Research Article

Azzh Saad Alshehry, Humaira Yasmin*, Ahmed A. Khammash, and Rasool Shah

Numerical analysis of dengue transmission model using Caputo–Fabrizio fractional derivative

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Abstract: This study demonstrates the use of fractional calculus in the field of epidemiology, specifically in relation to dengue illness. Using noninteger order integrals and derivatives, a novel model is created to examine the impact of temperature on the transmission of the vector–host disease, dengue. A comprehensive strategy is proposed and illustrated, drawing inspiration from the first dengue epidemic recorded in 2009 in Cape Verde. The model utilizes a fractional-order derivative, which has recently acquired popularity for its adaptability in addressing a wide variety of applicable problems and exponential kernel. A fixed point method of Krasnoselskii and Banach is used to determine the main findings. The semi-analytical results are then investigated using iterative techniques such as Laplace-Adomian decomposition method. Computational models are utilized to support analytical experiments and enhance the credibility of the results. These models are useful for simulating and validating the effect of temperature on the complex dynamics of the vector–host interaction during dengue outbreaks. It is essential to note that the research draws on dengue outbreak studies conducted in various geographic regions, thereby providing a broader perspective and validating the findings generally. This study not only demonstrates a novel application of fractional calculus in epidemiology but also casts light on the complex relationship between temperature and the dynamics of dengue transmission. The obtained results serve as a foundation for enhancing our understanding of the complex interaction between environmental factors and infectious diseases, leading the way for enhanced prevention and control strategies to combat global dengue outbreaks.

Keywords: fractional calculus, epidemic model, Caputo–Fabrizio derivative, numerical method, dynamical behavior

*** Corresponding author: Humaira Yasmin,** Department of Basic Sciences, General Administration of Preparatory Year, King Faisal University, Al-Ahsa, 31982, Saudi Arabia, e-mail: hhassain@kfu.edu.sa
Azzh Saad Alshehry: Department of Mathematical Sciences, Faculty of Sciences, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia, e-mail: asalshihry@pnu.edu.sa
Ahmed A. Khammash: Department of Mathematics, Deanship of Applied Sciences Umm Al-Qura University, Makkah, Saudi Arabia, e-mail: aakhammash@uqu.edu.sa
Rasool Shah: Department of Mathematics, Abdul Wali University, Mardan, Pakistan, e-mail: shahrasool26@gmail.com

1 Introduction

The Dengue transmission model using the Caputo–Fabrizio fractional derivative has potential applications in several areas of physics and epidemiology. Physically, the model contributes to the understanding of infectious disease dynamics, specifically in the context of dengue transmission. It aids in predicting the spread and impact of the disease within a population, considering the fractional-order nature of the derivative. The application extends to public health and epidemiology, assisting in the development of strategies for disease control and prevention. Furthermore, the fractional derivative introduces a mathematical tool that allows for a more nuanced representation of complex phenomena, enabling researchers to capture noninteger order dynamics inherent in certain systems. This mathematical framework has broader implications for studying various physical processes characterized by fractional dynamics, such as anomalous diffusion or complex fluid flow, beyond the immediate context of infectious disease modeling [1–5]. In recent years, various scientific studies have delved into multidisciplinary domains, exploring diverse aspects from vaccination effectiveness assessment using theoretical models [6] to iterative algorithms for solving sparse problems in video technology [7]. The dynamic shifts in corporate social responsibility efficiency amidst the COVID-19 pandemic have been meticulously studied in the Chinese food industry [8], along with insightful analyses of virus disease models and transmission

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trend predictions [9]. These studies reflect the broad spectrum of research endeavors addressing critical issues ranging from public health implications to technological advancements, presenting a comprehensive landscape for scientific exploration and problem-solving [10].

Dengue fever has been more common in recent decades, putting an estimated 40% of the world's population at risk. This extraordinary population expansion, expanding urbanization without appropriate home water sources, increasing transfer of the virus among people, and a lack of efficient mosquito control have all contributed to this global pandemic [11–17]. Infected Aedes mosquitoes, especially *Aedes aegypti* bite people and spread the dengue virus. Once a mosquito has been infected, it will carry the virus, infecting anybody it feeds on who is vulnerable to the disease. Due to the lack of a vaccine, the sole method of combating dengue fever is the elimination of potential vectors. Disease transmission mechanisms may be better understood with the use of suitable mathematical models [18–20]. Standard epidemiological models use classical derivatives of integer order. We suggest generalized fractional derivatives for this purpose. Despite a lengthy history as a pure field of mathematics, fractional calculus (calculus of noninteger order) has only lately been demonstrated to be valuable as a practical tool [21]. In this study, we argue that fractional calculus may be a useful tool for building epidemiological model. We start by thinking about a basic epidemiological model that may be used to describe a dengue fever outbreak. The remainder of the study is devoted to explaining fractional derivatives in the Riemann–Liouville sense, recasting the dynamics of the classical model in terms of fractional derivatives and then using a new approximation approach to calculate numerical solutions to the fractional model [22–26]. Comparing these alternative models with the traditional systems through numerical simulations reveals that the former may be a more accurate representation of reality.

To accurately describe and effectively halt the spread of (epidemic/pandemic) illnesses, more in-depth knowledge of mathematical models is required. The spread of vector-borne illnesses poses a serious risk to both the health of humans and animals. Many demographic, ecological, and societal variables come into play when determining the geographic range of a disease vector. More than 700,000 fatalities each year are attributed to vector-borne illnesses, which account for about 17% of all viral infections. Dengue fever is a severe, flu-like disease that mostly affects people living in urban and semi-urban regions in nations with subtropical and tropical climates [27,28]. Although there is currently no cure for dengue fever, it can be effectively treated, and the fatality rate can be brought down to around 1% by early diagnosis and intervention. Female *Aedes aegypti* mosquitoes are primarily responsible for transmitting the dengue virus from infected vectors to susceptible hosts *via* their bites. A mosquito infected with the dengue virus may spread the disease for the entirety of its life after just 410 days of incubation [28]. An adult mosquito completes the life cycle after four stages (egg, larva, pupa, and adult). The larvae, pupae, and adult all spend their time in water, but the adult is a lively, flying bug. Only the female mosquito will bite, and only if she needs to feed on human or animal blood.

Some strong and suitable mathematical analyses have been proposed [18,20,29–38] for maintaining dengue control. A model for the dynamic analysis of the spread of dengue fever using a nonlinear rate of recovery was created by Abdelrazec *et al.* [38] to examine the transfer and control of the illness. Esteva and Yang [32] developed a mathematical model for dengue to trace the spread of two epidemic illnesses across distinct human populations. Stability analysis provides an explanation for the reproduction number R as an epidemic threshold quantity. According to their simulations, ecological management as a means of vector control is insufficient at best and at worst would only serve to stall the progression of infectious diseases temporarily. Using a vaccination may provide concurrent control against certain serogroups. The assumptions for parameter threshold values and control methods in deterministic models of dengue transmission are reviewed by Andraud *et al.* [39]. The epidemiological influence of seasonal variations in temperature and other climatic factors on the transmission dynamics of dengue infections has been the subject of recent experimental research (*e.g.*, see [40]) and mathematical calculations [41–44]. These sources conduct their mathematical analysis by using compartmental integer-order epidemic models, which include an ODE system. However, in most cases, memory is not required in integer-order systems [45–49].

2 Evaluation of dengue fever

Let us say N_h and N_v stand for the total number of hosts and vectors, respectively. The male mosquito N_h population is divided into the classes of susceptibility $A(t)$, partial immunity $B(t)$, infection $C(t)$, carrier $D(t)$, and recovery $E(t)$ in the model's formulation, while the female mosquito N_v population is divided into the classes of susceptibility $F(t)$ and infectious $G(t)$ in the same way. A bite from an infection caused by mosquitoes, for instance, may transmit dengue illness to an unwary person. The susceptibility of hosts, the infectiousness of illnesses, the bitten frequency of vectors, and the likelihood of transmission all play a role in establishing the infectiousness of a given host and vector population. In order to calculate the rate of infection per vector that is susceptible $F(t)$ and host $A(t)$, we use the formulae $\left(\frac{b\beta_v}{N_h}(C + D)\right)$ and $\left(\frac{b\beta_{h1}}{N_h}C\right)$. In contrast, per susceptible $B(t)$, we use the formula $\left(\frac{b\beta_v}{N_h}G\right)$ with $\beta_{h2} < \beta_{h1}$. We hypothesize that some infected individuals are symptom-free carriers (asymptomatic) and that some recovered individuals (R_h) become vulnerable to the illness once again. Assuming a negligible death rate induced by infection on hosts, the natural birth rate of the vector and host, denoted as μ_v and μ_h, respectively, are taken into account. The ordinary differential equations (ODEs) describing dengue infection are given as follows:

$$\begin{cases} \dfrac{\mathrm{d}A(t)}{\mathrm{d}t} = \mu_h N_h - \dfrac{\beta h1^b}{N_h}AG - pA - \mu_h A \\ \dfrac{\mathrm{d}B(t)}{\mathrm{d}t} = vE - \dfrac{\beta h2^b}{N_h}BG - \mu_h B \\ \dfrac{\mathrm{d}C(t)}{\mathrm{d}t} = (1-\psi) - \dfrac{\beta h1^b}{N_h}AG + (1-\psi)\dfrac{\beta h2^b}{N_h}BG \\ -(\mu_h + \tau + \gamma)C \\ \dfrac{\mathrm{d}D(t)}{\mathrm{d}t} = \psi\dfrac{\beta_{h1}^b}{N_h}AG + \psi\dfrac{\beta_{h1}^b}{N_h}BG - (v + \mu_h)R_h \\ \dfrac{\mathrm{d}E(t)}{\mathrm{d}t} = pA + \gamma(C + D) + \tau C - (\psi + \mu_h) - E \\ \dfrac{\mathrm{d}F(t)}{\mathrm{d}t} = \mu_v N_v - \dfrac{\beta_v^b}{N_h}(C + D)F - \mu_v F \\ \dfrac{\mathrm{d}G(t)}{\mathrm{d}t} = \dfrac{\beta_v^b}{N_h}(C + D)F - \mu_v G, \end{cases} \quad (1)$$

