Research Article

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Pulsed excitation of a quantum oscillator: A model accounting for damping

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Abstract: This article proposes a model that accounts for damping of a quantum oscillator (QO) during pulsed excitation. Our model is based on the Schwinger formula, which calculates oscillator's excitation probability through the energy of an associated classical damped oscillator. We utilize this model to describe the influence of damping on temporal and spectral dependences of QO excitation, induced by electromagnetic pulses with exponential and double exponential envelopes. The oscillator excitation is analyzed in terms of transition probability between stationary states after pulse termination. Here, we present an analytical description of these dependences, along with numerical results. Specifically, we derive analytical expressions that depict the saturation effect during pulsed excitation, taking into account the damping of a QO. The evolution of the temporal dependence of the excitation probability with a change in the damping constant is numerically traced. We demonstrate that the number of maxima in this dependence is determined by the values of pulse parameters and the damping constant.

Keywords: ultrashort pulses, transition probability, saturation

1 Introduction

The development of the ultrashort laser pulses (USP) generation technique (Nobel Prize 2023 [1]) necessitates a more detailed exploration of the theory of USP–matter interaction. In particular, it is interesting to examine how this interaction depends on USP parameters such as duration, amplitude, carrier frequency, and envelope. One of the most important models of a quantum system interacting with an electromagnetic (EM) pulse is a quantum oscillator (QO) [2]. This model can be applied to a wide range of objects, *e.g.*, photons, phonons, vibrons, plasmons, electrons in a parabolic potential, a magnetic field, a micromechanical oscillator, and so on. A unique feature of a QO model is its ability to provide an exact description of its excitation by external force with any amplitude [3,4].

The QO pulsed excitation has been investigated by Hassan and co-authors [5–7], who used the solution of Heisenberg's equations for the creation and annihilation operators. They studied time dependences of an average number of excited quanta and transient spectra of fluorescence for different pulse envelopes and initial states of a QO.

In the study by Hassan *et al.* [7], QO damping is accounted for by adding a dissipative term to the Hamiltonian. This addition results in a complex eigenfrequency, with an imaginary part equal to the damping constant, appearing in the Heisenberg equations for the creation and annihilation operators. Within this framework, a transient spectrum of the pulsed-driven QO was calculated for different pulse shapes and damping constants.

Arkhipov *et al.* considered the USP–QO interaction using the sudden perturbation approximation [8]. They represented the probability of the QO excitation through the electric area of a unipolar subcycle pulse. By using this approximation, they calculated dependences of the QO excitation probability between stationary states on the pulse duration. In particular, it was demonstrated that with an increase in the electric field strength of the pulse, the central maximum in these dependences is transformed into a minimum, with the appearance of two side maxima. The approximation of the electric pulse area was utilized by Arkhipov *et al.* [9] to investigate the population difference gratings produced on vibrational transitions by subcycle THz pulses. In this article, an analytical approach was validated by numerical calculations in a nonperturbative regime for a three-level system.

Makarov [10,11] used a QO model to describe the quantum entanglement of a coupled harmonic oscillator.

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The corresponding expression for entanglement was derived and analyzed. In particular, it was established that entanglement depends on a single parameter with a clear physical meaning, namely, the reflection coefficient.

We analytically and numerically investigated the excitation of an undamped QO by both multicycle and subcycle EM pulses with different envelopes, after pulse termination [12,13]. The basic laws of QO excitation were established for various parameters of exciting pulses in nonperturbative regime, and the modes of weak and strong excitation were studied in detail, including the criteria and features of their manifestation.

This article aims to present a simple model for describing the pulsed excitation of a QO. This model takes damping into account and examines how damping affects the spectral and temporal dependences of the excitation probability.

2 Methods

2.1 Model

To describe the excitation of a QO by an EM pulse, we start with the Schwinger formula [3]. This formula calculates the probability of transition between two stationary states n and m:

$$W_{mn} = \frac{m!}{n!}\nu^k |L_n^k(\nu)|^2 e^{-\nu},\ k = |n - m|, \tag{1}$$

where $L_n^k(\nu)$ is the Laguerre polynomial and ν is a dimensionless parameter, which depends on the EM pulse and the oscillator characteristics.

In the following, we consider the excitation probability of a QO after the termination of an EM pulse.

To describe the excitation of a QO by an EM pulse, we use an approach based on the connection between movements of quantum and classical oscillators [4]. Thus, we consider a classical oscillator that is associated with a quantum one. This means it has the same parameters: eigenfrequency ω_0, mass m, charge q, and damping constant γ. Furthermore, it obeys a well-known equation in the electric field $E(t)$:

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \frac{q}{m}E(t). \tag{2}$$

As previously shown in the studies by Husimi [4] and Astapenko and Sakhno [12], the key parameter ν can be expressed as follows:

$$\nu = \frac{\Delta\varepsilon_{\mathrm{clas}}}{\hbar\omega_0}, \tag{3}$$

where $\Delta\varepsilon_{\mathrm{clas}}$ is the excitation energy of a classical oscillator that is initially at rest and is associated with a quantum one.

