

Research Article

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Natural convection and flow patterns of Cu–water nanofluids in hexagonal cavity: A novel thermal case study

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Abstract: The purpose of the current research is to inspect the free convection of the nanofluid (Cu–water) within a hexagonal cavity containing a square obstacle with isothermal vertical walls at T_h and T_c , and insulated horizontal walls. The aim of this study is to analyze the interaction between the Rayleigh number ($10^3 < Ra < 10^5$), obstacle's position (top, bottom, and center), and volume fraction of the nanoparticles ($0 < \varnothing < 0.2$) on the thermal behavior within the enclosure. Simulations were performed using COMSOL Multiphysics software based on the finite element method. The obtained results were demonstrated using streamlines, isotherms, and average Nusselt numbers. It is concluded that the increase in the Rayleigh quantity Ra and nanoparticle concentration \varnothing increases the average Nusselt Nu_{av} , which expresses the rate of heat flow in the studied enclosure. Furthermore, the position of the inner obstacle in the middle of the cavity has a more significant thermal efficiency than the other cases.

Keywords: convection, nanofluid (Cu–water), hexagonal enclosure, Rayleigh number, Galerkin's finite element technique

1 Introduction

The study of heat exchange within enclosed containers is of tremendous interest since it occurs in a variety of energy systems, such as the cooling of electronic equipment, particularly microcontrollers, heat producers in computers, cooling of electronic components, and improving heat transfer in thermal devices. Convection is the primary heat transmission method in these systems, and temperature management is critical for their operation. These thermal systems must be highly efficient and remain small in size. Several strategies, such as changing the exchanging surfaces or employing a fluid with a high thermal conductivity instead of standard fluids (water, oil, etc.), can be employed to accomplish this goal. To generate an effective heat transition fluid, we combined very tiny (nanosized) nanomaterials with a base fluid, resulting in nanofluids. Various forms of nanofluids have been thoroughly studied, and their thermophysical characteristics have been documented in the literature. Considering that their thermal conductivity is higher than that of fluids, these nanofluids boost the thermodynamic efficiency of various energy systems, including heat exchangers and solar collectors. After successfully enhancing the thermophysical characteristics of nanofluids, numerous industries and research institutions have attempted to utilize them in a wide range of applications, including medical, transportation, conditioning systems, and semiconductors. Nanofluids are of great importance in various sectors, including advanced science, chemical engineering and technology, atomic engineering, and biomedicine. They are created by dispersing nanoparticles such as copper (Cu), alumina (Al_2O_3), or silver (Ag) in a base fluid, typically a conventional liquid. Because these liquids increase the thermal conductance and ameliorate the heat transfer competence of

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unadulterated fluids, they also play an important role in several industrial and manufacturing sectors, including vehicle cooling, microchip technology, biomedical strategies, nuclear system conservation, and control generators [1,2].

Several studies have addressed the natural convection of nano-liquids in square cavities using nanofluid particles to perform free convective heat transference inside the enclosed space and reported that the nano-molecule fractional size variation is accompanied by an increase in the heat transfer coefficient [3–5]. The Khanafer model is used to assess the heat transfer efficiency of nanofluids inside a confined space while considering the dispersed nature of the solid particles [6]. The transfer formulas are represented using a stream function-vorticity framework and resolved using a computerized methodology called the finite-difference technique. A computational investigation was conducted to explore the natural convection of water-based nanofluids in a square cavity [7]. The cavity was partially heated at the bottom wall and filled with nanofluids containing various nanomaterials. The remaining wall of the cage was maintained at a low temperature. The computational technique used in this study was based on the Galerkin weighted residual approach of finite element formulations. The study [8] gave a quantitative investigation of the process of natural convection cooling of a heat source that is placed inside the lower wall of a container that is loaded with nanofluids. The upper and vertical surfaces of the enclosure were maintained at comparatively cool temperatures. The specific characteristics of the nanomaterials, as well as the length and position of heat generation, were shown to have a substantial impact on the highest possible temperature of heat generation. Several studies have investigated the influence of the enclosure shape on the nanofluid thermal flow [9–11], and their results show that changing the geometry design has a positive influence on the flow pattern, and the heat rate has been well probed and discussed in recent years. Examinations [12,13] numerically investigated the free convection liquid flowing inside a square enclosed space packed with (Cu–water nano-liquid), giving an adiabatic square block at its center. Their observations revealed an optimal size range for nanoparticles as the Rayleigh number increased. This suggests a corresponding increase in the thermal flow rate.