given the vector's proper initial condition

$$F(0) \geq (0), \quad G(0) \geq 0$$

and the host's proper initial condition

$$A(0) \geq 0, \quad B(0) \geq 0, \quad C(0) \geq 0, \quad D(0) \geq 0, \quad E(0) \geq 0,$$

moreover, the host and vector's strengths are described as follows:

$$N_v = F + G, \quad N_h = A + B + C + D + E.$$

Incorporating both current and historical data into fractional-order models has been proven to accurately depict the nonlocal behavior of biological systems. We use a fractional-order Liouville-derivative framework to describe the dynamical system underlying dengue infection transmission. Caputo [50–52] more correctly described their fractional system by retaining a constant dimension on both sides of the system; we did the same to offer a more accurate description. So, the previously described dengue system's fractional Liouville–Caputo derivative is

$$\begin{cases} {}_0^{LC}D_t^{\vartheta}A(t) = \mu_{h^{\vartheta}}N_h - \dfrac{\beta h1^{b^{\vartheta}}}{N_h}AG - p^{\vartheta}A - \mu_h^{\vartheta}A \\ {}_0^{LC}D_t^{\vartheta}B(t) = v^{\vartheta}E - \dfrac{\beta h2^{b^{\vartheta}}}{N_h}BG - \mu_h^{\vartheta}B \\ {}_0^{LC}D_t^{\vartheta}C(t) = (1-\psi) - \dfrac{\beta h1^{b^{\vartheta}}}{N_h}AG + (1-\psi)\dfrac{\beta h2^{b^{\vartheta}}}{N_h} \\ BG - (\mu_h^{\vartheta} + \tau^{\vartheta} + \gamma^{\vartheta})C \\ {}_0^{LC}D_t^{\vartheta}D(t) = \psi\dfrac{\beta_{h1}^{b^{\vartheta}}}{N_h}AG + \psi\dfrac{\beta_{h1}^{b^{\vartheta}}}{N_h}BG - (v^{\vartheta} + \mu_{h^{\vartheta}})D \\ {}_0^{LC}D_t^{\vartheta}E(t) = p^{\vartheta}A + \gamma^{\vartheta}(C + D) + \tau^{\vartheta}C - (\psi^{\vartheta} + \mu_h^{\vartheta}) \\ \quad - E \\ {}_0^{LC}D_t^{\vartheta}F(t) = \mu_v^{\vartheta}N_v - \dfrac{\beta_v^{b^{\vartheta}}}{N_h}(C + D)F - \mu_v^{\vartheta}F \\ {}_0^{LC}D_t^{\vartheta}G(t) = \dfrac{\beta_v^{b^{\vartheta}}}{N_h}(C + D)F - \mu_v^{\vartheta}G, \end{cases} \quad (2)$$

where ${}_0^{LC}D_t^{\vartheta}$ represents fractional derivative (of Liouville and Caputo) of ϑ and ϑ represents the memory index of the system.

3 Basic definitions

Within the domain of this research area, we will elucidate a range of essential concepts.

Definition 1. Suppose $\xi \in \mathcal{H}^1(a, b)$ with $b > a$ and $\wp \in (0, 1)$; under these conditions, the provided Caputo–Fabrizio fractional derivative (CFFD) can be expressed as [53,54]:

$${}_0^{CF}D_t^{\wp}\xi(t) = \frac{\kappa(\wp)}{1-\wp}\int_a^t \xi'(\Phi)\exp\left[-\frac{t-\Phi}{1-\Phi}\right]\mathrm{d}\Phi. \quad (3)$$

The function $\kappa(\wp)$ in Eq. (3) is chosen such that $\kappa(1) = \kappa(0) = 1$. Furthermore, if ξ does not belong to $\mathcal{H}^1(a, b)$, the equation undergoes a transformation, resulting in

$${}_0^{CF}D_t^{\wp}\xi(t) = \frac{\kappa(\wp)}{1-\wp}\int_a^t \xi(t) - \xi(\Phi)\exp\left[-\frac{t-\Phi}{1-\Phi}\right]\mathrm{d}\Phi.$$

Definition 2. Consider $\wp \in (0, 1]$ and let us denote the integral of the function ξ to the fractional order $\wp$ as [53,54]:

$$ {}_{0}^{CF}I_t^{\wp}\xi(t) = \frac{(1-\wp)}{\kappa(\wp)}\xi(t) + \frac{\wp}{\kappa(\wp)}\int_0^t \xi(\Phi)\mathrm{d}\Phi. $$

Lemma 1. *An issue that arises with the CFFD is* [53,54]

$$ \begin{cases} {}_{0}^{CF}D_t^{\wp}\xi(t) = z(t), \quad 0 < \wp \le 1, \\ \xi(0) = \xi_0, \quad \text{where } \xi \text{ is real constant}. \end{cases} $$

Alternatively, this can be expressed as being equivalent to the integral as follows:

$$ \xi(t) = \xi_0 + \frac{1-\wp}{\kappa(\wp)}\xi(t) + \frac{\wp}{\kappa(\wp)}\int_0^t \xi(\Phi)\mathrm{d}\Phi. $$

Definition 3. [37,55] CFFD's Laplace transform is ${}_{0}^{CF}D_t^{\wp}$ and $\wp \in (0, 1]$ of $M(t)$ is given as follows:

$$ L[{}_{0}^{CF}I_t^{\wp}M(t)] = \frac{sL[M(t)] - M(0)}{s + \wp(1-s)}. $$

4 Dengue fever model of fractional order: existence and uniqueness results

To establish the existence of at least one solution to the model, we utilize the theorems of Banach and Krasnoselskii.

$$ \begin{cases} f_1(t, A, B, C, D, E, F) = \mu_h N_h - \dfrac{\beta h1^{b}}{N_h}AG - pA - \mu_h A \\ f_2(t, A, B, C, D, E, F) = \upsilon^{\vartheta}E - \dfrac{\beta h2^{b^{\vartheta}}}{N_h}BG - \mu_h^{\vartheta}B \\ f_3(t, A, B, C, D, E, F) = (1-\psi) - \dfrac{\beta h1^{b^{\vartheta}}}{N_h}AG \\ + (1-\psi)\dfrac{\beta h2^{b^{\vartheta}}}{N_h}BG - (\mu_h^{\vartheta} + \tau^{\vartheta} + \gamma^{\vartheta})C \\ f_4(t, A, B, C, D, E, F) = \psi\dfrac{\beta_{h1}^{b^{\vartheta}}}{N_h}AG + \psi\dfrac{\beta_{h1}^{b^{\vartheta}}}{N_h}BG \\ -(\nu^{\vartheta} + \mu_{h^{\vartheta}})D \\ f_5(t, A, B, C, D, E, F) = p^{\vartheta}A + \gamma^{\vartheta}(C + D) \\ + \tau^{\vartheta}C - (\psi^{\vartheta} + \mu_h^{\vartheta}) - E \\ f_6(t, A, B, C, D, E, F) = \mu_v^{\vartheta}N_v - \dfrac{\beta_v^{b^{\vartheta}}}{N_h}(C + D)F - \mu_v^{\vartheta}F \\ f_7(t, A, B, C, D, E, F) = \dfrac{\beta_v^{b^{\vartheta}}}{N_h}(C + D)F - \mu_v^{\vartheta}G, \end{cases} \tag{4} $$

where

$$ A(0) = N_1, \quad B(0) = N_2, \quad C(0) = N_3, \quad D(0) = N_4, $$
$$ E(0) = N_5, \quad F(0) = N_6, \quad G(0) = N_7. $$

So our problem becomes

$$ \begin{cases} {}_{0}^{LC}D_t^{\vartheta}A(t) = f_1(t, A, B, C, D, E, F), \\ {}_{0}^{LC}D_t^{\vartheta}B(t) = f_2(t, A, B, C, D, E, F), \\ {}_{0}^{LC}D_t^{\vartheta}C(t) = f_3(t, A, B, C, D, E, F), \\ {}_{0}^{LC}D_t^{\vartheta}D(t) = f_4(t, A, B, C, D, E, F), \\ {}_{0}^{LC}D_t^{\vartheta}E(t) = f_5(t, A, B, C, D, E, F), \\ {}_{0}^{LC}D_t^{\vartheta}F(t) = f_6(t, A, B, C, D, E, F), \\ {}_{0}^{LC}D_t^{\vartheta}G(t) = f_7(t, A, B, C, D, E, F), \end{cases} \tag{5} $$

where

$$ A(0) = N_1, \quad B(0) = N_2, \quad C(0) = N_3, \quad D(0) = N_4, $$
$$ E(0) = N_5, \quad F(0) = N_6, \quad G(0) = N_7. $$

Here, we consider

$$ W(t) = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{bmatrix}, \quad W_0 = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{bmatrix}, $$

$$ F(t, W(t)) = \begin{bmatrix} f_1(t, W(t)) \\ f_2(t, W(t)) \\ f_3(t, W(t)) \\ f_4(t, W(t)) \\ f_5(t, W(t)) \\ f_6(t, W(t)) \\ f_7(t, W(t)) \end{bmatrix}. $$

Hence, we can express system (5) as follows:

$$ \begin{cases} {}_{0}^{LC}D_t^{\vartheta}W_t = F(t, W(t)), \quad 0 < \wp \le 1, \\ W(0) = W_0, \end{cases} \tag{6} $$

Lemma 1 provides the solution to Eq. (6) if and only if the right side evaluates at zero.

$$ W(t) + W_0 + XF(t, W(t)) + \bar{X}\int_0^t F(\xi, W(\Phi))\mathrm{d}\Phi \quad \text{where} $$

$$ X = \frac{1-\wp}{\kappa(\wp)}, $$

$$\text{and} \quad \bar{X} = \frac{\wp}{\kappa(\wp)}.$$

To facilitate further analysis, we will define the Banach space $\mathcal{D} = L[0, T]$ by specifying the norm of $\mathcal{D}$ on the interval $0 < t \leq T < \infty$.

