Research Article

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Dynamical and physical characteristics of soliton solutions to the (2+1)-dimensional Konopelchenko–Dubrovsky system

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Abstract: Soliton solutions of the Konopelchenko–Dubrovsky (KD) equation using four analytical methods are established. The KD system is used to study the portrays in physics with weak dispersion. The investigated results are obtained in different forms such as trigonometric, hyperbolic, and exponential functions. For the physical behavior of the concerned nonlinear system, some solutions are plotted graphically via assigning the certain values to the parameters. Mathematica software 11.11 is used to handle all results as well as figures. Hence, searched results have rewarding recompenses in nonlinear science.

Keywords: (2+1)-dimensional Konopelchenko–Dubrovsky system, analytical solutions

1 Introduction

In the past decennium, with the enhancement of emergent mathematical formulations and approximations, researchers and scientists are eternally endeavoring to establish incipient solutions of nonlinear evolution equations (NLEEs) using various schemes [1–6]. The applications of these equations are wide and prodigious [7–11]. These approaches are instigated directly or indirectly in applied physics and mathematics, nonlinear optics, traffic flow, and many more [12–17]. Sequentially, the solution to NLEEs is seeking a great deal of contemplation in the research municipal nowadays. The analytical solutions of NLEEs are momentous in solving mathematical and physical models [18–23]. A large number of researchers and mathematicians have enveloped numerous effective methods for nonliear partial differential equations (NLPDEs), the tanh function method [24], Hirota's bilinear method [25,26] the Jacobi elliptic function expansion method [27], the Kudryashov method [28], the (G'/G)-expansion method [29], the Darboux transformation method [30], the Backlund transformation method [31], the inverse scattering method [32], Lie symmetry analysis [33], the general exponential rational function method [34–37], and much more [38–43].

Let Konopelchenko–Dubrovsky (KD) system as [44]:

$$\begin{aligned} &U_y = V_x, \\ &U_t - U_{xxx} - 6\beta UU_x + \frac{3}{2}(\alpha^2 U^2 U_x) - 3V_y + 3\alpha U_x V = 0. \end{aligned} \tag{1}$$

Several researchers have used specific fruitful approaches to explore the wave solutions in Eq. (1). Shah et al. [45] used one dimensional fuzzy fractional partial differential equations. Kumar and Tiwari [46] acquired exact solutions of the KD system by applying the similarity transformation techniques with arbitrary choice of functions. The bifurcation theory approach is proficiently used by Tian-lan He [47] in 2008 to investigate the bounded traveling wave solutions of the (2+1)-dimensional KD system. In 2019, Rizvi et al. [48] used the modified simplest equation method and B-spline method to the KD equation. Recently, Younas et al. [49] presented the modified auxiliary equation method to this system to catch traveling wave solutions. Ren et al. [50] in 2016 acquired the non-local symmetries for the KD equation with the truncated Painleve method and the Mobius conformal invariant forms. Seadawy et al. [51] in 2019 derived wave solutions via modified extended direct algebraic scheme. Song et al. [52] attained the exact solutions of the KD system using the extended Riccati equation rational expansion schemes. But we have established soliton solutions of Eq. (1) by applications of four mathematical methods, namely, the extended simple equation method [53], the modified extended auxiliary mapping method [54], the (G'/G)-expansion method [55], and the $\mathrm{Exp}(-\Psi(\xi))$-expansion method [56].

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The arrangement of this work is given as follows: in Section 2, the proposed mathematical methods are explained. In Section 3, wave solutions of Eq. (1) are constructed. In Section 4, conclusion of the work is mentioned.

2 Proposed methods

Let nonlinear PDEs including three variables as:

$$M_1(U, V, U_x, V_x, U_y, V_y, U_t, V_t, \ldots) = 0. \tag{2}$$

Let

$$U = U(\xi), \quad V = V(\xi), \quad \text{and} \quad \xi = k_1x + k_2y - k_3t. \tag{3}$$

Substituting (3) into (2),

$$M_2(U, V, k_1U', k_1V', k_2U', k_2V', -k_3U', -k_3V', \ldots) = 0. \tag{4}$$

2.1 Extended simple equation method

Let Eq. (4) has the solution such as,

$$U(\xi) = \sum_{i=-N}^{N} A_i\Psi^i(\xi). \tag{5}$$

Let Ψ satisfy

$$\Psi' = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3. \tag{6}$$

Put Eq. (5) with Eq. (6) in Eq. (4). Solve the achieved system for the required solution of Eq. (2).

2.2 Modified extended auxiliary equation mapping method

Let solution of Eq. (4) be

$$U = \sum_{i=0}^{N} A_i\Psi^i + \sum_{i=-1}^{-N} B_{-i}\Psi^i + \sum_{i=2}^{N} C_i\Psi^{i-2}\Psi' + \sum_{i=1}^{N} D_i\left(\frac{\Psi'}{\Psi}\right)^i. \tag{7}$$

Let Ψ satisfy

$$\Psi' = \sqrt{\beta_1\Psi^2 + \beta_2\Psi^3 + \beta_3\Psi^4}. \tag{8}$$

Put Eq. (7) with Eq. (8) in Eq. (4), and solve the obtained system for the required destination of Eq. (2).

2.3 (G'/G)-expansion method

Let Eq. (4) has the solution:

$$U = A_0 + \sum_{i=1}^{N} A_i\left(\frac{G'}{G}\right). \tag{9}$$

Let

$$G'' = -\lambda G' - \mu G. \tag{10}$$

Put Eq. (9) with Eq. (10) in Eq. (4), and solve the obtained system for the required destination of Eq. (2).

2.4 $\text{Exp}(\Psi(\phi))$-expansion method

Suppose Eq. (4) has the solution:

$$U = A_N(Ex(-\Psi(-\phi)))^N + \ldots. \tag{11}$$

Let

$$\Psi' = \exp(-\Psi(\phi)) + \mu\exp(\Psi(\phi)) + \lambda. \tag{12}$$

Put Eq. (11) with Eq. (12) in Eq. (4), and solve the obtained system for the required destination of Eq. (2).

3 Applications

3.1 Application of the extended simple equation method

Putting Eq. (3) into Eq. (1), we have

$$\begin{aligned} k_2U'(\xi) &= k_1V'(\xi) - 6k_1\beta UU' - k_1^2U^3 - k_3U' \\ &\quad + \frac{3}{2}\alpha^2k_1U^2U' - 3k_2V' + 3\alpha k_1VV' = 0. \end{aligned} \tag{13}$$

Integrating the first equation,

$$k_2U(\xi) = k_1V(\xi). \tag{14}$$

Substituting Eq. (14) into second Eq. (14),

$$-\frac{1}{2}\alpha^2k_1U^3 + \frac{3}{2}U^2(2\beta k_1 - \alpha k_2) + k_1^3U'' + \left(\frac{3k_2^2}{k_1} + k_3\right)U = 0. \tag{15}$$

Let Eq. (15) has solution:

$$U = A_1\Psi + \frac{A_{-1}}{\Psi} + A_0. \tag{16}$$

Put Eq. (16) with Eq. (6) in Eq. (15).