Enhancing heat transfer within established cavities remains a challenge, prompting numerous recent attempts to achieve necessary modifications. Some tactics employed include adding bendable or firm separates to the holes (to divide them into different portions), thickening the boundaries, adding fins to the holes, or changing the shapes or hole orientations. Each technique has the potential to increase or

decrease heat transport. Several empirical and theoretical studies have been conducted on heat transfer in various cavity forms. The preponderance of research has focused on increasing the thermal effectiveness of the heat-transfer rate in different cavity structures. Some studies have focused on analyzing the thermal flow in different cavity shapes. Saleh *et al.* [14] elaborated a numerical study to investigate the enhancement of thermal behavior for nanofluid flow within a trapezoidal cavity containing Al_2O_3 –water and Cu–water. Hussein *et al.* [15] numerically investigated the natural convective in a trapezoid bottom heated hollow included (Cu– H_2O) nanofluids. The investigation was conducted to study the influence of the position of the side walls for different angles ($\theta = 0^\circ, 30^\circ, 45^\circ$), and for nanoparticles fractional varying between $0 \leq \varnothing \leq 0.2$. They noticed that the thermal flow rate coefficient (Nu_{avg}) becomes optimal at $\theta = 0^\circ$ for pure fluids and nanofluids; however, at positions $\theta = 30^\circ$ and $\theta = 45^\circ$, the thermal rates are almost equal for pure fluids and nanofluids. Rostami and Abadi [16] explored the thermal flow of nanofluid within the triangular enclosure, and the study was conducted for different angle (0° to 120°) inclination of the triangle. They observed that the thermal flow rate increases with increasing the angular position. Dogonchi *et al.* [17] evaluated the heat transfer and natural convection of Cu– H_2O inside a triangular enclosure containing a semicircular lower side. The study observed a correlation between a reduction in the radius of the semicircle and the intensification of heat transfer within the enclosed space. Other authors have investigated the natural convection of nanoliquid flows in a novel cavity shape. Mahmoodi and Hashemi [18] investigated the thermal behavior of nanofluids within a C-shaped shell. Yuan *et al.* [19] discussed the numerical study of Al_2O_3 – H_2O or TiO_2 – H_2O inside a U-shaped hollow.

This study aims to analyze the steady-state laminar free convection behavior of a Cu–water nanofluid within a novel hexagonal cavity geometry. The influence of a square block located at various positions within the cavity is also investigated. The results of this problem were obtained at Rayleigh numbers of 10^3 – 10^5 and nanoparticle volume fractions ($0.1 < \varnothing < 0.2$). The results are presented in terms of streamline contours, isotherm contours, and average Nusselt number.

2 Definition of model

The problem considered is the two-dimensional characterization of heat transfer conducted by free convection inside a novel cavity shape. As shown in Figure 1, the

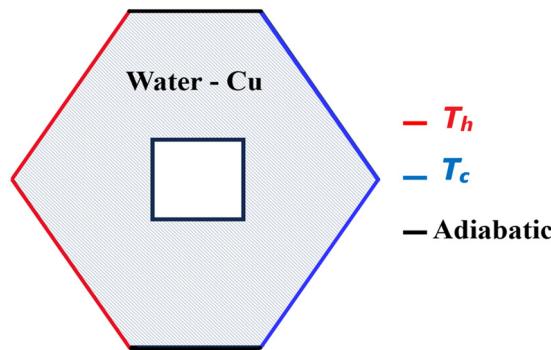


Figure 1: Physical model.

physical model of this study consists of a hexagonal cavity filled with Cu–water nanofluid (Table 1). The left and right sides were assumed to be hot and cold, respectively, at T_h and T_c , while the remaining sides were insulated.

