Research Article

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Bayesian estimation of equipment reliability with normal-type life distribution based on multiple batch tests

https://doi.org/10.1515/phys-2023-0188
received June 20, 2023; accepted January 10, 2024

Abstract: The test of new equipment is usually carried out in multiple batches according to the task schedule and test results. Constrained by the test environment, cost, and other factors, the amount of reliability test data in each batch is relatively limited, which brings difficulties to the accurate equipment reliability estimation work. For the reliability simulation tests conducted before each batch tests, it is particularly important to make full use of each batch tests information and simulation tests information to estimate the reliability of the equipment for small sample tests. This study takes the common normal-type life distribution equipment as the research object, and selects the normal-inverse gamma distribution as the equipment life parameters prior distribution based on the Bayesian method. Combined with the system contribution, the fusion weights of each batch tests information are determined and all the batch tests information is fused. Finally, the estimation of equipment reliability based on multiple batch tests is completed. The research results show that this method can integrate the information of each batch test and simulation test, overcome the problem of insufficient information of single batch tests, and provide an effective analytical tool for equipment reliability estimation.

Keywords: equipment reliability, normal-type life distribution, multiple batches tests, system contribution, Bayesian estimation

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1 Introduction

New equipment may have reliability design defects in the initial stage of development and design, and most of these defects can be exposed through strict reliability test programs. The equipment in the initial batch tests is difficult to meet the established reliability target, so it needs to undergo reliability improvement in multiple batch tests, so as to ultimately bring reliability improvement to the equipment. The multiple batches reliability test programs are a set of carefully designed programs and procedures for exposing the reliability problems and failures through testing, combined with corrective actions and design improvements to improve equipment reliability throughout the design and test phases. The existing estimation of equipment reliability is mainly studied from the perspective of single batch tests, only the reliability test data of the latest batch is utilized, with less use of previous batch test information [1–3]. Multiple batch tests consist of several different batch tests, which are combinations of single batch tests. The same batch tests contain several equipment with the same reliability, and the equipment reliability of different batch tests is different. Therefore, it is neither possible to divide single batch tests into multiple batches nor to aggregate multiple batch tests into single batch. Fusion of batch tests data is essential to address the problem of insufficient information for reliability estimation of single batch tests. For the small sample tests of equipment reliability, the reliability estimation results obtained by using only single batch tests information are more risky. Therefore, there is an urgent need to find an equipment reliability estimation method that can integrate multiple batch tests information.

According to engineering practice, it is known that different equipment will have different life distribution forms due to different failure mechanisms. The normal distribution is a relatively common distribution of equipment life, *e.g.*, most mechanical equipment subjected to cyclic loading, such as fatigue testing, will exhibit a normal distribution of service life. In addition, the normal distribution has better theoretical properties and computational convenience, and its probability density function and cumulative distribution function have relatively simple expressions. Compared with Weibull distribution and other distributions, normal distribution is more convenient for theoretical analysis and calculation. Partial lognormal distribution and Weibull distribution models can also be analyzed by approximating them with the normal distribution [4,5]. Considering the mathematical superiority of normal distribution model and its convenience in reliability analytical modeling, the study focuses on the reliability of equipment with normal-type life distribution.

Existing research on equipment reliability estimation has emerged with many advanced methods, including neural networks, big data analysis, Bayesian networks, Bayesian statistical inference, and other intelligent algorithms. Guo *et al.* [6] proposed a deep feature learning method that combines convolutional neural network-convolutional block attention module and transformer network as a parallel channel method to predict the remaining useful life of drilling pumps, which overcame the problem of insufficient measured data and effectively improved the prediction accuracy. Chen *et al.* [7] proposed an equipment reliability estimation method based on deep learning, which uses time series data for equipment reliability analysis. In view of the characteristics of small sample and high reliability test of aerospace valves, Wang *et al.* [8] integrated multiple sources information for reliability estimation. Guo *et al.* [9] integrated dynamic Bayesian network and XGBoost in an evaluation framework to assess the equipment operational reliability. Wang [10] proposed a reliability evaluation method for accelerated degradation test of electromechanical products. Jia *et al.* [11] evaluated the reliability of DC power distribution system in intelligent buildings based on big data analysis. Dai *et al.* [12] proposed a new reliability evaluation model based on wavelet kernel net, bi-directional gated recurrent unit, and wiener process model, which effectively solves the problem caused by the lack of measured data. Alex *et al.* [13] estimated the reliability of degraded all-terminal network equipment based on deep neural networks and Bayesian methods. Zahra *et al.* [14] estimated the reliability of Weibull distributed products based on intuitionistic fuzzy life data. Most of the existing studies only study the estimation of equipment reliability based on single batch tests, fail to consider the characteristics of multiple batch tests, and are unable to make full use of the information of each batch tests and simulation tests.

Fusion of each batch tests information is the key to equipment reliability estimation of multiple batch tests. Bayesian methods are capable of fusing multiple sources information in product research, and the results of the batch tests before each batch tests can be considered as prior information for Bayesian fusion, so as to achieve equipment reliability estimation of multiple batch tests. Noting that the information closer to the latest batch tests is more dominant in the fusion process, so the fusion weights of information from different batch tests need to be derived. The fusion weights reflect the contribution degree of different batch tests information to equipment reliability estimation. The system contribution is commonly used to measure the contribution degree of weapon equipment to combat capability in the combat system, and this theory is introduced to calculate the information fusion weights of different batch tests [15,16].

Combining the above analysis, this study takes the normal-type life distribution equipment as the research object, and studies the Bayesian estimation of equipment reliability based on multiple batch tests in the equipment design stage. First, the probability distribution function, unreliability function, reliability function, failure rate function, and sample estimates of reliability parameters for reliability tests of equipment with normal-type life distribution are given. Second, the process of Bayesian estimation of equipment reliability with normal-type life distribution is given, including three parts: prior information consistency test, Bayesian estimation of equipment reliability based on single batch tests, and Bayesian estimation of equipment reliability based on multiple batch tests. This part of the content selects the normal-inverse gamma distribution as the prior distribution of equipment life parameters, determines the prior distribution of parameters, and adopts the Bayesian method combined with the contribution degree of the system to integrate the information of each batch tests, and solves the problem. Finally, the results of Bayesian estimation of equipment reliability with normal-type life distribution based on multiple batch tests are derived through example analysis, and compared with the existing single batch test methods to highlight the superiority of the methodology of this study.