A consistent quantum mechanical derivation of formulas (1) and (3) assumes that there is no damping of the oscillator (*i.e.*, the damping constant is equal to zero: $\gamma = 0$). In this case, the excitation energy of a classical oscillator is given by ref. [12]:

$$\Delta\varepsilon_{\mathrm{clas}} = \frac{q^2E_0^2}{2m}|\tilde{E}(\omega = \omega_0)|^2, \tag{4}$$

where $\tilde{E}(\omega) = E(\omega)/E_0$ is the Fourier transform of the electric field strength, which is normalized by the amplitude of the field in the EM pulse E_0.

The average number of excited quanta $\bar{n}$ is linked to the excitation energy of the classical oscillator (4), as described by relation [4]:

$$\bar{n} = \frac{\Delta\varepsilon_{\mathrm{clas}}}{\hbar\omega_0} + n_0, \tag{5}$$

where n_0 is the number of QO initial state.

The main assumption of our model is this: we will incorporate the excitation energy of the classical oscillator with nonzero damping into expressions (1) and (3):

$$\Delta\varepsilon_{\mathrm{clas}}(\gamma = 0) \to \Delta\varepsilon_{\mathrm{clas}}(\gamma \neq 0). \tag{6}$$

Taking into account damping, we have the following expression for the excitation energy instead of Eq. (4):

$$\Delta\varepsilon_{\mathrm{clas}} = \frac{q^2E_0^2}{2m}\int_0^\infty d\omega|\tilde{E}(\omega)|^2\frac{4\omega^2\gamma/\pi}{(\omega^2 - \omega_0^2)^2 + 4\omega^2\gamma^2}. \tag{7}$$

By using formulas (5) and (7), we obtain the average number of excited quanta for the excitation from ground state ($n_0 = 0$):

$$\bar{n} = \Omega_0^2\int G_{\mathrm{osc}}(\omega,\gamma)|\tilde{E}(\omega,\omega_c,\tau)|^2 d\omega, \tag{8}$$

where

$$\Omega_0 = \frac{qE_0}{\sqrt{2m\hbar\omega_0}} \tag{9}$$

is the Rabi frequency, which determines the strength of EM interaction and in the case of a two-level system (TLS) describes the frequency of oscillations between energy levels, where

$$\tilde{E}(\omega,\omega_c,\tau) = \int\tilde{E}(t,\omega_c,\tau)\exp(i\omega t)dt. \tag{10}$$

is the Fourier transform of the normalized electric field strength in the pulse, and

$$G_{\mathrm{osc}}(\omega) = \frac{1}{\pi}\frac{4\omega^2\gamma}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\gamma^2} \tag{11}$$

is the “oscillator” shape of a line. Within the limits of small damping,

$$\gamma \ll \omega_0. \tag{12}$$

The function $G_{osc}(\omega)$ coincides with the Lorentzian:

$$G_{osc}(\omega) \simeq G_L(\omega) = \frac{1}{\pi}\frac{\gamma}{(\omega_0 - \omega)^2 + \gamma^2}. \tag{13}$$

Thus, instead of formula (1), we have the following expression for the excitation probability of a damped QO:

$$W_{mn}(\gamma) = \frac{m!}{n!}\bar{n}(\gamma)^k |L_n^k(\bar{n}(\gamma))|^2 \exp(-\bar{n}(\gamma)), \quad k = |n - m|. \tag{14}$$

Note that for small $\bar{n}(\gamma) \ll 1$, we have from (14):

$$W_{0\to 1}(\gamma) \simeq \bar{n}(\gamma). \tag{15}$$

On the other hand, we have the following expression for the probability of the TLS excitation with eigenfrequency ω_0 and spectral profile $G(\omega, \gamma)$ within the framework of the perturbation theory:

$$W^{(TLS)} = \omega_0\, \Omega_0^2 \int \frac{G(\omega, \gamma)}{\omega} |\tilde{E}(\omega, \omega_c, \tau)|^2 d\omega. \tag{16}$$

By comparing Eqs. (15) and (16) and taking into account Eqs. (8) and (13), we conclude that:

$$W_{0\to 1}(\gamma) \simeq W^{(TLS)} \tag{17}$$

if the spectral profile of the TLS is the Lorentzian and the inequality (12) is met.

Thus, our model corresponds to the description of the TLS excitation within the framework of the perturbation theory if the TLS has a Lorentzian spectral profile.