$$||W|| = \sup_{t\epsilon[0,T]} \{|W(t)| : W \in \mathcal{D}\} \tag{7}$$

Theorem 1. (Krasnoselskii fixed point theorem) *Suppose $\mathcal{D} \subset \mathcal{X}$ is a convex and closed subset, and consider two operators $\mathcal{A}$ and $\mathcal{B}$ such that*
(1) $\mathcal{A}W_1 + \mathcal{B}W_2 \in \mathcal{D}$;
(2) *$\mathcal{B}$ is continuous and compact, while $\mathcal{A}$ is contraction*;
(3) *$\exists$ at least one fixed point W, such that $\mathcal{A}W + \mathcal{B}W = W$ hold.*

The following statement holds:
($\mathcal{H}_1$) Assume $\kappa_F > 0$ is a given constant, then

$$|F(t, W(t)) - F(t, \overline{W}(t))| \leq \kappa_F|W - \overline{W}|,$$

($\mathcal{H}_2$) Let $C_F > 0$ and $\mathcal{M}_F > 0$ be two constants. In this case, we have the following relationship:

$$|F(t, W)| \leq C_F|W| + \mathcal{M}_F.$$

Theorem 2. *Thanks to Theorem* 4.1, *If $G\kappa_F < 1$, then the problem defined by Eqs* (4) *and* (7) *has at least one solution.*

Proof. Suppose we intend to define the set D as a set that possesses both compact and closed. $\mathcal{D} = \{W \in \mathcal{X} : ||W|| \leq r\}$. Consider the operators $\mathcal{A}$ and $\mathcal{B}$, then we have the following:

$$\begin{aligned} \mathcal{A}W(t) &= W_0 + GF(t, W(t)) \\ \mathcal{B}W(t) &= \overline{G}\int_0^t F(\xi, W(\xi))d\xi. \end{aligned} \tag{8}$$

Let W and $\overline{W}$ belong to $\mathcal{X}$ for the contraction condition of $\mathcal{A}$ defined in Eq. (8). In this case, we can observe the following relationship:

$$\begin{aligned} ||\mathcal{A}W - \mathcal{A}\overline{W}|| &= \sup_{t\in[0,T]} |\mathcal{A}W(t) - \mathcal{A}\overline{W}(t)| \\ &= \sup_{t\in[0,T]} G|F(t, W(t)) - F(t, \overline{W}(t))| \\ &\leq G\kappa_F[||W - \overline{W}||], \end{aligned} \tag{9}$$

thus $\mathcal{A}$ is contraction. For compactness of $\mathcal{B}$, consider

$$\begin{aligned} |\mathcal{B}W(t)| &= \left|\overline{G}\int_0^t F(\Phi, W(\Phi))d\Phi\right| \\ &\leq \overline{G}\int_0^t |F(\Phi, W(\Phi))|d\Phi. \end{aligned} \tag{10}$$

Taking max of Eq. (10), we have

$$\begin{aligned} ||\mathcal{B}W|| &\leq \overline{G} \sup_{t\in[0,T]} \int_0^t |F(\Phi, W(\Phi))|d\Phi \\ &\leq \overline{G} \sup_{t\in[0,T]} \int_0^t [C_F||W|| + \mathcal{M}_F]d\Phi \\ &\leq \overline{G}T(C_F r + \mathcal{M}_F). \end{aligned} \tag{11}$$

Consequently, $\mathcal{B}$ is bounded in Eq. (11). Assuming the domain of t is $t_1 < t_2$, we obtain the following:

$$\begin{aligned} |\mathcal{B}W(t_2) - \mathcal{B}W(t_1)| &= \left|\overline{G}\int_0^{t_2} F(\Phi, W)d\Phi - \overline{G}\int_0^{t_1} F(\Phi, W)d\Phi\right| \\ &= \left|\overline{G}\int_0^{t_2} F(\Phi, W)d\Phi + \overline{G}\int_{t_1}^0 F(\Phi, W(\Phi))d\Phi\right| \\ &\leq \overline{G}\int_{t_1}^{t_2} |F(\Phi, W)|d\Phi \\ &\leq \overline{G}(C_F r + \mathcal{M}_F). \end{aligned} \tag{12}$$

Upon $t_2 \to t_1$, the right side of Eq. (12) tends to zero. In addition, $\mathcal{B}$ is uniformly bounded, so

$$|\mathcal{B}W(t_2) - \mathcal{B}W(t_1)| \to 0.$$

Thus, satisfying all the assumptions of Theorem 1, the analyzed model (Eq. (6)), possesses at least one solution due to the complete continuity of $\mathcal{B}$. □

Theorem 3. *Considering ($\mathcal{H}_1$), if $\overline{G}\mathcal{F}(1 + T) < 1$ is satisfied, then there exists a unique solution to the problem presented in Eq.* (6). *Consequently, the model* (4) *possesses at most one solution.*

Proof. Let $\mathcal{P} : \mathcal{X} \to \mathcal{X}$ be an operator defined as follows:

$$\mathcal{P}W(t) = W_0 + GF(t, W(t)) + \overline{G}\int_0^{t_1} F(\Phi, W(\Phi))d\Phi.$$

Let $\mathcal{W}, \overline{W} \in \mathcal{X}$, then

$$\begin{aligned}||\mathcal{P}(W)-\mathcal{P}(\overline{W})|| &= \sup_{t\in[0,T]}|\mathcal{P}(W)(t)-\mathcal{P}(\overline{W})(t)|\\ &\le \sup_{t\in[0,T]} G|F(t,W(t))-F(t,\overline{W}(t))|\\ &\quad+\overline{G}\sup_{t\in[0,T]}|\int_0^t|(F(\Phi,W(\Phi)))\\ &\quad-F(\Phi,(\overline{W}(\Phi)))|\mathrm{d}\Phi\\ &\le \overline{G}\kappa_F||W-\overline{W}||+GK_FT||W-\overline{W}||,\end{aligned}$$

which suggests that

$$||\mathcal{P}(W(-\mathcal{P}(\overline{W})))||\le\overline{G}\kappa_F(1+T)||W-\overline{W}||. \tag{13}$$

Therefore, the problem stated in Eq. (6) can have a maximum of one solution, implying that model (5) possesses a unique solution. □

5 Developing a generic algorithm to solve the model under consideration

Setting $\kappa(\wp)=1$ and applying the Laplace transform yields the series-type solution to the issue. It is thus possible to construct the following algorithm:

$$\begin{cases}\dfrac{s\mathcal{L}[A(t)]-A(0)}{s+\wp(1-s)}=\left[\mu_{h^\vartheta}N_h-\dfrac{\beta h1^{b^\vartheta}}{N_h}AG-p^\vartheta A-\mu_h^\vartheta A\right]\\ \dfrac{s\mathcal{L}[B(t)]-B(0)}{s+\wp(1-s)}=\left[v^\vartheta E-\dfrac{\beta h2^{b^\vartheta}}{N_h}BG-\mu_h^\vartheta B\right]\\ \dfrac{s\mathcal{L}[C(t)]-C(0)}{s+\wp(1-s)}=\left[(1-\psi)-\dfrac{\beta h1^{b^\vartheta}}{N_h}AG\right.\\ \qquad\left.+(1-\psi)\dfrac{\beta h2^{b^\vartheta}}{N_h}BG\right]\\ \qquad-[(\mu_h^\vartheta+\tau^\vartheta+\gamma^\vartheta)C]\\ \dfrac{s\mathcal{L}[D(t)]-D(0)}{s+\wp(1-s)}=\left[\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}AG+\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}BG\right.\\ \qquad\left.-(v^\vartheta+\mu_{h^\vartheta})D\right]\\ \dfrac{s\mathcal{L}[E(t)]-E(0)}{s+\wp(1-s)}=[p^\vartheta A+\gamma^\vartheta(C+D)+\tau^\vartheta C\\ \qquad-(\psi^\vartheta+\mu_h^\vartheta)-E]\\ \dfrac{s\mathcal{L}[F(t)]-F(0)}{s+\wp(1-s)}=\left[\mu_v^\vartheta N_v-\dfrac{\beta_v^{b^\vartheta}}{N_h}(C+D)F-\mu_v^\vartheta F\right]\\ \dfrac{s\mathcal{L}[G(t)]-G(0)}{s+\wp(1-s)}=\left[\dfrac{\beta_v^{b^\vartheta}}{N_h}(C+D)F-\mu_v^\vartheta G\right]\end{cases} \tag{14}$$

$$\begin{cases}\mathcal{L}[A(t)]=\dfrac{A(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}\left[\mu_{h^\vartheta}N_h-\dfrac{\beta h1^{b^\vartheta}}{N_h}AG\right.\\ \qquad\left.-p^\vartheta A-\mu_h^\vartheta A\right]\\ \mathcal{L}[B(t)]=\dfrac{B(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}\left[v^\vartheta E-\dfrac{\beta h2^{b^\vartheta}}{N_h}BG\right.\\ \qquad\left.-\mu_h^\vartheta B\right]\\ \mathcal{L}[C(t)]=\dfrac{C(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}\left[(1-\psi)\right.\\ \qquad\left.-\dfrac{\beta h1^{b^\vartheta}}{N_h}AG+(1-\psi)\dfrac{\beta h2^{b^\vartheta}}{N_h}BG\right]\\ \mathcal{L}[D(t)]=\dfrac{D(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}\left[\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}AG\right.\\ \qquad\left.+\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}BG-(v^\vartheta+\mu_{h^\vartheta})D\right]\\ \mathcal{L}[E(t)]=\dfrac{E(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}[p^\vartheta A+\gamma^\vartheta(C+D)\\ \qquad+\tau^\vartheta C-(\psi^\vartheta+\mu_h^\vartheta)-E]\\ \mathcal{L}[F(t)]=\dfrac{F(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}\left[\mu_v^\vartheta N_v\right.\\ \qquad\left.-\dfrac{\beta_v^{b^\vartheta}}{N_h}(C+D)F-\mu_v^\vartheta F\right]\\ \mathcal{L}[G(t)]=\dfrac{G(0)}{S}+\dfrac{s+\wp(1-s)}{S}\mathcal{L}\left[\dfrac{\beta_v^{b^\vartheta}}{N_h}(C+D)F\right.\\ \qquad\left.-\mu_v^\vartheta G\right].\end{cases} \tag{15}$$

