g.*, both the LT and LTA. In fact, when considering the same physical situation from the moving frame S', we find that the one way light speed is still the invariant c for the LT, while it is no longer invariant and is $\approx c + v$ (or $\approx c - v$) for the LTA. Yet, the interesting feature of the transformations (1) emerges when passing from the formal description to the measure of an observable from frame S'. It turns out that the observable does no longer depend on v and, therefore, we are unable to measure the relative velocity $\approx v$ of the preferred frame S, which is then not identifiable. Actually, if we try to measure the one-way speed of light in S', we synchronize clocks using the procedure of clock transport (or any other internal synchronization) on S' [6], and we find that the resulting light speed is again c. The reason for this result is that, as shown by Mansouri and Sexl [6] and other physicists [7,8,33,36], any internal synchronization turns out to be equivalent to Einstein synchronization. Therefore, according to the conventionalist thesis, the LT and LTA are physically equivalent and, by using any of the transformations (1) to describe physical reality from the moving frame S', or predict an experimental result (*e.g.*, the result of the Michelson-Morley experiment), we always obtain the same observable result foreseen by the LT, regardless of the synchronization adopted [6,8,33,36].

One of the criticisms to some of the approaches presented in the literature made by physicists adhering to the conventionalist thesis, is that what is measurable, or has been measured, is the average round-trip light speed c and not the one-way speed. An example where the supposed measurement of the one-way light speed turns out to be the measurement of the average round-trip light speed c, which is given by the experiment realized by Greaves *et al.* [38]. About this experiment, we point out the comment made by Finkelstein [39], who, citing Reichenbach [2], argues that the mentioned experiment actually measures the average round-trip light speed c, not the one-way light speed, because the light signal performs a round-trip in the experimental set up. In fact, Greaves *et al.* do not specify what synchronization has been adopted for light propagation along the sections of the optical fiber, which forms a closed contour in their experiment.

Other examples related to the measurement of the one-way light speed and the possibility to test space anisotropy are discussed by Anderson *et al.* [36] and, in the context of possible violation of the relativity principle, by Spavieri *et al.* [40]. The conclusion and arguments are applied for the interpretation of the experiment of Greaves *et al.* [38] apply to a large class of experiments [40], such as those of Cahill and Brotherton [41], and the one of Krisher *et al.* [42] on space isotropy. Thus, since "round-trip" optical experiments are conceptually unsuitable for detecting light speed anisotropy or testing Lorentz invariance, the value of c measured with high precision in various experiments is not the one-way light speed but corresponds to the average value of the speed of light. Hence, when dealing with the topic of the one-way light speed, it is essential to consider the related issue of clock synchronization, as done for the tests of the one-way light speed proposed in previous studies [25,27,28], where the topic has been discussed in detail.

In any case, many physicists do not agree with the conventionalist thesis because, as discussed in previous studies [13–31] and considered below, there are several physical situations where problems surge with the use of the LT, while these problems disappear if we use the LTA [8,13–31], suggesting that the LT and LTA are not physically equivalent after all.

In the evolution of special relativity, physicists have discovered and formulated many paradoxes. Although there is no consensus about the solutions of all paradoxes, most of them are ascribed unanimously to the nonconservation of simultaneity of the LT. Instead, no paradoxes arise with the LTA. Of the many "paradoxes" of the theory, we mention here the important Selleri [13–15] paradox related to the Sagnac [43] optical effect, carried out in 1913 and shown in Figure 1(a). Later, other optical effects of the Sagnac type have been discovered (such as the linear effect of Figure 1(b) [9,10]) and physicists favoring conservation of simultaneity [13–15,17–31] have been questioning the conventionalism of Mansouri and Sexl and the validity of the "equivalence" between relative (LT) and absolute simultaneity (LTA).

The study introduces a one-way internal synchronization applicable to closed contours (at rest or in motion), makes a revision of the effects of the Sagnac type including

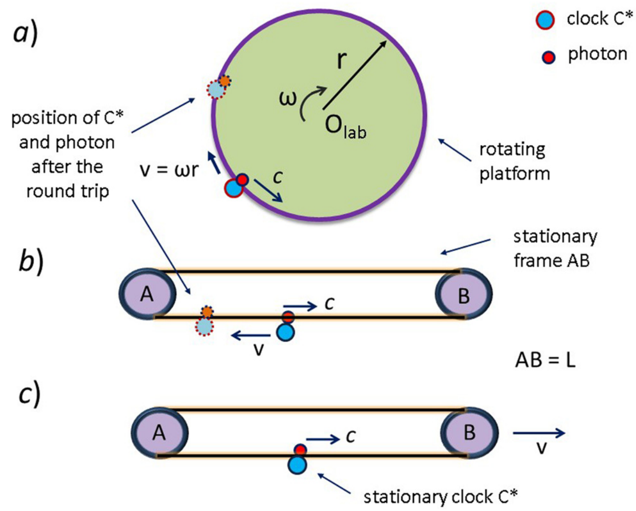

Figure 1: (a) On the rotating platform of the circular Sagnac effect, the clock C^* located on the circumference emits two counter-propagating photons (only a single photon is shown) traveling along the rim. C^* measures the difference ΔT in the photons' arrival times after a round trip. The position of C^* after the round trip indicates that the photon has covered the distance $2\pi r - vT$, as seen from the lab reference frame. However, as seen from the moving frame of the clock, the round-trip distance covered along the circumference is $2\pi r$. (b) In the linear Sagnac effect, the two photons are emitted by C^*, which is moving with velocity v relative to the stationary frame AB. The counter-propagating photons travel in an optical fiber that slides frictionless around pulley A and B. Again, as for the circular effect, the distance covered by a photon is not the same when seen from the lab frame or the clock frame. (c) In the reciprocal Sagnac effect, the clock C^* emitting the counter-propagating photons, is stationary, while the frame AB moves with velocity v relative to C^*. While the interval T is invariant for the Lorentz transforms in the circular and linear effects, it is no longer in variant in the reciprocal linear effect.

the recent reciprocal linear effect [11,12], and presents some of the most important arguments of supporters and detractors of the LT. Note that, as mentioned earlier, high-precision experimental confirmation of the relativistic effects of standard special relativity (equally foreseen by both the LT and LTA) involves the average light speed c, as in the Michelson-Morley experiment, and not the one-way light speed, proven to be not invariant by the Sagnac effects. We show that the approach of Mansouri and Sexl, with the contended equivalence between relative (LT) and absolute simultaneity (LTA), has no general validity, and we believe that it may be limited to the following special case:

"The arbitrary synchronization involves two spatially separated clocks and makes use of the two-way Einstein synchronization procedure, without any reference to other relatively moving inertial frames."

By considering a single propagating photon along a closed contour, as shown in the Sagnac effects of Figure 1, we show in Sections 2–4, that, in general, Einstein synchronization, the LT, and Mansouri and Sexl's approach are unfeasible, and we discuss the following cases:

– For the recent reciprocal linear Sagnac effect [11,12], the LT and LTA foresee strikingly different values for some observables and the reciprocity is maintained in the scenario where the relativity principle holds with the LTA, but not with the LT. Hence, the reciprocal linear Sagnac effect can be used for testing Lorentz and light speed invariance. This result invalidates the conventionalist claim of the validity of the physical equivalence between the LT and LTA. Other examples of nonequivalence, such as the Thomas precession, are given.
– As well known [8,12–23,29], Einstein synchronization and the LT fail when applied to the closed contour of the Sagnac effects. The failure of the LT indicates that these transformations are not suitable for describing these aspects of physical reality and, thus, have limited validity. Instead, LTAs seem to possess a more general validity because are applicable to these optical effects and foresee the correct result.
– If the local light speed is c in one section of the linear effect (Figure 1(b)), the local speed in the other section cannot be c. In fact, in the observed round-trip interval T and at the local speed c in both sections, a counterpropagating photon covers only the distance $2L(1 - v/c)$, which is shorter than the closed contour $2L$. The "missing" uncovered path indicates a breach in spacetime continuity and is related to the "time gap" of relative simultaneity arising when using the LT. Instead, spacetime continuity is preserved using the LTA.
– Considering the role of coordinates transformations within the mere kinematical perspective, the one-way synchronization and the effects of the Sagnac type single out the corresponding correct synchronization parameter (LTA with $\varepsilon = 0$) that leads to a consistent interpretation of relativistic effects in a scenario preserving the relativity principle with transformations based on conservation of simultaneity.

2 The circular and linear Sagnac effects and their interpretations

In discussing optical experiments, we may consider electromagnetic light waves or photons. For the purpose of checking light speed invariance from different reference frames in relative motion, we find it more convenient to describe the velocity of propagation of light as that of photons instead of electromagnetic waves.

In his experiment, shown in a schematic idealized form in Figure 1(a), Sagnac measured the one-way speed of light with an interferometer (or clock C^*) on the rim of a circular rotating platform (or disk) where light (or a photon) travels along an optical fiber. The linear Sagnac effect [9], discussed below, is shown in Figure 1(b). In these effects, two counter propagating photons are traveling on the fiber moving at speed v, where the device C^*, fixed to the fiber, is co-moving with it. Then, C^* measures the time interval $\Delta T = T_+ - T_-$, where T_+ and T_- represent the round-trip proper time of the co- and counter-moving photons along the contour of perimeter $2\pi r$, or $2L$, in the circular and linear effects, respectively.

Following Post [44], for the linear effect, ΔT can be written as follows:

$$\Delta T = T_+ - T_- = \frac{2L}{\gamma(c - v)} - \frac{2L}{\gamma(c + v)} = \frac{4\gamma v L}{c^2}. \qquad (2)$$

For the circular Sagnac effect, with $v = \omega r$, result (2) with $2L = 2\pi r$, is usually expressed [44] as $\Delta T = 4\,\boldsymbol{\omega} \cdot \mathbf{A}/c^2$, where ω is the platform angular velocity and $\mathbf{A}$ is the area enclosed by the light path. Since the Sagnac effect is of the first-order in v/c, in some cases, for simplicity, we may take the factor γ as $\gamma = (1 - v^2/c^2)^{-1/2} \simeq 1$ in (2).