It is interesting to consider two types of EM pulse envelopes, namely, the exponential pulse (an asymmetrical in time pulse with a steep front), for which an analytical description of the QO excitation is possible:

$$E_{exp}(t, \omega_c, \tau) = E_0 \theta(t) \exp(-t/\tau) \cos(\omega_c t). \tag{18}$$

The double exponential pulse (a symmetrical in time pulse):

$$E_{2\,exp}(t, \omega_c, \tau) = E_0 \exp(-|t|/\tau) \cos(\omega_c t). \tag{19}$$

In formula (18), $\theta(t)$ is the Heaviside step function. Excitation of the oscillator by pulses (18) and (19) has its own characteristic features due to their different time symmetry, which makes it possible to capture a wider spectrum of the oscillator’s response to pulse action.

2.2 Average number of oscillator quanta after excitation

Analytical results for the average quanta number can be obtained within the framework of the following approximations: $\omega_c \tau \gg 1$ (multi-cycle pulse), $\omega_0 \gg \gamma$. The average number of oscillator quanta excited by *exponential* pulse is equal to:

$$\bar{n}_{exp}(\gamma) = \frac{1}{4}\Omega_0^2 \tau \frac{\gamma + 1/\tau}{(\omega_c - \omega_0)^2 + (\gamma + 1/\tau)^2}, \tag{20}$$

and for the double exponential pulse:

$$\bar{n}_{2\,exp}(\gamma) = \frac{1}{8}\Omega_0^2 \tau \frac{(\omega_c - \omega_0)^2 \gamma + (\gamma + 1/\tau)^2(\gamma + 2/\tau)}{[(\omega_c - \omega_0)^2 + (\gamma + 1/\tau)^2]^2}. \tag{21}$$

It is interesting to note that the dependence of the average number of excited quanta on the carrier frequency in the case of an exponential pulse (20) is described by the Lorentzian, and the line width is equal to the sum of the damping constant and the inverse pulse duration.

Within the framework of our model, these average numbers should be substituted into expression (14) to calculate the probability of the QO excitation by exponential and double exponential pulses, taking damping into account.

In the following, we consider the QO excitation from the ground state, which simplifies the expression (14) to the form:

$$W_{0\to n}(\gamma) = \frac{\bar{n}(\gamma)^n}{n!} \exp(-\bar{n}(\gamma)). \tag{22}$$

This formula implies that the *extrema condition* for the excitation of a $0 \to n$ transition in a QO is the following:

$$\bar{n}(\gamma, \omega_c, \tau, \Omega_0) = n. \tag{23}$$

We further use Eq. (23) to determine the extrema of the excitation probability as a function of pulse parameters (carrier frequency, duration, and the Rabi frequency) for different values of the damping constant γ.

2.3 Excitation spectrum of QO with damping

First, we consider the dependence of the QO excitation spectrum on the damping constant. The excitation spectrum is understood as the dependence of the excitation probability on the carrier frequency of the EM pulse.

As established in the previous studies [12,13], there are two regimes of the QO excitation, namely, weak and strong regimes, which depend on the value of the Rabi frequency.

In the case of the weak regime, there is a single spectral maximum of the excitation probability when the carrier frequency of the EM pulse is equal to eigenfrequency of the QO. With an increase in the Rabi frequency, this maximum transforms into a minimum, and two side maxima appear. The positions of these maxima are determined by Eq. (23). This transformation signifies the transition to the strong regime of excitation. It is analogous to the saturation effect in a TLS, we refer to the corresponding value of the Rabi frequency as the saturation one.

The positions of spectral maxima for the excitation probability of the $0 \to n$ transition in a QO by an *exponential pulse* can be calculated from Eq. (23). They are given by the corresponding detunings $\Delta_{\max}$ of the carrier pulse frequency from eigenfrequency of a QO:

$$\Delta_{\max}^{(\exp)}(\gamma) = \pm\sqrt{(1+\gamma\tau)\left(\frac{\Omega_0^2}{4n} - \frac{1+\gamma\tau}{\tau^2}\right)}; \quad (24)$$

$$\Delta = \omega_c - \omega_0.$$

From Eq. (24), it follows that the saturation Rabi frequency for the excitation of a QO with damping by an exponential pulse is given as follows:

$$\Omega_0^{(\mathrm{sat})}(\gamma) = \frac{2\sqrt{n}}{\tau}\sqrt{1+\gamma\tau}. \quad (25)$$

