Research Article

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Propagation of traveling wave solution of the strain wave equation in microcrystalline materials

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Abstract: This study focuses on the propagation behavior of traveling wave solution in microcrystalline materials using the polynomial complete discriminant system method. By establishing a complete discriminant system, we systematically analyze the formation and evolution process of traveling wave solution in microcrystalline materials. Specifically, we apply the cubic polynomial extension to the strain wave equation to obtain more accurate analytical solutions. Additionally, two-dimensional, three-dimensional, and contour plots are generated to visually illustrate the characteristics of the obtained solutions, facilitating a more intuitive understanding of their physical significance. These findings not only help reveal the propagation mechanism of traveling wave solution but also provide a theoretical foundation for the application of microcrystalline materials.

Keywords: traveling wave solution, microcrystalline materials, strain wave equation, polynomial complete discriminant system method

1 Introduction

In recent years, research on microcrystalline materials has attracted widespread attention due to their unique properties at the nanoscale [1–3]. These materials exhibit different characteristics from their macroscopic counterparts, and their optical properties may vary with different grain sizes and arrangements, making them an interesting subject for materials science and engineering research. A crucial aspect of studying microcrystals is analyzing the traveling wave solution within their structure [4–7]. Traveling wave solutions are self-reinforcing waves that propagate as localized energy packets in nonlinear media with stable shapes and speeds, and they are widely used in fields such as optical communication, fiber lasers, and superconductivity. The importance of the traveling wave solution in physics is evident. Nonlinear partial differential equation (NLPDE) [8–14] plays a crucial role in describing the behavior of microcrystals. These equations are widely used to describe the behavior and evolution of complex systems in various fields such as physics, aerodynamics, fluid dynamics, atmospheric and ocean physics, explosion physics, chemistry, physiology, biology, and ecology. They provide the necessary mathematical tools for understanding and solving natural phenomena and engineering problems. Traveling wave solutions of NLPDE are widely used in the theory and applications of nonlinear science. Additionally, NLPDEs consider various factors that affect the response of microcrystalline materials, capturing the complex interactions at the nanoscale. Obtaining the traveling wave solution of these equation is essential for accurately predicting and analyzing the behavior of microcrystalline materials in different environments.

Recently, the polynomial complete discriminant system method [15] provides a systematic approach to analyzing NLPDE. This method offers a comprehensive framework for discerning the complex dynamics of microcrystals and their response to external stimuli. In this article, we extend the polynomial complete discriminant system method to the strain wave equation and classify its solutions to obtain more accurate analytical solutions of traveling waves. Usually, the strain wave equation in micro-crystalline solids is described as follows [16]:

$$\mathcal{T}_{tt} - \mathcal{T}_{xx} - a[l_1(\mathcal{T}^2)_{xx} - l_2\mathcal{T}_{xxxx} + l_3\mathcal{T}_{xxtt}] = 0, \quad (1.1)$$

where $\mathcal{T} = \mathcal{T}(x, t)$ represents the microstrain wave function. a stands for the coefficient related to elastic strain, and l_i ($i = 1, 2, 3$) represent the nonzero real numbers. In this article, Eq. (1.1) is commonly used to describe the mathematical model of microcrystalline solids, which are

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composed of many small grains or microcrystals. Microcrystalline solid materials are one of the most important materials worth paying attention to in materials physics. The propagation model of waves in microcrystalline materials is usually simulated using Eq. (1.1), and the study of the traveling wave solution of this equation is crucial. Currently, research methods for the strain wave equation include the modified exp-function method [17], generalized Jacobi elliptic equation method [18], $(\frac{G'}{G^2})$-expansion method [19], modified extended mapping method [20], generalized (G'/G)-expansion method [21], exponential expansion method [22]. Although many predecessors have used different methods to study the traveling wave solution of Eq. (1.1), research on the strain wave equation is still ongoing. Particularly, for the study of the Jacobian function solution of Eq. (1.1), we still need to construct it. As early as 2010, Professor Liu [23] proposed the complete discriminant system method and used it to study the traveling wave solutions of NLPDE. This article is based on the fully discriminative system method to study the traveling wave solution of the strain wave equation. Using this method, the rational number solutions, trigonometric function solutions, hyperbolic function solutions, and Jacobian elliptic function solutions of the Strain wave equation are obtained.

The remaining sections of this article are arranged as follows: in Section 2, the traveling wave transform was applied to Eq. (1.1) to obtain a third-order polynomial. In Section 3, the solutions of Eq. (1.1) are constructed. In Section 4, the results through plotted graphs are presented. Finally, a brief summary is provided in Section 5.