CASE 1: $c_3 = 0$ (Figure 1)

FAMILY-I

$$A_{-1} = 0, \quad A_1 = \frac{2c_2k_1}{\alpha}, \quad A_0 = \frac{c_1k_1}{\alpha}, k_2 = \frac{2\beta k_1}{\alpha}, \quad \text{and} \quad k_3 = \frac{\alpha^2 c_1^2 k_1^3 - 4\alpha^2 c_0 c_2 k_1^3 - 24\beta^2 k_1}{2\alpha^2}. \tag{17}$$

Put (17) in (16),

$$U_1 = \frac{c_1k_1}{\alpha} - \left(\frac{(2c_2k_1)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{\alpha(2c_2)}\right), \; 4c_0c_2 > c_1^2. \tag{18}$$

From Eq. (14), we have

$$V_1 = \frac{k_2}{k_1}\left(\frac{c_1k_1}{\alpha} - \left(\frac{(2c_2k_1)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{\alpha(2c_2)}\right)\right), \; 4c_0c_2 > c_1^2. \tag{19}$$

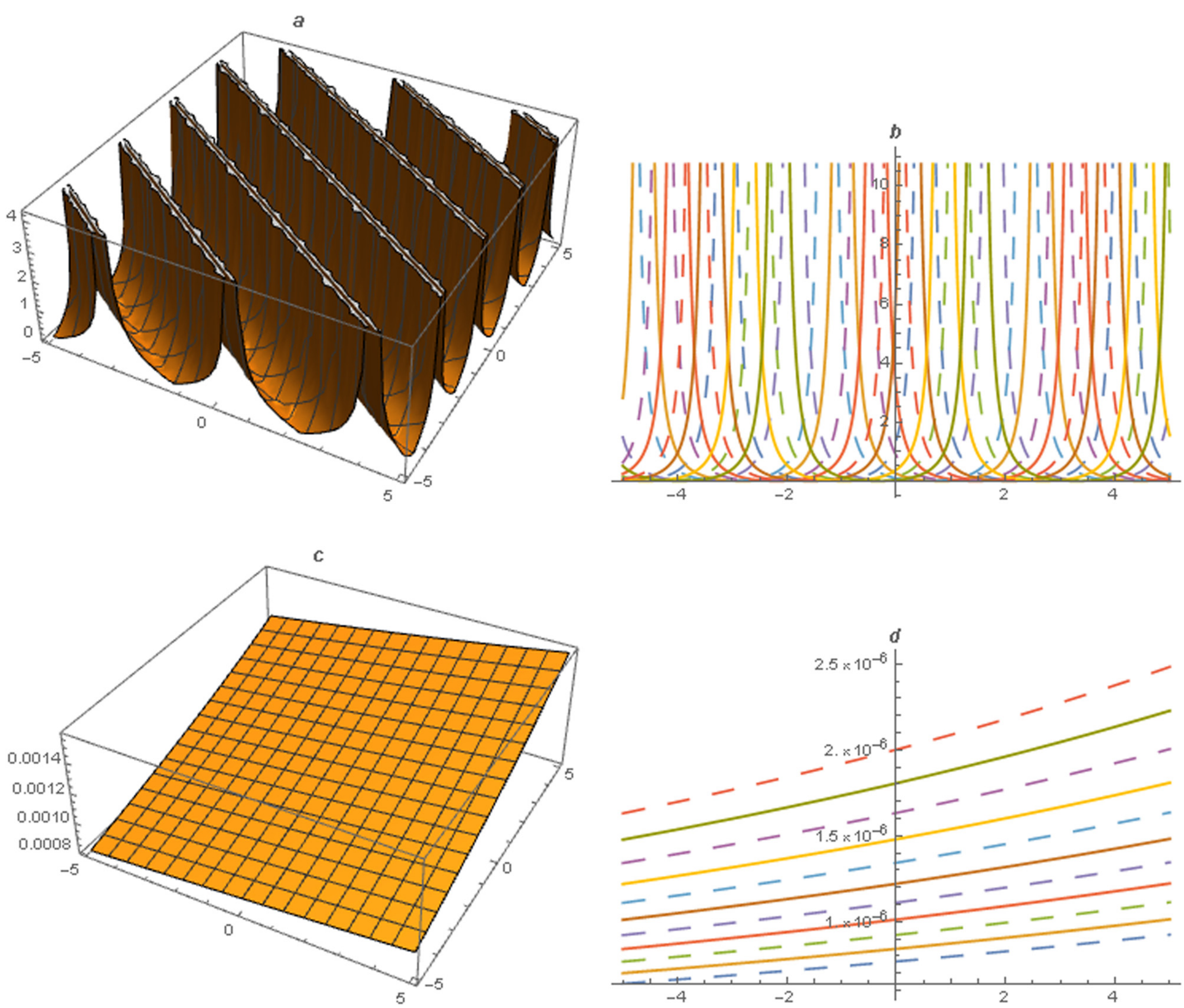

Figure 1: Solutions U_1 (a and b) and V_1 (c and d) with $\alpha = 3.01$, $\beta = 0.1$, $c_0 = 3$, $c_1 = 0.2$, $c_2 = 1$, $\xi_0 = 1$, $k_1 = 0.5$, and $y = 1$, and $\alpha = 20.01$, $\beta = 9.1$, $c_0 = c_1 = 1.2$, $c_2 = 1$, $\xi_0 = 1$, $k_1 = 0.01$, and $y = 1$, respectively.

FAMILY-II

$$A_{-1} = -\frac{2c_0k_1}{\alpha},\ A_1 = 0, A_0 = -\frac{c_1k_1}{\alpha},\ k_2 = \frac{2\beta k_1}{\alpha},\ \text{and}\quad k_3 = \frac{\alpha^2c_1^2k_1^3 - 4\alpha^2c_0c_2k_1^3 - 24\beta^2k_1}{2\alpha^2}. \tag{20}$$

Substituting (20) into (16),

$$U_2 = \left(\frac{2c_0k_1}{\frac{\alpha\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{2c_2}} - \frac{c_1k_1}{\alpha}\right),\ 4c_0c_2 > c_1^2, \tag{21}$$

$$V_2 = \left(\frac{k_2\left(\frac{2c_0k_1}{\frac{\alpha\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{2c_2}} - \frac{c_1k_1}{\alpha}\right)}{k_1}\right),\ 4c_0c_2 > c_1^2. \tag{22}$$

FAMILY-III

$$A_{-1} = -\frac{2c_0k_1}{\alpha},\ A_1 = -\frac{2c_2k_1}{\alpha},\ A_0 = -\frac{2c_1k_1}{\alpha},\quad k_2 = \frac{k_1(2\beta + \alpha c_1k_1)}{\alpha},\quad k_3 = \frac{4k_1(3\beta^2 + \alpha k_1(3\beta c_1 + \alpha(c_1^2 - c_0c_2)k_1))}{\alpha^2} \tag{23}$$