Side maxima in the excitation probability spectrum appear when the following saturation condition is met:

$$\Omega_0 > \Omega_0^{(\mathrm{sat})}(\gamma). \quad (26)$$

For the *double exponential* pulse, the spectral maximum positions are given by the following formula:

$$\Delta_{\max}^{(2\exp)}(\gamma) = \pm\sqrt{\frac{\Omega_0}{2\sqrt{n}}\sqrt{\frac{\Omega_0^2\gamma^2\tau^2}{64n} + \left(\gamma + \frac{1}{\tau}\right)^2} + \frac{\Omega_0^2\gamma\tau}{16n} - \left(\gamma + \frac{1}{\tau}\right)^2}. \quad (27)$$

The saturation Rabi frequency corresponds to a zero value of the expression under the square root in formula (27). It is equal to:

$$\Omega_0^{(\mathrm{sat})}(\gamma) = \frac{2\sqrt{2n}}{\tau}\frac{1+\gamma\tau}{\sqrt{2+\gamma\tau}}. \quad (28)$$

3 Results and discussion

The following graphs depict the splitting of spectral maxima during the excitation of the $0 \to 1$ transition in the QO (strong excitation regime) as a function of the pulse duration for different values of the damping constant. This is when the QO is excited by an exponential pulse with a low Rabi frequency (Figure 1) and a double exponential pulse with a higher Rabi frequency (Figure 2).

In all calculations, we assume that $\omega_0 = 1$ in relative units, and all quantities are measured in relative units.

From Figures 1 and 2, it can be inferred that in the case of short durations of exciting pulses, the splitting of the spectral maxima increases with an increase in τ for both envelopes and all the considered values of the damping constant. In the case of sufficiently long pulses, the dependence of the spectral splitting on the duration varies for different envelopes and damping constants. Specifically, the spectral splitting can either increase or decrease with an increase in τ.

Figures 3 and 4 demonstrate the evolution of the QO excitation spectrum for the $0 \to 1$ transition with a change in the damping constant for different pulse durations and

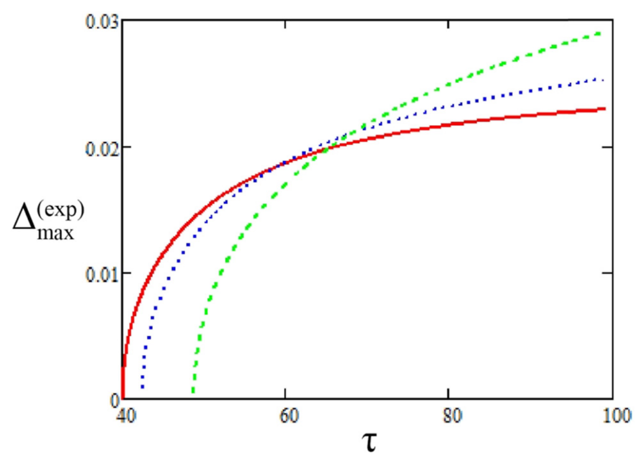

Figure 1: Exponential pulse excitation: solid line – $\gamma = 0$, dotted line – $\gamma = 0.003$, dashed line – $\gamma = 0.01$, $\Omega_0 = 0.05$.

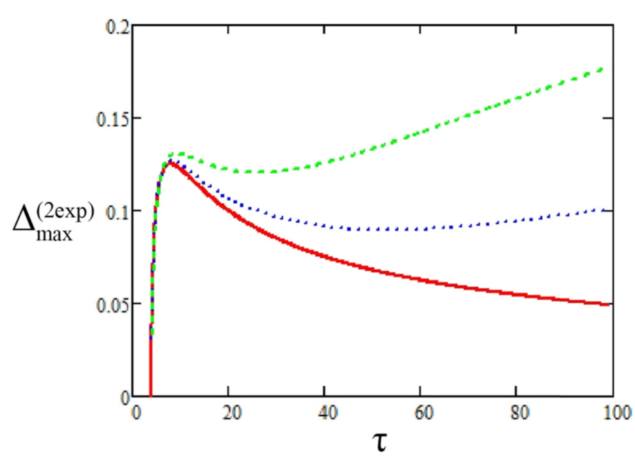

Figure 2: Double exponential pulse excitation: solid line – $\gamma = 0$, dotted line – $\gamma = 0.001$, dashed line – $\gamma = 0.03$, $\Omega_0 = 0.5$.