2 Reliability test of equipment with normal-type life distribution

The equipment life distribution mainly includes normal distribution, exponential distribution, Weibull distribution, lognormal distribution, *etc.* [17–20]. The normal distribution is more common and the model is simple, and partial lognormal distribution and Weibull distribution models can also be approximated by normal distribution. This research focuses on the reliability of equipment with normal life distribution, and the probability distribution of equipment life can be expressed as follows:

$$f(t)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(t-u)^2}{2\sigma^2}\right\}. \quad (1)$$

where u is the mean value of equipment life, and σ is the standard deviation. When $\sigma<0.3u$, $\int_{-\infty}^{0}f(t)\mathrm{d}t<0.0005$, it can be considered that when $t<0$, $f(t)=0$.

The equipment in the process of use will experience wear and degradation, when the use of time exceeds its service life, the equipment will fail and loose its function. The process of equipment degradation is shown in Figure 1.

The cumulative distribution function of equipment life distribution function is the equipment unreliability function, which can be expressed as follows:

$$F(t)=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{t}\exp\left\{-\frac{(t-u)^2}{2\sigma^2}\right\}\mathrm{d}t. \quad (2)$$

The equipment reliability function can be expressed as follows:

$$R(t)=1-F(t)=\frac{1}{\sqrt{2\pi}\sigma}\int_{t}^{+\infty}\exp\left\{-\frac{(t-u)^2}{2\sigma^2}\right\}\mathrm{d}t. \quad (3)$$

The equipment failure rate function can be expressed as:

$$\lambda(t)=\frac{f(t)}{R(t)}=\frac{\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(t-u)^2}{2\sigma^2}\right\}}{\frac{1}{\sqrt{2\pi}\sigma}\int_{t}^{+\infty}\exp\left\{-\frac{(t-u)^2}{2\sigma^2}\right\}\mathrm{d}t}. \quad (4)$$

The key to reliability test research of equipment with normal life distribution is to determine the parameters u and σ^2. u and σ^2 are equipment life distribution parameters or reliability parameters. The estimated values of equipment reliability parameters u and σ^2 can be obtained by using the equipment reliability field test samples data t_i, $i=1,2,\cdots n$.

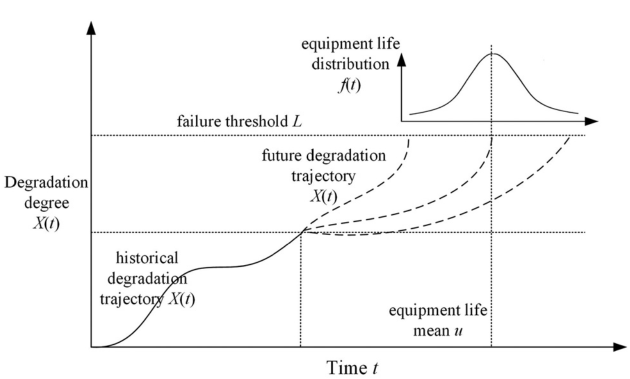

Figure 1: The process of equipment degradation.

$$\begin{cases}\hat{u}=\bar{t}_i=\frac{1}{n}\sum_{i=1}^{n}t_i\\ \hat{\sigma}^2=S^2=\frac{1}{n-1}\sum_{i=1}^{n}(t_i-\bar{t}_i)^2.\end{cases} \quad (5)$$

The error degree of parameter estimates $\hat{u}$ and $\hat{\sigma}^2$ are calculated by using the sample mean square error (MSE) as follows:

$$\begin{cases}\mathrm{MSE}(\hat{u})=\frac{\sigma^2}{n}\\ \mathrm{MSE}(\hat{\sigma}^2)=\frac{2\sigma^4}{n-1}.\end{cases} \quad (6)$$

It can be seen that when the samples' data volume n of equipment reliability field tests are small, the error of parameters estimation values obtained by using only samples' data are larger. Therefore, it is necessary to find a reliability estimation method that can integrate multiple sources information and reduce the error degree of parameters estimation.

3 Bayesian estimation of equipment reliability with normal-type life distribution

3.1 Prior information consistency test

In order to use Bayesian method for information fusion, consistency test should first be carried out to ensure that all prior information and field data belong to the same whole [21–23]. The rank sum test, Mood test, and hypothesis test can all be tested by using sample data directly, but there are higher requirements for the amount of equipment reliability data [24–26]. Therefore, considering the limited amount of test sample data, Bayesian confidence interval method is adopted for testing, and the confidence interval of parameters is obtained by combining single information source with field data [27]. If the parameter points estimated value of the field data falls within the confidence interval under the conditional of prior information, the information source is considered compatible with the field data, *i.e.*, it passes the consistency test.

The prior distribution $g(u,\sigma^2)$ of parameters u and σ^2 is obtained from the information sources (u_j,σ_j^2), $j=1,2,3,\cdots,m$. Combined with field data t_i, $i=1,2,3,\cdots,n$, the posterior distribution $g(u,\sigma^2|t)$ of parameters u and σ^2 is obtained, then the confidence interval under significance level α is $\{(u,\sigma^2)_{\mathrm{L}},(u,\sigma^2)_{\mathrm{U}}\}$, and the calculation formulas are as follows:

$$\begin{cases} \iint\limits_{(u,\,\sigma^2)<(u,\,\sigma^2)_L} g(u,\ \sigma^2|t)\mathrm{d}u\mathrm{d}\sigma^2 = \frac{\alpha}{2} \\ \iint\limits_{(u,\,\sigma^2)>(u,\,\sigma^2)_U} g(u,\ \sigma^2|t)\mathrm{d}u\mathrm{d}\sigma^2 = \frac{\alpha}{2} \end{cases}. \tag{7}$$

In the formula, $(u, \sigma^2)_U$ and $(u, \sigma^2)_L$ are the upper and lower limits of (u, σ^2), respectively.

According to the field data $(t_1, t_1, \cdots, t_n)$, the Bayesian estimation value of parameters (u, σ^2) under the non-information prior are $(\hat{u}, \hat{\sigma}^2)$. If $(\hat{u}, \hat{\sigma}^2)$ are within the confidence interval $\{(u, \sigma^2)_L,\ (u, \sigma^2)_U\}$, there is no significant difference between the information source and the field data on the whole, and the information source (u_j, σ_j^2) passes the consistency test.

3.2 Bayesian estimation of equipment reliability based on single batch tests

3.2.1 Selection of prior distribution of equipment reliability parameters

Bayesian methods are commonly used in small sample tests evaluation research, which can be used for small sample data equipment reliability tests with high value and complex test environment [28–30]. The life parameters u and σ^2 of equipment with normal life distribution are estimated by Bayesian methods. The Bayesian method regards the parameters as random variables for estimation, and marks the joint prior distribution of parameters u and σ^2 as $g(u, \sigma^2)$, then

$$g(u, \sigma^2) = g(u|\sigma^2)g(\sigma^2). \tag{8}$$

where $g(\sigma^2)$ is the prior distribution of σ^2, and $g(u|\sigma^2)$ is the prior distribution of u under the condition σ^2.