Substituting (23) into (16),

$$U_3 = \left(\frac{(2c_2k_1)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{\alpha(2c_2)}\right) + \left(\frac{2c_0k_1}{\frac{\alpha\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{2c_2}}\right) - \left(\frac{2c_1k_1}{\alpha}\right), 4c_0c_2 > c_1^2. \tag{24}$$

$$V_3 = \left(\frac{k_2\left(\frac{(2c_2k_1)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{\alpha(2c_2)} + \frac{2c_0k_1}{\frac{\alpha\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{2c_2}} - \frac{2c_1k_1}{\alpha}\right)}{k_1}\right), 4c_0c_2 > c_1^2. \tag{25}$$

CASE 2: $c_0 = c_3 = 0$ (Figure 2)

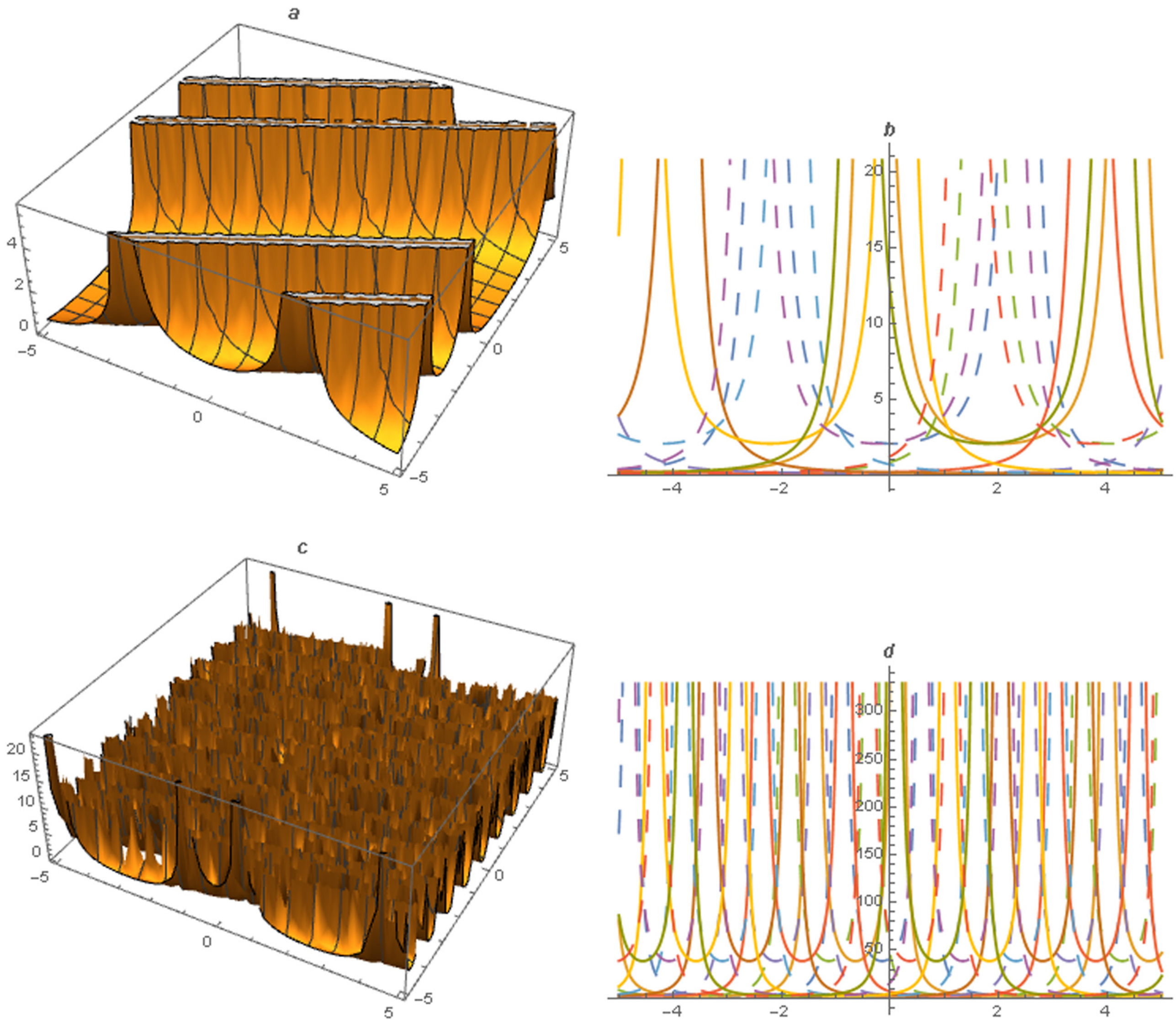

Figure 2: Solutions U_3 (a and b) and V_3 (c and d) with $\alpha = 1.1$, $\beta = 0.3$, $c_0 = 2$, $c_1 = 3$, $c_2 = 3$, $\xi_0 = 7$, $k_1 = 0.1$, and $y = 1$, and $\alpha = 1.1$, $\beta = 0.3$, $c_0 = 2$, $c_1 = 3$, $c_2 = 3$, $\xi_0 = 0.7$, $k_1 = 0.3$, and $y = 1$, respectively.

$$A_{-1} = 0,\ A_1 = \frac{2c_2k_1}{\alpha},\ A_0 = \frac{2c_1k_1}{\alpha},\ k_2 = \frac{2\beta k_1 - \alpha c_1 k_1^2}{\alpha},$$
$$k_3 = -\frac{4(\alpha^2 c_1^2 k_1^3 - 3\alpha\beta c_1 k_1^2 + 3\beta^2 k_1)}{\alpha^2}. \tag{26}$$

Put (26) in (16),

$$U_4 = \left(\frac{(2c_2k_1)(c_1\exp(c_1(\xi + \xi_0)))}{\alpha(1 - c_2\exp(c_1(\xi + \xi_0)))} + \frac{2c_1k_1}{\alpha}\right),\ c_1 > 0, \tag{27}$$

$$V_4 = \left(\frac{k_2\left(\frac{(2c_2k_1)(c_1\exp(c_1(\xi + \xi_0)))}{\alpha(1 - c_2\exp(c_1(\xi + \xi_0)))} + \frac{2c_1k_1}{\alpha}\right)}{k_1}\right),\ c_1 > 0, \tag{28}$$

$$U_5 = \left(\frac{(2c_2k_1)(-c_1\exp(c_1(\xi + \xi_0)))}{\alpha(c_2\exp(c_1(\xi + \xi_0)) + 1)} + \frac{2c_1k_1}{\alpha}\right),\ c_1 < 0, \tag{29}$$

$$V_5 = \left(\frac{k_2\left(\frac{(2c_2k_1)(-c_1\exp(c_1(\xi + \xi_0)))}{\alpha(c_2\exp(c_1(\xi + \xi_0)) + 1)} + \frac{2c_1k_1}{\alpha}\right)}{k_1}\right),\ c_1 < 0. \tag{30}$$