Therefore, the boundary constraints can be specified as $U^* = V^* = 0$ for surfaces:

on the hot wall, $T^* = 1$,

on the cold wall, $T^* = 0$,

at the insulated wall, $\frac{\partial T^*}{\partial x} = 0$.

3 Mathematical model

3.1 Dimensional equation [3]

We consider the fluid to be Newtonian and incompressible, and the shape and size of the nanoparticles to be uniform. Furthermore, considering the above assumptions, the dimensional governing equations with the Boussinesq approximation are expressed as follows:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0. \quad (1)$$

Momentum equation [20]:

$$\rho_{\text{nf}} \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = - \frac{\partial P}{\partial X} + \mu_{\text{nf}} \left[\left(\frac{\partial^2 U}{\partial X^2} \right) + \left(\frac{\partial^2 U}{\partial Y^2} \right) \right], \quad (2)$$

Table 1: Physical Properties of the base fluid and nanoparticles [20]

Properties	Base fluid (water)	Copper (Cu)
ρ (kg/m ³)	997.1	8,933
C_p (J/kg K)	4,179	385
k (W/mK)	0.613	401

$$\rho_{\text{nf}} \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = - \frac{\partial P}{\partial Y} + \mu_{\text{nf}} \left[\left(\frac{\partial^2 V}{\partial X^2} \right) + \left(\frac{\partial^2 V}{\partial Y^2} \right) \right] + g(\rho\beta)_{\text{nf}} \Delta T. \quad (3)$$

Energy equation [20]:

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = a_{\text{nf}} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right). \quad (4)$$

3.2 Dimensionless equation [20]

Dimensionless variables for converting the above-mentioned governing equations:

$$\begin{aligned} X^* &= \frac{X}{L}, & Y^* &= \frac{Y}{L}, & U^* &= \frac{UL}{a_f}, & V^* &= \frac{VL}{a_f}, \\ P^* &= \frac{PL^2}{\rho_{\text{nf}} a_f^2}, & T^* &= \frac{T - T_c}{T_h - T_c}. \end{aligned} \quad (5)$$

As a result, the dimensionless is

Continuity equation:

$$\frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} = 0. \quad (6)$$

Momentum equation:

$$\begin{aligned} \left(U^* \frac{\partial U^*}{\partial X^*} + V^* \frac{\partial U^*}{\partial Y^*} \right) &= - \frac{\rho_f}{\rho_{\text{nf}}} \left(\frac{\partial P^*}{\partial X^*} \right) \\ &+ \frac{\mu_{\text{nf}}}{\rho_{\text{nf}} a_f} \left[\left(\frac{\partial^2 U^*}{\partial X^{*2}} \right) + \left(\frac{\partial^2 U^*}{\partial Y^{*2}} \right) \right]. \end{aligned} \quad (7)$$

$$\begin{aligned} \left(U^* \frac{\partial V^*}{\partial X^*} + V^* \frac{\partial V^*}{\partial Y^*} \right) &= - \frac{\rho_f}{\rho_{\text{nf}}} \left(\frac{\partial P^*}{\partial Y^*} \right) \\ &+ \frac{\mu_{\text{nf}}}{\rho_{\text{nf}} a_f} \left[\left(\frac{\partial^2 V^*}{\partial X^{*2}} \right) + \left(\frac{\partial^2 V^*}{\partial Y^{*2}} \right) \right] \\ &+ \frac{(\rho\beta)_{\text{nf}}}{\rho_{\text{nf}} \beta_f} \text{RaPr} T^*. \end{aligned} \quad (8)$$

Energy equation [4]:

$$U^* \frac{\partial T^*}{\partial X^*} + V^* \frac{\partial T^*}{\partial Y^*} = \frac{a_{\text{nf}}}{a_f} \left(\frac{\partial^2 T^*}{\partial X^{*2}} + \frac{\partial^2 T^*}{\partial Y^{*2}} \right). \quad (9)$$