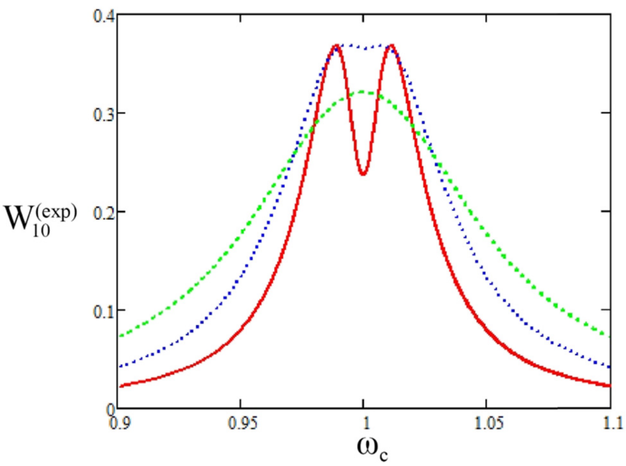

Figure 3: Excitation spectra by exponential pulse for τ = 100, Ω_0 = 0.03; solid line – γ = 0, dotted line – γ = 0.01, dashed line – γ = 0.03.

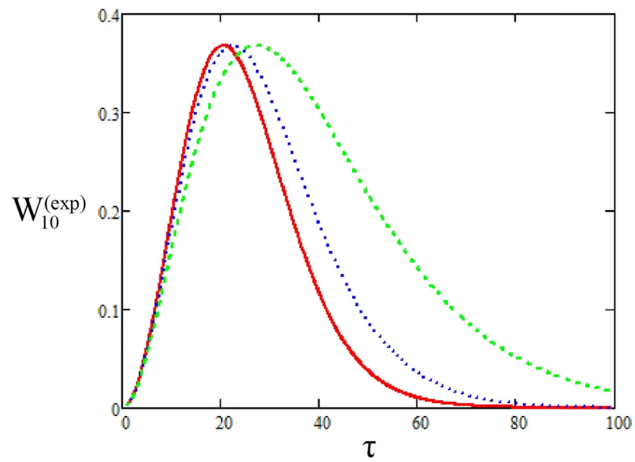

Figure 5: Exponential pulse excitation of QO, near-resonance case ω_c = 1.01, Ω_0 = 0.1; solid line – γ = 0.001, dotted line – γ = 0.01, dashed line – γ = 0.03.

the Rabi frequencies. Thus, in Figure 3, one can observe the transformation of a strong excitation regime of the QO with zero damping to a weak excitation regime with an increase in the damping constant value. Figure 4 shows the case of strong saturation when the QO is excited by a double exponential pulse. For the given parameters, the splitting of spectral maxima increases with an increase in the damping constant.

3.1 Dependence of excitation probability on EM pulse duration (τ-dependence)

To determine the extrema of the QO excitation probability as a function of the pulse duration, Eq. (23) must be resolved for the τ variable. This implies that in the case of an exponential pulse, there is a third degree equation, and in the case of a double exponential pulse, there is a fifth degree equation. Thus, it is not possible to derive simple expressions for the extreme values of the exciting pulse duration for the damped QO. It is possible to obtain a simple approximate equality for the pulse duration at the maximum of the function $W_{n0}(\tau)$ only for an exponential pulse and for a small frequency detuning of the carrier frequency ω_c from the eigenfrequency ω_0:

$$\tau_{\max}^{(\exp)}(|\Delta| < \Omega_0/2) \approx \frac{2}{\sqrt{\gamma^2 + \Omega_0^2/n} - \gamma}. \tag{29}$$

In the limit $\gamma \ll \Omega_0$, the expression (29) coincides with the corresponding formula obtained in the paper [13] for

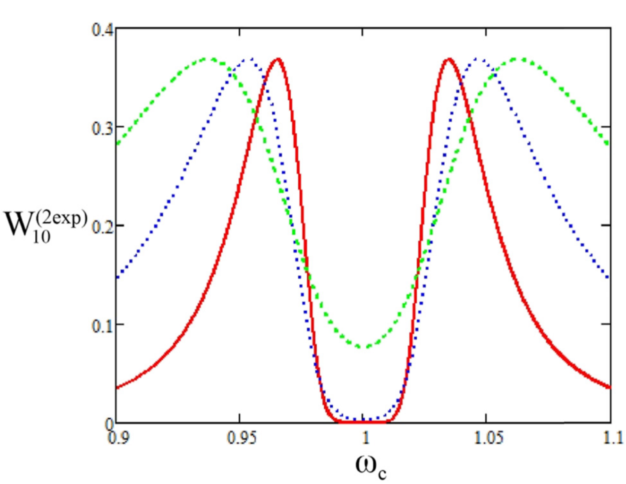

Figure 4: Excitation spectra by double exponential pulse for τ = 50, Ω_0 = 0.15; solid line – γ = 0.001, dotted line – γ = 0.01, dashed line – γ = 0.03.