CASE 3: $c_1 = 0$, $c_3 = 0$,

FAMILY-I

$$A_{-1} = 0,\ A_1 = -\frac{2c_2k_1}{\alpha},\ A_0 = 0,\ k_2 = \frac{2\beta k_1}{\alpha},$$
$$k_3 = -\frac{2(\alpha^2 c_0 c_2 k_1^3 + 6\beta^2 k_1)}{\alpha^2}. \tag{31}$$

Put (31) in (16),

$$U_6 = \left(-\frac{(2c_2k_1)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right),\ c_0c_2>0, \quad (32)$$

$$V_6 = \frac{k_2}{k_1}\left(-\frac{(2c_2k_1)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right),\ c_0c_2>0, \quad (33)$$

$$U_7 = \left(\frac{(2c_2k_1)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right),\ c_0c_2<0, \quad (34)$$

$$V_7 = \frac{k_2}{k_1}\left(\frac{(2c_2k_1)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right),\ c_0c_2<0. \quad (35)$$

FAMILY-II

$$A_{-1} = -\frac{2c_0k_1}{\alpha},\ A_1=0,\ A_0=0,\ k_2=\frac{2\beta k_1}{\alpha},\ \text{and}\quad k_3 = -\frac{2(\alpha^2c_0c_2k_1^3+6\beta^2k_1)}{\alpha^2}. \quad (36)$$

Put (36) in (16),

$$U_8 = \left(-\frac{2c_0k_1}{\frac{\alpha(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{c_2}}\right),\ c_0c_2>0, \quad (37)$$

$$V_8 = \frac{k_2}{k_1}\left(-\frac{2c_0k_1}{\frac{\alpha(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{c_2}}\right),\ c_0c_2>0, \quad (38)$$

$$U_9 = \left(-\frac{2c_0k_1}{\frac{\alpha(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2}}\right),\ c_0c_2<0, \quad (39)$$

$$V_9 = \frac{k_2}{k_1}\left(-\frac{2c_0k_1}{\frac{\alpha(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2}}\right),\ c_0c_2<0. \quad (40)$$

FAMILY-III

$$A_{-1} = \frac{2c_0k_1}{\alpha}, A_1 = \frac{2c_2k_1}{\alpha}, A_0 = \frac{2\sqrt{2}\sqrt{c_0}\sqrt{c_2}k_1}{\alpha},\quad k_2 = -\frac{2(\sqrt{2}\alpha\sqrt{c_0}\sqrt{c_2}k_1^2-\beta k_1)}{\alpha},\quad k_3 = -\frac{4(8\alpha^2c_0c_2k_1^3-6\sqrt{2}\alpha\beta\sqrt{c_0}\sqrt{c_2}k_1^2+3\beta^2k_1)}{\alpha^2}. \quad (41)$$

Put (41) in (16) (Figure 3)

$$U_{10} = \left(\frac{(2c_2k_1)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right) + \left(\frac{2c_0k_1}{\frac{\alpha(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{c_2}}\right) + \left(\frac{2\sqrt{2}\sqrt{c_0}\sqrt{c_2}k_1}{\alpha}\right),\ c_0c_2>0, \quad (42)$$

$$V_{10} = \frac{k_2}{k_1}\left(\frac{(2c_2k_1)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right) + \frac{k_2}{k_1}\left(\frac{2c_0k_1}{\frac{\alpha(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{c_2}}\right) + \frac{k_2}{k_1}\left(\frac{2\sqrt{2}\sqrt{c_0}\sqrt{c_2}k_1}{\alpha}\right),\ c_0c_2>0, \quad (43)$$

$$U_{11} = \left(\frac{(2c_2k_1)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right) + \left(\frac{2c_0k_1}{\frac{\alpha(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2}}\right) + \left(\frac{2\sqrt{2}\sqrt{c_0}\sqrt{c_2}k_1}{\alpha}\right),\ c_0c_2<0, \quad (44)$$

$$V_{11} = \frac{k_2}{k_1}\left(\frac{(2c_2k_1)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{\alpha c_2}\right) + \frac{k_2}{k_1}\left(\frac{2c_0k_1}{\frac{\alpha(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2}}\right) + \frac{k_2}{k_1}\left(\frac{2\sqrt{2}\sqrt{c_0}\sqrt{c_2}k_1}{\alpha}\right),\ c_0c_2<0. \quad (45)$$

3.2 Application of the modified extended auxiliary equation mapping method

Let Eq. (15) has the following solution:

$$U = A_1\Psi + A_0 + \frac{B_1}{\Psi} + D_1\left(\frac{\Psi'}{\Psi}\right). \quad (46)$$