Sagnac's experiment indicates that the average one-way speed of light along the rotating fiber is approximately $c \pm v$, where v is the peripheral speed of the disk. In the following, we shall consider a single propagating photon, usually the counter-propagating one. The expression for the proper time interval T_- can be written as follows:

$$\begin{aligned} T_- = T = \frac{2L}{\gamma(c + v)} = \frac{2\gamma L}{\gamma^2(c + v)} \simeq \frac{2L}{c + v} \quad \text{or} \\ \frac{2\pi r}{c + v} = \frac{2\gamma L(1 - v/c)}{c} \simeq \frac{2L(1 - v/c)}{c} \quad \text{or} \\ \frac{2\pi r(1 - v/c)}{c}, \end{aligned} \qquad (3)$$

where the terms with $2\pi r$ refers to the circular effect.

For the circular and linear Sagnac effects, let us consider the interpretation of the last terms in the first and second line of expression (3) where we take $\gamma \simeq 1$. To avoid discussions on the inertiality of the moving clock C^* in the circular effect, we may consider the linear effect only, where C^* can be always in uniform motion during the interval T. Still, considering that the circular and linear are equivalent effects, the argument of Sagnac presented below should hold for both the circular and linear effects.

According to any reference frame, such as S_{lab} (where the arm AB is stationary), which sees clock and photon counter-moving in Figure 1(a) and (b), the spatial distance covered is different from that observed by measuring it along the fiber. In fact, the shorter "spatial" distance $2L(1 - v/c) \simeq 2L - vT$, differs from the longer fiber "ground" length $2L$, by $v2L/c$, as shown in Figure 1(a) and (b). For an observer instantaneously co-moving with the fiber and C^* (Figure 1(b)), the last term in the first line of (3) indicates that, in the interval T, the photon has covered at the average one-way speed $c + v$, the "ground" distance $2L$ along the fiber. Instead, the last term in the second line indicates that, in the same interval T, the photon has covered at the average speed c, the shorter "spatial" distance $2L(1 - v/c)$. From the mere kinematical perspective, in the same time interval T, different distances must be covered at different speeds. Hence, the "ground" light speed along the fiber cannot be the same as the "spatial" light speed.

If we assume space isotropy on $S_{\rm lab}$ where the one-way light speed is c, in the case of a fiber in uniform motion and on account of symmetry, the average speed $c + v$ measured by C^* along $2L$ must coincide with the uniform "ground" local speed along the whole fiber. Thus, according to Sagnac [43], Selleri [13–15], and many physicists, [17–31], light speed invariance is invalidated by the experiment because it is c for the observer $S_{\rm lab}$ and $c + v$ locally along the fiber upper (or lower) section for an observer on the inertial frame of Figure 1(b) co-moving with the fiber. Sagnac's result and interpretation are in agreement with coordinate transformations based on absolute simultaneity, such as the LTA, which foresee the one-way light speed $c + v$ along the fiber, and not the invariant c foreseen by the LT.

Objections and rebuttals on the interpretation of the Sagnac experiment can be found in the literature and some are given in previous studies [12–23]. Historically, the main objection to Sagnac's interpretation is that the observer on the rotating platform is on an accelerated frame and not on an inertial frame. Thus, some physicists suggested that the kinematical problem inherent to the circular Sagnac effect requires to be interpreted within the framework of General Relativity. Actually, rather than solving the simple kinematical problem, this suggestion can be taken as an indication that the standard special relativity theory is incomplete, and a different theory is needed for its solution. This polemic objection about inertiality became obsolete after the discovery of the linear Sagnac effect (Wang *et al.* [9,10]), which is considered to be equivalent to the circular effect. In the linear effect, the interferometer, or clock, can be always in uniform motion during the round-trip time interval T and, thus, on an inertial frame. The one-way light speed along a closed contour can be measured by a single clock and, since it is $c \pm v$ locally along the fiber, detractors of the LT claim that the Lorentz and light speed invariance do not hold and fail [8,12–23,29] in this case.

Practically, all physicists (supporters or detractors of the LT) agree that, from the kinematical perspective, the LTA (or, for $\gamma = 1$, the Galilean transforms) correctly interpret the Sagnac effects. Yet, the physical interpretation of special relativity through more than a century has been evolving and, currently, the supporters of the LT argue that although the LTAs provide the correct interpretation, it does not invalidate the LT because, on account of the arbitrariness of synchronization, the LTs are equivalent to the LTA [6,8,33,37]. We shall consider in detail the arguments of the detractors and supporters of the theory about this contended point on "LT-LTA equivalence" and show, instead, why the LT and LTA are not equivalent and represent different physical realities.

3 Standard two-way Einstein synchronization and one-way synchronization in relativistic theories

Epistemologists [1–5] claim that the basic postulates of a meaningful physical theory must be falsifiable (*i.e.*, testable). Then, if one of its basic postulates is not falsifiable, it may be argued that the theory is not physically meaningful. If the LT (with relative simultaneity) are equivalent to the LTA (with absolute simultaneity) and the speed of light is conventional [6], the *standard* theory of special relativity based on Einstein synchronization and the LT, has a drawback because its fundamental postulate of the one-way light speed invariance cannot be tested.

Advances in the alternative formulations of relativistic theories that make use of synchronization procedures not equivalent to Einstein's and that can lead to the measurement of the one-way light speed may succeed in eliminating the drawback of conventionalism in physics, thus restoring physical meaning to the foundations of a physical theory by making it a viable and falsifiable theory.

As mentioned earlier, Einstein synchronization fails when applied to moving closed contours. The problems inherent to Einstein synchronization can be solved by introducing, as we do in the next section, the one-way clock synchronization procedure.

3.0.1 Standard two-way Einstein synchronization procedure

Referring to Figure 2, let $s' = s_g$ be the "ground" distance traversed by the photon, as measured along the moving optical fiber (refractive index $n = 1$) starting from clock C^*. When $v = 0$, the curvilinear distance s' coincides with the corresponding distance s, measured from the lab inertial rest frame S_{lab}, where O_{lab} and the arm AB are at rest. Two clocks, C^* and C^0 (not shown), are placed at the same point in Figure 2(a) and (b), but spatially separated in Figure 2(c). By $\Delta s'$, we denote the distance C^*C^0 measured along the fiber from C^* to C^0. We may apply Einstein synchronization to the usual linear path C^*C^0 of the rod of rest length $\Delta s' = \Delta_0$ of Figure 2(c), and also to the paths C^*C^0 of the circular or linear effects of Figure 2(a) and (b), where C^0 coincides with C^*.

When C^* is stationary ($v = 0$, $\Delta s' = \Delta s$), for the two-way photon round-trip from C^* to C^0 (out trip) and, after changing direction, back from C^0 to C^* (return trip), the proper time interval measured by C^* is given by,

$$T_{\text{two-way}} = T_{\text{out}} + T_{\text{ret}} = \frac{2\Delta s'}{c} = \frac{\Delta s'}{c_{\text{out}}} + \frac{\Delta s'}{c_{\text{ret}}}, \tag{4}$$

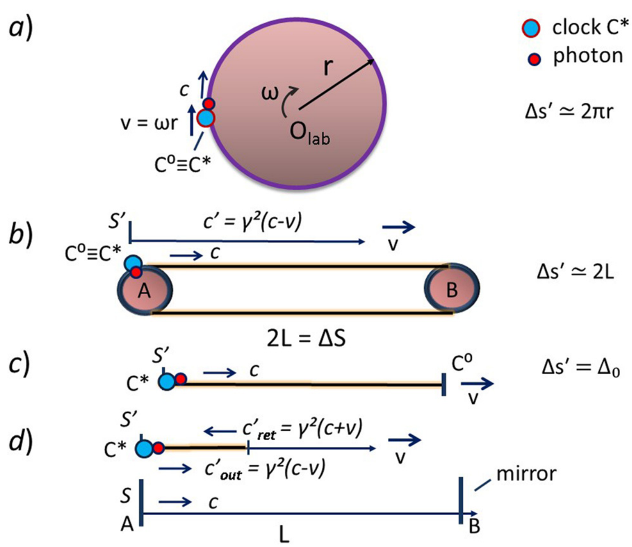

Figure 2: Comparing Einstein and one-way synchronizations: For the circular (a) and linear (b) effects, when $v = 0$ and clock C^* is stationary on S_{lab}, with Einstein two-way synchronization the photon starts from C^*, travels along the contour and returns to C^0. Then, changes direction, travels along the contour and gets back to C^*. With the one-way synchronization, the photon starts from C^*, travels along the contour and returns to C^0 only. If $v = 0$, the two synchronizations coincide. When C^* is in motion ($v \neq 0$), for the effects (a) and (b), the one-way synchronization foresees the light speed $\approx c - v$, which does not coincide with the speed c of Einstein synchronization. (c) With Einstein synchronization, in the rod frame S', the one-way light speed is undetermined. (d) Two mirrors, one at A and the other at B, are placed in the preferred frame S, where the one-way light speed is c. Adopting the one-way synchronization, the analogy with 2(b) for the photon round trip determines the one-way light speed c' along the rod co-moving with frame S', giving $c'_{\text{out}}(v) = \gamma^2(c + v)$ and $c'_{\text{ret}}(v) = \gamma^2(c + v)$.

where c is the invariant average two-way light speed. Assuming space isotropy on frame S_{lab}, the one-way light speed on S_{lab} is c and $c_{\text{out}} = c_{\text{ret}} = c$.

When C^* (and the optical fiber, to which C^* is fixed) is moving with uniform speed v relative to S_{lab}, the "ground" length $\Delta s' = C^*C^0 = C^0C^*$ of the fiber is the same, as measured locally along the fiber, regardless of whether in relative motion, and the expression (4) for the invariant average two-way light speed is valid for all the instances of Figure 2. By using Einstein synchronization procedure, we may arbitrarily synchronize the clock at C^0 by setting,

$$(T_{\text{out}})_E = (T_{\text{ret}})_E = \frac{\Delta s'}{c}, \tag{5}$$

in agreement with the LT.