By referring to the Bayesian conjugate prior distribution family, it is known that the conjugate prior distribution under the condition that the characteristic parameters u and σ^2 of normal distribution are unknown is normal-inverse gamma distribution [31,32]. Then, there are

$$\begin{cases} u|\sigma^2 \sim N\left(u_0, \frac{\sigma^2}{k_0}\right) \\ \sigma^2 \sim IGa\left(\frac{v_0}{2}, \frac{v_0\sigma_0^2}{2}\right) \\ g(u, \sigma^2) = g(u|\sigma^2)\pi(\sigma^2) \\ = N\left(u_0, \frac{\sigma^2}{k_0}\right)IGa\left(\frac{v_0}{2}, \frac{v_0\sigma_0^2}{2}\right) \end{cases} \tag{9}$$

where $u_0, k_0, v_0, \sigma_0^2$ are the hyperparameters of the prior distribution.

After obtaining the equipment reliability field test samples data $(t_1, t_1, \cdots, t_n)$, the likelihood function $L(t|u, \sigma^2)$ of the test samples can be calculated, and the posterior distribution of parameters u and σ^2 can be obtained by using the Bayesian formula to integrate the prior information and samples information.

$$g(u, \sigma^2|t) = \frac{L(t|u, \sigma^2)g(u|\sigma^2)g(\sigma^2)}{\iint L(t|u, \sigma^2)g(u|\sigma^2)g(\sigma^2)\mathrm{d}u\mathrm{d}\sigma^2}. \tag{10}$$

where $\iint L(t|u, \sigma^2)g(u|\sigma^2)g(\sigma^2)\mathrm{d}u\mathrm{d}\sigma^2$ is the edge density function and its value is a constant. Formula (10) can be simplified to

$$\begin{aligned} g(u, \sigma^2|t) &\propto L(t|u, \sigma^2)g(u|\sigma^2)g(\sigma^2) \\ &\propto (\sigma^2)^{-\left(\frac{v_n}{2}+\frac{3}{2}\right)} \exp\left\{-\frac{v_n\sigma_n^2 + k_n(u - u_n)^2}{2\sigma^2}\right\}. \end{aligned} \tag{11}$$

where $u_n, k_n, v_n, \sigma_n^2$ are posterior distribution parameters. The posterior distribution $g(u, \sigma^2|t)$ of equipment life can be represented by prior distribution and samples data, and the posterior distribution parameters are as follows:

$$\begin{cases} k_n = k_0 + n \\ v_n = v_0 + n \\ u_n = \frac{k_0}{k_0 + n}u_0 + \frac{n}{k_0 + n}\bar{t} \\ v_n\sigma_n^2 = v_0\sigma_0^2 + (n - 1)S_0^2 + \frac{k_0 n}{k_0 + n}(u_0 - \bar{t})^2. \end{cases} \tag{12}$$

where S_0 is the variance of field equipment life test data.

3.2.2 Determination of priori distribution hyperparameters of equipment reliability parameters

Determining the hyperparameters of the prior distribution is the key to calculating the posterior distribution. Let $\hat{\alpha}$ be the estimator of $\frac{v_0}{2}$ and $\hat{\beta}$ be the estimator of $\frac{v_0\sigma_0^2}{2}$. According to the prior information parameters samples data (u_j, σ_j^2), $j = 1, 2, \cdots, m$, the maximum likelihood estimation of the hyperparameters $\hat{\alpha}$ and $\hat{\beta}$ are solved, and the solution steps are shown as follows:

1) Establish the likelihood function of inverse gamma distribution:

$$L(\alpha,\ \beta) = \frac{\beta^{\alpha m}}{\Gamma^m(\alpha)}\left(\prod_{j=1}^{m}\sigma_j^2\right)^{-(\alpha+1)} \exp\left(-\sum_{j=1}^{m}\frac{\beta}{\sigma_j^2}\right). \tag{13}$$

2) Obtain the first-order partial derivative of the likelihood function:

$$\begin{cases}\dfrac{\partial \ln L(\alpha,\beta)}{\partial\alpha} = m\ln\beta - m\dfrac{(\Gamma(\alpha))'}{\Gamma(\alpha)} - \sum\limits_{j=1}^{m}\ln\sigma_j^2 \\ \dfrac{\partial \ln L(\alpha,\beta)}{\partial\beta} = \dfrac{\alpha m}{\beta} - \sum\limits_{j=1}^{m}\dfrac{1}{\sigma_j^2}\end{cases}. \quad (14)$$

3) Let the first-order partial derivatives be 0 to establish the likelihood equations, and obtain the maximum likelihood estimates of the parameters.

$$\begin{cases}\ln(\hat{\alpha}_{\mathrm{MLE}}) - \dfrac{(\Gamma(\hat{\alpha}_{\mathrm{MLE}}))'}{\Gamma(\hat{\alpha}_{\mathrm{MLE}})} \\ = \dfrac{\sum_{j=1}^{m}\ln(\sigma_j^2)}{m} + \ln\left(\sum\limits_{j=1}^{m}\dfrac{1}{\sigma_j^2}\right) - \ln(m) \\ \hat{\beta}_{\mathrm{MLE}} = \dfrac{m\hat{\alpha}_{\mathrm{MLE}}}{\sum_{j=1}^{m}\frac{1}{\sigma_j^2}}\end{cases}. \quad (15)$$

$\Gamma(\alpha)$ is the gamma function, and

$$\frac{(\Gamma(\alpha))'}{\Gamma(\alpha)} = \frac{\partial \ln(\Gamma(\alpha))}{\partial\alpha} = \frac{\int_0^{+\infty}\exp\{-t\}t^{\alpha-1}\ln t\,\mathrm{d}t}{\int_0^{+\infty}\exp\{-t\}t^{\alpha-1}\mathrm{d}t}. \quad (16)$$

Take the mean value of parameter samples σ_j^2 as the estimated value of σ^2.

$$\hat{\sigma}^2 = \frac{\sum_{j=1}^{m}\sigma_j^2}{m}. \quad (17)$$

According to formula (8), $u|\sigma^2 \sim N\left(u_0, \frac{\sigma^2}{k_0}\right)$ can be obtained, then the estimated values of parameters u_0 and $\frac{\sigma^2}{k_0}$ can be obtained based on previous data.

$$\begin{cases}\hat{u}_0 = \dfrac{\sum_{j=1}^{m}u_j}{m} \\ (\sigma^2/k_0)' = \dfrac{\sum_{j=1}^{m}[u_j - (\sum_{j=1}^{m}u_j)/m]^2}{m}\end{cases}. \quad (18)$$

The value of parameter k_0 can be obtained by combining formulas (17) and (18):

$$k_0 = \frac{\hat{\sigma}^2}{(\sigma^2/k_0)'} = \frac{\sum_{j=1}^{m}\sigma_j^2}{\sum_{j=1}^{m}[u_j - (\sum_{j=1}^{m}u_j)/m]^2}. \quad (19)$$

3.2.3 Solution of posterior distribution of equipment reliability parameters based on single batch tests

The posterior distribution of parameters can be obtained through Bayesian formula calculation after obtaining the estimated value of the prior distribution hyperparameters. According to the principle of maximizing the probability of actual sampling occurrence, the maximum posterior estimation of parameters u and σ^2 is selected as the Bayesian estimation of parameters.