Put (46) with (8) in (16),

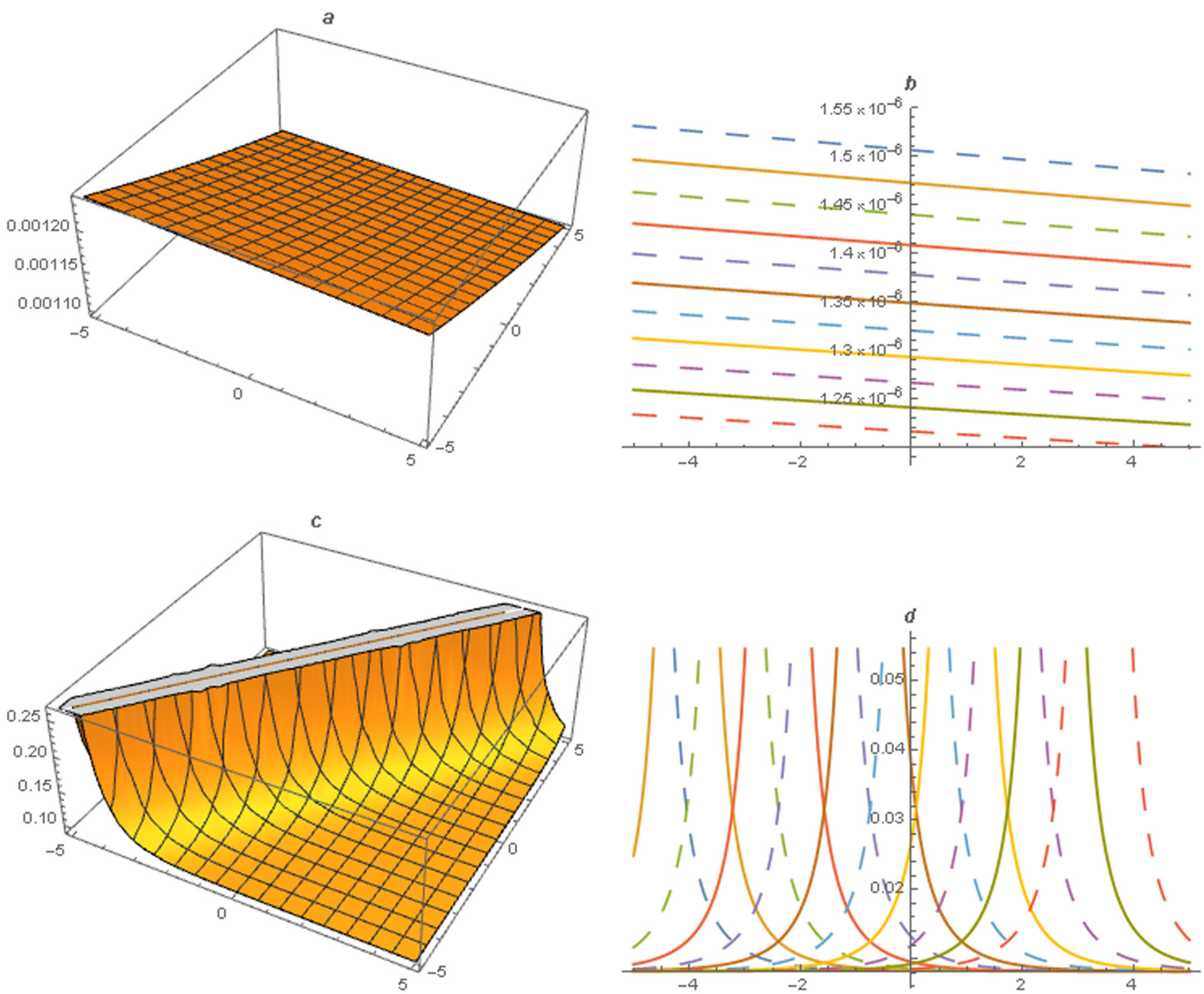

Figure 3: Solutions U_9 (a and b) and V_9 (c and d) with $\alpha = 3.01$, , $c_0 = 1.3$, $c_2 = 0.3$, $\xi_0 = 0.6$, $k_1 = 0.001$, and $y = 1$, and $\alpha = 3.01$, $\beta = 0.3$, $c_0 = 0.3$, $c_2 = 1.3$, $\xi_0 = 0.6$, $k_1 = 1.1$, and $y = 1$, respectively.

$$A_0 = 0,\ A_1 = -\frac{\sqrt{\beta_3}k_1}{\alpha},\ D_1 = \frac{k_1}{\alpha},\ B_1 = 0,\ k_3 = \frac{\alpha^2\beta_1k_1^3 - 24\beta^2k_1}{2\alpha^2},\ \text{and}\ k_2 = \frac{2\beta k_1}{\alpha}. \tag{47}$$

Put Eq. (47) in Eq. (46), (Figure 4)

CASE I:

$$U_{12} = \left(\frac{k_1\left(\beta_1^{3/2}\varepsilon\,\text{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right)\right)}{\frac{\alpha\left((2\beta_2)\left(-\beta_1\left(\varepsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)\right)}{\beta_2}}\right) - \left(\frac{(\sqrt{\beta_3}k_1)\left(-\beta_1\left(\varepsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right) + 1\right)\right)}{\alpha\beta_2}\right),\ \beta_1 > 0,\ \beta_2^2 - 4\beta_1\beta_3 = 0. \tag{48}$$

$$V_{12} = \frac{k_2}{k_1}\left(\left(\left(\frac{k_1\left(\beta_1^{3/2}\varepsilon\,\text{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right)\right)}{\frac{\alpha\left((2\beta_2)\left(-\beta_1\left(\varepsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)\right)}{\beta_2}}\right) - \left(\frac{(\sqrt{\beta_3}k_1)\left(-\beta_1\left(\varepsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right) + 1\right)\right)}{\alpha\beta_2}\right)\right)\right),\ \beta_1 > 0,\ \beta_2^2 - 4\beta_1\beta_3 = 0. \tag{49}$$