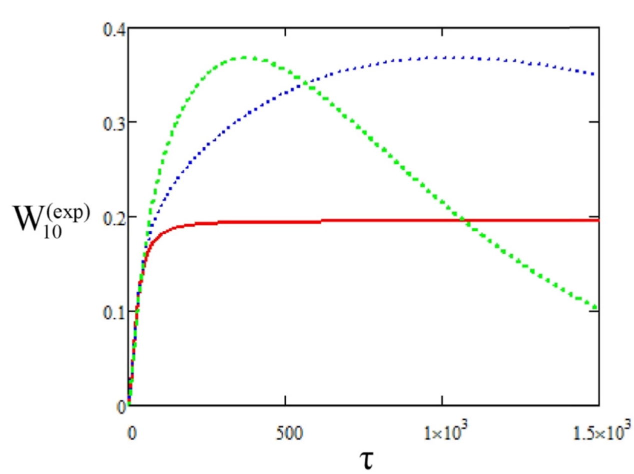

Figure 6: Exponential pulse excitation of QO, ω_c = 1.03, low Rabi frequency Ω_0 = 0.03; solid line – γ = 0, dotted line – γ = 0.003, dashed line – γ = 0.01.

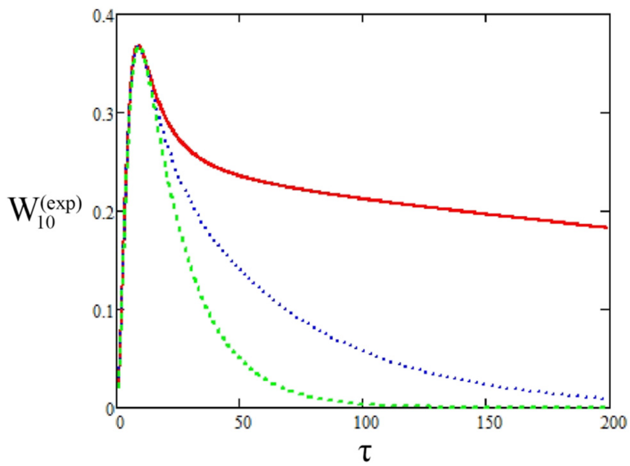

Figure 7: Exponential pulse excitation of QO, off-resonance case $\omega_c = 1.1$, high Rabi frequency $\Omega_0 = 0.3$; solid line – $\gamma = 0.001$, dotted line – $\gamma = 0.01$, dashed line – $\gamma = 0.03$.

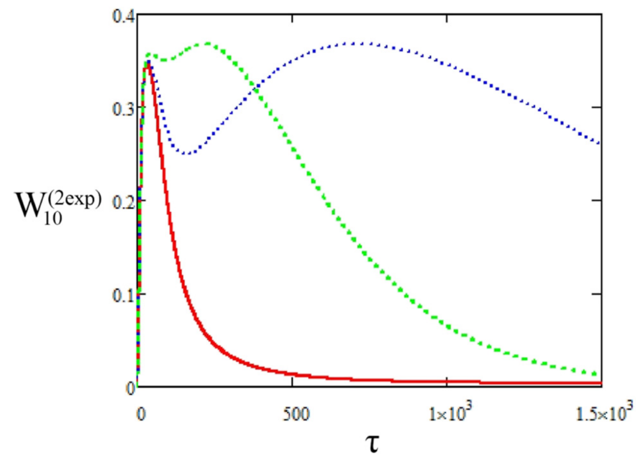

Figure 9: Double exponential pulse excitation of QO, $\omega_c = 1.03$, $\Omega_0 = 0.1$; solid line – $\gamma = 0$, dotted line – $\gamma = 0.001$, dashed line – $\gamma = 0.003$.

the undamped QO in the resonance case ($\omega_c = \omega_0$). From formula (29), it can be seen that the pulse duration at the maximum of the excitation probability increases monotonically with an increase in the damping constant.

The results of numerical calculations of the QO excitation probability as a function of the pulse duration are shown in Figures 5–10 for different values of the damping constant and EM pulse parameters. Figure 5 demonstrates a slight shift in the maximum of the QO excitation probability by a near-resonance exponential pulse ($|\Delta| \ll \Omega_0$) to larger τ with an increase in the damping constant, according to formula (29).

Figure 6 shows the case $|\Delta| = \Omega_0$ for the QO excitation by an exponential pulse. One can observe a strong influence of the damping constant on the $W_{n0}(\tau)$ function. The monotonically increasing dependence transforms into a function with a maximum as γ grows. This maximum shifts into a shorter pulse range with an increase in the damping constant.

Figure 7 demonstrates that if $\gamma \ll \Omega_0$, $|\Delta|$ the maximum of τ-dependence is not affected by the damping constant γ, there is a sharper decrease of the function $W_{n0}(\tau)$ in the long pulse range. For the QO double exponential pulse excitation and under similar conditions as in Figure 7, the function $W_{n0}(\tau)$ strongly depends on the value of the damping constant (Figure 8). Here, the shift of the maximum to a smaller τ with an increase in γ contrasts with the graphs shown in Figure 5.

