Research Article

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Optimal auxiliary function method for analyzing nonlinear system of coupled Schrödinger–KdV equation with Caputo operator

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Abstract: The optimal auxiliary function method (OAFM) is introduced and used in the analysis of a nonlinear system containing coupled Schrödinger–KdV equations, all within the framework of the Caputo operator. The OAFM, known for its efficiency in solving nonlinear issues, is used to obtain approximate solutions for the coupled equations' complicated dynamics. Numerical and graphical assessments prove the suggested method's correctness and efficiency. This study contributes to the understanding and analysis of coupled Schrödinger–KdV equations and their many applications by providing insights into the behavior of nonlinear systems within mathematical physics.

Keywords: optimal auxiliary function method, nonlinear system of Schrödinger–KdV equation, Caputo operator, fractional calculus

1 Introduction

Fractional partial differential equations (FPDEs) have arisen as a strong mathematical framework for explaining complex systems that show anomalous diffusion, memory effects, and nonlocal interactions. These equations incorporate fractional-order derivatives derived within the context of fractional calculus and so generalize traditional partial differential equations [1–5]. Due to their capacity to simulate complex dynamics that conventional integer-order derivatives cannot fully represent, FPDEs are used in many scientific fields, including biology, engineering, and finance [6–8]. It is possible to trace the theoretical roots of fractional calculus to the pioneering work of mathematicians like Riemann, Liouville, and Caputo [9–11]. However, the promise of FPDEs to provide more precise and realistic representations of diverse natural and artificial systems has only recently attracted substantial attention [12–14]. This introduction's goal is to give a general understanding of fractional calculus's core ideas and how they apply to FPDEs while also identifying important sources that have shaped this field's development [14–18].

The study of nonlinear systems has revealed complex dynamics crucial to several scientific fields. A notable area of study among them is the coupling of the Schrödinger and Korteweg–de Vries (KdV) equations. These equations are important tools for understanding a wide range of physical events because they capture key aspects of wave propagation and soliton processes. The combination of the nonlinear dispersive KdV equation with the quantum mechanical Schrödinger equation results in a coupled system that exhibits intricate interactions between linear and nonlinear processes [19–21]. In addition to offering a more comprehensive framework for comprehending wave dynamics, this coupling also creates opportunities for investigating fascinating phenomena that result from their interdependence. To acquire an understanding of the interaction between quantum and nonlinear dynamics and its larger ramifications across scientific fields, this work investigates the analysis of the nonlinear system of coupled Schrödinger–KdV equations in all of its complexity [22–24].

Understanding the complex behavior of many physical events requires understanding nonlinear systems, which offers an enthralling panorama. The investigation

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of coupled Schrödinger–KdV equations is one especially fascinating direction in this area. The Schrödinger equation, which regulates the behavior of wave functions in quantum mechanics, and the Korteweg–de Vries equation, which is well known for its role in explaining nonlinear wave propagation and soliton production, are combined in these equations. A new system that captures both linear quantum effects and nonlinear dispersive dynamics is created by linking these equations, offering a singular opportunity to investigate the interactions between these several physical phenomena [25].

In addition to enhancing our knowledge of wave processes, this coupling also has potential applications in several other disciplines, including fluid dynamics and nonlinear optics [26–28]. Understanding the deep links between quantum effects and nonlinear interactions would help researchers understand the complex mechanisms that influence wave behavior in challenging physical contexts [29–31]. This work attempts to reveal the basic insights that result from the coupling of the nonlinear system of coupled Schrödinger–KdV equations, providing a greater understanding of the complicated dynamics and interdependencies that underpin complex wave propagation situations [32,33]. We want to contribute to the better knowledge of nonlinear systems and their importance in various scientific areas via thorough analysis and numerical research [34–36].

The search for efficient methods for resolving nonlinear equations and systems has taken center stage in many scientific fields. The optimal auxiliary function method (OAFM) has emerged as a potential strategy to solve these issues by providing a structured and adaptable framework. The OAFM, which has its roots in mathematical analysis, aims to approximate solutions by carefully including auxiliary functions that improve convergence characteristics. The method's applicability to various nonlinear situations gives it versatility and makes it useful in disciplines including physics, engineering, and applied mathematics. Using examples from various situations, we examine the theoretical underpinnings, benefits, and practical application of the OAFM in this study [37–39].