The thermophysical characteristics of the nanofluids were evaluated using Brinkman's equation, formulated as follows [20]:

$$\mu_{\text{nf}} = \mu_f (1 - \varnothing)^{-2.5}, \quad (10)$$

$$\rho_{\text{nf}} = (1 - \varnothing) \rho_f + \varnothing \rho_s, \quad (11)$$

$$(\rho C_p)_{\text{nf}} = (1 - \varnothing) (\rho C_p)_f + \varnothing (\rho C_p)_s, \quad (12)$$

$$(\rho\beta)_{\text{nf}} = (1 - \varnothing) (\rho\beta)_f + \varnothing (\rho\beta)_s, \quad (13)$$

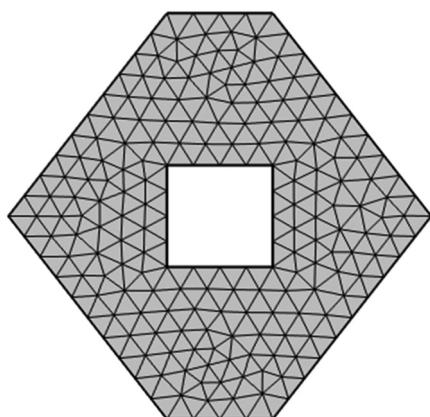


Figure 2: Triangular mesh distribution in the enclosure.

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\emptyset(k_f - k_s)}{k_s + 2k_f - \emptyset(k_f - k_s)}, \quad (14)$$

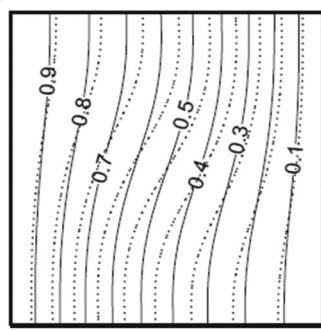
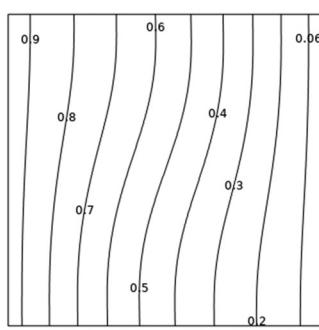
$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad (15)$$

$$Ra = \frac{\rho_f g \beta_f (T_h - T_c) L^3}{\mu_f \alpha_f}, \quad (16)$$

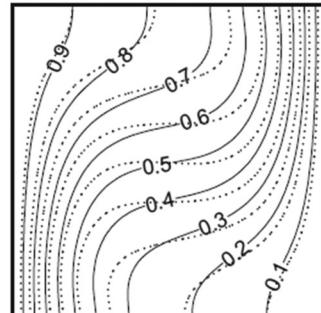
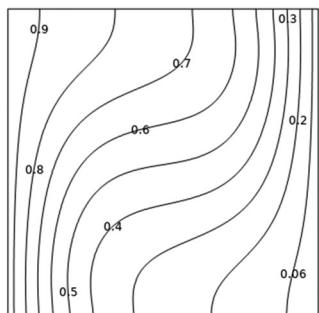
$$Pr = \frac{\vartheta_f}{\alpha_f}. \quad (17)$$

The local Nusselt number is defined as

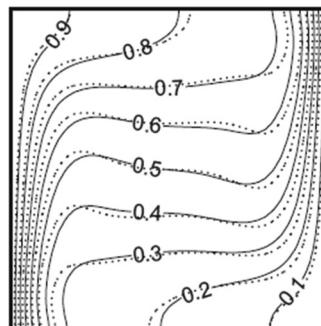
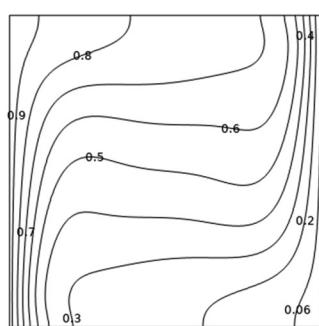
$$Nu_x = -\frac{k_{nf}}{k_f} \frac{dT}{dx}, \quad (18)$$



$$Ra = 10^3$$



$$Ra = 10^4$$



$$Ra = 10^5$$

Figure 3: Comparison between our results and those of ref. [3] for Cu–water nanofluids with various Rayleigh numbers ($Ra = 10^3$ – 10^5) in terms of isothermal patterns.