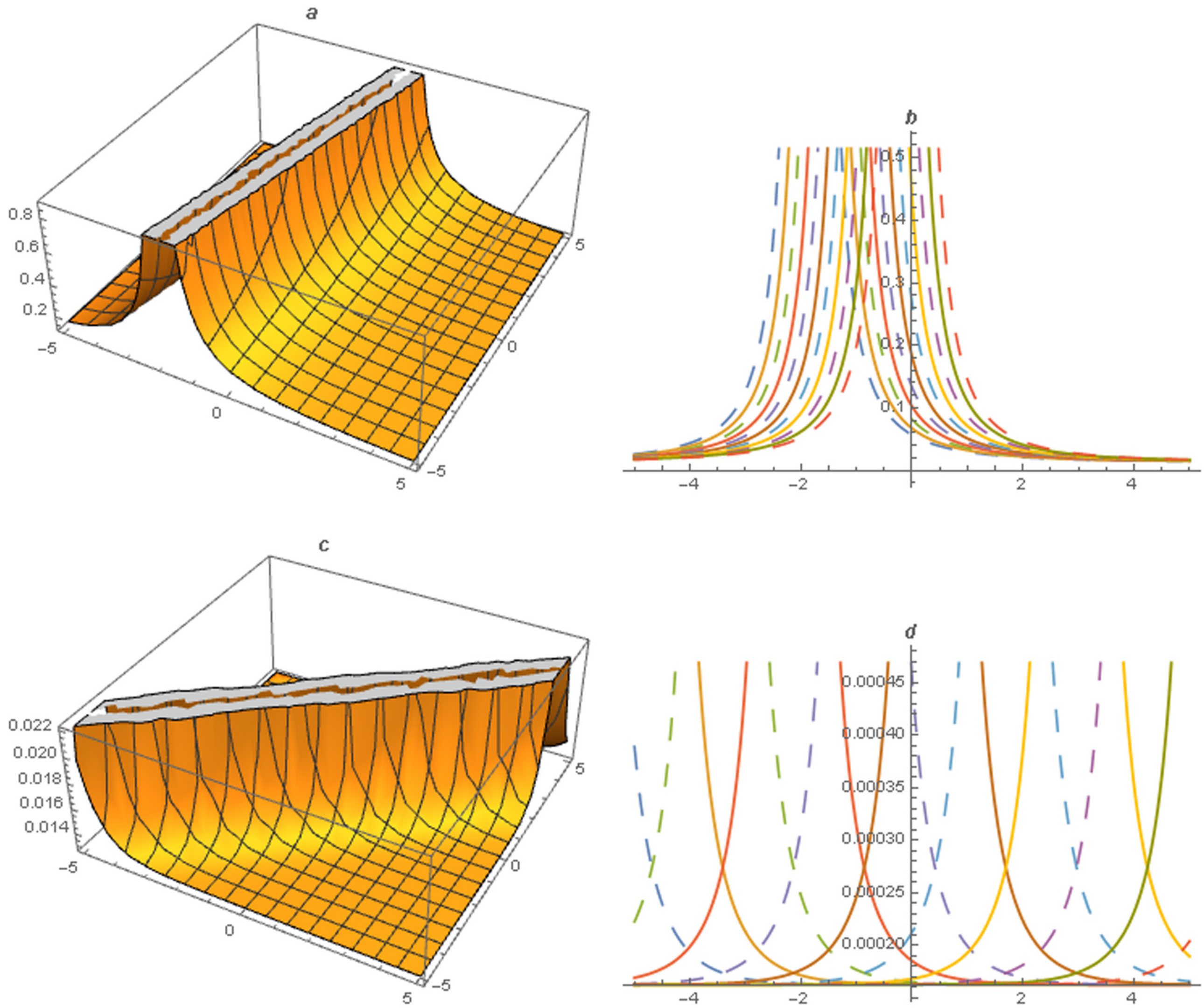

Figure 4: Solutions U_{12} (a and b) and V_{12} (c and d) with $\alpha = 5.01$, $\beta_1 = 1$, $\beta_2 = 4$, $\beta_3 = 4$, $\beta = 0.1$, $\xi_0 = 0.6$, $k_1 = 0.6$, $y = 1$, and $\varepsilon = 1$, and $\alpha = 5.01$, $\beta_1 = 1$, $\beta_2 = 4$, $\beta_3 = 4$, $\beta = 0.1$, $\xi_0 = 0.6$, $k_1 = 1.6$, $y = 1$, and $\varepsilon = 1$, respectively.

CASE II:

$$U_{13} = \left(\frac{k_1\left(\sqrt{\frac{\beta_1}{\beta_3}}\left(-\left(\frac{\sqrt{\beta_1}\varepsilon\cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta} - \frac{\sqrt{\beta_1}\varepsilon\sinh^2(\sqrt{\beta_1}(\xi+\xi_0))}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta)^2}\right)\right)\right)}{\alpha\left(2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\varepsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)\right)}\right) - \left(\frac{(\sqrt{\beta_3}k_1)\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\varepsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)}{\alpha}\right), \beta_1 > 0,\ \beta_3 > 0,\ \beta_2 = (4\beta_1\beta_3)^{1/2}. \tag{50}$$

$$V_{13} = \frac{k_2}{k_1}\left(\frac{k_1\left(\sqrt{\frac{\beta_1}{\beta_3}}\left(-\left(\frac{\sqrt{\beta_1}\varepsilon\cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta} - \frac{\sqrt{\beta_1}\varepsilon\sinh^2(\sqrt{\beta_1}(\xi+\xi_0))}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta)^2}\right)\right)\right)}{\alpha\left(2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\varepsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)\right)}\right) - \left(\frac{k_2}{k_1}\right) \times \left(\frac{(\sqrt{\beta_3}k_1)\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\varepsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)}{\alpha}\right), \beta_1 > 0,\ \beta_3 > 0,\ \beta_2 = (4\beta_1\beta_3)^{1/2}. \tag{51}$$

CASE III:

$$U_{14} = \left(\frac{k_1\left(-\left(\beta_1\left(\frac{\sqrt{\beta_1}\varepsilon\cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\varepsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P)}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1})^2}\right)\right)\right)}{\frac{\alpha\left(\beta_2\left(-\beta_1\left(\frac{\varepsilon(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1}}+1\right)\right)\right)}{\beta_2}}\right) - \left(\frac{(\sqrt{\beta_3}k_1)\left(-\beta_1\left(\frac{\varepsilon(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1}}+1\right)\right)}{\alpha\beta_2}\right), \beta_1 > 0. \tag{52}$$

$$V_{14} = \frac{k_2}{k_1}\left(\frac{k_1\left(-\left(\beta_1\left(\frac{\sqrt{\beta_1}\varepsilon\cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\varepsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P)}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1})^2}\right)\right)\right)}{\frac{\alpha\left(\beta_2\left(-\beta_1\left(\frac{\varepsilon(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1}}+1\right)\right)\right)}{\beta_2}}\right) - \frac{k_2}{k_1}\left(\frac{(\sqrt{\beta_3}k_1)\left(-\beta_1\left(\frac{\varepsilon(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{P^2+1}}+1\right)\right)}{\alpha\beta_2}\right), \beta_1 > 0. \tag{53}$$

3.3 Application of (G'/G)-expansion method

Let Eq. (15) has the following solution,

$$U = A_0 + A_1\left(\frac{G'}{G}\right). \tag{54}$$

Put (54) with (10) in (16),

$$A_0 = -\frac{\lambda k_1}{\alpha},\ A_1 = -\frac{2k_1}{\alpha},\ k_2 = \frac{2\beta k_1}{\alpha},\ \text{and}\quad k_3 = \frac{\alpha^2\lambda^2k_1^3 - 4\alpha^2k_1^3\mu - 24\beta^2k_1}{2\alpha^2}. \tag{55}$$

Put (55) in (54).
CASE I: $\lambda^2 - 4\mu > 0$

$$U_{15} = -\left(\frac{\lambda k_1}{\alpha}\right) - \left(\frac{(2k_1)\left(\frac{\xi P_1 \sinh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right) + \xi P_2 \cosh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)}{\xi P_2 \sinh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right) + \xi P_1 \cosh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)} - \frac{\lambda}{2}\right)}{\alpha}\right), \tag{56}$$

$$V_{15} = \frac{k_2}{k_1}\left(-\left(\frac{\lambda k_1}{\alpha}\right) - \left(\frac{(2k_1)\left(\frac{\xi P_1 \sinh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right) + \xi P_2 \cosh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)}{\xi P_2 \sinh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right) + \xi P_1 \cosh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)} - \frac{\lambda}{2}\right)}{\alpha}\right)\right). \tag{57}$$

CASE II: $\lambda^2 - 4\mu < 0$

$$U_{16} = \left(-\frac{\lambda k_1}{\alpha} - \frac{(2k_1)\left(\frac{\xi P_2 \cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right) - \xi P_1 \sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)}{\xi P_2 \sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right) + \xi P_1 \cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)} - \frac{\lambda}{2}\right)}{\alpha}\right), \tag{58}$$

$$V_{16} = \frac{k_2}{k_1}\left(-\frac{\lambda k_1}{\alpha} - \frac{(2k_1)\left(\frac{\sqrt{4\mu - \lambda^2}\left(\xi P_2 \cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right) - \xi P_1 \sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\right)}{2\left(\xi P_2 \sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right) + \xi P_1 \cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\right)} - \frac{\lambda}{2}\right)}{\alpha}\right). \tag{59}$$

CASE III: $\lambda^2 - 4\mu = 0$ (Figure 5)

$$U_{17} = \left(\frac{(-(2k_1))\left(\frac{P_2}{\xi P_2 + P_1}\right) - \left(\frac{\lambda}{2}\right)}{\alpha}\right) - \left(\frac{\lambda k_1}{\alpha}\right), \tag{60}$$

$$V_{17} = \left(\frac{(-(2k_1))\left(\frac{P_2}{\xi P_2 + P_1}\right) - \left(\frac{\lambda}{2}\right)}{\alpha}\right) - \left(\frac{\lambda k_1}{\alpha}\right). \tag{61}$$