Using the initial conditions of system (2):

$$\begin{cases} \mathcal{L}[A(t)] = \dfrac{N_1}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}\left[\mu_{h^\vartheta}N_h - \dfrac{\beta h1^{b^\vartheta}}{N_h}AG \right. \\ \qquad \left. - p^\vartheta A - \mu_h^\vartheta A\right] \\ \mathcal{L}[B(t)] = \dfrac{N_2}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}\left[v^\vartheta E - \dfrac{\beta h2^{b^\vartheta}}{N_h}BG - \mu_h^\vartheta B\right] \\ \mathcal{L}[C(t)] = \dfrac{N_3}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}\left[(1-\psi) - \dfrac{\beta h1^{b^\vartheta}}{N_h}AG \right. \\ \qquad \left. + (1-\psi)\dfrac{\beta h2^{b^\vartheta}}{N_h}BG\right] \\ \mathcal{L}[D(t)] = \dfrac{N_4}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}\left[\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}AG + \psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}BG \right. \\ \qquad \left. - (v^\vartheta + \mu_{h^\vartheta})D\right] \\ \mathcal{L}[E(t)] = \dfrac{N_5}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}[p^\vartheta A + \gamma^\vartheta(C + D) \\ \qquad + \tau^\vartheta C - (\psi^\vartheta + \mu_h^\vartheta) - E] \\ \mathcal{L}[F(t)] = \dfrac{N_6}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}\left[\mu_v^\vartheta N_v \right. \\ \qquad \left. - \dfrac{\beta_v^{b^\vartheta}}{N_h}(CF + DF) - \mu_v^\vartheta F\right] \\ \mathcal{L}[G(t)] = \dfrac{N_7}{s} + \dfrac{s + \wp(1-s)}{S}\mathcal{L}\left[\dfrac{\beta_v^{b^\vartheta}}{N_h}(CF + DF) - \mu_v^\vartheta G\right]. \end{cases} \tag{16}$$

Now we calculate the series form solutions is given as, *i.e.*,

$$A(t) = \sum_{n=0}^{\infty} A_n(t), \quad B(t) = \sum_{n=0}^{\infty} B_n(t), \quad C(t) = \sum_{n=0}^{\infty} C_n(t),$$

$$D(t) = \sum_{n=0}^{\infty} D_n(t), \quad E(t) = \sum_{n=0}^{\infty} E_n(t),$$

$$F(t) = \sum_{n=0}^{\infty} D_n(t), \quad G(t) = \sum_{n=0}^{\infty} G_n(t).$$

The nonlinear terms AG, BG, CF, and DF decomposed in terms of Adomian polynomial as follows:

$$A(t)G(t) = \sum_{n=0}^{\infty} R_n(t),$$

where

$$R_n = \frac{1}{\Upsilon(n+1)}\frac{\mathrm{d}^n}{\mathrm{d}\varrho^n}\left[\left(\sum_{k=0}^{n}\varrho^k A_k\right)\left(\sum_{k=0}^{n}\varrho^k G_k\right)\right]\Bigg|_{\varrho=o}$$

$$\begin{aligned} n = 0 &: R_0 = A_0G_0 \\ n = 1 &: R_1 = A_0G_1 + A_1G_0 \\ n = 2 &: R_2 = A_0G_2 + A_1G_1 + A_2G_0 \\ n = 3 &: R_3 = A_0G_3 + A_1G_2 + A_2G_1 + A_3G_0 \\ n = 4 &: R_4 = A_0G_4 + A_1G_3 + A_2G_2 + A_3G_1 + A_4G_0 \\ &\vdots \\ n = n &: R_n = A_0G_n + A_1G_{n-1} + \cdots + A_{n-1}G_1 + A_nG_0 \end{aligned}$$

$$B(t)G(t) = \sum_{n=0}^{\infty} M_n(t),$$

where

$$M_n = \frac{1}{\Upsilon(n+1)}\frac{\mathrm{d}^n}{\mathrm{d}\varrho^n}\left[\left(\sum_{k=0}^{n}\varrho^k B_k\right)\left(\sum_{k=0}^{n}\varrho^k G_k\right)\right]\Bigg|_{\varrho=o}$$

$$\begin{aligned} n = 0 &: M_0 = B_0G_0 \\ n = 1 &: M_1 = B_0G_1 + B_1G_0 \\ n = 2 &: M_2 = B_0G_2 + B_1G_1 + B_2G_0 \\ n = 3 &: M_3 = B_0G_3 + B_1G_2 + B_2G_1 + B_3G_0 \\ n = 4 &: M_4 = B_0G_4 + B_1G_3 + B_2G_2 + B_3G_1 + B_4G_0 \\ &\vdots \\ n = n &: M_n = B_0G_n + B_1G_{n-1} + \cdots + B_{n-1}G_1 + B_nG_0 \end{aligned}$$

$$C(t)F(t) = \sum_{n=0}^{\infty} Q_n(t),$$

where

$$Q_n = \frac{1}{\Upsilon(n+1)}\frac{\mathrm{d}^n}{\mathrm{d}\varrho^n}\left[\left(\sum_{k=0}^{n}\varrho^k C_k\right)\left(\sum_{k=0}^{n}\varrho^k F_k\right)\right]\Bigg|_{\varrho=o}$$

$$\begin{aligned} n = 0 &: Q_0 = C_0F_0 \\ n = 1 &: Q_1 = C_0F_1 + C_1F_0 \\ n = 2 &: Q_2 = C_0F_2 + C_1F_1 + C_2F_0 \\ n = 3 &: Q_3 = C_0F_3 + C_1F_2 + C_2F_1 + C_3F_0 \\ n = 4 &: Q_4 = C_0F_4 + C_1F_3 + C_2F_2 + C_3F_1 + C_4F_0 \\ &\vdots \\ n = n &: Q_n = C_0F_n + C_1F_{n-1} + \cdots + C_{n-1}F_1 + C_nF_0 \end{aligned}$$

$$D(t)F(t) = \sum_{n=0}^{\infty} N_n(t),$$

where

$$N_n = \frac{1}{\Upsilon(n+1)}\frac{\mathrm{d}^n}{\mathrm{d}\varrho^n}\left[\left(\sum_{k=0}^{n}\varrho^k D_k\right)\left(\sum_{k=0}^{n}\varrho^k F_k\right)\right]\Bigg|_{\varrho=o}$$

$$\begin{aligned} n = 0 &: N_0 = D_0F_0 \\ n = 1 &: N_1 = D_0F_1 + D_1F_0 \\ n = 2 &: N_2 = D_0F_2 + D_1F_1 + D_2F_0 \\ n = 3 &: N_3 = D_0F_3 + D_1F_2 + D_2F_1 + D_3F_0 \\ n = 4 &: N_4 = D_0F_4 + D_1F_3 + D_2F_2 + D_3F_1 + D_4F_0 \\ &\vdots \\ n = n &: N_n = D_0F_n + D_1F_{n-1} + \cdots + D_{n-1}F_1 + D_nF_0. \end{aligned}$$

Considering these values, model develops as follows:

$$\begin{cases} \mathcal{L}\left[\sum_{k=0}^{\infty} A_k(t)\right] = \frac{N_1}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[\mu_{h^\vartheta} N_h - \frac{\beta h1^{b^\vartheta}}{N_h}\sum_{k=0}^{\infty} R_k(t) - p^\vartheta \sum_{k=0}^{\infty} A_k(t) \right. \\ \left. - \mu_h^\vartheta \sum_{k=0}^{\infty} A_k(t)\right] \\ \mathcal{L}\left[\sum_{k=0}^{\infty} B_k(t)\right] = \frac{N_2}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[\upsilon^\vartheta \sum_{k=0}^{\infty} E_k(t) - \frac{\beta h2^{b^\vartheta}}{N_h}\sum_{k=0}^{\infty} M_k(t) - \mu_h^\vartheta \sum_{k=0}^{\infty} B_k(t)\right] \\ \mathcal{L}\left[\sum_{k=0}^{\infty} C_k(t)\right] = \frac{N_3}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[(1-\psi) - \frac{\beta h1^{b^\vartheta}}{N_h}\sum_{k=0}^{\infty} R_k(t) + (1-\psi)\frac{\beta h2^{b^\vartheta}}{N_h}\sum_{k=0}^{\infty} M_k(t)\right] \\ \mathcal{L}\left[\sum_{k=0}^{\infty} D_k(t)\right] = \frac{N_4}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[\psi \frac{\beta_{h1}^{b^\vartheta}}{N_h}\sum_{k=0}^{\infty} R_k(t) + \psi \frac{\beta_{h1}^{b^\vartheta}}{N_h}\sum_{k=0}^{\infty} M_k(t) \right. \\ \left. - (\nu^\vartheta + \mu_{h^\vartheta})\sum_{k=0}^{\infty} D_k(t)\right] \\ \mathcal{L}\left[\sum_{k=0}^{\infty} E_k(t)\right] = \frac{N_5}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[p^\vartheta \sum_{k=0}^{\infty} A_k(t) + \gamma^\vartheta\left(\sum_{k=0}^{\infty} C_k(t) + \sum_{k=0}^{\infty} D_k(t)\right) + \tau^\vartheta \sum_{k=0}^{\infty} C_k(t) \right. \\ \left. - (\psi^\vartheta + \mu_h^\vartheta) - \sum_{k=0}^{\infty} E_k(t)\right] \\ \mathcal{L}\left[\sum_{k=0}^{\infty} F_k(t)\right] = \frac{N_6}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[\mu_v^\vartheta N_v - \frac{\beta_v^{b^\vartheta}}{N_h}\left(\sum_{k=0}^{\infty} Q_k(t) + \sum_{k=0}^{\infty} N_k(t)\right) - \mu_v^\vartheta \sum_{k=0}^{\infty} F_k(t)\right] \\ \mathcal{L}\left[\sum_{k=0}^{\infty} G_k(t)\right] = \frac{N_7}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L} \\ \left[\frac{\beta_v^{b^\vartheta}}{N_h}\left(\sum_{k=0}^{\infty} Q_k(t) + \sum_{k=0}^{\infty} N_k(t)\right) - \mu_v^\vartheta \sum_{k=0}^{\infty} G_k(t)\right]. \end{cases} \tag{17}$$