Einstein synchronization for the rod of Figure 2(c). Since we are unable to measure T_{out} without first synchronizing the two spatially separated clocks, in this case, the one-way light speed remains undetermined because of the arbitrariness of the clock synchronization procedure. Then, the corresponding one-way light speed c_{out} along the path $C^*C^0 = \Delta s' = \Delta_0$ can be conventionally chosen [6] (*e.g.*, $c_{\text{out}} = c$ or $c_{\text{out}} = \gamma^2(c - v)$), as long as (4) is verified (*e.g.*, with $c_{\text{ret}} = c$ or $c_{\text{ret}} = \gamma^2(c + v)$).

Determining the synchronization parameter ε for the Sagnac effects of Figure 2(a) and (b) with the one-way synchronization. When the fiber is in motion, Sagnac's experiments show that the one-way light speeds, $c_{\text{out}} = c_{\text{out}}(v)$ in the out trip, and $c_{\text{ret}} = c_{\text{ret}}(v)$ in the return trip, are different from c. Nevertheless, the interval $T_{\text{two-way}}$ in (4) may remain invariant if the average c is invariant because, on average, the different one-way out trip interval T_{out} from C^* to C^0 may be balanced by the one-way return trip interval T_{ret} from C^0 to C^* along the same path.

For the Sagnac effects, there is no arbitrariness because $T_{\text{out}} = T \simeq (T_{\text{out}})_{\text{lab}}$ can be measured by the single clock C^*. Then, on account of (2), for a co-propagating photon, the observable interval,

$$T_{\text{one-way}} = T = \frac{\Delta s'}{c'} = \frac{\Delta s'}{\gamma^2(c - v)}, \tag{6}$$

with the known $\Delta s' = \Delta s_g = 2\gamma L \simeq 2L$, provides the correspondent average one-way ground speed $c' = c_g = \gamma^2(c - v) = c/(1 + v/c)$ along the optical fiber.

Let us consider an inertial frame S' instantaneously co-moving with the fiber, with $c' = dx'/dt'$ the differential local speed along an elementary "ground" section $dx' = ds' = ds_g$ of the fiber. For the circular effect of Figure 2(a), we have to consider the set of inertial frames instantaneously co-moving with the fiber at the adjacent infinitesimal sections ds'_1, $ds'_2, \ldots$, etc. [24], while for the linear effect of Figure 2(b), we may consider just two inertial frames, S' co-moving with the fiber upper section and S'' (not shown) co-moving with the lower section, which has velocity $-v$ relative to S_{lab}. Assuming space to be isotropic and the one-way light speed to be c on frame S_{lab}, symmetry implies that any observer instantaneously co-moving with the fiber "sees" the same ground local light speed, which coincides with the average one-way ground speed. Thus, the ground local light speed $c' = c_g$ along the upper section of the co-moving fiber seen by an observer on S' in Figure 2(b) is the same as the ground local light speed $c'' = c_g$ seen by an observer on S'' along the lower section of the fiber. Hence, for one-dimensional light propagation along the contour, symmetry allows us to express [24] the transformations (1) in terms of the one-dimension curvilinear coordinate $s' = s_g$ and write $t' = t/\gamma - \varepsilon s'/c^2$, $s' = \gamma(s - vs/c^2)$, $c'(\varepsilon) = \mathrm{d}s'/\mathrm{d}t'$. Integrating $\mathrm{d}t' = \mathrm{d}s'/c'(\varepsilon)$ over $\mathrm{d}s'$ along C^*C^0, with the help of (1) and (6), we find

$$T = \int \mathrm{d}t' = \frac{\int \mathrm{d}s'}{c'(\varepsilon)} = \frac{\Delta s'}{c'(\varepsilon)} = \frac{\Delta s'}{\gamma^2(c - v)} \qquad (7)$$

$$c'(\varepsilon) = \frac{c}{1 + v/c - \varepsilon/c} = \gamma^2(c - v) \quad \Rightarrow \varepsilon = 0.$$

Thus, the synchronization parameter is determined ($\varepsilon = 0$) and singles out the LTA based on conservation of simultaneity, invalidating the LT and relative simultaneity. The same result is obtained in Section 4 using Cartesian coordinates in the form of (1) applied to the linear effect.

Result (7) reflects the well-known problem inherent to Einstein synchronization when applied to a moving closed contour. In fact, if we apply Einstein synchronization and set the local ground speed to be $c' = c$, after integrating $\mathrm{d}t' = \mathrm{d}s'/c$ over $\mathrm{d}s'$ along C^*C^0, in agreement with (5), the clock at C^0 is foreseen to display the reading,

$$(T_{\mathrm{out}})_E = \frac{\Delta s'}{c} \simeq \frac{2L}{c} \neq T_{\mathrm{out}} = T, \qquad (8)$$

which, compared with (6), implies that the clock $C^0 \equiv C^*$ is out of synchrony with itself [12–23,29]. Moreover, since also $(T_{\mathrm{ret}})_E = \Delta s'/c \simeq 2L/c$, we have

$$\Delta T = (T_{\mathrm{ret}})_E - (T_{\mathrm{out}})_E \simeq 0, \qquad (9)$$

and there is no Sagnac effect. Hence, as recognized also by physicists adhering to the conventionalist view [6,8,29,33–37], Einstein synchronization and the LT fail to interpret the Sagnac effects, when observed along the fiber from the moving device C^*.

We show in Section 4 that the use of the LT in the linear effect leads to a spacetime continuity breach and, in the case of the reciprocal linear Sagnac effect, the LT and LTA foresee different observable results, confirming that they are not equivalent. In any event, by itself, Einstein synchronization does not determine the one-way light speed. Thus, results (5), (8), and (9) point out the limited validity of Lorentz and light speed invariance based on Einstein synchronization, which is not applicable to the Sagnac effects.

Extending the one-way internal synchronization procedure to the rod of Figure 2(c) and (d)

We denote as "one-way synchronization" the procedure, alternative to Einstein synchronization and applicable to the effects of Figure 2(a) and (b), that we wish to extend to the rod of Figure 2(c) and the example of Figure 2(d), as described below. For our procedure, we consider in Figure 2(d) two generic reference frames, S' and S, in relative motion and, without introducing the synchronization parameter ε of (1), we make use directly of the results (2) and (3). We assume first that the one-way light speed is c on frame S of Figure 2(d), where the section AB = L has a mirror placed at A and another at B (as in the Fabry-Perot laser cavity). With the clock C^* on frame S' moving at the velocity v relative to S, when A is passing by C^*, a photon is sent from the position of C^* toward the mirror at B and, after being reflected, travels back to reach C^*. The round-trip interval derived from S provides the same result as in (3),

$$T = \frac{T_{\mathrm{lab}}}{\gamma} = \frac{2\gamma L}{\gamma^2(c + v)} = \frac{2\gamma L(1 - v/c)}{c}, \qquad (10)$$

where $\gamma^{-2} = (1 - v^2/c^2) = (1 + v/c)(1 - v/c)$.

Let us now consider the analogy between the physical situation of Figure 2(d) and that of Figure 2(b). For the example in Figure 2(b), the result (10) corresponds to the counter-propagating photon emitted by C^*, traveling initially along the fiber's lower section from $A \equiv C^*$ to B and returning from B to C^* on the upper section. For this photon, the local ground speed along the moving fiber is $c_g = \gamma^2(c + v)$, which, on frame S' co-moving with the fiber, represents the local one-way return speed from B to C^* on the fiber's upper section. However, the motion of this photon is indistinguishable from that of the photon performing a round trip in Figure 2(d). Regardless of whether an experiment analogous to that in Figure 2(b) is actually performed or hypothetically thought of for Figure 2(d), physical reality is the same for both Figure 2(b) and (d) and the invariant round-trip interval T is the same in both cases. Hence, in frame S' of Figure 2(b) and (d), we must have $c'_{\mathrm{ret}}(v) = c'_g = \gamma^2(c + v)$ for the return ground speed of the photon moving from B to C^*. Furthermore, since in frame S' the two-way average light speed is c, on account of (4), the one-way light speed in the opposite "out" direction from C^* to B, is found to be $c'_{\mathrm{out}}(v) = \gamma^2(c - v)$, which is the speed of the photon seen by S' in Figure 2(b) and (d) when traveling from $A \equiv C^*$ to B.

By exploiting the analogy between Figure 2(d) and (b) and assuming S to be the preferred frame, we have determined the one-way light speed in S'. Analogous results are obtained by assuming the one-way light speed to be c in S', which is now the preferred frame. Hence, what the linear Sagnac effect implies is that, if the light speed is c in the chosen preferred frame S, the one-way light speed in the relatively moving frame S' is $c'(v) = \gamma^2(c \pm v)$, as foreseen by transforms (1) with $\varepsilon = 0$. Although, in principle, the choice of the preferred inertial frame is arbitrary, symmetry considerations may indicate the more convenient choice. For example, for the description of the circular effect of Figure 2(a), the frame S_{lab} can be conveniently chosen as representing the preferred frame.

Once the one-way light speed $c'(v) = c_{\text{out}}(v) = \gamma^2(c - v)$ is known, for a clock at a point P along the fiber at the distance $s' = s'_g$ from C^* in Figure 2(b) or along the $x' = s'$ axis of S' in Figure 2(d), the one-way synchronization procedure requires setting the clock readings to,

$$t' = \frac{s'}{c'(v)} = \frac{s'}{\gamma^2(c - v)}, \tag{11}$$

in agreement with experimental evidence (6). The synchronization can be applied along any finite section of frame S' of Figure 2(b) and (d), where the rod of Figure 2(c) forms a section of frame S'. However, using curvilinear coordinates, the synchronization can be applied along the whole moving fiber s_g.

The synchronization procedure described earlier represents an “internal” synchronization [6] because the ground one-way speed c'_g is determined without any reference to the readings of the external synchronized clocks on the preferred frame S_{lab}. If we use these S_{lab} readings to synchronize the clocks on S', we are performing an “external” absolute synchronization [6], which turns out to coincide with our one-way internal synchronization. Synchronization (11) supports the LTAs, which are the only transformations that can interpret consistently the Michelson-Morley and Sagnac experiments [8,12–19].