Figure 9 presents a case of the QO near-resonance excitation by a double exponential pulse. From Figure 9,

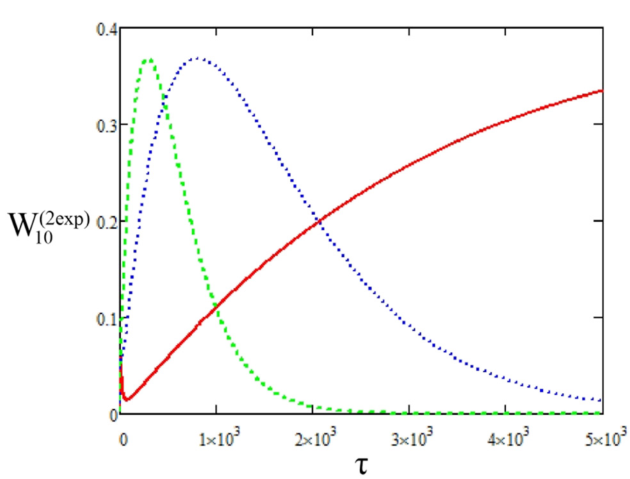

Figure 8: Double exponential pulse excitation of QO, off-resonance case $\omega_c = 1.1$, low Rabi frequency $\Omega_0 = 0.1$; solid line – $\gamma = 0.001$, dotted line – $\gamma = 0.01$, dashed line – $\gamma = 0.03$.

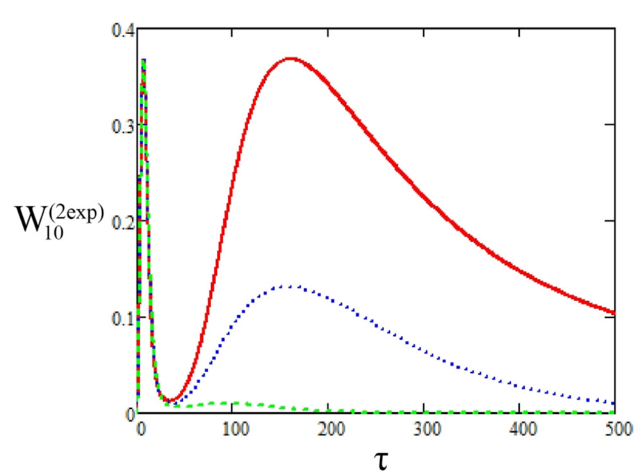

Figure 10: Double exponential pulse excitation of QO, $\omega_c = 1.03$, $\Omega_0 = 0.3$; solid line – $\gamma = 0$, dotted line – $\gamma = 0.001$, dashed line – $\gamma = 0.003$.

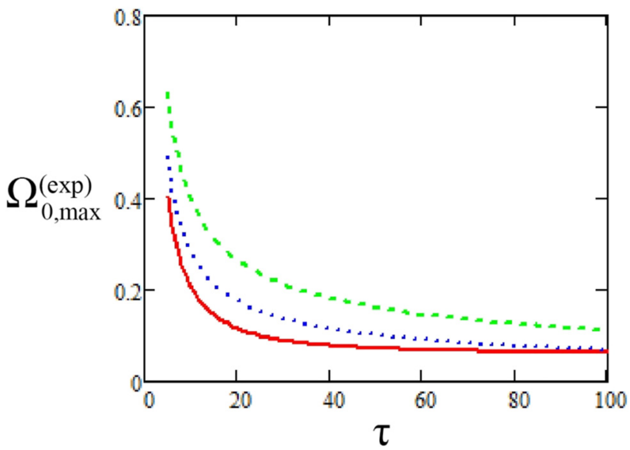

Figure 11: Exponential pulse excitation of QO, $\omega_c = 1.03$; solid line – $\gamma = 0$, dotted line – $\gamma = 0.1$, dashed line – $\gamma = 0.3$.

it can be seen that with an increase in the damping constant, two maxima appear in the τ-dependence of the excitation probability instead of one maximum for $\gamma = 0$. The situation drastically changes with an increase in the Rabi frequency as shown in Figure 10. Then, the two maxima in τ-dependence transform into one maximum with an increase in γ.