Comparing the terms of Eq. (17), the following complications arise;

Case 1: If we set $n = 0$, then

$$\begin{cases} \mathcal{L}[A_0(t)] = \frac{N_1}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L}[\mu_{h^\vartheta} N_h] \\ \mathcal{L}[B_0(t)] = \frac{N_2}{s} \\ \mathcal{L}[C_0(t)] = \frac{N_3}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L}[(1-\psi)] \\ \mathcal{L}[D_0(t)] = \frac{N_4}{s} \\ \mathcal{L}[E_0(t)] = \frac{N_5}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L}(\psi^\vartheta + \mu_h^\vartheta) \\ \mathcal{L}[F_0(t)] = \frac{N_6}{s} + \frac{s + \wp(1-s)}{s}\mathcal{L}[\mu_v^\vartheta N_v] \\ \mathcal{L}[G_0(t)] = \frac{N_7}{s}. \end{cases} \tag{18}$$

Taking inverse Laplace transformation, and we obtain

$$\begin{cases} A_0(t) = N_1 + (\mu_{h^\vartheta} N_h)[1 + \wp(t-1)] \\ B_0(t) = N_2 \\ C_0(t) = N_3 + (1-\psi)[1 + \wp(t-1)] \\ D_0(t) = N_4 \\ E_0(t) = N_5 + (\psi^\vartheta + \mu_h^\vartheta)[1 + \wp(t-1)] \\ F_0(t) = N_6 + (\mu_v^\vartheta N_v)[1 + \wp(t-1)] \\ G_0(t) = N_7. \end{cases} \tag{19}$$

Case 2: If we set $n = 1$, then

$$\begin{cases} \mathcal{L}[A_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}\left[-\dfrac{\beta h1^{b^\vartheta}}{N_h}A_0(t)G_0(t) - p^\vartheta A_0(t) - \mu_h^\vartheta A_0(t)\right] \\ \mathcal{L}[B_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}\left[v^\vartheta E_0(t) - \dfrac{\beta h2^{b^\vartheta}}{N_h}B_0(t)G_0(t) - \mu_h^\vartheta B_0(t)\right] \\ \mathcal{L}[C_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}\left[-\dfrac{\beta h1^{b^\vartheta}}{N_h}A_0(t)G_0 + (1-\psi)\dfrac{\beta h2^{b^\vartheta}}{N_h}B_0(t)G_0(t)\right] \\ \mathcal{L}[D_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}\left[\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}A_0(t)G_0(t) + \psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}B_0(t)G_0(t) - (v^\vartheta + \mu_{h^\vartheta})D_0(t)\right] \\ \mathcal{L}[E_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}[p^\vartheta A_0(t) + \gamma^\vartheta(C_0(t) + D_0(t)) + \tau^\vartheta C_0(t) - (\psi^\vartheta + \mu_h^\vartheta) - E_0(t)] \\ \mathcal{L}[F_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}\left[-\dfrac{\beta_v^{b^\vartheta}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^\vartheta F_0(t)\right] \\ \mathcal{L}[G_1(t)] = \dfrac{s + \wp(1-s)}{s}\mathcal{L}\left[\dfrac{\beta_v^{b^\vartheta}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^\vartheta G_0(t)\right] \end{cases} \tag{20}$$

Taking inverse Laplace transformation, we obtain

$$\begin{cases} A_1(t) = \left[-\dfrac{\beta h1^{b^\vartheta}}{N_h}A_0(t)G_0(t) - p^\vartheta A_0(t) - \mu_h^\vartheta A_0(t)\right][1 + \wp(t-1)] \\ B_1(t) = \left[v^\vartheta E_0(t) - \dfrac{\beta h2^{b^\vartheta}}{N_h}B_0(t)G_0(t) - \mu_h^\vartheta B_0(t)\right][1 + \wp(t-1)] \\ C_1(t) = \left[-\dfrac{\beta h1^{b^\vartheta}}{N_h}A_0(t)G_0 + (1-\psi)\dfrac{\beta h2^{b^\vartheta}}{N_h}B_0(t)G_0(t)\right][1 + \wp(t-1)] \\ D_1(t) = \left[\psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}A_0(t)G_0(t) + \psi\dfrac{\beta_{h1}^{b^\vartheta}}{N_h}B_0(t)G_0(t) - (v^\vartheta + \mu_{h^\vartheta})D_0(t)\right][1 + \wp(t-1)] \\ E_1(t) = \left[-\dfrac{\beta_v^{b^\vartheta}}{N_h}(C_0(t)F_0(t) + N_0(t)) - \mu_v^\vartheta F_0(t)\right][1 + \wp(t-1)] \\ F_1(t) = \left[-\dfrac{\beta_v^{b^\vartheta}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^\vartheta F_0(t)\right][1 + \wp(t-1)] \\ G_1(t) = \left[\dfrac{\beta_v^{b^\vartheta}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^\vartheta G_0(t)\right][1 + \wp(t-1)]. \end{cases} \tag{21}$$

Case 3: If we set $n = 2$, then

$$\begin{cases} \mathcal{L}[A_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}\left[-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_1(t)G_1(t) - p^{\vartheta}A_1(t) - \mu_h^{\vartheta}A_1(t)\right] \\ \mathcal{L}[B_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}\left[v^{\vartheta}E_1(t) - \frac{\beta h2^{b^{\vartheta}}}{N_h}B_1(t)G_1(t) - \mu_h^{\vartheta}B_1(t)\right] \\ \mathcal{L}[C_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}\left[-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_1(t)G_1 + (1-\psi)\frac{\beta h2^{b^{\vartheta}}}{N_h}B_1(t)G_1(t)\right] \\ \mathcal{L}[D_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}\left[\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}A_1(t)G_1(t) + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}B_1(t)G_1(t) - (v^{\vartheta} + \mu_{h^{\vartheta}})D_1(t)\right] \\ \mathcal{L}[E_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}[p^{\vartheta}A_1(t) + \gamma^{\vartheta}(C_1(t) + D_1(t)) + \tau^{\vartheta}C_1(t) - (\psi^{\vartheta} + \mu_h^{\vartheta}) - E_1(t)] \\ \mathcal{L}[F_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}\left[-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_1(t)F_1(t) + D_1(t)F_1(t)) - \mu_v^{\vartheta}F_1(t)\right] \\ \mathcal{L}[G_2(t)] = \frac{s+\wp(1-s)}{s}\mathcal{L}\left[\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_1(t)F_1(t) + D_1(t)F_1(t)) - \mu_v^{\vartheta}G_1(t)\right]. \end{cases} \tag{22}$$

Taking inverse Laplace transformation, we obtain (23)

$$\left\{\begin{aligned}
A_2(t) &= \Bigg[-\frac{\beta h1^{b^{\vartheta}}}{N_h}\left(\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) - p^{\vartheta}A_0(t) - \mu_h^{\vartheta}A_0(t)\right)\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}G_0(t)\right) - p^{\vartheta}\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) - p^{\vartheta}A_0(t)\right.\\
&\quad \left. - \mu_h^{\vartheta}A_0(t)\right) - \mu_h^{\vartheta}\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) - p^{\vartheta}A_0(t) - \mu_h^{\vartheta}A_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]\\
B_2(t) &= \Bigg[\upsilon^{\vartheta}\left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + N_0(t)) - \mu_v^{\vartheta}F_0(t)\right) - \frac{\beta h2^{b^{\vartheta}}}{N_h}\left(\upsilon^{\vartheta}E_0(t) - \frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - \mu_h^{\vartheta}B_0(t)\right)\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}G_0(t)\right)\\
&\quad - \mu_h^{\vartheta}\left(\upsilon^{\vartheta}E_0(t) - \frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - \mu_h^{\vartheta}B_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]\\
C_2(t) &= \Bigg[-\frac{\beta h1^{b^{\vartheta}}}{N_h}\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) - p^{\vartheta}A_0(t) - \mu_h^{\vartheta}A_0(t)\right)\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}G_0(t)\right)\\
&\quad + (1-\psi)\frac{\beta h2^{b^{\vartheta}}}{N_h}\left(\upsilon^{\vartheta}E_0(t) - \frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - \mu_h^{\vartheta}B_0(t)\right)\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}G_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]\\
D_2(t) &= \Bigg[\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) - p^{\vartheta}A_0(t) - \mu_h^{\vartheta}A_0(t)\right)\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}G_0(t)\right)\\
&\quad + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}\left(\upsilon^{\vartheta}E_0(t) - \frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - \mu_h^{\vartheta}B_0(t)\right)\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t))\right.\\
&\quad \left. - \mu_v^{\vartheta}G_0(t)\right) - (\nu^{\vartheta} + \mu_{h^{\vartheta}})\left(\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - (\nu^{\vartheta} + \mu_{h^{\vartheta}})D_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]\\
E_2(t) &= \Bigg[p^{\vartheta}\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) - p^{\vartheta}A_0(t) - \mu_h^{\vartheta}A_0(t)\right) + \gamma^{\vartheta}\Bigg(\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0 + (1-\psi)\frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t)\right)\\
&\quad + \left(\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - (\nu^{\vartheta} + \mu_{h^{\vartheta}})D_0(t)\right)\Bigg) + \tau^{\vartheta}\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0 + (1-\psi)\frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t)\right) - (\psi^{\vartheta} + \mu_h^{\vartheta})\\
&\quad - \left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + N_0(t)) - \mu_v^{\vartheta}F_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]\\
F_2(t) &= \Bigg[-\frac{\beta_v^{b^{\vartheta}}}{N_h}\Bigg(\left(-\frac{\beta h1^{b^{\vartheta}}}{N_h}A_0(t)G_0 + (1-\psi)\frac{\beta h2^{b^{\vartheta}}}{N_h}B_0(t)G_0(t)\right)\left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}F_0(t)\right)\\
&\quad + \left(\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - (\nu^{\vartheta} + \mu_{h^{\vartheta}})D_0(t)\right)\left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}F_0(t)\right)\Bigg)\\
&\quad - \mu_v^{\vartheta}\left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}F_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]\\
G_2(t) &= \Bigg[\frac{\beta_v^{b^{\vartheta}}}{N_h}\Bigg(C_1(t)\left(\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - (\nu^{\vartheta} + \mu_{h^{\vartheta}})D_0(t)\right)\left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}F_0(t)\right)\\
&\quad + \left(\psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}A_0(t)G_0(t) + \psi\frac{\beta_{h1}^{b^{\vartheta}}}{N_h}B_0(t)G_0(t) - (\nu^{\vartheta} + \mu_{h^{\vartheta}})D_0(t)\right)\left(-\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}F_0(t)\right)\Bigg)\\
&\quad - \mu_v^{\vartheta}\left(\frac{\beta_v^{b^{\vartheta}}}{N_h}(C_0(t)F_0(t) + D_0(t)F_0(t)) - \mu_v^{\vartheta}G_0(t)\right)\Bigg]\left[1 + 2\wp(t-1) + \wp^2\left(\frac{t^2}{2!} - 2t + 1\right)\right]
\end{aligned}\right. \tag{23}$$