4 The linear Sagnac effect, its reciprocal, and the principles of Einstein and Galilean relativity

4.1 The reciprocal linear Sagnac effect

More evidence supporting the fact that the LT and LTA are not physically equivalent comes from our recent publications [11,12] where we consider a new optical effect (denoted by some physicists as the “Spavieri-Haug effect”), which may be considered as a kind of reciprocal linear Sagnac effect.

In the standard linear effect (Figure 1(b)), the invariant interval T is independent of whether the device C^* stays, or not, on the same section (upper or lower) during the interval T. If the principle of relativity is valid, concerning the invariant proper time interval T, there should be no difference between the situation when the clock C^* moves back and forth relative to the stationary contour as in the standard effect (Figure 1(b)), or the situation when the contour moves back and forth relative to the stationary clock on frame S, as in the reciprocal effect of Figure 1(c) and 3. Actually, calculations show [11] that, for the two counter-propagating photons, ΔT in (2) is always invariant, the same in the reciprocal linear effect as in the standard linear effect, thus confirming the reciprocity of the two effects for ΔT.

However, for the LT and with $X = AC^*$ in Figure 3, in the reciprocal effect the one-way round-trip interval T for a single photon is X-dependent ($T = T(X)$) when, in changing the direction of motion, the contour has the device C^* first on one section and then on the other. On the contrary, if the LTAs are adopted and the one-way light speed is c on the rest frame S_{count} of the contour, both ΔT and T are always invariant and T is independent of X [11].

It is surprising that in the case of the reciprocal linear Sagnac effect, the reciprocity is fully preserved using the LTA but not the LT. We know that in many physical situations, the relativity principle is compatible and in complete agreement with the LT, indicating that there is a vast class of physical phenomena where the relativity principle and the LT are compatible and, moreover, the LT and LTA are equivalent.

Nevertheless, in the wider class of phenomena that includes other effects, *e.g.*, the reciprocal linear Sagnac effect, the compatibility requires the use of the LTA, rather than the LT. Thus, to indicate the limited class of phenomena in which the reciprocity is compatible with the LT, we introduce tentatively the term “weak form” of the relativity principle when dealing with the LT. In fact, in this case, for the reciprocal linear Sagnac effect, only the quantity ΔT is invariant.

Still, for the same reciprocal linear Sagnac effect, the traditional form of the relativity principle will keep holding when using the LTA since the reciprocity is maintained because both the quantities ΔT and T are now invariant.

In the framework of the LTA, the relativity principle and the reciprocity of the linear effect hold if the contour

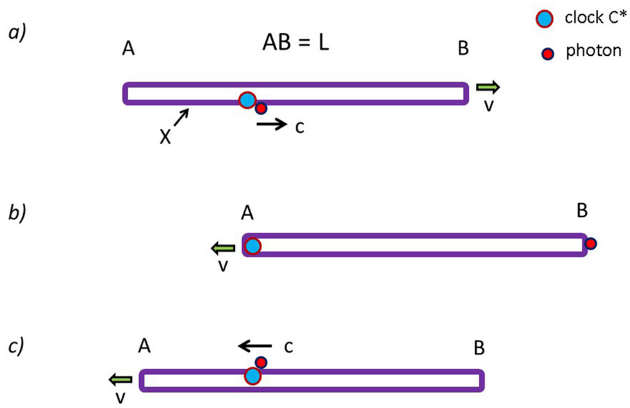

Figure 3: Determining th interval T in the reciprocal effect: (a) Clock C^* is stationary and initially located on the contour lower section, while the arm AB of the contour is moving at the speed v relative to C^*. (b) The initial distance X of A from C^*, is such that the photon emitted from C^* reaches B at the same moment when A reaches C^*. At this instant, the contour changes the direction of motion and the photon moves toward C^*. (c) The photon completes its round trip and reaches C^* on the upper section after the interval T.

inertial frame S_{count} represents, regardless of changing or not direction of motion, the preferred frame where Maxwell's equations are valid and the electromagnetic waves travel at speed c. The property can be extended to the Michelson-Morley interferometer and other electromagnetic phenomena involving interferometry. In particular, the Michelson-Gale experiment [45] of 1925, which provided a nonnull result, can be another indication that the contour of the interferometer can be linked to a preferred frame, as considered below about the problem of synchronization in the global positioning system.

To prove the nonequivalence of the LT and LTA, we show below that, in the reciprocal effect, T depends on X for the LT, but is X-independent for the LTA.

(a) **Showing that the LT foresee $T = T(x)$ for the reciprocal linear effect**

The calculations for a generic value of X are made in ref. [11]. If, during the interval T, C^* is always on the same (lower or upper) section, T is invariant. Here, we consider in Figure 3 the case when the stationary C^* emits the counter-propagating photon initially on the lower section (Figure 3(a)) and, after the contour changes its direction of motion (Figure 3(b)), receives the photon traveling on the upper section (Figure 3(c)). To simplify calculations, we assume that the original position $X = AC^*$ is such that the photon reaches B when, simultaneously, point A reaches C^*. At this moment (Figure 3(b)), simultaneously on the inertial clock frame S, the whole contour changes the direction of motion in the interval η. We also assume, for simplicity, that the rest length L of the arm AB of the contour is quite large, relative to the dimension R of the pulleys, $L \gg R$. In this hypothetically ideal linearized case, $T \gg \eta$, we can neglect the details of the motion taking place during the interval η.

With reference to Figure 3(a) and considering that B is moving with velocity v relative to the clock frame S, using the LT, we calculate the interval T_{LT} for the photon round-trip. Starting from C^* and taking into account the length contraction of the arm AB, the photon reaches B after the time interval determined by the equation, $ct = (L - X)/\gamma + vt$, while the moving point A, initially at X/γ, reaches C^* after the interval determined by the equation, $-X/\gamma + vt = 0$. Then, with $X/\gamma = vt$, the equation for the photon becomes $ct = L/\gamma$, and $T_{\text{out}} = L/\gamma c$, while $X = (v/c)L$. With the photon at B as shown in Figure 3(b), the return time is obtained from the equation $L/\gamma - ct = 0$, providing $T_{\text{ret}} = L/\gamma c$. Note that, in the return trip, T_{ret} is independent of whether the contour changes its direction of motion. Hence, instead of the expected invariant $T = 2\gamma^{-1}L/(c + v)$(3) of the standard linear effect, in the reciprocal effect, the round-trip time interval is

$$T_{\text{LT}} = T(X)_{X=(v/c)L} = T_{\text{out}} + T_{\text{ret}} = \frac{2L}{\gamma c}. \tag{12}$$

Different values for $T_{\text{LT}} = T(X)$ are obtained for different X [11]. By taking into account the effect of relative simultaneity with the LT, the same result (12) is obtained if derived from the contour frame S_{count}, where the light speed is c when S_{count} is in uniform motion before and after the interval η.

(b) **T is X-independent when derived in the framework of the LTA**

Assuming conservation of simultaneity, according to the relativity principle the result is the same whether the clock is moving relative to the contour or *vice versa*. In the standard linear effect, the one-way light speed is assumed to be c on the stationary contour frame S_{count}. Therefore, assuming again that the one-way light speed is c on the moving S_{count}, before and after the small interval η, C^* is seen from S_{count} to be in relative motion and, when calculated from the contour frame S_{count}, the result for the invariant T is just the one derived for the standard linear effect.

Let us confirm that T is invariant by deriving it also from the frame S of the stationary clock. We derive T from the frame S of the clock, using, for simplicity, the first-order approximation in v/c. The exact result, calculated from the frame S to all orders in v/c, can be derived without problems assuming that the contour frame S_{count} is the preferred frame where LTAs are adopted (before and after the negligible interval η). Since the one-way speed of light along AB is c on the moving preferred frame S_{count}, according to the LTAs, which now coincide with the

Galilean transformations, the corresponding light speed on frame S is $c + v$. Then, the photon reaches B at the time determined by the equation, $(c + v)t = (L - X) + vt$, while A reaches C^* at the time determined by the equation, $-X + vt = 0$. With $X = vt$, the equation for the photon becomes, $(c + v)t = L$, and we have $T_{\text{out}} = L/(c + v)$. According to the LTA, on the clock frame S, the return light speed from B to C^* is again $c + v$. Then, with the photon at B, as shown in Figure 3(b), the return time is obtained from the equation $L - (c + v)t = 0$, providing $T_{\text{ret}} = L/(c + v)$. Hence, for the reciprocal effect, LTAs foresee the approximated round-trip time invariant interval,

$$T = T_{\text{out}} + T_{\text{ret}} \simeq \frac{2L}{c + v}, \tag{13}$$

the same as for the standard linear effect, as foreseen by the relativity principle, but different from the result (12) foreseen by the LT.

The difference between (12) and (13) is due to the "time gap" $\delta t'$ of relative simultaneity between the two frames in relative motion, as shown explicitly in the examples below.

4.2 Measuring the round-trip interval T with the clock C^* fixed to the moving contour, which changes velocity relative to the inertial frame S

(a) **Deriving T from the rest frame S_{count} in the framework of the LTA.**

In this case, we derive the round-trip interval T, measured by C^*, from the rest frame S_{count} of the contour, where clock C^* is fixed at the distance X from point A, as shown in Figure 4. S_{count} and the inertial frame S are in relative motion before and after the small interval η.

Assuming the local light speed to be c on S_{count}, the counter-propagating photon leaves C^* in the direction of B and performs the round-trip C^*B, BA, AC^* to return to C^*. The arm AB and frame S are in relative motion and, if the relative velocity does not change, the round-trip interval measured by C^* is $T = 2L/c$. However, when the photon reaches point B, the contour changes the direction of motion relative to S. Therefore, the contour is moving with velocity v relative to the inertial frame S during the out trip from C^* to B, and with velocity $-v$ during the return trip from B to C^*. Vice versa, frame S is seen in relative motion from S_{count}, as shown in Figure 4 (before and after the small interval η).

Since conservation of simultaneity is assumed, the relative change of motion occurs simultaneously on both S_{count} and S. As usual, we neglect the interval η taken by the contour to change velocity, assuming that $T \gg \eta$. Then, before and after changing velocity, the contour is an inertial frame, S_{count}, on which the one-way light speed is c.