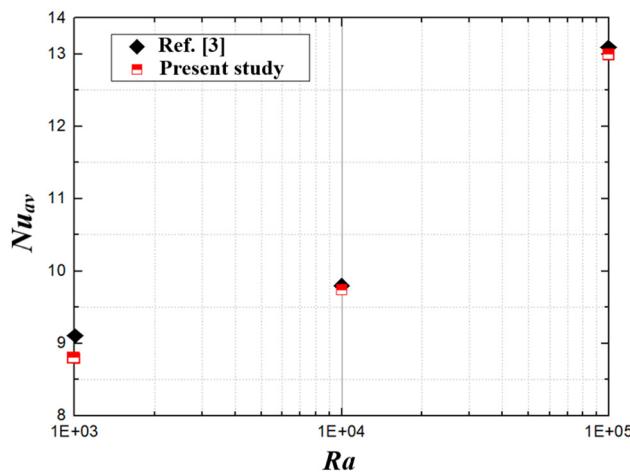


Figure 4: Validation of heat transfer rate (Nusselt average) versus Rayleigh numbers (10^3 – 10^5) for $\varnothing = 0.1$ [3].

The average Nusselt number is computed by integrating the local Nusselt number along the wall:

$$Nu_{av} = \int Nu_x dl. \quad (19)$$

4 Solution procedure

In the current study, the principal equation was discretized by employing the finite element approach using the COMSOL Multiphysics code. The solution elements of these governing equations (6–9) were generated by a triangular mesh. Figure 2 illustrates the computational accomplishment of the numerical simulation.