1) Establish the kernel density function of the posterior distribution.

$$G(u,\sigma^2) = (\sigma^2)^{-\left(\frac{v_n}{2}+\frac{3}{2}\right)}\exp\left\{-\frac{v_n\sigma_n^2 + k_n(u-u_n)^2}{2\sigma^2}\right\}. \quad (20)$$

2) Calculate the first-order partial derivatives of the kernel density function, and let the first-order partial derivatives be 0.

$$\begin{cases}\dfrac{\partial G(u,\ \sigma^2)}{\partial u} = 0 \Rightarrow u - u_n = 0 \\ \dfrac{\partial G(u,\ \sigma^2)}{\partial\sigma^2} = 0 \Rightarrow \dfrac{v_n\sigma_n^2}{\sigma^2} - v_n - 3 = 0\end{cases}. \quad (21)$$

The maximum posterior estimates of parameters u and σ^2 can be obtained.

$$\begin{cases}u_{\mathrm{MD}} = u_n \\ \sigma_{\mathrm{MD}}^2 = \dfrac{2v_n\sigma_n^2}{v_n+3}\end{cases}. \quad (22)$$

Combining formulas (11) and (21)

$$\begin{cases}u_{\mathrm{MD}} = \dfrac{ku_0 + n\bar{t}}{k_0+n} \\ \sigma_{\mathrm{MD}}^2 = \dfrac{v_0\sigma_0^2 + (n-1)S_0^2 + \frac{k_0 n}{k_0+n}(u_0-\bar{t})^2}{v_0+n+3}\end{cases}. \quad (23)$$

The equipment reliability of the fused prior information can be estimated by bringing the Bayesian maximum posterior estimates u_{MD} and σ_{MD}^2 into formulas (1)–(4).

3.3 Bayesian estimation of equipment reliability based on multiple batch tests

3.3.1 Calculation of fusion weights based on system contribution

Different from single batch and overall equipment reliability test research, multiple batch tests need to consider the test information of each batch, so it needs to be analyzed gradually from the initial batch tests. The multiple batch tests information fusion process is as follows (Figure 2):

System contribution is commonly used to measure the contribution degree of weapon and equipment to the comprehensive combat capability of the combat system, and can also be used to calculate the contribution degree of evaluation elements to the system [33–35]. System

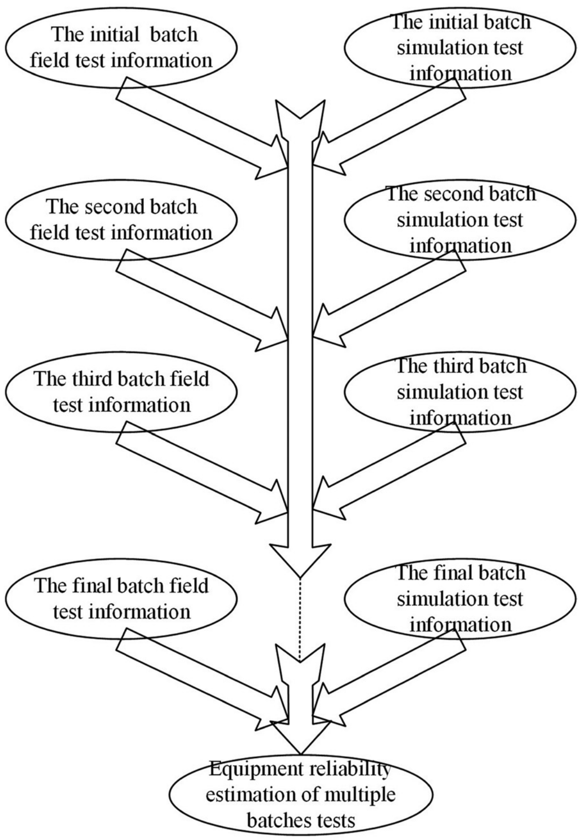

Figure 2: Multiple batch equipment reliability test information fusion process.

contribution is introduced to measure the influence degree of multiple sources prior information on the equipment reliability estimation system, and the influence is mainly determined by its reliability. The reliability of equipment reliability data sources is evaluated by analyzing the information from different sources, and the system contribution is taken as the Bayesian fusion weight. The higher the reliability, the greater the system contribution assigned, *i.e.*, the greater the fusion weight. The analysis process of system contribution fusion weights is as follows:

The equipment reliability information sources of the ith batch tests are mainly composed of three parts: the reliability information of the batch field tests, the reliability information of the $i-1$th batch tests, and the information of the ith batch simulation tests. The prior information is composed of the equipment reliability information of the $i-1$th batch tests and the information of the ith batch simulation tests. The information fusion weights of these two parts are calculated using the system contribution. $\phi = \{\delta_1, \delta_2\}$ is used to represent the prior information set of equipment reliability, where δ_1 is the equipment reliability information of the $i-1$ th batch tests and δ_2 is the information of the ith batch simulation tests. $\omega_{\delta_i}^d$, $i = 1, 2$ are used to represent the reliable membership degree of information δ_1 identified by experts ψ_d, $d = 1, 2, \cdots, m$. $\upsilon_{\delta_i}^d$, $i = 1, 2$ are used to represent the unreliable membership degree of information δ_i identified by experts ψ_d. The membership degree of equipment reliability information δ_i is expressed by intuitionistic fuzzy membership function method [36].

$$\Theta_{\delta_i}^d = \{\delta_i, (\omega_{\delta_i}^d, \upsilon_{\delta_i}^d)\}. \tag{24}$$

In the formula, $0 \le \omega_{\delta_i}^d \le 1, 0 \le \upsilon_{\delta_i}^d \le 1$, and $\omega_{\delta_i}^d + \upsilon_{\delta_i}^d \le 1$.