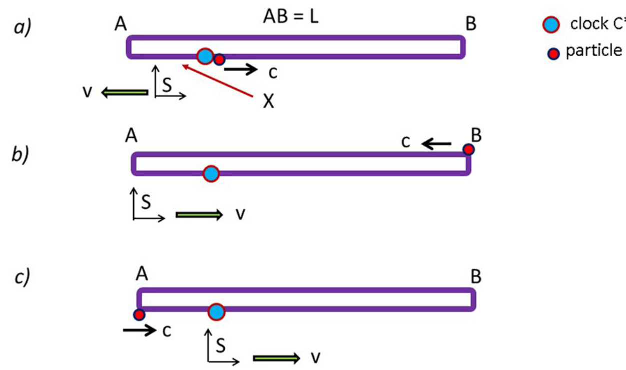

Figure 4: Interval T calculated from the contour rest frame S_{cont} when C^* is fixed to the contour and the inertial frame S is moving at the velocity v. (a) Clock C^* is fixed at the distance X from A. The photon emitted from S^* moves at speed c toward B. (b) When the photon reaches B, the contour changes its direction of motion relative to S. After the negligible interval η, the contour rest frame is still an inertial frame and the photon moves at speed c from B toward A. (c) After reaching A, the photon moves toward C^* and completes its round trip in the interval T.

Therefore, from Figure 3(a), on $(S_{\text{count}})_{\text{before}}$, we have $T_{\text{out}} = (L - X)/c$, and, from Figure 3(b) and (c), on $(S_{\text{count}})_{\text{after}}$ we find $T_{\text{ret}} = (L + X)/c$, with the expected invariant result,

$$T = T_{\text{out}} + T_{\text{ret}} = \frac{2L}{c}, \tag{14}$$

corresponding to the proper time measured by C^* for the one-way round trip of the photon, independent of its state of motion.

Together with the invariance of the clock's proper time, even when accelerating, the invariant result (14) should be holding for circular or other shapes of contours and, in general, for models of elementary particles or atomic systems where a particle performs a closed trajectory in the interval T. The invariant result (14) implies that S_{count} (before and after the interval η) stands as a preferred frame where Maxwell's equations are valid and the electromagnetic waves are confined to traveling at the one-way speed c along the closed contour. Consequently, if the LTA and conservations of simultaneity are valid, the invariant T of (14) must be foreseen from any other inertial frame of reference. However, we show that result (14) is not foreseen by the LT.

(b) **Deriving T from frame S assuming light speed invariance with the LTs**

The calculations performed from the inertial frame S are straightforward and, with $\gamma^{-2} = (1 + v/c)(1 - v/c)$, the results are given as follows:

$$\begin{aligned}(T(X))_S &= (T_{\text{out}} + T_{\text{ret}})_S = \frac{L - X}{\gamma(c - v)} \\ &\quad + \frac{L}{\gamma(c - v)} + \frac{X}{\gamma(c + v)} \\ T_{\text{LT}}(X) &= \frac{(T(X))_S}{\gamma} = \frac{L - X}{\gamma^2(c - v)} \\ &\quad + \frac{L}{\gamma^2(c - v)} + \frac{X}{\gamma^2(c + v)} \\ &= \frac{2L}{c} + \frac{2v}{c}\frac{L - X}{c} = \frac{2L}{c} + 2\delta t',\end{aligned} \tag{15}$$

where, in (15), $(T(X))_S$ represents the time interval derived from frame S and $T(X)_{\text{LT}} = \gamma^{-1}(T(X))_S$ represents the proper time reading of C^* foreseen by the LT. The term $\delta t' = v(L - X)/c^2$ in (15) corresponds to the "time gap" between the two frames (S and $(S_{\text{count}})_{\text{before}}$) due to relative simultaneity (see also Section 4, Eqs (23) and (32)) foreseen by the time transform of the LT (1). As in the case of the reciprocal effect, the result T in (14), foreseen by the LTA, differs from $T_{\text{LT}}(X)$ in (15) foreseen by the LT.

Let us consider the case when $X = 0$. Then, according to the LT, clock C^* measures the following intervals,

$$\begin{aligned}T_{\text{LT}} &= (T_{\text{out}})_{\text{LT}} + (T_{\text{ret}})_{\text{LT}} = \frac{L}{c}\left(1 + \frac{v}{c}\right) + \frac{L}{c}\left(1 + \frac{v}{c}\right) \\ &= \frac{2L}{c}\left(1 + \frac{v}{c}\right) = \frac{2L}{c} + 2\delta t',\end{aligned} \tag{16}$$

indicating that, due to the effect of relative simultaneity, as seen from clock C^* on S_{count}, the photon covers in the proper time interval T_{LT} the distance $2L + v2L/c > 2L$, which is greater by $2c\delta t'$ than the size $2L$ of the contour.

The difference between results (14) and (16), represents the Spavieri-Haug relative-simultaneity effect foreseen by standard special relativity when the contour reverses its velocity. The corresponding space, $2c\delta t'$, and time, $2\delta t'$, variations represent a spacetime discontinuity foreseen by the LT, but not by the LTA. The analogous spacetime discontinuity, or breach in spacetime continuity, for the standard linear Sagnac effect, is discussed below.

In conclusion, the difference between (12) and (13) (and between (15) and (14)) is related to the "time gap" $\delta t'$ of relative simultaneity. Since the difference is observable, it implies that relative (LT) and absolute simultaneity (LTA) can be discriminated experimentally. Possible experimental setups for the test of Lorentz and light speed invariance using the reciprocal effect are described in refs [11,12]. The unexpected results of the reciprocal effect confirm that the predictions of the LT and the LTA for the round-trip interval T are different, and that, in general, the LT and LTA are by no means equivalent and reflect different physical realities. The conventionalist thesis of Mansouri and Sexl is valid in the special cases when the two-way Einstein synchronization between spatially separated clocks is applicable and the one-way light speed is undetermined, as shown in Figure 2(c). However, in the case of the Sagnac effects, the one-way light speed is determined by the one-way internal synchronization that, for a closed contour, relies on the use of a single clock. Hence, the strategy to adopt the LTA for "solving" the paradoxes that arise with the "equivalent" LT, implemented for decades by supporters of the LT, fails with the Sagnac effects, particularly in the case of the reciprocal linear effect. Unfortunately, this flawed strategy has delayed considerably the correct interpretation of the various paradoxes and optical experiments.

4.3 The linear Sagnac effect: Spacetime continuity requires to adopt conservation of simultaneity with the corresponding local speed $\simeq c \pm v$ along the moving optical fiber

We revise the linear Sagnac effect of Figure 1(b), focusing on the special case when the device C^* moves from the lower to the upper section in the interval T', as discussed in detail in previous studies [23,24,26]. We consider the case of a counter-propagating photon that leaves the clock C* and returns to it after the round-trip time interval T. To simplify the calculations, we assume as usual that the interval η, taken by C^* to move around the pulley of radius R, while moving from the lower to the upper fiber section, is negligible and much less than T, which implies $L \gg R$.

Let us denote by S'' the inertial rest frame of C^* when on the fiber lower section and by S' the rest frame when on the upper section. The round-trip interval $T(3)$ measured by C^* co-moving with the fiber, is,

$$T = \frac{2\gamma L}{\gamma^2(c + v)} = \frac{2\gamma L(1 - v/c)}{c}, \tag{17}$$

where $\gamma^{-2} = (1 - v^2/c^2) = (1 + v/c)(1 - v/c)$. We gave above the interpretation of the two terms of (17), and we consider it once more below.

We denote by $c'' = c''_g$ the "ground" local light speed on S''. Then, c''_g represents the "ground" local light speed along the fiber ground section that is at rest on S'' on the lower section. Similarly, we denote by $c' = c'_g$ the ground local light speed on S′. *A priori*, c'' and c' do not necessarily coincide, depending on the theory and the kinematical relation revealed by experimental evidence. As a way to check the consistency and completeness of the theory, with the LT or the LTA, we need to verify what is the ground local speed on both the lower and upper sections and the ground fiber total length covered by the photon in the interval T. For this purpose, it is convenient to consider the following case where, in the interval T, C^* moves from the lower to the upper section.

Single frame description with the LT from S'' With C^* initially on the frame S'' of the lower section (Figure 1(b)), the initial position of C^* relative to A can be chosen in such a way (AC^*=$X = (v/c)L/\gamma$) that the counter-propagating photon leaving C^* reaches B when, simultaneously, A reaches C^*, as indicated in Figure 5. Assuming $c'' = c$ as seen from C^* on the clock frame S'', the time interval taken by the photon to reach B is,

$$T''_{\rm out} = T_{\rm out} = \frac{L''}{c''} = \frac{L}{\gamma c}, \tag{18}$$

which is the same time interval $T_{\rm out} = X/v$ taken by A to reach C^*. Since L'' and c'' are "ground" kinematical quantities measured on S'', L'' represents the fiber "ground" section covered by the photon at the local "ground" speed c''. Hence, the fiber ground length covered at speed $c'' = c$ by the photon in the out trip $T_{\rm out}$ from C^* to B, is $L'' = \gamma^{-1}L$.