Table 1: Interpretation of the parameters and its values

Parameters	Description of parameters	Values	Source
β_{h1}	Transmission from vectors to susceptible A	0.75	[55]
β_{h2}	Vector-to-susceptible B transmission probability	0.375	Assume
μ_v	Vector's birth and death rates	0.0295	Assume
ϑ	Fractional memory index	0.5	Assume
τ	Rate of individual recovery	Variable	Assume
ν	Rate at which the host's immunity declines	0.05	Assume
β_v	Probability of transmission from humans to vectors	0.75	[55]
γ	Rate at which the host recovers	0.32883	[37]
ψ	Asymptomatic carrier proportion	Variable	Assume
b	Vector's biting rate	0.5	[55]
p	Vaccination fraction for type A susceptible hosts	0.3	Assume
μ_h	Human's birth and death rates	0.000046 to 0.004500	[37]

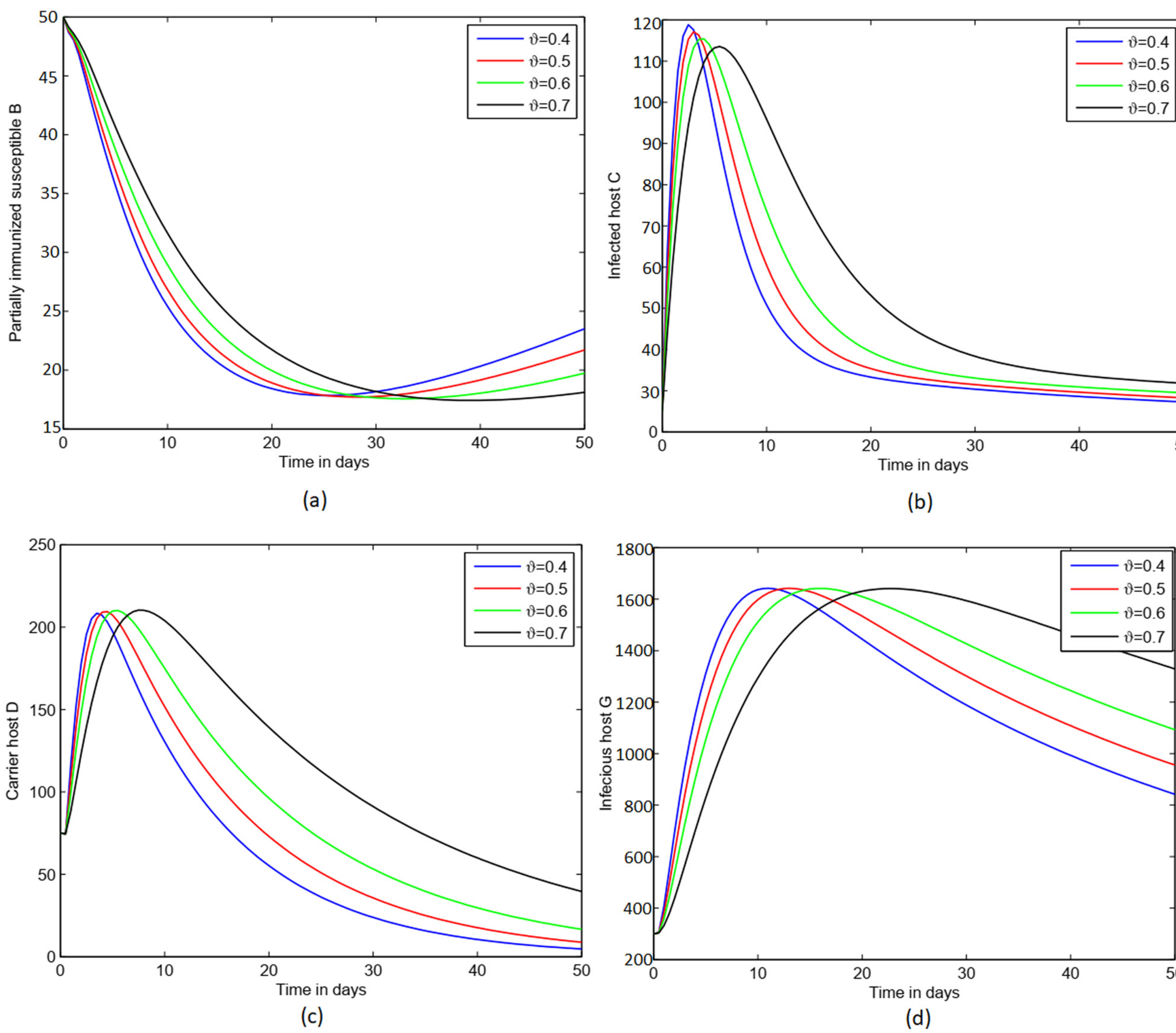

Figure 1: Illustration for depicting the interaction between host and vector populations, where (a) ϑ = 0.4, (b) 0.5, (c) 0.6, and (d) 0.7.

and so forth. To find more terms in the series solution, this method might be used. Consequently, we arrive at the solution as follows:

$$\begin{cases} A(t) = A_0(t) + A_1(t) + A_2(t) + \cdots \\ B(t) = B_0(t) + B_1(t) + B_2(t) + \cdots \\ C(t) = C_0(t) + C_1(t) + C_2(t) + \cdots \\ D(t) = D_0(t) + D_1(t) + D_2(t) + \cdots \\ E(t) = E_0(t) + E_1(t) + E_2(t) + \cdots \\ F(t) = F_0(t) + F_1(t) + F_2(t) + \cdots \\ G(t) = G_0(t) + G_1(t) + G_2(t) + \cdots \end{cases} \tag{24}$$

6 Computational results

In this section of the study, we present the outcomes relating to the approximate series solution of the proposed model. To obtain these results, we utilize the approximate values of the parameters specified in Table 1. Considering these parameter values, we generate plots that illustrate the solution up to five terms. Figures 1–6 correspond to different fractional orders within the model.

For different values of the parameters given in Table 1, we run simulations using the model 2, inspecting the time