3.2 Optimal Rabi frequency

By using Eq. (23), one can derive the expressions for Rabi frequency, which corresponds to the maximum of the damped QO excitation probability at the $0 \to n$ transition for given values of other parameters of an EM pulse. In the case of an exponential pulse, we have the following expression for the optimal Rabi frequency:

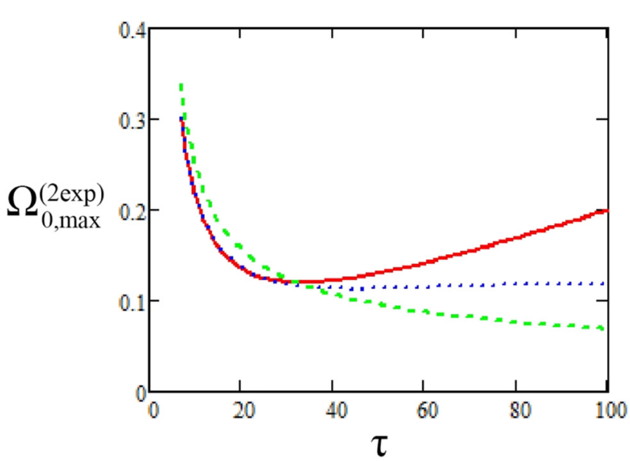

Figure 12: Double exponential pulse excitation of QO, $\omega_c = 1.03$; solid line – $\gamma = 0$, dotted line – $\gamma = 0.003$, dashed line – $\gamma = 0.03$.

$$\Omega^{(\mathrm{exp})}_{0,\mathrm{max}} = 2\sqrt{n\frac{\Delta^2 + (\gamma + 1/\tau)^2}{1 + \gamma\tau}}. \tag{30}$$

Figure 11 shows the τ-dependence of the optimal Rabi frequency for an exponential pulse for different values of the damping constant and a given value of the carrier frequency. It can be observed that the optimal Ω_0 value is higher for a larger damping constant. Calculations show that with an increase in spectral detuning for sufficiently long pulses, the situation can be reversed.

For a double exponential pulse, the optimal Rabi frequency is expressed as follows:

$$\Omega^{(2\,\mathrm{exp})}_{0,\mathrm{max}} = 2\sqrt{2n}\,\frac{\Delta^2 + (\gamma + 1/\tau)^2}{\sqrt{\Delta^2\gamma\,\tau + (2 + \gamma\tau)(\gamma + 1/\tau)^2}}. \tag{31}$$

The results of the calculations using formula (31) are presented in Figure 12. In this case, the character of τ-dependence is significantly determined by the value of the damping constant. For zero and small values, the corresponding curve has a minimum, while for larger γ, it monotonically decreases with an increase in pulse duration.

4 Conclusions

We proposed a simple model to account for the damping of a QO during its excitation by an EM pulse. This model aligns with the exact solution in the case of zero damping and corresponds to the description of a TLS excitation in the case of a small EM perturbation.

Within the framework of the proposed model, we conducted both analytical and numerical investigations into the influence of the damping constant on the spectral and temporal (pulse duration) dependences of the QO excitation probability. These investigations were carried out using both the exponential and double exponential pulses in weak and strong excitation regimes.

We derived an analytical description of the QO excitation spectrum for different values of the EM pulse parameters, including the positions of spectral maxima, expressions of the Rabi frequency saturation, and the optimal Rabi frequency at which the probability has maximum. It was shown that in the case of an exponential pulse, the splitting of spectral maxima increases with an increase in the pulse duration for all values of the damping constant. However, for a double exponential pulse, this dependence alters its character with changes in the value of γ. As the damping constant increases, the excitation spectrum characteristic of a strong regime with two maxima is transformed into a weak regime spectrum with a single spectral maximum.

We numerically investigated the dependences of the QO excitation probability on pulse duration (τ-dependence) for different values of the damping constant. Our results show that both the carrier frequency of the EM pulse and the Rabi frequency significantly influence these dependences and their evolution with the change in the damping constant. In the case of the QO excitation by an exponential pulse, the maximum of the τ-dependence shifts with an increase in γ. In addition, the character of its increase can change from a monotonic rise to having a maximum at certain relations between the Rabi frequency and the detuning of the pulse's carrier frequency from the QO eigenfrequency.

A notable feature of the τ-dependence when a QO is excited by a double exponential pulse is that its transformation changes in the nature with an increase in the damping constant γ for different Rabi frequencies. For instance, at a relatively low Rabi frequency, the τ-dependence has one maximum for small γ and two maxima for large γ. Conversely, at a higher Rabi frequency, there are two clearly expressed maxima in the τ-dependence at small γ, while at large γ, the second maximum becomes nearly indistinguishable.

The τ-dependence for the optimal Rabi frequency is also determined by the EM pulse envelope. In the case of an exponential pulse, this dependence always decreases monotonically. However, for a double exponential pulse, it varies with the damping constant: for small γ, the corresponding curve has a minimum, while for larger γ, it decreases monotonically.

We believe that the findings of this article significantly enhance the current knowledge of pulsed excitation of a damped QO. These results can be applied in various uses of this fundamental model, especially when a detailed description of EM interaction is required.

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