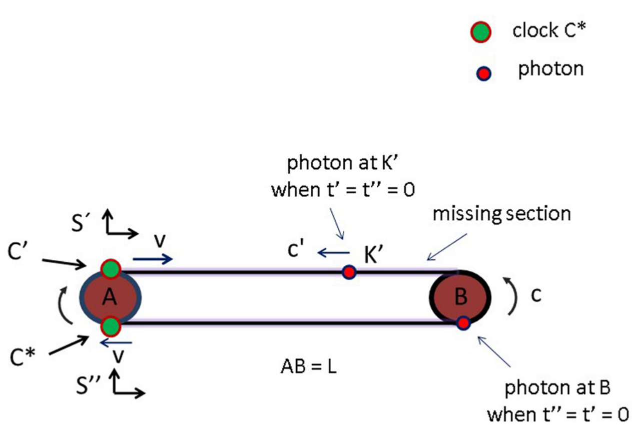

Figure 5: In the linear Sagnac effect, clock C^* is at rest on the inertial frame S'' and clock C' is at rest on the inertial frame S'. Frames S' and S'' are in motion with opposite velocities v relative to the frame AB of the contour and coincide at A at $t' = t'' = 0$. After being emitted earlier by C^* on the fiber lower section (Figure 1(b)), the photon reaches B when A reaches C^* and, as observed from C^* on frame S'', the photon has covered the distance L/γ in the interval $T''_{\rm out} = T_{\rm out} = L/(\gamma c)$ measured by clock C^*. We can look now at the physical situation as seen from frame S' at $t' = t'' = 0$. According to the LT, the photon is at K' at $t' = 0$ and covers the shorter distance $\gamma L(1 - v/c)^2$ in the return trip in the interval $T_{\rm ret} = T'_{\rm ret} = \gamma L(1 - v/c)^2/c$ measured by clock C'. The missing section $K'B = c\delta t' = 2\gamma(v/c)L$ has not been covered for $t' > 0$ in the observed interval $T'_{\rm ret}$. According to the LTA, there are no missing sections because the photon is at B at $t' = 0$, when C' is at A and covers the whole distance $\gamma_w L/\gamma$ in the return trip.

To avoid considerations about the noninertiality of the circular ends (pulleys A and B in Figures 1(a) and 5) of the linear effect, we highlight that the pulleys A and B can be ideally replaced by mirrors that guide the photon when moving from the lower section to the upper section. Moreover, the relevant time intervals can be measured by two clocks always in uniform motion, C^* co-moving with the lower section and, for the photon return trip on the upper section, we introduce, as shown in Figure 5, the second clock C' co-moving on the upper section with the inertial frame S' moving with velocity v relative to the arm AB. This way, the effect is completely linearized, and light propagation and clocks measurements can be consistently described from inertial frames in standard flat spacetime.

Clock C' is set at $t' = t'' = 0$ at point A when coinciding with C^*. Of course, the time intervals measured by C' after $t' = 0$ are the same intervals that C^* would measure after having moved to the upper section. In any case, it should be clear that we are dealing with time intervals measured on inertial frames, corresponding to the invariant proper time intervals of C^*, within the approximations made.

Denoting by $w \simeq 2v$ the relative velocity between S'' and S', the corresponding LT [23,24] and some of its relations with the AB frame S, are,

$$\begin{gathered} x' = \gamma_w(x'' - wt'') \quad t' = \gamma_w\left(t'' - \frac{wx''}{c^2}\right) \\ w = 2v/(1 + v^2/c^2) \quad \gamma_w^{-2} = (1 + w^2/c^2)^{1/2}, \end{gathered} \tag{19}$$

$$\begin{gathered} \gamma_w = \gamma^2(1 + v^2/c^2) \\ \gamma_w(1 + w/c) = \gamma^2(1 + v/c)^2 = \frac{1 + v/c}{1 - v/c}. \end{gathered} \tag{20}$$

The return trip time interval seen from S'' is obtained from the equation $wt'' = L/\gamma - ct''$, and,

$$\begin{gathered} T''_{\rm ret} = \frac{L}{\gamma(c + w)} = \frac{\gamma_w L(1 - v/c)}{\gamma c(1 + v/c)} \\ T_{\rm ret} = T'_{\rm ret} = \frac{T''_{\rm ret}}{\gamma_w} = \frac{\gamma L(1 - v/c)^2}{c}, \end{gathered} \tag{21}$$

where the proper time interval $T'_{\rm ret} = \gamma_w^{-1}T''_{\rm ret}$ is equally foreseen by the time transforms (1) of the LTA and LT (at $x'_{C'} = 0$). The S'' spatial distance covered by the photon in the return trip is,

$$cT''_{\rm ret} = \frac{L}{\gamma(1 + w/c)} = \frac{\gamma_w L(1 - v/c)}{\gamma(1 + v/c)} \simeq L(1 - 2v/c) < L, \tag{22}$$

less than L because, as seen from S'', clock $C^{*\prime}$ is moving at speed w toward the photon approaching at speed c. Then, as seen from the single frame S'', in the round trip T (17) the photon covers, at speed c, the total distance,

$$cT_{\rm out} + cT''_{\rm ret} \simeq L + L(1 - 2v/c) \simeq 2L - c\delta t' < 2L. \tag{23}$$

In (23), the term $\delta t' = \gamma_w \gamma^{-1} wL/c^2 = 2\gamma vL/c^2$ is the "time gap" from S' to S'' due to relative simultaneity foreseen by the time transform of the LT in (19). Thus, as seen from S'' or any other single frame, the spatial distance covered is $2L - c\delta t'$, less than the total ground fiber length $\simeq 2L$, and the uncovered path $\simeq 2\gamma vL/c = c\delta t' \simeq vT$ is shown for the circular and linear effects in Figure 1(a) and (b).

Description with the LT involving the two frames S'' **and** S'. In the out trip, the ground distance covered is $L'' = \gamma^{-1}L$, as given by (18). The return trip T'_{ret} is given by (21). According to the LT, the return light speed is c on S' and $T'_{\text{ret}} = L'/c$. For the observer on S',

$$L' = cT'_{\text{ret}} = \gamma L(1 - v/c)^2 \simeq L - c\delta t' < L, \tag{24}$$

indicating that the photon does not cover the whole path L in the return trip. The total ground path covered at speed c by the photon, L'' on S'' and L' on S', is exactly,

$$L'' + L' = \gamma^{-1}L + \gamma L(1 - v/c)^2 = 2\gamma L - c\delta t' < 2L, \tag{25}$$

essentially as in (23).

In conclusion and summarizing, if Einstein synchronization were applicable along the whole contour of length $2L$ of the linear Sagnac effect, the expected resulting round-trip interval would be $(T)_{\text{Einstein}} \simeq 2L/c$, with c the local ground speed along the whole contour of length $\simeq 2L$. However, $(T)_{\text{Einstein}}$ differs from the physically observed result, which is $T = T_{\text{out}} + T_{\text{ret}} = 2L/(c + v) \simeq 2L(1 - v/c)/c$, as in (17). Therefore, Einstein synchronization fails when applied to the Sagnac effect. In fact, in the observed interval $T \simeq 2L(1 - v/c)/c$ and at the local light speed c, the derived result (25) foreseen by the LT, indicates that the photon does not traverse the whole contour $2L$, but covers the shorter distance $\simeq 2L(1 - v/c) = 2L - c\delta t'$ only, being $c\delta t'$ the "time gap" from S'' to S' due to relative simultaneity.

Hence, both observers S'' and S' concur that, at the ground local speed c, the photon fails to cover the whole fiber length $\simeq 2L$ in the round-trip interval T and agree that the missing path $2\gamma vL/c = c\delta t'$ has not been traversed in the interval T. This result, foreseen by the LT, implies a breach by $c\delta t'$ in spacetime continuity.

From a physical point of view, the breach is a consequence of imposing the constraint of light speed invariance along the whole moving contour, as required by the LT based on the two-way Einstein synchronization. As pointed out by Sagnac, Selleri, Landau and Lifshitz (quoted in Section 4.5, point 3), and many other authors, the failure of Einstein synchronization becomes apparent when applied to the closed contour of the effects of the Sagnac type where the round-trip interval T is measured by a single clock.

Yet, as shown in the next section, spacetime continuity is preserved if the "natural" one-way synchronization of the LTA, not equivalent to Einstein's, is used in interpreting the optical effects of the Sagnac type.

4.4 Imposing spacetime continuity in deriving T

In the return trip from B to C^* on the upper section, in terms of "ground" kinematical quantities, clock C'^* measures the observable interval $T_{\text{ret}} = L'/c'$ from the instant $t' = 0$, when it coincides with A, to the moment when the photon reaches it. Assuming c' undetermined, also the ground distance L' is undetermined, but light propagation along the closed contour imposes a constraint: spacetime continuity requires the total ground length of the fiber, covered by the photon in the round trip interval T, to be $2\gamma L$. Since the distance $L'' = \gamma^{-1}L$ has been covered in the out trip, the remaining distance,

$$L' = 2\gamma L - L'' = \gamma^2\left(1 + \frac{v^2}{c^2}\right)\frac{L}{\gamma} = \gamma_w \gamma^{-1} L,$$

must be covered at speed c' in the return trip.

With the help of (21) and (20),

$$\begin{aligned} T_{\text{ret}} &= \frac{L'}{c'} = \frac{\gamma_w \gamma^{-1} L}{c'} = \frac{\gamma L(1 - v/c)^2}{c} \\ \frac{\gamma_w}{c'} &= \frac{1 - v/c}{c(1 + v/c)} = \frac{1}{c\gamma_w(1 + w/c)}, \end{aligned} \tag{26}$$

and, from (26), we find c' to be,

$$c' = \gamma_w^2(c + w) = \frac{c}{1 - w/c}. \tag{27}$$

According to the approach of Mansouri and Sexl [6] of (1), for the transformations from S'' to S' in terms of the synchronization parameter ε, we have [6], [26],

$$\begin{aligned} &t' = t''/\gamma - \varepsilon x'/c^2 \\ &x' = \gamma(x'' - wt'') \quad y' = y'' \quad z' = z'' \\ &c' = c'(\varepsilon) = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{c}{1 + w/c - \varepsilon/c}, \end{aligned} \tag{28}$$

where we have set $c'' = \mathrm{d}x''/\mathrm{d}t'' = c$ on frame S''. For counter-propagation $c'(\varepsilon) = c(1 - w/c + \varepsilon/c)^{-1}$ and, by setting $c' = c'(\varepsilon)$ in (27), the equation determines the value $\varepsilon = 0$, implying that the resulting synchrony, reflecting the interpretation of the linear Sagnac effect consistent with spacetime continuity, is that of the LTA with absolute simultaneity. With c' given by (27) on S', $c'' = c$ on S'', and the help of (20), the total ground length covered is $cT_{\text{out}} + c'T_{\text{ret}} = \gamma^{-1}L + \gamma_w\gamma^{-1}L = \gamma^{-1}L + \gamma(1 + v^2/c^2)L = 2\gamma L$, as expected.