Due to its potential for solving challenging nonlinear issues, the OAFM has attracted much interest lately. The OAFM provides a systematic framework for approximation solutions that could otherwise be difficult to acquire by efficiently integrating components of perturbation theory, auxiliary functions, and optimization approaches. The OAFM was used by Lu et al. [40] to tackle nonlinear computational intelligence systems, demonstrating its capacity to extract multiscale characteristics from mixed picture and text input. In the study of Yin et al. [41], a novel end-to-end lake boundary prediction model. This demonstrates how the approach may be used in various fields and highlights its promise as a tool for studying challenging real-world circumstances.

Furthermore, Chen et al. [42], who used the technique to create a generic linear free-energy relationship for forecasting partition coefficients in organic compounds, highlighted the OAFM's potency in handling mathematical models. This application highlights the OAFM's ability to draw important correlations from complex mathematical formulations. Furthermore, Lu et al. [43] discussed attention processes in the context of multi-modal fusion in visual question responding, emphasizing how the OAFM might illuminate the intricate interaction of diverse data sources.

The OAFM has shown promise in various applications, from feature extraction to predictive modeling. To contribute to the expanding body of knowledge on efficient nonlinear problem-solving strategies, this work clarifies the method's theoretical foundations, investigates its benefits, and offers insights into its practical application.

2 Preliminaries

Definition. The fractional Caputo derivative of a function $U(\zeta, \tau)$ of order α is given as follows:

$$^{C}D_t^{\alpha}U(\zeta, t) = J_t^{m-\alpha}U^m(\zeta, t), \ m-1 < \alpha \le m, \ t > 0. \quad (1)$$

Definition. The formula for the Riemann fractional integral is as follows:

$$J_t^{\alpha}U(\zeta, t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-r)^{\alpha-1}U(\zeta, r)\mathrm{d}r \quad (2)$$

Lemma. *For* $n-1 < \alpha \le n$, $p > -1$, $t \ge 0$, *and* $\lambda \in R$, *we have*:

1. $D_t^{\alpha}t^p = \frac{\Gamma(\alpha+1)}{\Gamma(p-\alpha+1)}t^{p-\alpha}$
2. $D_t^{\alpha}\lambda = 0$
3. $D_t^{\alpha}I_t^{\alpha}U(\zeta, t) = U(\zeta, t)$
4. $I_t^{\alpha}D_t^{\alpha}U(\zeta, t) = U(\alpha, t) - \sum_{i=0}^{n-1}\partial^i U(\zeta, 0)\frac{t^i}{i!}$.

3 General procedure of OAFM

To elucidate the fundamental concept of the OAFM, we shall dissect a general nonlinear equation represented as follows:

$$L(u) + N(u) + h(\varphi) = 0. \quad (3)$$

This equation is accompanied by the given initial/boundary conditions:

$$B(u(\varphi), \frac{\partial u(\varphi)}{\partial \varphi}) = 0. \tag{4}$$

In this context, L pertains to the linear term, N denotes the nonlinear term, and h is a given function. The approach involves the approximation of the solution for Eq. (3), which can be expressed as follows:

$$u^*(\varphi, C_i) = u_0(\varphi) + u_1(\varphi, C_n), \quad n = 1, 2, 3, 4 \ldots s. \tag{5}$$

To initiate this approximation process, we derive the initial and first approximations for Eq. (3) by introducing Eq. (5) into Eq. (3), yielding

$$L(u_0(\varphi) + u_1(\varphi, C_n)) + N(w_0(\varphi) + u_1(\varphi, C_n)) + h(\varphi) = 0. \tag{6}$$

The initial approximation, denoted as $u_0(\varphi)$, can be obtained from the linear term, leading to

$$L(u_0(\varphi)) + h(\varphi) = 0, \quad B\left(u_0, \frac{\mathrm{d}u_0}{\mathrm{d}\varphi}\right) = 0. \tag{7}$$

The linear operator L relies on the given initial/boundary conditions, while the function $h(\phi)$ remains variable.