In the process of determining the membership degree of equipment reliability parameters information by using intuitionistic fuzzy membership function method, it is noted that experts will hesitate when they consider whether the source of equipment reliability information is reliable. The intuitionistic index is introduced to represent the degree of expert hesitation, and the expression is as follows:

$$\tau_{\delta_i}^d = 1 - \omega_{\delta_i}^d - \upsilon_{\delta_i}^d. \tag{25}$$

In determining the weight of prior information, half of the hesitation degree is used for correction, and the membership expression of information δ_i can be obtained [37].

$$\varepsilon_{\delta_i} = \frac{1}{m}\sum_{d=1}^{m}(\omega_{\delta_i}^d - \upsilon_{\delta_i}^d) + \frac{1 - \frac{1}{m}\sum_{d=1}^{m}\omega_{\delta_i}^d - \frac{1}{m}\sum_{d=1}^{m}\upsilon_{\delta_i}^d}{2}. \tag{26}$$

The fusion weight of information δ_i can be obtained by normalization processing.

$$\varepsilon_{\delta_i}^{\tau} = \frac{\varepsilon_{\delta_i}}{\varepsilon_{\delta_1} + \varepsilon_{\delta_2}}. \tag{27}$$

Based on the fusion weight $\varepsilon_{\delta_i}^{\tau}$, the prior distribution of equipment reliability parameters of the ith batch tests can be obtained.

$$g(u, \sigma^2) = \varepsilon_{\delta_1}^{\tau} g_1(u, \sigma^2) + \varepsilon_{\delta_2}^{\tau} g_2(u, \sigma^2). \tag{28}$$

where $g_1(u, \sigma^2)$ is the probability distribution of equipment reliability parameters of the $i-1$ th batch tests; $g_2(u, \sigma^2)$ is the probability distribution of equipment reliability parameters of the i th batch simulation tests.

3.3.2 Estimation of equipment reliability parameters based on multiple batch tests

1) Equipment reliability estimation of the initial batch tests

The initial batch tests only contain the initial batch field tests reliability information $L(t^{(1)}|u^{(1)}, \sigma^{2(1)})$ and the initial batch simulation tests information $g(u^{(1)}, \sigma^{2(1)})$. The equipment reliability parameters distribution of the initial

Table 1: Equipment reliability tests information

	Field reliability tests information	Simulation tests information
	$(t_1, t_2, t_3, t_4, t_5)$	$\left(u_0, \frac{\sigma^2}{k_0}, \frac{v_0}{2}, \frac{v_0\sigma_0^2}{2}\right)$
The initial batch tests	(34, 36, 26, 30, 26)	(29, 7, 2.3, 18)
The second batch tests	(33, 40, 35, 32, 30)	(32, 4.5, 2.5, 16.2)
The third batch tests	(40, 44, 36, 37, 38)	(40, 3, 2.8, 15)

batch tests can be obtained by fusing the above information with Bayesian method as follows:

$$g(u^{(1)}, \sigma^2(1)|t^{(1)}) = \frac{L(t^{(1)}|u^{(1)}, \sigma^{2(1)})g_2(u^{(1)}, \sigma^{2(1)})}{\iint L(t^{(1)}|u^{(1)}, \sigma^{2(1)})g_2(u^{(1)}, \sigma^{2(1)})\mathrm{d}u^{(1)}\mathrm{d}\sigma^{2(1)}}. \quad (29)$$

The Bayesian maximum estimates $u_{MD}^{(1)}$ and $\sigma_{\mathrm{MD}}^{2(1)}$ of the initial batch test parameters are obtained from the parameters distribution functions. The equipment reliability of the initial batch tests can be estimated by bringing the estimated value into formulas (1)–(4).

2) Equipment reliability estimation of the second batch tests.

After the initial batch tests, the physical properties of the equipment were improved and the second batch tests were conducted. The second batch tests need to take the initial batch tests information into account when estimating equipment reliability. Therefore, the second batch tests contain the information of the second batch field tests' reliability information $L(t^{(2)}|u^{(2)}, \sigma^{2(1)})$, the second batch simulation tests' information $g_2(u^{(2)}, \sigma^{2(2)})$, and the initial batch tests' equipment reliability information $g(u^{(1)}, \sigma^{2(1)}|t^{(1)})$. Compared with the initial batch tests, the second batch tests information is added to the initial batch tests information. The information fusion weights are determined according to the system contribution method proposed in Section 3.3.1, and the equipment reliability parameters distribution of the second batch tests is obtained by Bayesian method.

$$g(u^{(2)}, \sigma^2(2)|t^{(2)}) = \frac{L(t^{(2)}|u^{(2)}, \sigma^{2(2)})g(u^{(2)}, \sigma^{2(2)})}{\iint L(t^{(2)}|u^{(2)}, \sigma^{2(2)})g(u^{(2)}, \sigma^{2(2)})du^{(2)}d\sigma^{2(2)}}. \quad (30)$$

where $g(u^{(2)}, \sigma^2(1))$ is the prior information of the equipment reliability of the second batch tests, and

$$g(u^{(2)}, \sigma^{2(2)}) = \varepsilon_{\delta_1}^{\tau} g(u^{(1)}, \sigma^2(1)|t^{(1)}) + \varepsilon_{\delta_2}^{\tau} g_2(u^{(2)}, \sigma^{2(2)}). \quad (31)$$

3) Equipment reliability estimation of the ith batch tests

The ith batch tests contain the information of the ith batch field tests reliability information $L(t^{(i)}|u^{(i)}, \sigma^2(i))$, the ith batch simulation tests information $g_2(u^{(i)}, \sigma^{2(i)})$, and the $i-1$ th batch tests equipment reliability information $g(u^{(i-1)}, \sigma^{2(i-1)}|t^{(i-1)})$. The equipment reliability parameters distribution of the i th batch tests can be obtained by Bayesian method as follows:

$$g(u^{(i)}, \sigma^2(i)|t^{(i)}) = \frac{L(t^{(i)}|u^{(i)}, \sigma^{2(i)})g(u^{(i)}, \sigma^{2(i)}}{\iint L(t^{(i)}|u^{(i)}, \sigma^{2(i)})g(u^{(i)}, \sigma^{2(i)})du^{(i)}d\sigma^{2(i)}}. \quad (32)$$

The prior distribution $g(u^{(i)}, \sigma^{2(i)})$ of the equipment reliability parameters of the i th batch tests is expressed as follows:

Table 2: Prior information experts membership score

Expert serial number	The i–1th batch tests information δ_1		The ith batch simulation tests information δ_2	
	Reliable membership degree $\omega_{\delta_1}^d$	Unreliable membership degree $\upsilon_{\delta_1}^d$	Reliable membership degree $\omega_{\delta_2}^d$	Unreliable membership degree $\upsilon_{\delta_2}^d$
1	0.7	0.3	0.6	0.2
2	0.7	0.3	0.9	0.1
3	0.8	0.2	0.7	0.3
4	0.6	0.4	0.8	0.17
5	0.7	0.3	0.7	0.1

Table 3: Bayesian estimation results of equipment reliability based on methods 1 and 2