2 Mathematical analysis

First, we make the transformation

$$\mathcal{T}(x,t) = \mathcal{T}(\xi), \quad \xi = kx + \omega t, \tag{2.1}$$

where k and ω are the real numbers.

Substituting transformation (2.1) into Eq. (1.1), we obtain

$$(k^2 - \omega^2)\mathcal{T}'' - al_1k^2(\mathcal{T}^2)'' + ak^2(l_2k^2 - l_3\omega^2)\mathcal{T}'''' = 0. \tag{2.2}$$

Integrating both sides of Eq. (2.2) simultaneously twice, we have

$$(k^2 - \omega^2)\mathcal{T} - al_1k^2\mathcal{T}^2 + ak^2(l_2k^2 - l_3\omega^2)\mathcal{T}'' = c_1, \tag{2.3}$$

where c_1 is the second integration constant.

Integrating Eq. (2.3) once, we can obtain

$$(\mathcal{T}')^2 = a_3\mathcal{T}^3 + a_2\mathcal{T}^2 + a_1\mathcal{T} + a_0, \tag{2.4}$$

where $a_3 = \frac{2}{3}\frac{l_1}{l_2k^2 - l_3\omega^2}$, $a_2 = -\frac{k^2-\omega^2}{ak^2(l_2k^2 - l_3\omega^2)}$, a_1 and a_0 are arbitrary constants.

Next, we make the assumptions

$$\psi = (a_3)^{\frac{1}{3}}\mathcal{T}, \quad \Upsilon_2 = a_2(a_3)^{-\frac{2}{3}}, \quad \Upsilon_1 = a_1(a_3)^{-\frac{1}{3}}, \quad \Upsilon_0 = a_0. \tag{2.5}$$

Substituting Eq. (2.5) into Eq. (2.4) yields

$$(\psi')^2 = \psi^3 + \Upsilon_2\psi^2 + \Upsilon_1\psi + \Upsilon_0. \tag{2.6}$$

Its integral form is

$$\pm(a_3)^{\frac{1}{3}}(\xi - \xi_0) = \int\frac{\mathrm{d}\psi}{\sqrt{\psi^3 + \Upsilon_2\psi^2 + \Upsilon_1\psi + \Upsilon_0}}, \tag{2.7}$$

where ξ_0 is the integration constant.

3 Traveling wave solution of Eq. (1.1)

First, we provide an assumption that

$$f(\psi) = \psi^3 + \Upsilon_2\psi^2 + \Upsilon_1\psi + \Upsilon_0. \tag{3.1}$$

Next, based on the complete discriminant system method (see Ref. [23]), we can obtain a third-order discriminant system as follows:

$$\begin{aligned} \Delta &= -27\left(\frac{2\Upsilon_2^3}{27} + \Upsilon_0 - \frac{\Upsilon_1\Upsilon_2}{3}\right)^2 - 4\left(\Upsilon_1 - \frac{\Upsilon_2^2}{3}\right)^3, \\ D_1 &= \Upsilon_1 - \frac{\Upsilon_2^2}{3}. \end{aligned} \tag{3.2}$$

Case 1. $\Delta = 0$ and $D_1 < 0$

In the case, the polynomial (3.1) can be written as $f(\psi) = (\psi - \rho_1)^2(\psi - \rho_2)$, where $\rho_1 \neq \rho_2$. If $\psi > \rho_2$, Eq. (2.7) can be simplified as

$$\begin{aligned} \pm(\xi - \xi_0) &= \int\frac{\mathrm{d}\psi}{(\psi - \rho_1)\sqrt{\psi - \rho_2}} \\ &= \begin{cases} \frac{1}{\sqrt{\rho_1 - \rho_2}}\ln\left|\frac{\sqrt{\psi - \rho_2} - \sqrt{\rho_1 - \rho_2}}{\sqrt{\psi - \rho_2} + \sqrt{\rho_1 + \rho_2}}\right|, \\ \rho_1 > \rho_2, \\ \frac{2}{\sqrt{\rho_2 - \rho_1}}\arctan\sqrt{\frac{\psi - \rho_2}{\vartheta_2 - \vartheta_1}}, \quad \rho_1 < \rho_2. \end{cases} \end{aligned} \tag{3.3}$$