3.4 Application of the $\exp(\Psi(\phi))$-expansion method

Let Eq. (15) has the solution,

$$U = A_0 + A_1 \exp(-\Psi(\phi)). \tag{62}$$

Put (6) with (12) in (16),

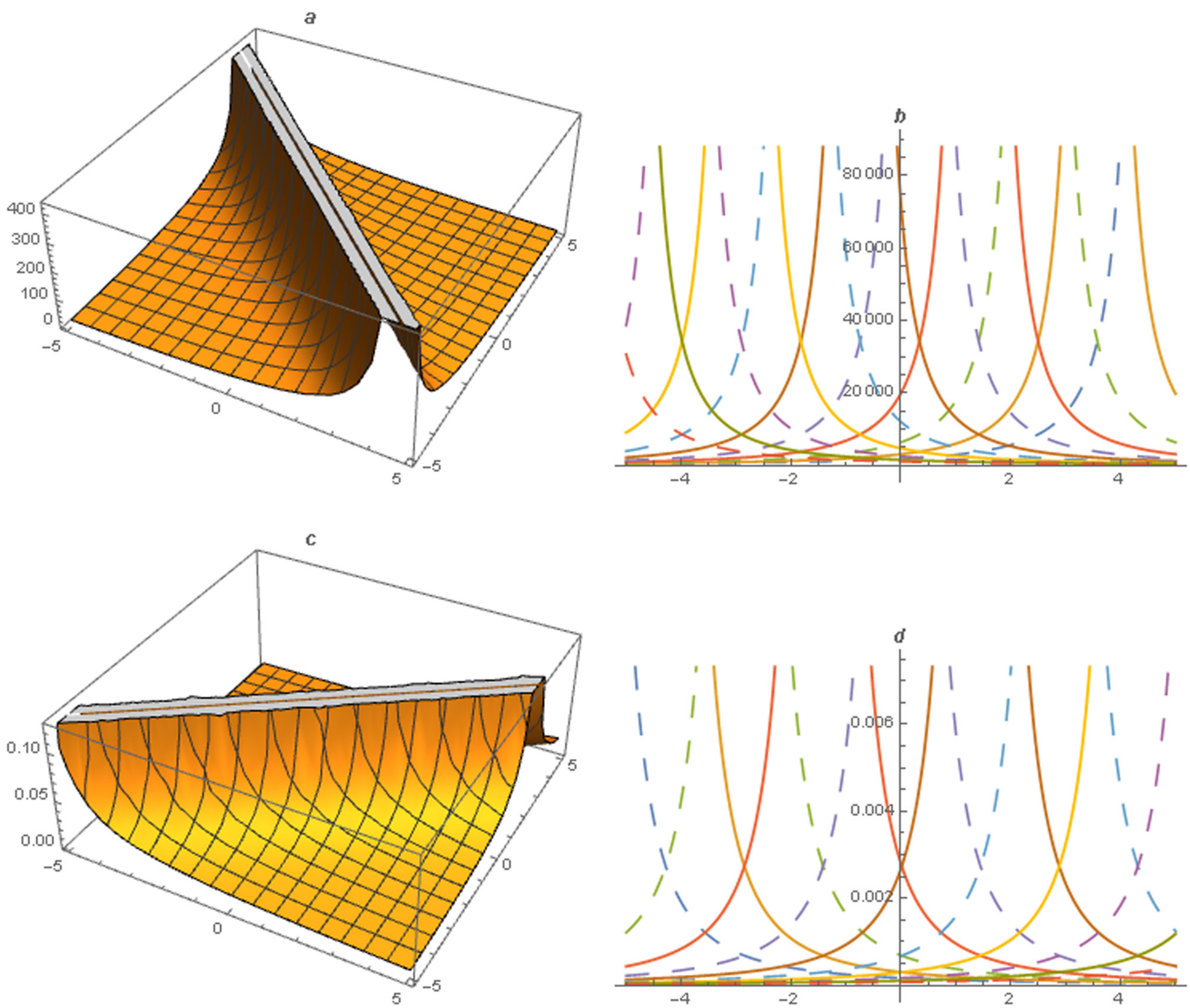

Figure 5: Solutions U_{17} (a and b) and V_{17} (c and d) with $\alpha = 0.01$, $\beta = 0.003$, $\lambda = 2$, $k_1 = 0.5$, $\mu = 1$, $P_1 = 0.05$, $P_2 = 0.7$, and $y = 1$, and $\alpha = 4$, $\beta = 0.3$, $\lambda = 4$, $k_1 = 0.5$. $\mu = 1$, $P_1 = 5$, $P_2 = 7.7$, and $y = 1$, respectively.

$$
\begin{aligned}
A_0 &= \frac{\sqrt{\alpha^2\lambda^2k_1^2 - 4\alpha^2k_1^2\mu} + \alpha\lambda k_1}{\alpha^2},\\
A_1 &= \frac{2k_1}{\alpha},\\
k_2 &= \frac{2\beta k_1 - k_1\sqrt{\alpha^2k_1^2(\lambda^2 - 4\mu)}}{\alpha},\\
k_3 &= \frac{4(3\beta k_1\sqrt{\alpha^2k_1^2(\lambda^2 - 4\mu)} - \alpha^2\lambda^2k_1^3 + 4\alpha^2k_1^3\mu - 3\beta^2k_1)}{\alpha^2}.
\end{aligned}
\tag{63}
$$

CASE I: $\lambda^2 - 4\mu > 0,\ \mu \neq 0$

$$
U_{18} = \left(\frac{\sqrt{\alpha^2\lambda^2k_1^2 - 4\alpha^2k_1^2\mu} + \alpha\lambda k_1}{\alpha^2}\right) + \left(\frac{(2k_1)\log\left(\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}(\xi + \xi_0)\sqrt{\lambda^2 - 4\mu}\right) - \lambda}{2\mu}\right)}{\alpha}\right),
\tag{64}
$$