		$\bar{t}$	S^2	u_n	k_n	v_n	$v_n\sigma_n^2$	u_{MD}	σ^2_{MD}
The Bayesian estimation method of equipment reliability based on multiple batch tests (method 1) in this study	The initial batch tests	30.4	16.6	29.9	7.5	9.6	119	29.9	10.52
	The second batch tests	34	11.6	32.7	9.32	12.4	125	32.7	8.6
	The third batch tests	39	8	38	10.17	13.32	108	38	7.14
The Bayesian estimation method of equipment reliability based on single batch tests (method 2)	The third batch tests	39	8	39.3	7.8	9.3	116	39.3	10.82

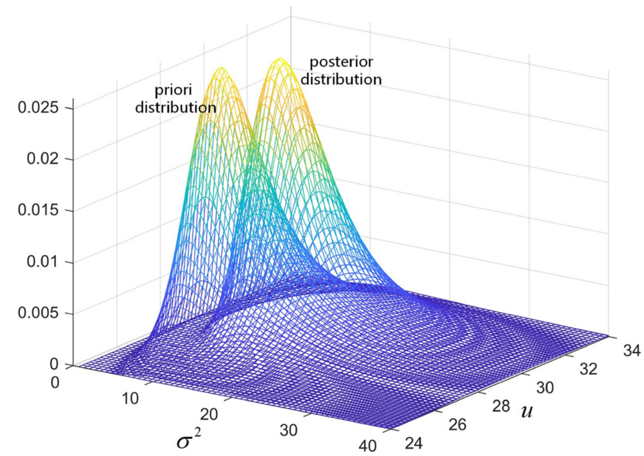

Figure 3: Distribution of equipment reliability parameters u, σ^2 of the initial batch tests.

$$g(u^{(i)}, \sigma^{2(i)}) = \varepsilon^{\tau}_{\delta_1} g(u^{(i-1)}, \sigma^{2(i-1)} | t^{(i-1)}) + \varepsilon^{\tau}_{\delta_2} g_2(u^{(i)}, \sigma^{2(i)}). \tag{33}$$

Constraint relationship can be constructed as follows:

$$\begin{cases} E(u^{(i)}) \geq E(u^{(i-1)}) \\ E(\sigma^{2(i)} \geq E(\sigma^{2(i-1)}) \end{cases}. \tag{34}$$

The constraint relationship indicates that the equipment reliability of the ith batch tests is better than the $i-1$th batch tests, indicating that the equipment reliability has been improved. According to the sampling results, if it is found that the constraint relationship is not satisfied, the equipment reliability of the ith batch tests is less than that of the $i-1$th batch tests, indicating that the improvement of the equipment reliability of tests before this batch

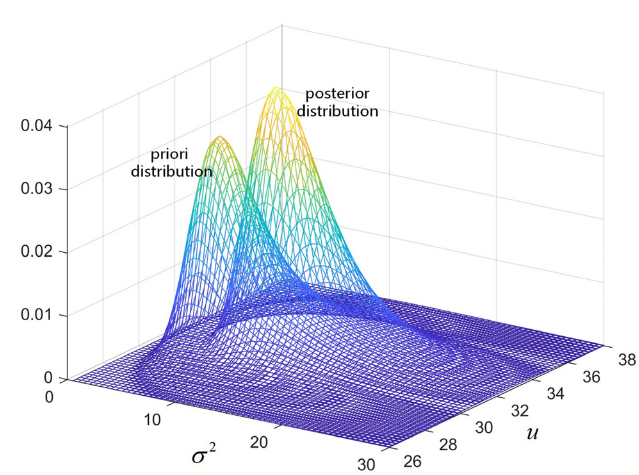

Figure 4: Distribution of equipment reliability parameters u, σ^2 of the second batch tests.

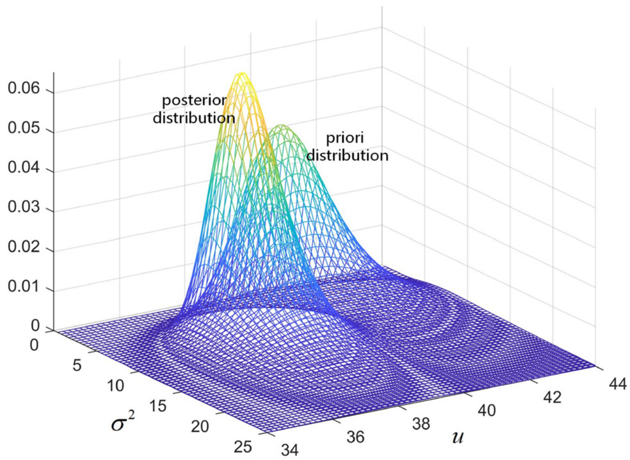

Figure 5: Distribution of equipment reliability parameters u, σ^2 of the third batch tests.

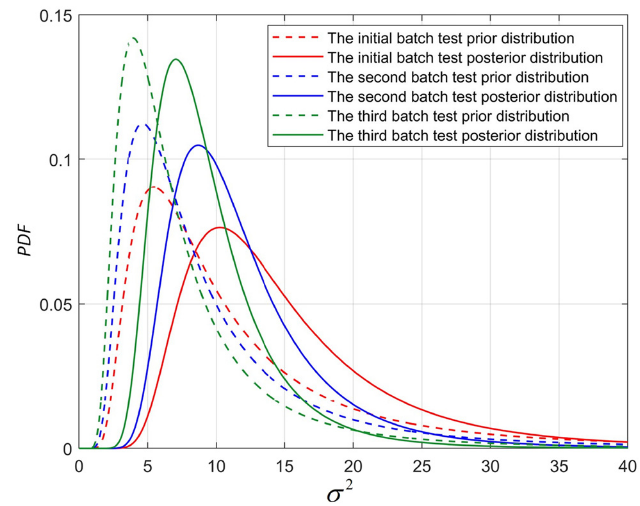

Figure 7: Equipment reliability parameter σ^2 distribution of each batch tests.

tests is not ideal, and it is necessary to further study the equipment reliability.

4 Experimental analysis

In the test activity of the normal life distribution equipment, three batch tests were carried out successively, with five pieces of equipment tested in each batch tests. The field equipment reliability tests information and simulation tests information of these three batches were obtained. The field tests information t_i, $i = 1, 2, \cdots, 5$ is the time that the equipment reaches failure. The information obtained from simulation tests is the distribution information of equipment reliability parameters $u^{(i)}$ and $\sigma^{2(i)}$. The information results are shown in Table 1.

According to the scoring rules of system contribution, five experts in the same field were invited to score the membership degree of the equipment reliability information δ_1 of the $i-1$th batch tests and the simulation tests information δ_2 of the ith batch tests. The scoring results are shown in Table 2.