Integrating Eq. (3.6), the solution of Eq. (1.1) can be constructed

$$\mathcal{T}_1(x,t)=\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{-\frac{1}{3}}\left\{(\rho_1-\rho_2)\tanh^2\left[\frac{\sqrt{\rho_1-\rho_2}}{2}\times\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{\frac{1}{3}}(kx+\omega t-\xi_0)\right]+\rho_2\right\},\quad \rho_1>\rho_2,$$

$$\mathcal{T}_2(x,t)=\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{-\frac{1}{3}}\left\{(\rho_1-\rho_2)\coth^2\left[\frac{\sqrt{\rho_1-\rho_2}}{2}\times\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{\frac{1}{3}}(kx+\omega t-\xi_0)\right]+\rho_2\right\},\quad \rho_1>\rho_2,$$

$$\mathcal{T}_3(x,t)=\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{-\frac{1}{3}}\left\{(-\rho_1+\rho_2)\tan^2\left[\frac{\sqrt{-\rho_1+\rho_2}}{2}\times\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{\frac{1}{3}}(kx+\omega t-\xi_0)\right]+\rho_2\right\},\quad \rho_1<\rho_2.$$

Case 2. $\Delta=0$ and $D_1=0$

Suppose that

$$f(\psi)=(\psi-\rho_3)^3, \tag{3.4}$$

where ρ_3 is the root of equation $f(\psi)=0$. Substituting Eq. (3.4) into Eq. (2.7), then the solution of Eq. (1.1) can be constructed

$$\mathcal{T}_4(x,t)=4\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{-\frac{2}{3}}(kx+\omega t-\xi_0)^{-2}+\rho_3.$$

Case 3. $\Delta>0$ and $D_1<0$

Assume that

$$f(\psi)=(\psi-\varrho_1)(\psi-\varrho_2)(\psi-\varrho_3), \tag{3.5}$$

where ϱ_1, ϱ_2, and ϱ_3 are the roots of equation $f(\psi)=0$ satisfying $\varrho_1<\varrho_2<\varrho_3$.

When $\varrho_1<\psi<\varrho_3$, we assume that $\psi=\varrho_1+(\varrho_2-\varrho_1)\sin^2\vartheta$. From Eq. (2.7), we have

$$\begin{aligned}\pm(\xi-\xi_0)&=\int\frac{d\psi}{\sqrt{f(\psi)}}\\&=\int\frac{2(\varrho_2-\varrho_1)\sin\vartheta\cos\vartheta d\vartheta}{\sqrt{\varrho_3-\varrho_1}(\varrho_2-\varrho_1)\sin\vartheta\cos\vartheta\sqrt{1-\chi_1^2\sin^2\vartheta}}\\&=\frac{2}{\sqrt{\varrho_3-\varrho_1}}\int\frac{d\vartheta}{\sqrt{1-\chi_1^2\sin^2\vartheta}},\end{aligned} \tag{3.6}$$

where $\chi_1^2=\frac{\varrho_2-\varrho_1}{\varrho_3-\varrho_1}$.

Integrating Eq. (3.6), the solution of Eq. (2.7) is given as

$$\psi(\xi)=\varrho_1+(\varrho_2-\varrho_1)\operatorname{sn}^2\left(\frac{\sqrt{\varrho_3-\varrho_1}}{2}(kx+\omega t-\xi_0),\chi_1\right). \tag{3.7}$$

Therefore, the solution of Eq. (1.1) is given as

$$\begin{aligned}\mathcal{T}_5(x,t)=\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{-\frac{1}{3}}\Bigg[\varrho_1&+(\varrho_2-\varrho_1)\operatorname{sn}^2\Bigg(\frac{\sqrt{\varrho_3-\varrho_1}}{2}\\&\times\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{\frac{1}{3}}(kx+\omega t-\xi_0),\chi_1\Bigg)\Bigg].\end{aligned} \tag{3.8}$$

When $\psi>\varrho_3$, we suppose that $\psi=\frac{-\varrho_2\sin^2\vartheta+\varrho_3}{\cos^2\vartheta}$. Similarly, we obtain the solution of Eq. (1.1)

$$\begin{aligned}&\mathcal{T}_6(x,t)\\&=\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{-\frac{1}{3}}\\&\times\left[\frac{\varrho_3-\varrho_2\operatorname{sn}^2\left(\frac{\sqrt{\varrho_3-\varrho_1}}{2}\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{\frac{1}{3}}(kx+\omega t-\xi_0),\chi_2\right)}{\operatorname{cn}^2\left(\frac{\sqrt{\varrho_3-\varrho_1}}{2}\left(\frac{2}{3}\frac{l_1}{l_2k^2-l_3\omega^2}\right)^{\frac{1}{3}}(kx+\omega t-\xi_0),\chi_2\right)}\right],\end{aligned} \tag{3.9}$$

where $\chi_2^2=\frac{\varrho_2-\varrho_1}{\varrho_3-\varrho_1}$.