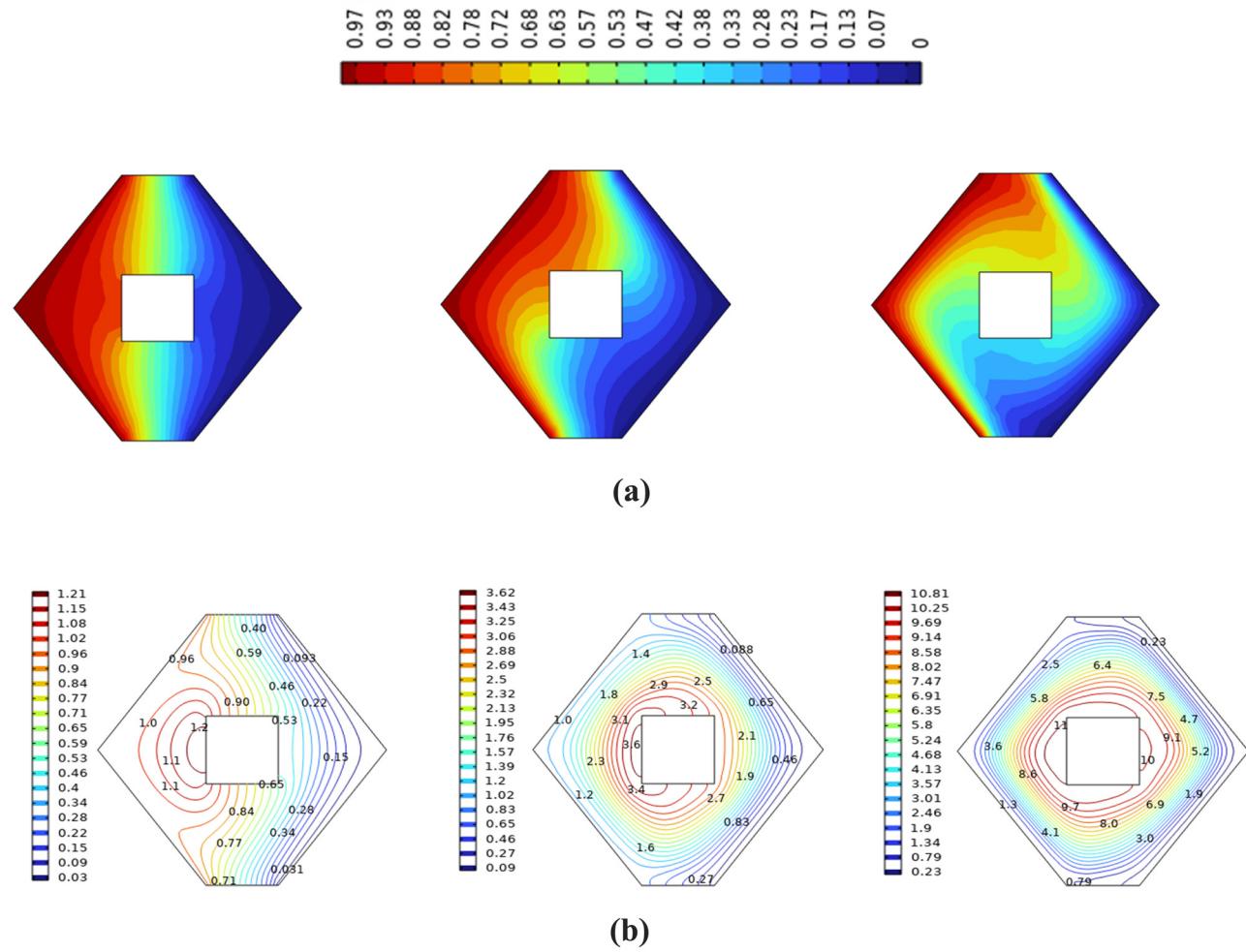


Figure 5: (a) Isotherm and (b) streamline contour distribution for different Rayleigh values.

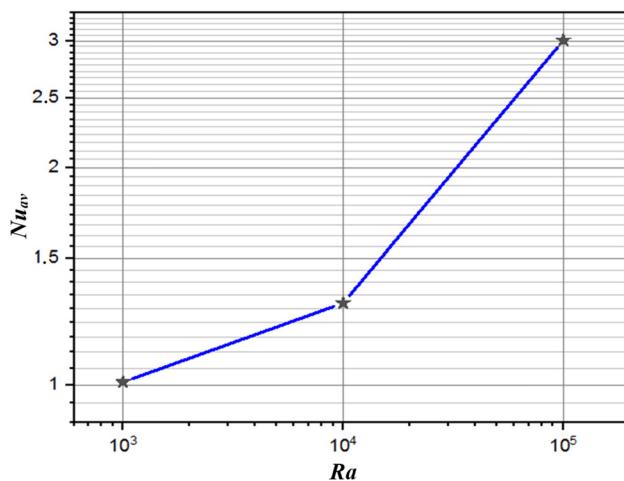


Figure 6: Variation in the Nusselt average as a function of Rayleigh number.

5 Validation

To verify and confirm the fragility and accuracy of the code calculation, we relied on research conducted by Basak and Chamkha [3]. The simulation of isotherm visualization was carried out under identical geometric and rheological circumstances [3] with a range of Ra values ($\text{Ra} = 10^3, 10^4, 10^5$). The fluid used was a water-based liquid containing copper nanomolecules (Cu–water). Figure 3 illustrates the isotherm results of our study for different Rayleigh numbers (10^3 – 10^5) compared with those of earlier investigations. The computational results in ref. [3] verified the accuracy and credibility of their findings.

The strong agreement between our results and the computational results in ref. [3] is further corroborated in Figure 4. The heat transfer rate was plotted against various Rayleigh numbers. This close correlation underscores the reliability of this model.

6 Results and discussion

6.1 Effects of Rayleigh numbers

Figure 5 illustrates the isotherms and streamlines for water and (Cu–water) nanofluids for different Rayleigh numbers $10^3 < \text{Ra} < 10^5$ from isotherm contours.