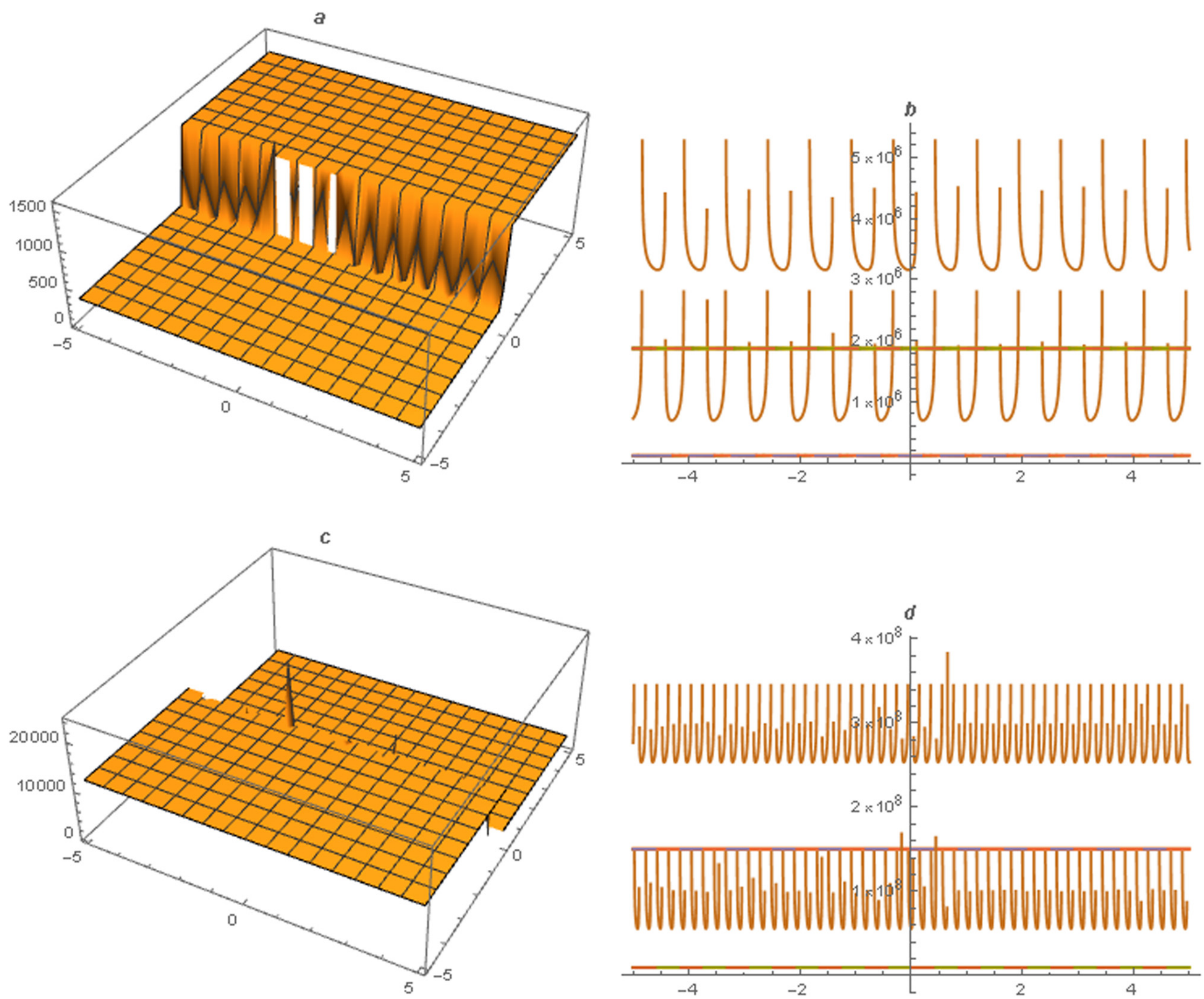

Figure 6: Solutions U_{22} (a and b) and V_{22} (c and d) with $\alpha = 0.01$, $\beta = 0.003$, $\xi_0 = 0.35$, $\lambda = 1$, $k_1 = 1.5$, $\mu = 8$, and $y = 1$, and $\alpha = 0.3$, $\beta = 0.3$, $\xi_0 = 8.35$, $\lambda = 1$, $k_1 = 5.5$, $\mu = 8$, and $y = 1$, respectively.

$$V_{18} = \frac{k_2}{k_1}\left(\left(\frac{\sqrt{\alpha^2\lambda^2k_1^2 - 4\alpha^2k_1^2\mu} + \alpha\lambda k_1}{\alpha^2}\right) + \left(\frac{(2k_1)\log\left(\frac{-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}(\xi+\xi_0)\sqrt{\lambda^2-4\mu}\right)-\lambda}{2\mu}\right)}{\alpha}\right)\right). \quad (65)$$

CASE II: $\lambda^2 - 4\mu > 0$, $\mu = 0$

$$U_{19} = \left(\frac{(2k_1)\left(-\log\left(\frac{\lambda}{\exp(\lambda(\xi+\xi_0))-1}\right)\right)}{\alpha}\right) + \left(\frac{2\lambda k_1}{\alpha}\right), \quad (66)$$

$$V_{19} = \frac{k_2}{k_1}\left(\left(\frac{(2k_1)\left(-\log\left(\frac{\lambda}{\exp(\lambda(\xi+\xi_0))-1}\right)\right)}{\alpha}\right) + \left(\frac{2\lambda k_1}{\alpha}\right)\right). \quad (67)$$

CASE III: $\lambda^2 - 4\mu = 0$, $\mu \neq 0$, $\lambda \neq 0$

$$U_{20} = \left(\frac{\sqrt{\alpha^2\lambda^2k_1^2 - 4\alpha^2k_1^2\mu} + \alpha\lambda k_1}{\alpha^2}\right) + \left(\frac{(2k_1)\log\left(\frac{2-2(\lambda(\xi+\xi_0))}{\lambda^2(\xi+\xi_0)}\right)}{\alpha}\right), \quad (68)$$

$$V_{20} = \frac{k_2}{k_1}\left(\left(\frac{\sqrt{\alpha^2\lambda^2k_1^2 - 4\alpha^2k_1^2\mu} + \alpha\lambda k_1}{\alpha^2}\right) + \left(\frac{(2k_1)\log\left(\frac{2-2(\lambda(\xi+\xi_0))}{\lambda^2(\xi+\xi_0)}\right)}{\alpha}\right)\right). \quad (69)$$

CASE IV: $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$

$$U_{21} = \left(\frac{(2k_1)\log(\xi + \xi_0)}{\alpha}\right), \quad (70)$$

$$V_{21} = \frac{k_2}{k_1}\left(\frac{(2k_1)\log(\xi + \xi_0)}{\alpha}\right). \quad (71)$$

CASE V: $\lambda^2 - 4\mu < 0$ (Figure 6)

$$U_{22} = \left(\frac{\sqrt{\alpha^2\lambda^2k_1^2 - 4\alpha^2k_1^2\mu} + \alpha\lambda k_1}{\alpha^2}\right) + \left(\frac{(2k_1)\log\left(\frac{-\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}(\xi+\xi_0)\sqrt{4\mu-\lambda^2}\right)-\lambda}{2\mu}\right)}{\alpha}\right), \quad (72)$$

$$V_{22} = \frac{k_2}{k_1}\left(\left(\frac{2(\alpha\delta\sqrt{\lambda^2 - 4\mu} + \alpha\delta\lambda)}{4\delta^2}\right. + \left.\frac{\alpha\log\left(\frac{-\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}(\xi+\xi_0)\sqrt{4\mu-\lambda^2}\right)-\lambda}{2\mu}\right)}{\delta}\right)\right). \quad (73)$$

4 Conclusion

In this article, we have constructed several diverse solutions of (2+1)-dimensional KD system by using the four analytical mathematical methods. Some solutions are plotted graphically in 2-dimensional and 3-dimensional by imparting the particular value to the parameters under the constrained condition on each disquiet solution. Hence, it shows that our proposed mathematical methods are powerful, melodious, and capable of using in supplementary works to originate novel results for NPDEs.

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