Adopting the LTA with conservation of simultaneity in the linear Sagnac effect

In the previous section, we assumed the one-way light speed to be c on the frame S'' of the fiber lower section. Nevertheless, considering that the relativity principle holds when the preferred frame is the contour frame S_{count}, let then be $S = S_{\text{count}}$ the preferred frame and introduce the LTA transformations from S to the frames S' and S''. The curvilinear transformations (already used in [24]) along the length s' of the optical fiber may be expressed as follows:

$$s' = \gamma(s - vt) \quad t' = \frac{t}{\gamma}. \tag{29}$$

As seen from frame S (Figure 2(b)), starting from the origin of S', light can travel along the fiber the distance $s = ct$ to the generic point on the upper or lower section. The corresponding distance seen from the fiber is $s' = \gamma(c - v)t = \gamma^2(c - v)t' = c't'$. In the circular Sagnac effect, after a counter-propagating photon covers the whole circumference traveling the distance $s' = \gamma 2\pi r$, we find $T' = T = s'/c' = \gamma 2\pi r/[\gamma^2(c + v)]$ in agreement with (17). For inertial frames in Cartesian coordinates, the corresponding LTA (1) can be written as follows:

$$\begin{aligned} x' &= \gamma(x - vt) \quad t' = \frac{t}{\gamma} \\ x'' &= \gamma(x + vt) \quad t'' = \frac{t}{\gamma}, \end{aligned} \tag{30}$$

where, relative to S, the equation of the first line is replaced by the equation of the second line when c changes sign and direction at B. In this case, with $V = \gamma^2 2v$ for the relative velocity, the LTAs between S'' and S' coincide with the Galilean transformations,

$$x' = x'' - V\ t'' \quad t' = t''. \tag{31}$$

From (30), we find the ground light speed $c' = c'' = \gamma^2(c - v)$, at which the photon covers the length $2\gamma L$ in the interval $T_{\text{one-way}}$ (6). No inconsistencies arise (such as spacetime breach) by describing the Sagnac effects with the simpler transformations LTA based on conservation of simultaneity.

4.5 Short review of the arguments in favor and against the LT

The current main argument against Sagnac's interpretation.

Supporters of standard special relativity agree that the LTAs interpret all the relativistic effects of the theory and that the LTA can be used, in lieu of the LT, to describe the Sagnac effect and "solve" the Selleri and other paradoxes [6,8,13–15]. Their main argument for claiming that the LT and LTA are equally valid is that they differ by the arbitrary synchronization parameter ε only [6], and thus, they are physically equivalent and interchangeable. Hence, according to them, LTs are still valid if the paradoxes of the LT can be "solved" with the LTA.

Arguments against the equivalence of the LT and the LTA, showing the limited validity of the LT

(1) For the result (17) of the linear Sagnac effect, we have the following two interpretations. As seen by the ground observer C^* co-moving with the fiber of ground length $L_{\text{ground}} \simeq 2L$, in the interval $T \simeq 2L/(c + v)$, the counter-moving photon covers at the ground local light speed $c_g \simeq c + v$ the whole fiber ground length L_{ground}. Instead, for the observer on the lab frame S_{lab} (where space is isotropic) the spatial distance covered by the particle is $L_{\text{space}} \simeq 2L(1 - v/c)$, as can be seen also for the circular effect of Figure 1(a), and, consequently, with $T \simeq 2L(1 - v/c)/c$, the one-way light speed is c. Hence, since T is the same and the two distances L_{ground} and L_{space} are different, also the relative local speeds of the photon must be different and are $c_g \simeq c + v$ locally along the moving fiber and c locally in the frame S_{lab} in relative motion. Hence, light speed invariance is invalidated, as Sagnac claimed.

(2) The reciprocal linear Sagnac effect indicates that the LT and LTA foresee different values for the observable T. In fact, $T = T(X)$ is X-dependent for the LT in (12), (15), and (16), while T is invariant and X-independent for the LTA (13). Then, the LT and LTA are not equivalent and represent different physical realities, invalidating the current main argument supporting the LT.

(3) In the case of the linear effect of Figure 5, the requirement of spacetime continuity for the photon covering the whole fiber length $2\gamma L$ in the interval T, supports conservation of simultaneity (LTA) and invalidates relative simultaneity (LT).

Note that the spacetime discontinuity of the LT has been pointed out more than 50 years ago by Landau and Lifshitz [46] by stating:

> However, synchronization of clocks along a closed contour turns out to be impossible in general. In fact, starting out along the contour and returning to the initial point, we would obtain for dx^0 a value different from zero...

In relation to Figure 5, the inconsistencies of the LT emerge when the effective "ground" section, covered by the photon in the round-trip interval T is pointed out.

– For the LT, the total ground length covered in the interval T by the photon at the local speed c, is (25),

$$2\gamma L - 2\gamma vL/c = 2\gamma L - c\delta t' < 2\gamma L.$$

Hence, the photon does not cover the "missing" section of Figure 5,

$$c\delta t' = 2\gamma vL/c, \tag{32}$$

where $\delta t' = 2\gamma(v/c^2)L$ represents the delay, or time gap, between S' and S'' due to relative simultaneity. In Figure 5, it seems as if the photon "jumps" from B to K' traversing the missing path at infinite speed.

– If the local ground speed is $c'' = c$ on the fiber's lower section, the local ground speed on the upper section can no longer be c, but $c' = \gamma_w^2(c + w)$, for the photon to be able to cover also the missing section in the return interval T_{ret}.

– If the differential local ground speed is c along the whole fiber length, after integrating $dt' = \mathrm{d}s'/c$ over s', we have

$$T_{c'=c} \simeq 2L/c$$

in disagreement with observation (3).

(4) The Thomas precession [47] related to the electron spin. In the study by Spavieri and Haug [11] we consider the different symmetries of the LT and LTA. Exploiting the symmetry of the transformations along the electron orbit, Jackson's [48] derivation shows that the Thomas precession is foreseen by the LT. Repeating the derivation using the LTA, we show that, due to the different symmetry, the LTAs foresee no Thomas precession [11]. Once more, the LT and LTA foresee different results.

(5) If the LTAs interpret consistently and solve the paradoxes of the LT, while the LTs do not, it is an indication that the LT and LTA are different and physically nonequivalent, rather than "equivalent." Conceptually, the fact that the LTAs do solve the paradox, does not change the reality that the LTs do not.

(6) GPS (global positioning system). We consider here the GPS argument by Gift [49], as described by Spavieri *et al.* [24], favoring absolute, rather than relative simultaneity.

"As considered by Gift [49], Ashby [50], and other authors, the existence of the ECI (Earth Centered Inertial frame) is supported by the fact that it clarifies the problem of clock synchronization on the Earth. Indeed, for achieving the clock synchronization with Einstein synchronization in the GPS and maintaining accuracy, the GPS must apply a Sagnac velocity correction to the propagation of its electromagnetic signals. This can be understood by considering that, if the speed of light is c locally in the ECI frame, it turns out to be $c \pm v$ on the rotating Earth surface (at the distance R from its center) because of the tangential velocity $v = \omega R$. Thus, the GPS algorithm seems to be supportive of the ECI frame and absolute synchronization for maintaining global accuracy among synchronized clocks. The result is a worldwide network of precisely synchronized clocks that are within 4 nanoseconds of 'perfect synchronization' with global simultaneity within the GPS [49]."

In support of Gift's argument, we may say that the non-null result of the Michelson-Gale experiment [45] is generally interpreted as a Sagnac effect capable of detecting the angular velocity of the Earth [51,52]. In fact, if the inertial frame of the Michelson-Gale interferometer is taken to be the ECI frame, the rotation of the interferometer coincides with Earth's rotation. Ideally, we can conceive a Michelson-Gale interferometer of the size πR^2 (the cross-section area of the Earth) perpendicular to the Earth's axis of rotation. Then, on account of the implications of the Sagnac effects described in Section 4, the ECI frame stands for the preferred frame where Maxwell's equations are valid and the one-way light speed is c and, on the surface of the rotating Earth the light speed must be $c \pm v$, as required for the GPS synchrony and foreseen by the LTA.

Additional considerations about the effects of the Sagnac type.

Bhadra *et al.* [53] state that in the Sagnac effect "unequal path lengths (are) traversed by the light rays in reaching the interferometer." We agree with the authors on the fact that in the Sagnac effects, shown in Figure 1(a) and (b), the paths of the counterpropagating light signals have different lengths when observed from the lab frame S where the one-way light speed is c. In fact, as observed from the lab frame S and to the first-order in v/c, in the round-trip interval T, the device has moved by vT, and thus, the paths of the two counterpropagating light signals differ by $2vT$.

However, when observed from the device in motion relative to S, the length of the optical path along the optical fiber is $2L$ (linear effect) or $2\pi R$ (circular effect), the same for the two counterpropagating light signals. Then, for an observer comoving with the device, a consistent interpretation requires different propagation velocities ($c + v$ and $c - v$, respectively) for the two signals.

For the circular Sagnac effect, the light speed along the rim of the rotating platform is calculated in the article of ref. [24]. For the linear Sagnac effect, the local light speed along each section of the contour can be calculated exactly with the approach used in Section 4.3.

Standing waves. Bhadra *et al.* [54] claim that the experimental detection of standing waves substantiates that the one-way velocity is equal to the round-trip velocity, implying the uniqueness of the Einstein synchronization convention.

However, in the derivation of the standing wave equation (4)* of the study by Bhadra *et al.* [54], we need to consider the invariance of the phase of the traveling waves (2)* (the symbol * indicates cross references of ref. [54]). If we consider the wave with the phase $Kx' - \omega t'$ in some inertial frame where one-way light speed is c and apply the transformation (1)* $t' = t - \xi x/c$, $x' = x$, we obtain $Kx - \omega t + \xi x/c$. Phase invariance requires to have $Kx' - \omega t' = Kx - \omega t$, which is possible if $\xi = 0$. Thus, with $\xi = 0$ in equation (4)*, we find $x' = x$, $t' = t$ and, as expected, the standing wave is invariant under transformation (1)*.