We observe that the flow is symmetric, presenting a regularity in the distribution of the isotherm contours for $\text{Ra} = 10^3$. This indicates that the flow movement inside the enclosure was small, demonstrating that the conduction became stronger and significantly dominant in this case. However, with an increase of Rayleigh number (10^4 – 10^5), the

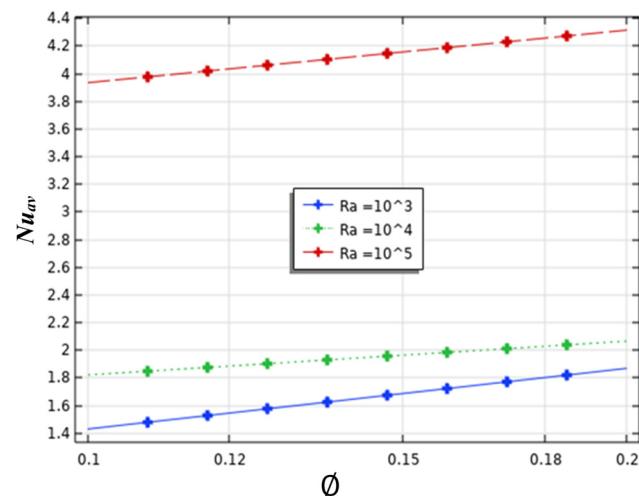


Figure 7: Average Nusselt numbers for different particle fractions ϕ and Ra numbers.

isotherm pattern changes direction, and the flow moves toward the cold wall. Furthermore, the streamlines for low values of $\text{Ra} = 10^3$ are generated inside the cavity, and starting with the hot wall, the increase of the Rayleigh value ($\text{Ra} = 10^4$ – 10^5) leads to the expansion of the moving zone inside the enclosure and is accompanied by an increase in the flow intensity. In addition, with higher Rayleigh numbers, the rotation over the inner block becomes stronger, and the convection becomes more dominant. As shown in Figure 6, which presents the variation in the Nusselt average *versus* different Rayleigh values ($\text{Ra} = 10^3$ – 10^5), an increase in the Rayleigh number increases the thermal transference rates Nu_{av} and the convection rates.

7 Effect of nanoparticles

In this section, the effect of the nanoparticle fraction in the range of $\phi = 0.1$ – 0.2 is investigated. Figure 7 illustrates the behavior of the Nusselt average number for increasing values of Rayleigh number and specific nanoparticle fraction. The analysis of these results revealed that the average Nusselt numbers increased. This augmentation is evident, indicating an improvement in the convective heat transfer within the cavity.

8 Effect of the position of the inner block

In this section, three different locations on the inner block ($c = 0, 0.25$, and -0.25) were examined. As shown in Figure 8, the streamline inside the enclosure, when the