Case 4. $\Delta<0$

Assume that

$$f(\psi)=(\psi-\rho)(\psi^2+p\psi+q), \tag{3.10}$$

where ρ is the only real root of $f(\psi)=0$. $p^2-4q<0$.

When $\psi>\rho$, we suppose that $\psi=\rho+\sqrt{\rho^2+p\rho+q}\tan^2\frac{\vartheta}{2}$. From Eq. (2.7), we have

$$\begin{aligned}\xi-\xi_0&=\int\frac{d\psi}{\sqrt{(\psi-\vartheta)(\psi^2+p\Phi+q)}}\\&=\int\frac{\sqrt{\rho^2+p\rho+q}\frac{\tan\frac{\vartheta}{2}}{\cos^2\frac{\vartheta}{2}}d\vartheta}{(\rho^2+p\rho+q)^{\frac{3}{4}}\frac{\tan\frac{\vartheta}{2}}{\cos^2\frac{\vartheta}{2}}\sqrt{1-\varrho^2\sin^2\vartheta}}\\&=\frac{1}{(\rho^2+p\rho+q)^{\frac{1}{4}}}\int\frac{d\vartheta}{\sqrt{1-\varrho^2\sin^2\vartheta}},\end{aligned} \tag{3.11}$$

where $\chi_3^2=\frac{1}{2}\left(1-\frac{\rho+\frac{p}{2}}{\sqrt{\rho^2+p\rho+q}}\right)$.

Assume that $\operatorname{cn}((\rho^2+p\rho+q)^{\frac{1}{4}}(\xi-\xi_0),\chi_3)=\cos\vartheta$. Moreover, we obtain

$$\cos\vartheta=\frac{2\sqrt{\rho^2+p\rho+q}}{\psi-\rho+\sqrt{\rho^2+p\rho+q}}-1. \tag{3.12}$$

When $\psi>\rho$, we have

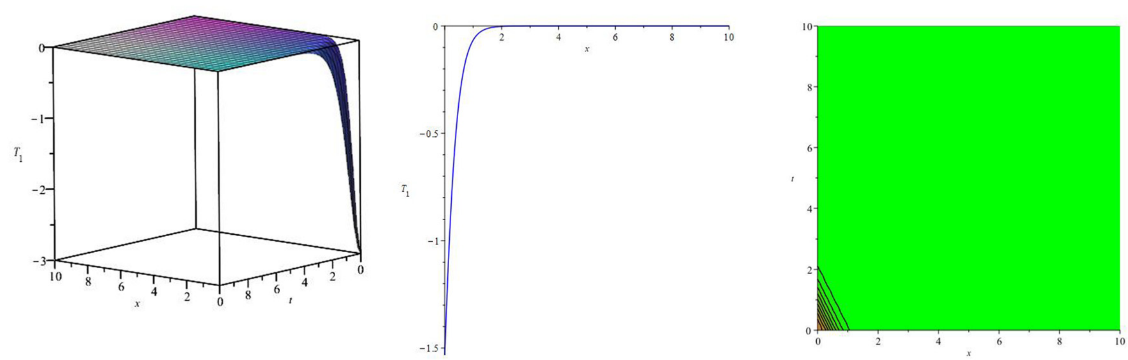
$$\mathcal{T}_7(x,t) = \left(\frac{2}{3}\frac{l_1}{l_2k^2 - l_3\omega^2}\right)^{-\frac{1}{3}} \times \left[\rho + \frac{2\sqrt{\rho^2 + p\rho + q}}{1 + \operatorname{cn}\left((\rho^2 + p\rho + q)^{\frac{1}{4}}\left(\frac{2}{3}\frac{l_1}{l_2k^2 - l_3\omega^2}\right)^{-\frac{1}{3}}(kx + \omega t - \xi_0), \chi_3\right)} - \sqrt{\rho^2 + p\rho + q}\right]. \tag{3.13}$$