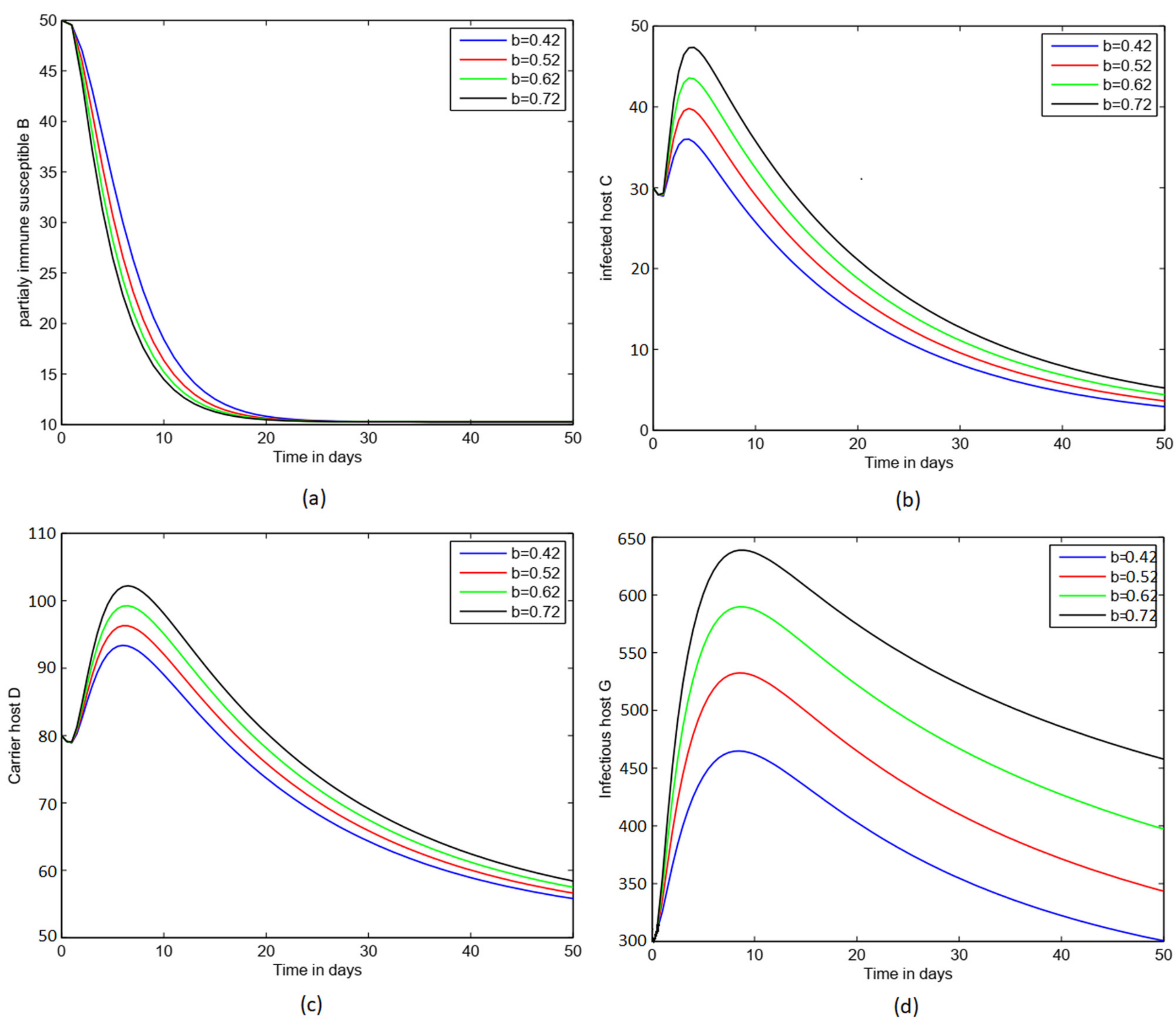

Figure 2: Illustration for depicting the interaction between host and vector populations, where (a) b = 0.42, (b) 0.52, (c) 0.62, and (d) 0.72.

series through the Laplace Adomian decomposition approach in order to better understand the proposed fractional model's dynamics. To see how the fractional order effects the system, we depict the dengue virus's behavior in Figure 1 by varying ϑ. It is shown that the index of memory may be lowered to reduce the prevalence of illness in a community. In order to reduce the prevalence of infections in the population as a whole, it is recommended that policymakers implement a strategy and procedure that reduce the memory index ϑ. Figure 2 shows the population and changes in biting rate, which can help you better understand the impact of the vectors biting rate. We found that the vector bite rate is crucial in the sense of increasing the virus infection level as a whole, and thus we can limit the dengue infection level by reducing the mosquito biting rate. Due to global warming, which creates favorable conditions for the proliferation of mosquitoes, the bite rate and, by extension, the infection rate will rise. In Figure 3, we change ψ, which is the asymptomatic fraction, to see how it affects the pace of virus spread and how we might slow it down. Infected host people may be profoundly impacted by the asymptomatic percentage, as we have shown. The asymptomatic subset, on the other hand, has been shown to infect and disseminate the dengue virus to areas where it is not prevalent. This suggests that asymptomatic carriers pose a greater threat, suggesting greater levels of control. In Figures 4 and 5, we see how the dynamics of system 2 change when the vaccination rate and treatment intensity are varied; this suggests that the vaccination rate has a little impact on the system overall,

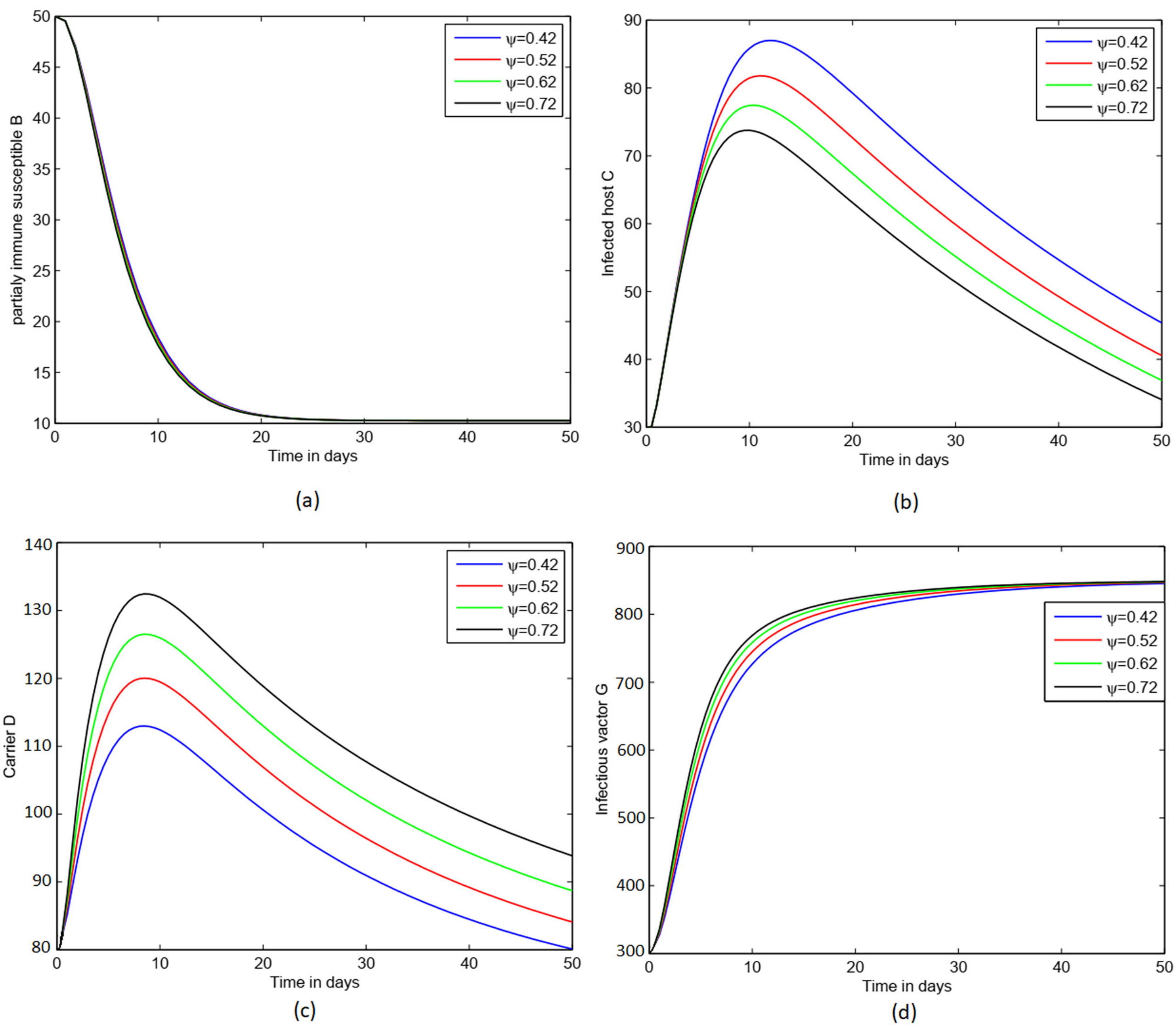

Figure 3: Illustration for depicting the interaction between host and vector populations, where (a) ψ = 0.42, (b) 0.52, (c) 0.62, and (d) 0.72.

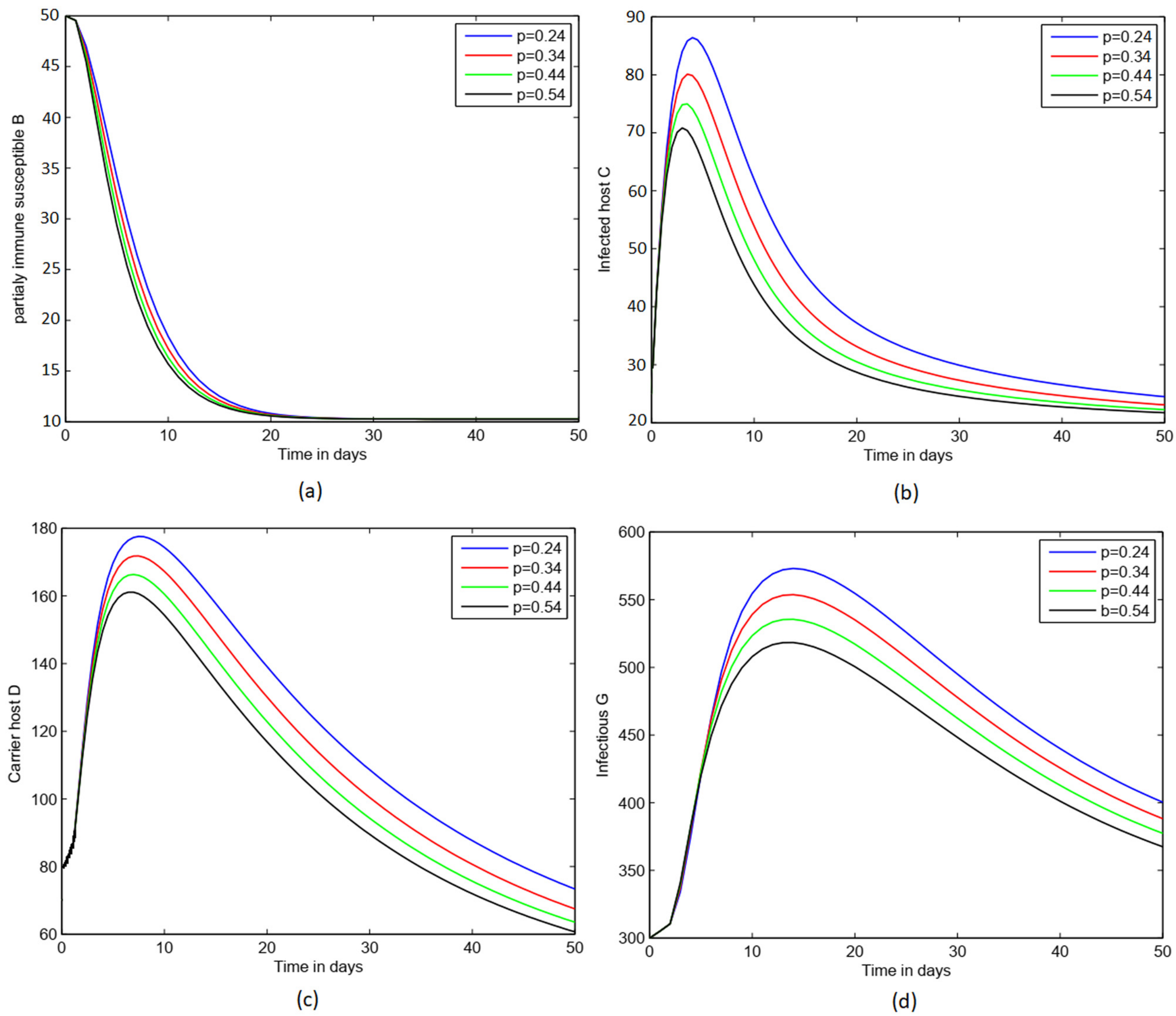

Figure 4: Illustration for depicting the interaction between host and vector populations, where (a) p = 0.24, (b) 0.34, (c) 0.44, and (d) 0.54.

but treatment has a significant role to play in reducing the number of infected people. Changing ν by a little amount in the second-to-last simulation (shown in Figure 6) showed that the widespread level was very sensitive to this input parameter. This suggests that dengue's partial immunity is crucial, prompting stronger measures of management.