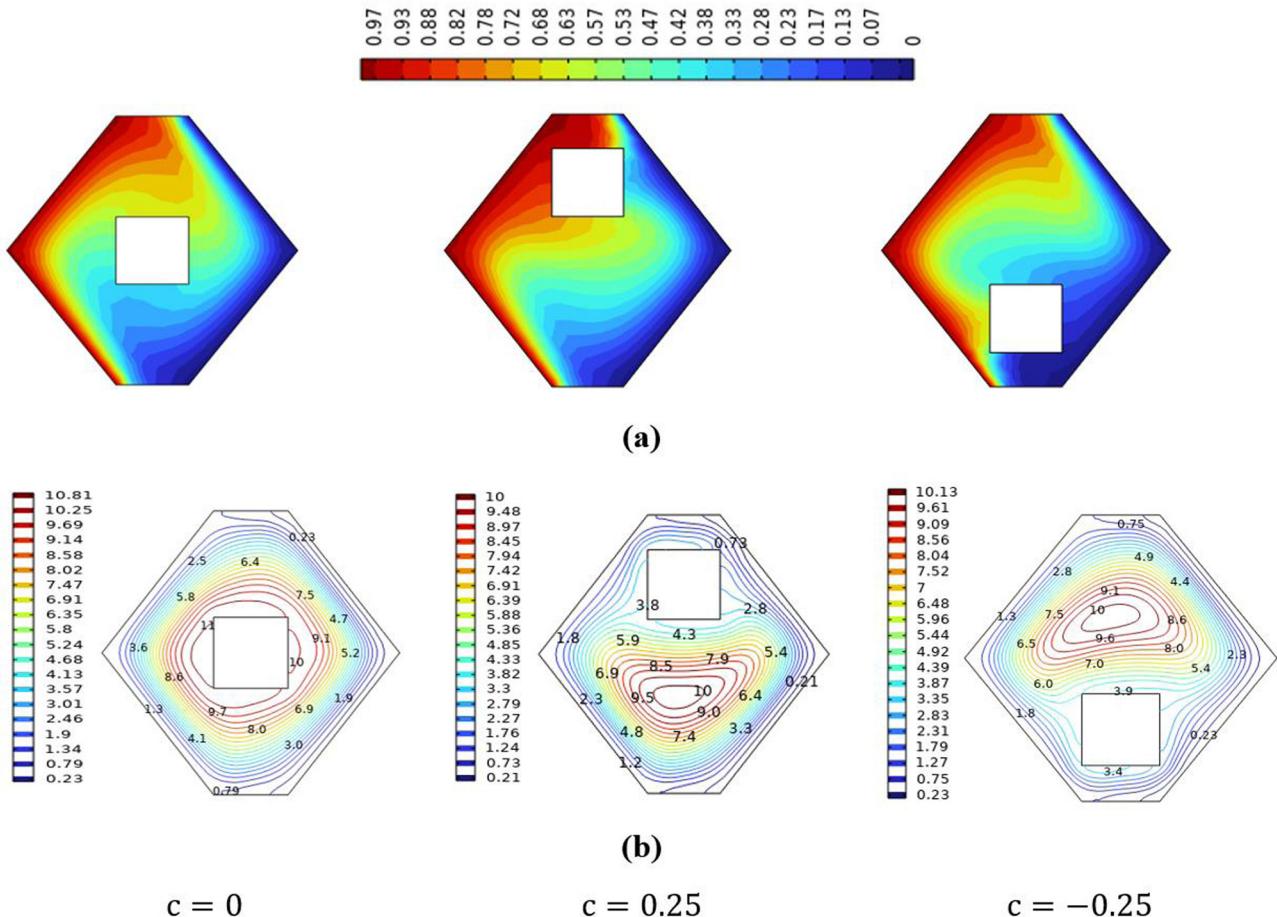


Figure 8: (a) Isotherm and (b) streamline distributions for different positions of inner obstacles inside the enclosure.

location of the obstacle is in the middle of the cavity ($c = 0$), presents the largest convective flow circulation inside the enclosure comparatively with the rest position. However, in the presence of an obstacle in the position $c = 0.25, -0.25$, the vertices appear above and below the side of the obstacle, with the presence of a stagnant region in the rest of the region.

9 Concluding remarks

In this study, a numerical investigation was conducted to examine the laminar natural convection flow of a copper–water nanofluid confined within a hexagonal enclosure. The study focused on the influence of several key parameters: the Rayleigh number (Ra) across a range of $10^3, 10^4$, and 10^5 ; the nanoparticle volume fraction ($\phi = 0.1–0.2$); and the location of the internal obstacle within the cavity. The results of this study show that the following:

- An increase in the Rayleigh number led to more vigorous flow behavior. This suggests that buoyancy forces, which

become stronger with increasing Rayleigh number, play a crucial role in driving nanofluid circulation. Consequently, we observed a significant positive correlation between the Rayleigh number and thermal performance. Notably, at a Rayleigh number of 10^5 , the heat-transfer rate increased by up to 300%.

- The introduction of nanoparticles into a cavity significantly enhances the heat transfer rate owing to their higher thermal conductivity and increased surface area for heat exchange.
- The location of the inner obstacle within the cavity significantly affected the thermal efficiency. Placing the obstacle in the middle of the cavity ($c = 0$) resulted in the highest thermal efficiency compared to the other positions. This suggests that the obstacle disrupts the flow pattern, potentially creating localized mixing zones that enhance heat transfer from the hot wall to the cool wall.

Finally, our study demonstrates that strategically manipulating the Rayleigh number, nanoparticle concentration, and inner obstacle location significantly alters the convective flow

behavior of the nanofluid within the cavity. These combined adjustments led to a more vigorous flow field, consequently enhancing the overall heat-transfer rate.

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