According to the expert membership scoring data, the fusion weights are calculated to obtain $\varepsilon^{\tau}_{\delta_1} = 0.40$ and $\varepsilon^{\tau}_{\delta_2} = 0.60$.

The tested prior data are within the confidence interval, and through the consistency test, there is no

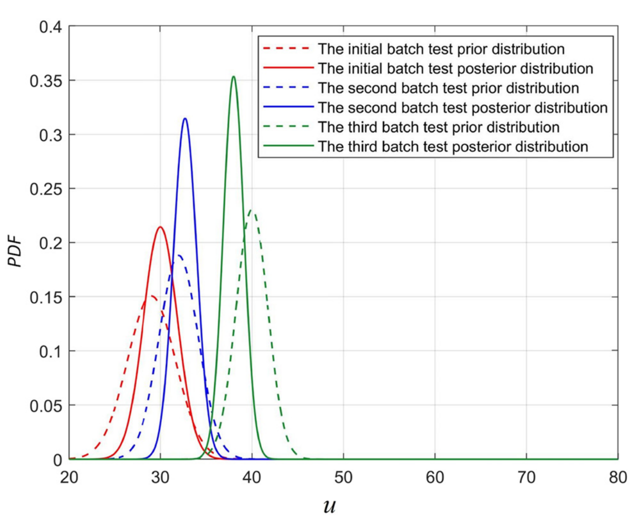

Figure 6: Equipment reliability parameter u distribution of each batch tests.

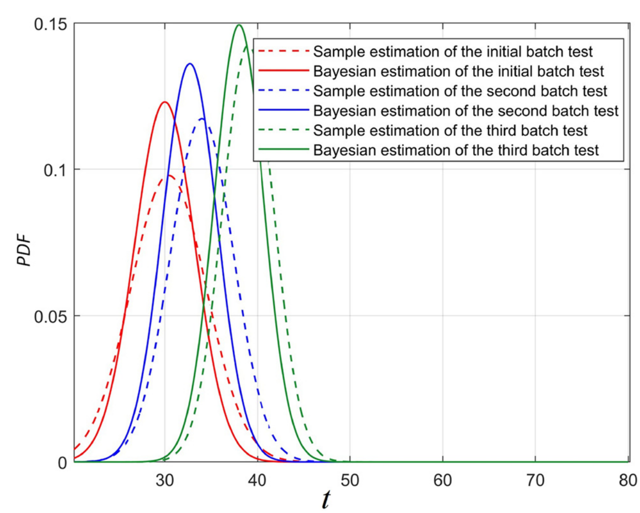

Figure 8: Equipment life distributions of each batch tests.

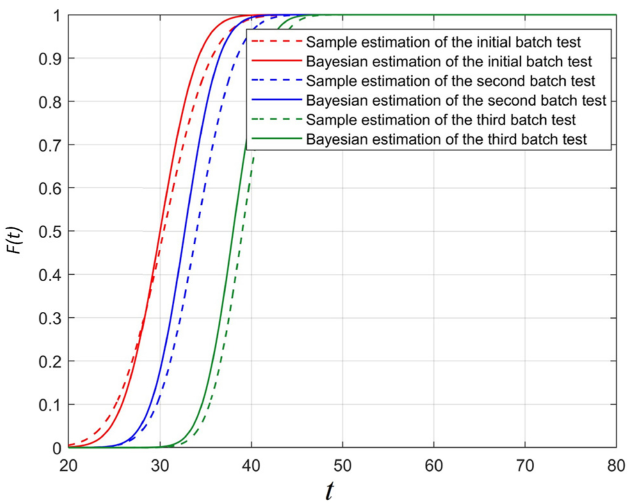

Figure 9: Equipment unreliability functions of each batch tests.

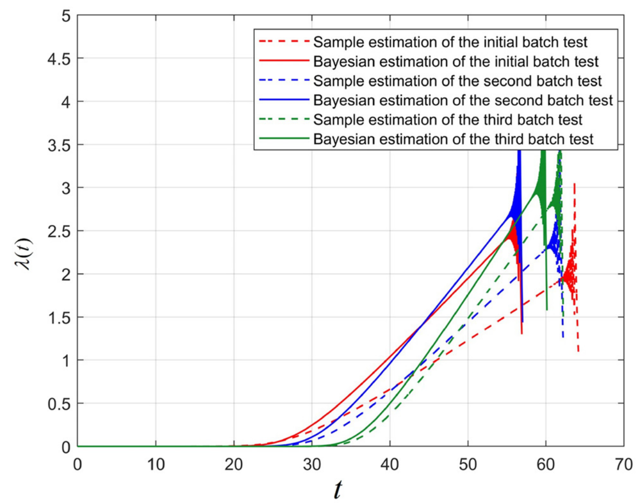

Figure 11: Equipment failure rate functions of each batch tests.

significant difference between the prior information and the field data.

In order to highlight the superiority of the method proposed in this study, the Bayesian estimation method of equipment reliability based on multiple batch tests (method 1) in this study is compared with the Bayesian estimation method of equipment reliability based on single batch tests (method 2). Field test information and prior information are used to calculate the distribution of equipment reliability parameters, and the calculation results of the two methods are obtained, as shown in Table 3.

The distribution diagrams of equipment reliability parameters u and σ^2 of each batch tests are plotted, as shown in Figures 3–5.

Figures 3–5 intuitively present the joint distribution of equipment reliability parameters u, σ^2. It can be found that the posterior distribution is slightly different from the prior distribution. The reason is that the posterior distribution integrates the prior information and the field reliability tests information, which is the result of the field reliability tests information modifying the prior distribution.

In order to more intuitively reflect the changes in reliability parameters u, σ^2 of each batch tests, the changes in parameters u, σ^2 with each batch tests are drawn respectively, as shown in Figures 6 and 7.

It can be seen from Figures 6 and 7 that the equipment reliability parameter u of each batch tests presents an increasing trend, while the parameter σ^2 presents a decreasing trend, which satisfies the constraint

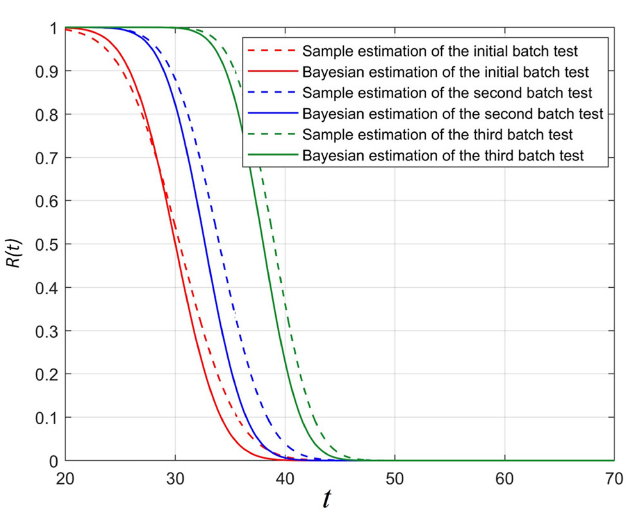

Figure 10: Equipment reliability functions of each batch tests.