7 Conclusion

The article focuses on the dengue virus, which is transmitted by mosquitoes and causes illness in humans. It presents a comprehensive overview of research concerning the virus, with a specific emphasis on the concerns of scientists regarding its potential for rapid spread and the growing risk of an epidemic. To gain a deeper understanding of the intricate dynamics of dengue fever transmission, the study proposes the use of a mathematical model that incorporates crucial factors such as vaccination and the application of fractional derivatives. This article represents a noteworthy advancement in our understanding of the transmission dynamics of dengue fever and serves as a crucial resource for individuals engaged in prevention, detection, and treatment efforts for the illness. The utilization of fractional

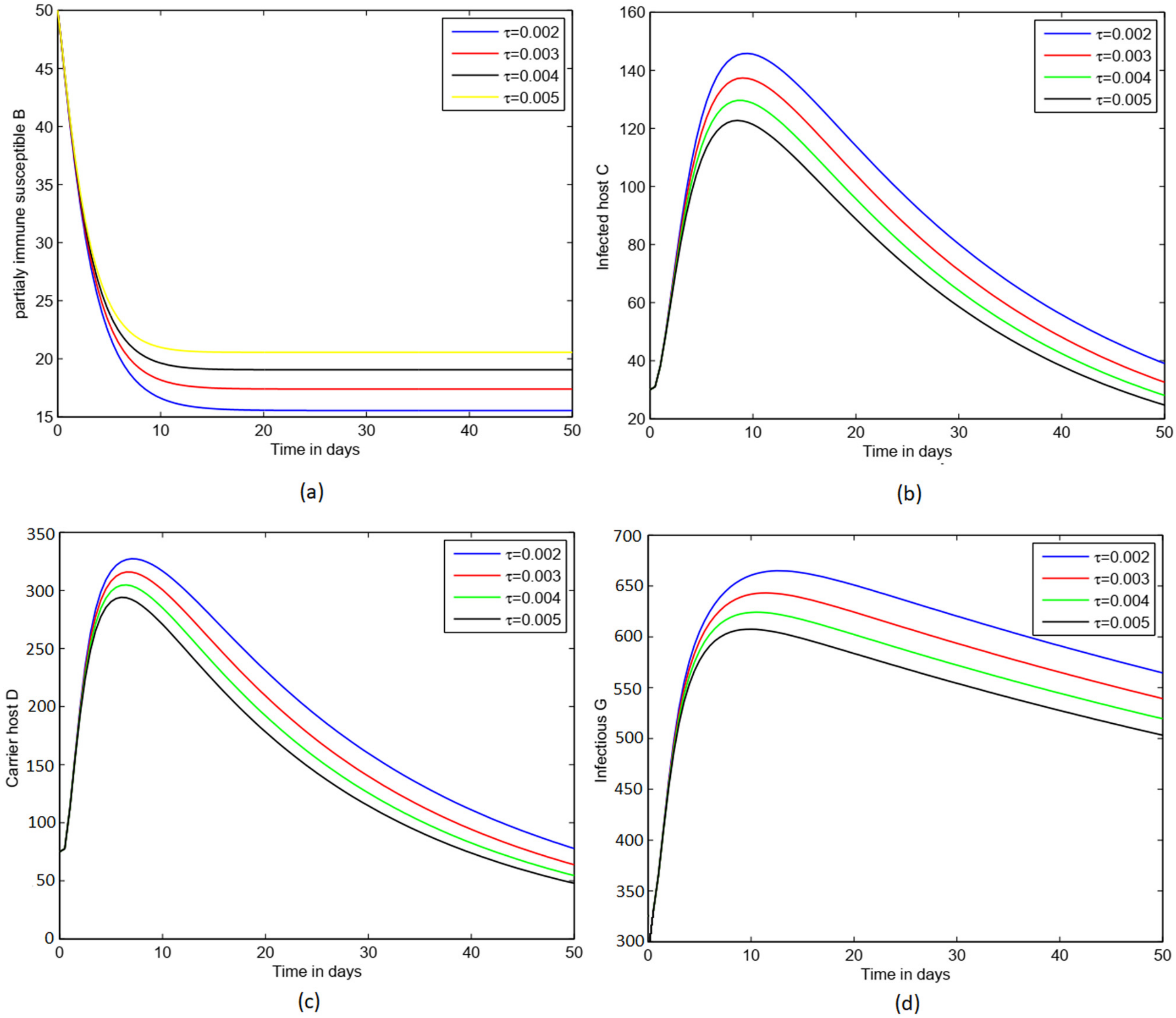

Figure 5: Illustration for depicting the interaction between host and vector populations, where (a) τ = 0.002, (b) 0.003, (c) 0.004, and (d) 0.005.

derivatives and diverse computational methodologies opens up new possibilities for improving management strategies and treatments for dengue fever. Furthermore, the research underscores the significance of collaborative endeavors in addressing complex infectious diseases and establishes a solid groundwork for future investigations into dengue fever.

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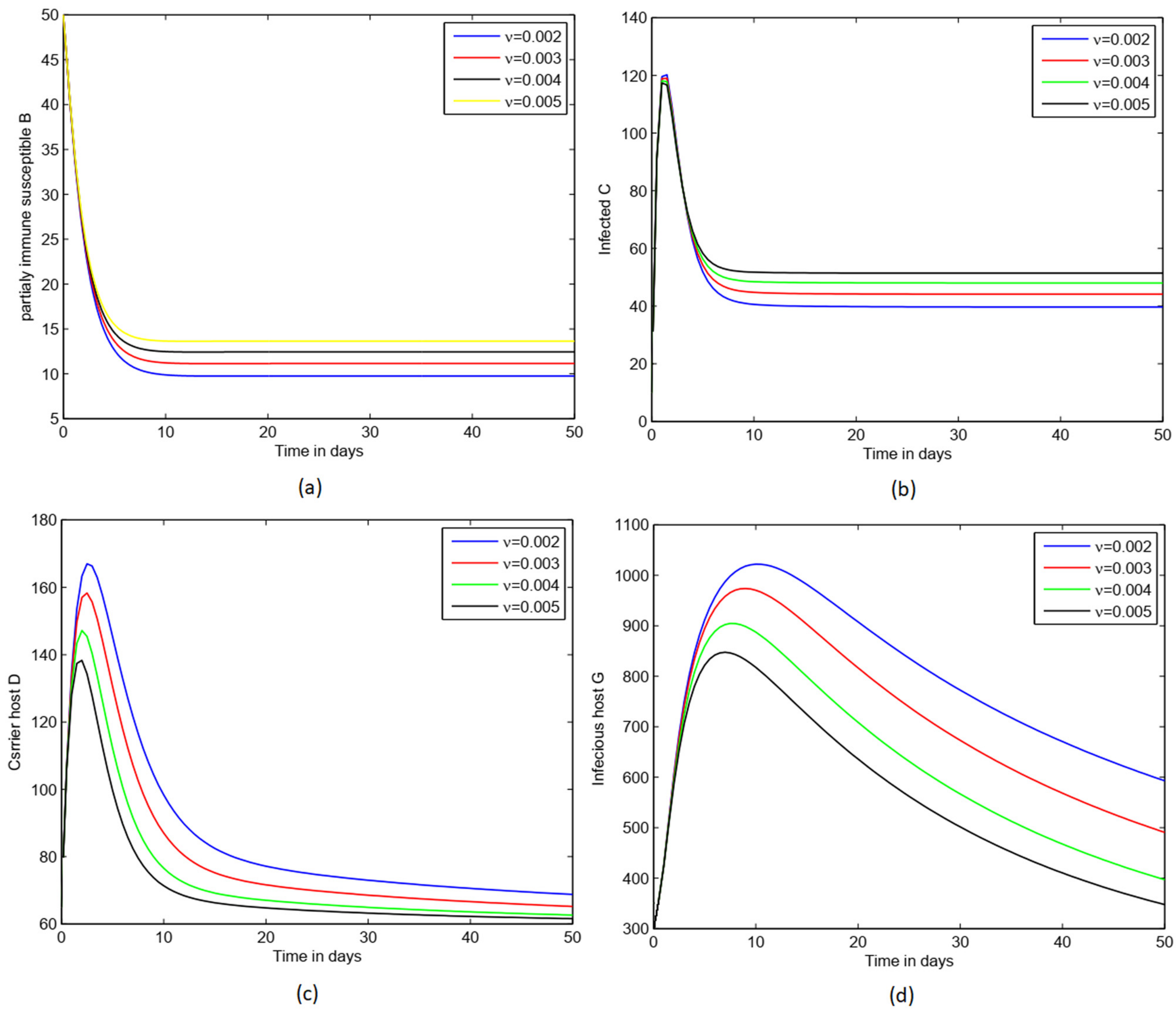

Figure 6: Illustration for depicting the interaction between host and vector populations, where (a) ν = 0.002, (b) 0.003, (c) 0.004, and (d) 0.005.

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