To determine the first approximation $u_1(\varphi)$, we take into account both the initial approximation and the nonlinear differential equation, along with the corresponding initial/boundary conditions, which results in:

$$L(u_1(\varphi, C_n)) + N(u_0(\varphi) + u_1(\varphi, C_n)) = 0, \tag{8}$$

accompanied by the following relevant initial/boundary conditions:

$$B\left(u_1(\varphi, C_n), \frac{\partial u_1(\varphi, C_n)}{\partial \varphi}\right) = 0. \tag{9}$$

Furthermore, the nonlinear term in Eq. (8) can be expanded as follows:

$$N(u_0 + u_1) = N(u_0) + \sum_{k=1}^{\infty} \frac{u_1^{(k)}}{k!} N^{(k)}(u_0(\varphi)). \tag{10}$$

This expansion, as delineated in Eq. (10), can be presented algorithmically to achieve the limiting solution.

In order to overcome the challenges associated with solving the nonlinear differential equation presented in Eq. (6) and expedite the convergence of the first approximation $u_1(\varphi, C_n)$, an alternate expression, as represented in Eq. (7), is introduced for Eq. (8). This expression is vital for controlling the issues encountered during the solution of nonlinear differential equations and enhancing the convergence of the first approximation.

Remark 1. A_1 and A_2 are considered auxiliary functions that are contingent on $u_0(\varphi)$ and unknown parameters C_n and C_m, where $n = 1, 2, 3, \ldots s$ and $m = s + 1, s + 2, s + 3 \ldots q$.

Remark 2. A_1 and A_2 are not fixed and may encompass $u_0(\varphi)$ or $N(u_0(\varphi))$, or even a combination of both, depending on the specific context.

Remark 3. The nature of A_1 and A_2 depends on whether $u_0(\varphi)$ or $N(u_0(\varphi))$ is a polynomial, trigonometric, or exponential function, resulting in corresponding summation forms. In the special case where $N(u_0(\varphi)) = 0$ and $u_0(\varphi)$ serves as the exact solution.

Remark 4. The determination of the values for the unknown parameters C_n and C_m can be achieved using various methods, such as the Ritz method, Collocation method, Least Square method, or Galerkin's method.

This comprehensive approach underlines the flexibility and adaptability of the OAFM in handling a wide range of nonlinear problems.

3.1 Problem 1

3.1.1 Implementation of OAFM

Consider the coupled Schrödinger–Kdv equation of fractional order

$$\begin{aligned} &D_t^{\mu} u(\varphi, t) - \partial_{\varphi\varphi} v(\varphi, t) - v(\varphi, t) w(\varphi, t) = 0, \\ &\text{where } 0 < \mu \le 1 \\ &D_t^{\mu} v(\varphi, t) + \partial_{\varphi\varphi} u(\varphi, t) + u(\varphi, t) w(\varphi, t) = 0 \\ &D_t^{\mu} w(\psi, t) + 6 w(\varphi, t) \partial_{\varphi} w(\varphi, t) + \partial_{\varphi\varphi\varphi} w(\zeta, t) \\ &\quad - 2u(\zeta, t) \partial_{\varphi} u(\varphi, t) - 2v(\varphi, t) \partial_{\varphi} v(\varphi, t) = 0 \end{aligned} \tag{11}$$

subject to the following initial conditions:

$$\begin{aligned} u(\varphi, 0) &= \tanh(\varphi) \cos(\varphi), \\ v(\varphi, 0) &= \tanh(\varphi) \sin(\varphi), \\ w(\varphi, 0) &= \frac{7}{8} - 2 \tanh^2(\varphi). \end{aligned} \tag{12}$$

Consider the following linear terms from Eq. (11):

$$\begin{aligned} L(u(\varphi, t)) &= D_t^{\mu} u(\varphi, t), \\ L(v(\varphi, t)) &= D_t^{\mu} u(\varphi, t), \\ L(w(\varphi, t)) &= D_t^{\mu} u(\varphi, t). \end{aligned} \tag{13}$$