Comparing the gauge transformations (1)* with the Mansouri and Sexl transformations (1) with $v = 0$, we find that the condition $\xi = 0$ implies $\varepsilon = 0$. Then, for any value of v and with the fixed value $\varepsilon = 0$, we have $t' = t/\gamma$ in (1), which represents the time transform of the LTA based on absolute simultaneity. Thus, we do not agree with the statement of Bhadra *et al.* [54] that the choice $\xi = 0 = \varepsilon$ represents Einstein synchronization. Considering that the choice of the preferred inertial frame is arbitrary for the LTA, we may say that the apparatus where the standing waves are formed represents a preferred frame where the one-way light speed is c for both the LT and LTA.

Yet, it would be interesting to see what happens when transforming between frames in relative motion using the Mansouri and Sexl transformations (1), both from the theoretical point of view and the available observation, if any, of standing waves in moving frames.

It is convenient to recall that, for two-way light propagation along a linear contour, such as in the case of the Michelson-Morley (M-M) experiment, what is measured is the average light speed c, as discussed in Section 3 in relation to Figure 2(c). Instead, for a contour such as the circular Sagnac effect, which encloses a finite area, even if both the contour and the detector are stationary, the counterpropagating waves (likely forming standing waves) may have different velocities. This fact has been verified with great accuracy by Schreiber *et al.* [51], Stedman [52], and other physicists. As discussed in point 6 of the previous section, it consists of detecting the Sagnac effect when contour and the detector are stationary in the lab frame of the Earth. In these experiments, for the observed velocity-dependent light speed $c = c(v) = c(\omega r)$, the quantity ω represents the Earth's angular velocity, while v is the velocity of an observer S* instantaneously comoving with the emitter-receiver detector relative to the ECI frame. We mention in point 6 above, that the same effect was detected by Michelson-Gale in 1925 [45] and can be interpreted as a Sagnac effect, which, differently from the M-M experiment, provides a nonnull result. The experiments were sensitive enough to be able to detect the variation of the speed $c = c(\omega r)$ for different values of r along the different sections of the contour.

We know that Einstein synchronization fails when applied to the contour of the Sagnac effect. It follows that the synchronization capable of interpreting consistently the aforementioned experiments is not that of Einstein with the LT. Thus, we agree with the study by Bhadra *et al.* [54] that the conventionality issue becomes nonexistent, and that the (unphysical) synchronization gauge freedom of relativistic theories can be eliminated. However, we stress that experimental evidence shows the correct natural synchronization parameter to be the one that corresponds to the conservation of simultaneity of the LTA.

Clock synchronization procedures that are not equivalent to Einstein synchronization:

Summarizing the arguments of conventionalism and those in favor of the LTA.

With the conventionalist thesis, the one-way light speed is not observable (conventional), synchronization is arbitrary, and any internal synchronization procedure is equivalent to Einstein's. Thus, the LT and LTA, which are physically equivalent, foresee the same observable results.

To rebut the arguments of conventionalism, we highlight the following points.

The main problem with Einstein synchronization (and all the other synchronizations equivalent to it, expressed by the equation (4)), is that it consists of a two-way procedure where the resulting measured average light speed c conceals the one-way velocity dependence of $c_{\text{out}} = c_{\text{out}}(v)$ or $c_{\text{ret}} = c_{\text{ret}}(v)$, where v is the velocity relative to a hypothetically preferred inertial frame. Therefore, with Einstein synchronization, the local one-way light speed becomes unobservable in measurements involving, for example, optical effects of the Sagnac type.

Nevertheless, in the literature, there are several approaches for measuring the one-way speed of light, and we cite here the ones of the previous studies [25,27,28] that we know well. It is true that we need synchronized spatially separated clocks to measure the local one-way speed of light propagating, for example, along the rod of length Δ_0 of Figure 2(c). Then, for measuring the one-way light speed, we must introduce and also make use of a synchronization procedure not equivalent to Einstein's. To achieve this goal, we may start with the well-known GPS synchronization, considered by many authors and discussed by Gift in ref. [16] and Spavieri *et al.* [24,27].

The GPS synchronization, which is accurate and successfully employed, is equivalent to the synchronization obtained with the LTA in the circular Sagnac effect and is implemented assuming that the one-way light speed is uniform and equal to c in the ECI frame. Hence, with two GPS synchronized clocks on Earth, spatially separated by Δ_0, the result for the local one-way light speed along Δ_0 is foreseen to be velocity dependent, $c' = c(v)$, in agreement with the LTA and not with the result $c' = c$ foreseen by the LT.

Moreover, our one-way synchronization procedure, presented in Section 3, is based on the linear Sagnac effect and is analogous to the one of GPS based on the circular Sagnac effect. Again, in agreement with the LTA and not with the LT, our one-way synchronization foresees the one-way light speed along the distance Δ_0 to be v-dependent, *i.e.*, $c_{\text{out}}(v) = \gamma^2(c - v)$ and $c_{\text{ret}}(v) = \gamma^2(c + v)$.

Furthermore, we may consider also the procedure introduced by Spavieri *et al.* [55,56]. Differently from the usual procedures that rely on the transport of information at the speed of light (or any other finite speed), Spavieri's procedure consists of an internal one-way synchronization performed by means of a physical system that possesses a built-in simultaneity. No information transport is needed because it is preset in the system. Thus, this procedure is definitely different from Einstein synchronization.

Since the three aforementioned procedures represent one-way synchronizations sensitive to the v-dependence of the one-way light speed, they are not equivalent to Einstein synchronization and, thus, are suitable for testing light speed invariance.

We have shown in Section 4 that the LT and LTA foresee different results for the reciprocal Sagnac effect and also shown that the LTs based on Einstein synchronization fail in interpreting the linear Sagnac effect. Not to mention the many paradoxes of the LT that do not exist with the LTA, as even Mansouri and Sexl [6] point out, the nonequivalence between the LTA and LT is corroborated also by the fact that the symmetry of the LT (Lie group) is different from that of the LTAs, which do not possess a symmetry group.

Finally, the theoretical discussions about the reciprocal Sagnac effect can be extended to the context of its related novel applications, as shown by Spavieri and Haug [12], where they discuss the one-way linear effect. For this effect, the predictions of the LTA, which differ from those of the LT, are in principle experimentally observable by means of standard interferometry, ring laser techniques, or high precision time-delay detectors of light pulses. Since the one-way linear effect is sensitive to velocity variations, it may have relevant applications in inertial guidance systems and related areas and can be used to confirm Lorentz invariance by testing relative simultaneity versus absolute simultaneity.

In short, considering the various inconsistencies of the LT in relation to the Sagnac effects and the many other arguments presented earlier, with regard to the "LT-LTA equivalence," we may say, by challenging the conventionalist thesis, that there is sufficient evidence showing that the LT and LTA are not equivalent.

Additional information: Some latest journal papers on synchronization controllers [57–59] have been added in the references to give readers a more up-to-date picture on the approaches to synchronization in other fields, such as neural networks.

5 Conclusion

The one-way internal synchronization procedure along a closed contour is viable and applicable either when the device C^* and the contour are at rest or in relative motion. In the case of relative motion, the validity of Einstein synchronization and the LT of standard special relativity is limited to the case considered by Mansouri and Sexl [6] where synchronization between two spatially separated clocks is arbitrary (Figure 2(c)). As well known, the LTs are not applicable, or "nonintegrable," on a moving closed contour, and the transformations that interpret consistently the invariant one-way round-trip interval T for light propagation along the moving contour are those based on conservation of simultaneity. Thus, generally speaking, the one-way internal synchronization and the Sagnac effects rule out the LT and favor the Lorentz transforms based on absolute simultaneity (LTA).

The nonequivalence between the LT and LTA becomes apparent even in the reciprocal linear effect (Figure 1(c)) where, relative to the inertial frame of the device C^*, the contour frame changes the direction of motion in the interval T. In this case, the LTs fail to foresee reciprocity for the one-way invariant interval T, which is now X-dependent ($T = T(x)$), suggesting a weak form only of the relativity principle. Instead, the relativity principle is completely feasible with the LTAs, which foresee full reciprocity for the invariant T and ΔT.

In the standard linear Sagnac effect, the use of the LT indicates that, in the interval T, the photon does not cover the "missing" section $c\delta t' = 2\gamma vL/c$ of the fiber length $2\gamma L$, where $\delta t'$ represents the delay, or relative simultaneity time gap, between the frames S' and S''. The existence of a missing section reveals a breach in spacetime continuity due to relative simultaneity. Instead, there is no missing path with the LTA based on conservation of simultaneity and spacetime continuity.

To solve the paradoxes and remove the unusual consequences of the LT in some physical situations [6,8,12–19], conventionalist physicists [6] adopt the LTA, claiming that the paradoxes do not invalidate the LT because the two transforms, LT and LTA, are physically equivalent and interchangeable on account of the arbitrariness of synchronization and the conventionality of the one-way light speed. Still, considering that the reciprocal linear Sagnac effect and other physical effects render apparent the nonequivalence of transforms with different synchronies (ε), the sole transforms capable of interpreting consistently the various paradoxes and light propagation along closed moving contours are the LTAs. Outside the limited conventionalist context where the LT and LTA can be considered equivalent, in the more general scenario of nonequivalence and by means of the reciprocal Sagnac effect, Lorentz and light speed invariance can be tested and the one-way speed of light is measurable in principle, as required by epistemologists.

In short, we may conclude that:

1) In Einstein's second postulate, what is constant is no longer the one-way light speed, but the observable round-trip speed of light (*i.e.*, the average light speed c during the round-trip from C^* to C^0 and then back to C^*) [8].
2) For the description of physical phenomena (*e.g.*, light propagation along a closed moving contour), a preferred (not absolute) frame S, where the one-way light speed is c and Maxwell's equations are valid, can be conveniently chosen. However, the one-way synchronization and spacetime continuity require the transformations from S to any other relatively moving frame, to be based on conservation of simultaneity (*e.g.*, LTA).

Optical experiments, supporting the LT and light speed invariance in 1905, seem to question today their validity and that of relative simultaneity. Since the LTAs interpret without paradoxes the optical and all the other experiments supporting standard special relativity [6], they might represent a viable alternative to the LT in the scenario where the principle of relativity is holding.

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