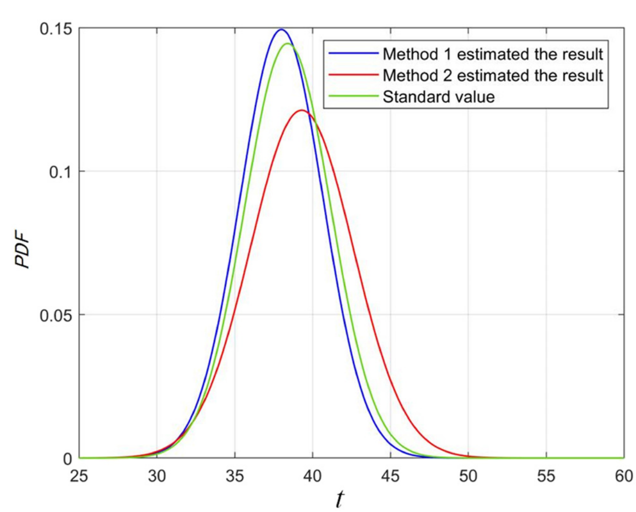

Figure 12: Distribution of equipment life corresponding to estimated and standard values.

relationship constructed by formula (34), indicating that the equipment reliability of each batch tests has been improved.

The maximum estimates u_{MD} and σ_{MD}^2 of the posterior distribution of parameters of each batch tests were taken as Bayesian estimates into the equipment reliability test function formulas (1)–(4). The changes in equipment reliability life distribution, unreliability function, reliability function, and failure rate function of each batch tests can be obtained. The results are shown in Figures 8–11.

The variation in equipment reliability functions obtained from field test samples information are also shown in Figures 8–11. Comparing the results in Figures 3–7, it can be found that compared with the gap between the Bayesian estimation results and the prior distribution, the Bayesian estimation results are closer to the field test samples information results, indicating that the field test samples information are more important, which is consistent with the actual situation.

These equipment tests give the change in the prior information, sample information, and Bayesian estimation results of each batch tests. The Bayesian results of the third batch tests contain the initial batch tests results, the second batch tests results, the third batch simulation tests information, and the field samples tests information. The Bayesian estimation results of the third batch tests contain more comprehensive information, so the Bayesian estimation results of the third batch tests are selected to estimate the reliability of equipment with normal life distribution.

Continue to increase the reliability test of the equipment to determine the standard values of the reliability parameters of the equipment: $u = 38.4$, $\sigma^2 = 7.62$. The distribution curves of equipment life corresponding to the estimated values calculated by the two parameter estimation methods and the standard value are shown in Figure 12.

From Figure 12, it can be found that the result value estimated by the Bayesian estimation method of equipment reliability based on multiple batch tests in this study is closer to the standard value, which indicates that this research method has better applicability at this time.

According to formula (6), the mean square error of the equipment reliability parameters estimation can be obtained, and the mean square error function image of parameter u can be plotted, as shown in Figure 13.

It can be seen that the mean square error of the equipment reliability Bayesian estimation method based on multi-batch tests in this study is small, while the mean square error of the equipment reliability Bayesian estimation method based on single batch tests is larger, indicating that the results obtained by this method are closer to the real value. The reason is that this research method uses the test information of three batches and more sample information, while the Bayesian estimation method based on single batch test only uses the test data of one batch, so there is a large risk of error in parameter estimation.

Bayesian estimation method of equipment reliability with normal-type life distribution based on multiple batch tests proposed in this study is applicable to the reliability research in the initial stage of equipment development and design, which consists of multiple batches reliability tests programs. When the number of reliability samples of each batch tests is small, it is more applicable to use the method of this study. Conversely, when the number of reliability samples of the final batch tests is large, it is sufficient to directly use the reliability sample data for estimation.

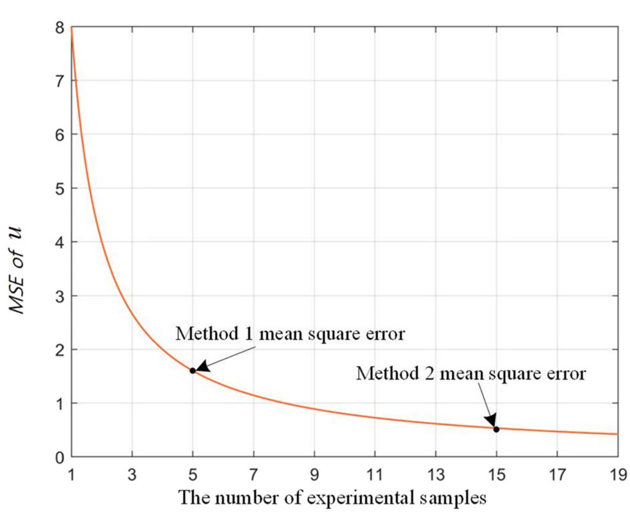

Figure 13: The mean square error function of parameter u.

5 Conclusion

The Bayesian estimation method of equipment reliability with normal life distribution based on multiple batch tests are proposed in this research, and takes the common equipment with normal-type life distribution as the research object, and uses Bayesian method and system contribution theory to make full use of the equipment reliability field tests information and simulation tests information of each batch. Compared with the existing reliability evaluation research, this research method uses more sufficient information, which can reduce the problem of large error of equipment reliability estimation caused by insufficient size of field tests samples.

In addition to the above advantages, there are some limitations in this study, for example, this study requires high reliability of the prior information of each batch tests. If the prior information is unreliable, it will affect the estimation accuracy of equipment reliability. Moreover, this study only investigates the reliability of equipment with normal-type life distribution and the reliability of equipment with Weibull distribution, lognormal distribution, and other distributions based on multiple batch tests will be further investigated in future research. The results of this study can provide reference for estimation of equipment reliability based on multiple batch tests.

Funding information: This research was funded by the National Natural Science Foundation of China (71501183).

Author contributions: conceptualization: H.L., T.H., and T.C.; methodology: H.L. and T.H.; software: H.L.; validation: H.L.; formal analysis: H.L.; investigation: T.H.; resources: T.H. and M.L.; data curation: K.D.; writing – original draft preparation: T.C.; writing – review and editing: T.C. and M.L.; visualization: H.L. and T.H.; supervision: M.L. and K.D.; project administration: M.L.; and funding acquisition: K.D. All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

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