The nonlinear terms can be defined as:

$$\begin{aligned} N(u(\varphi,t)) &= -\partial_{\varphi\varphi}v(\varphi,t) - v(\varphi,t)w(\varphi,t) \\ N(v(\varphi,t)) &= \partial_{\varphi\varphi}u(\varphi,t) + u(\varphi,t)w(\varphi,t) \\ N(w(\varphi,t)) &= 6w(\varphi,t)\partial_{\varphi}w(\varphi,t) + \partial_{\varphi\varphi\varphi}w(\varphi,t) \\ &\quad - 2u(\varphi,t)\partial_{\varphi}u(\varphi,t) - 2v(\varphi,t)\partial_{\varphi}v(\varphi,t) \end{aligned} \tag{14}$$

Zeroth order approximation

$$\begin{aligned} D_t^{\mu}u_0(\varphi,t) &= 0, \quad f_0(\varphi,0) = \tanh(\varphi)\cos(\varphi) \\ D_t^{\mu}v_0(\varphi,t) &= 0, \quad g_0(\varphi,0) = \tanh(\varphi)\sin(\varphi) \\ D_t^{\mu}w_0(\varphi,t) &= 0, \quad h_0(\zeta,0) = \frac{7}{8} - 2\tanh^2(\varphi). \end{aligned} \tag{15}$$

Using the inverse operator, we obtain the following solution:

$$\begin{aligned} u_0(\varphi,t) &= \tanh(\varphi)\cos(\varphi) \\ v_0(\varphi,t) &= \tanh(\varphi)\sin(\varphi) \\ w_0(\varphi,t) &= \frac{7}{8} - 2\tanh^2(\varphi). \end{aligned} \tag{16}$$

Using Eq. (16) in Eq. (14), the system of nonlinear term becomes

$$\begin{aligned} N[u_0(\zeta,t)] &= \frac{17}{8}\sin(\varphi)\tanh(\varphi) - 2\cos(\varphi)\,\mathrm{sech}^2(\varphi) \\ N[v_0(\zeta,t)] &= -\frac{17}{8}\cos(\varphi)\tanh(\varphi) - 2\sin(\varphi)\,\mathrm{sech}^2(\varphi) \\ N[w_0(\zeta,t)] &= 9\tanh(\varphi)\,\mathrm{sech}^2(\varphi), \end{aligned} \tag{17}$$

and we choose the auxiliary function A_1, A_2, and A_3 as follows:

$$\begin{aligned} A_1 &= c1\cos(\varphi)\tanh(\varphi) + c2\cos^3(\varphi)\tanh^3(\varphi), \\ A_2 &= c3\sin^4(\varphi)\tanh^4(\varphi) + c4\sin^5(\varphi)\tanh^5(\varphi), \\ A_3 &= c5\left(\frac{7}{8} - 2\tanh^2(\varphi)\right)^6 + c6\left(\frac{7}{8} - 2\tanh^2(\varphi)\right)^7. \end{aligned} \tag{18}$$

The first-order approximation according to OAFM procedure is discussed in Section 3, i.e.,

$$\begin{aligned} \frac{\partial^{\mu}u_1(\varphi,t)}{\partial t^{\mu}} &= -(A_1N[u_0(\varphi,t)]) \\ \frac{\partial^{\mu}v_1(\varphi,t)}{\partial t^{\mu}} &= -(A_2N[v_0(\varphi,t)]) \\ \frac{\partial^{\mu}w_1(\varphi,t)}{\partial t^{\mu}} &= -(A_3N[v_0(\varphi,t)]). \end{aligned} \tag{19}$$

Using Eqs. (17) and (18) in Eq. (19), we obtain

$$\begin{aligned} \frac{\partial^{\mu}u_1(\varphi,t)}{\partial t^{\mu}} &= -((c1\cos(\varphi)\tanh(\varphi) \\ &\quad + c2\cos^3(\varphi)\tanh^3(\varphi)) \\ &\quad \times \left(\frac{17}{8}\sin(\varphi)\tanh(\varphi)\right. \\ &\quad \left. - 2\cos(\varphi)\,\mathrm{sech}^2(\varphi)\right) \\ \frac{\partial^{\mu}v_1(\varphi,t)}{\partial t^{\mu}} &= -((c3\sin^4(\varphi)\tanh^4(\varphi) \\ &\quad + c4\sin^5(\varphi)\tanh^5(\varphi)) \\ &\quad \times \left(-\frac{17}{8}\cos(\varphi)\tanh(\varphi)\right. \\ &\quad \left. - 2\sin(\varphi)\,\mathrm{sech}^2(\varphi)\right) \\ \frac{\partial^{\mu}w_1(\varphi,t)}{\partial t^{\mu}} &= -9\tanh(\varphi)\,\mathrm{sech}^2(\varphi) \\ &\quad \times \left(c5\left(\frac{7}{8} - 2\tanh^2(\varphi)\right)^6\right. \\ &\quad \left. + c6\left(\frac{7}{8} - 2\tanh^2(\varphi)\right)^7\right), \end{aligned} \tag{20}$$