Similarly, the solutions of Eq. (1.1) is given as

$$\mathcal{T}_8(x,t) = \left(\frac{2}{3}\frac{l_1}{l_2k^2 - l_3\omega^2}\right)^{-\frac{1}{3}} \times \left[\vartheta + \frac{2\sqrt{\rho^2 + p\rho + q}}{1 + \operatorname{cn}\left((\rho^2 + p\rho + q)^{\frac{1}{4}}\left(\frac{2}{3}\frac{l_1}{l_2k^2 - l_3\omega^2}\right)^{-\frac{1}{3}}(kx + \omega t - \xi_0), \chi_3\right)} - \sqrt{\rho^2 + p\rho + q}\right]. \tag{3.14}$$

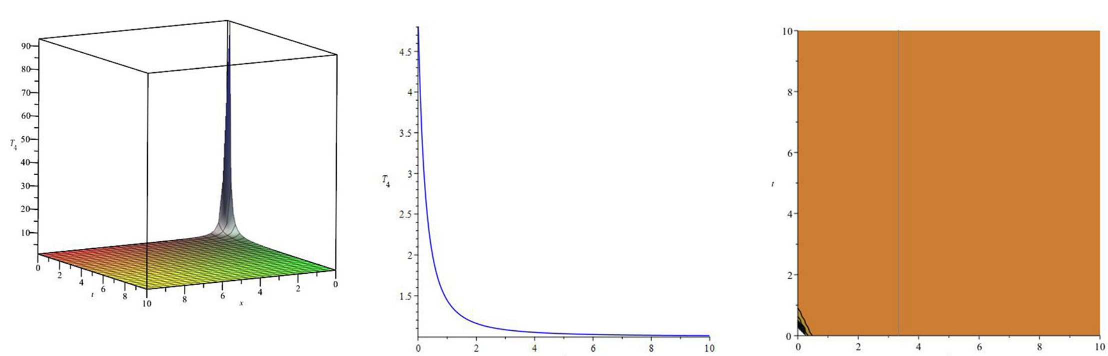
4 Numerical simulation

The solution of the strain wave equation plays a crucial role in the propagation of various waves in microstructured solids. In this section, we plotted the three-dimensional, two-dimensional, and density plots of the solution to Eq. (1.1). Obviously, when $k = 2$, $l_1 = 3$, $l_2 = \frac{3}{4}$, $l_3 = 1$, $a = -\frac{1}{8}$, $\omega = 1$, $\xi_0 = 0$, the solution $\mathcal{T}_1(t,x)$ is the hyperbolic function, solution, as shown in Figure 1. When $k = 2$, $l_1 = 3$, $l_2 = 1$, $l_3 = 2$, $a = \frac{1}{8}$, $\omega = 1$, and $\xi_0 = 0$, the solution $\mathcal{T}_4(t,x)$ is the rational function solution, as shown in Figure 2. From Figure 1, it can be seen that solution $\mathcal{T}_1(t,x)$ is a twisted solitary wave solution with an upper bound. However, it



(a) (b) (c)

Figure 1: Solution $\mathcal{T}_1(t,x)$ of Eq. (1.1) when $k = 2$, $l_1 = 3$, $l_2 = \frac{3}{4}$, $l_3 = 1$, $a = \frac{1}{8}$, $\omega = 1$, $\xi_0 = 0$: (a) 3D diagram, (b) 2D diagram, and (c) contour plot.



(a) (b) (c)

Figure 2: Solution $\mathcal{T}_4(t,x)$ of Eq. (1.1) when $k = 2$, $l_1 = 3$, $l_2 = 1$, $l_3 = 2$, $a = \frac{1}{8}$, $\omega = 1$, $\xi_0 = 0$: (a) 3D diagram, (b) 2D diagram, and (c) contour plot.

can be seen from Figure 2 that the solution $\mathcal{T}_4(t, x)$ is a function solution with a lower bound and no upper bound.

5 Conclusion

This study focuses on analyzing specific mathematical equations using analytical techniques, with a focus on studying the strain wave equation from the perspective of microcrystalline solids. Moreover, we successfully obtain the traveling wave solutions of solitary wave propagation in microcrystalline materials through the polynomial complete discriminant system method. We use symbolic computation to create 3D diagram, contour plot, and 2D diagram to enhance the visualization and analysis of mathematical solutions. The results show that the proposed strategy can provide a wide range of new wave solutions for various real-world nonlinear models. Compared with the study of Asghar et al. [16], the results obtained in this paper not only construct the trigonometric and hyperbolic function solutions of Eq. (1.1), but also obtain the Jacobian function solution. The Jacobian function solution obtained in this article has not been reported in the study of Asghar et al. [16]. This research enhances our understanding of nonlinear equations and solutions, laying the foundation for future studies.

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