by applying inverse operator to Eq. (20), we obtain

Table 1: Absolute error of $u(\varphi,t)$ at fractional orders $\mu = 0.7$, $\mu = 0.8$, and $\mu = 1$

ζ	Absolute error$_{(\mu=0.7)}$	Absolute error$_{(\mu=0.8)}$	Absolute error$_{(\mu=1)}$
0.	0.00004	0.00004	0.00004
0.1	0.00130227	0.000404499	0.000004655
0.2	0.00217469	0.000694956	0.0000421961
0.3	0.00236032	0.000759476	0.000053294
0.4	0.00197233	0.000635256	0.0000454302
0.5	0.0013183	0.000423718	0.0000290893
0.6	0.000679637	0.000217624	0.0000138151
0.7	0.000167597	0.0000534943	0.0000036002
0.8	0.00027163	0.0000866903	0.00000510754
0.9	0.00074588	0.000238765	0.00001506
1.	0.0012967	0.000416593	0.0000283522

Table 2: Absolute error of $v(\varphi,t)$ at fractional orders $\mu = 0.55$, $\mu = 0.75$, and $\mu = 1$

ζ	Absolute error$_{(\mu=0.55)}$	Absolute error$_{(\mu=0.75)}$	Absolute error$_{(\mu=1)}$
0.5	0.000577801	0.0000354637	0.0000281522
0.6	0.00191544	0.000181767	0.0000215926
0.7	0.0043588	0.000452243	0.00000599374807
0.8	0.00732438	0.000781972	0.0000145519
0.9	0.00921167	0.000992997	0.0000289521
1.	0.00812713	0.000874782	0.0000240854
1.1	0.00317805	0.00032797	0.0000006343112
1.2	0.0045951	0.000532017	0.0000554198
1.3	0.0123911	0.00139412	0.000104191
1.4	0.0169013	0.00189093	0.000130217
1.5	0.0159235	0.00177787	0.00011859

Table 3: Absolute error of $w(\varphi, t)$ at fractional orders $\mu = 0.6$, $\mu = 0.8$, and $\mu = 1$

ζ	Absolute error$_{(\mu=0.6)}$	Absolute error$_{(\mu=0.8)}$	Absolute error$_{(\mu=1)}$
1.3	0.000530974	0.0000269316	0.0000288141
1.37	0.000887848	0.0000693315	0.0000211942
1.44	0.00131683	0.000119502	0.0000129191
1.51	0.00178548	0.000173845	0.00000439
1.58	0.00225593	0.000228179	0.00000391539
1.65	0.00269217	0.000278529	0.000011588
1.72	0.00306528	0.000321714	0.0000182843
1.79	0.00335612	0.000355633	0.0000237875
1.86	0.00355569	0.000379302	0.0000280033
1.93	0.00366383	0.00039272	0.0000309441
2.	0.00368713	0.000396623	0.0000327027

$$
\begin{aligned}
u_1(\varphi, t) &= \frac{t^{\mu}}{8\Gamma(1+\mu)} \cos(\varphi)\tanh(\varphi)(c1 \\
&\quad + c2\cos^2(\varphi)\tanh^2(\varphi)) \\
&\quad \times (16\cos(\varphi)\,\mathrm{sech}^2(x) - 17\sin(\varphi)\tanh(\varphi)) \\
v_1(\varphi, t) &= \frac{t^{\mu}}{8\Gamma(1+\mu)} \sin^4(\varphi)\tanh^4(\varphi)(c3 \\
&\quad + c4\sin(\varphi)\tanh(\varphi)) \\
&\quad \times (17\cos(\varphi)\tanh(\varphi) + 16\sin(\varphi)\,\mathrm{sech}^2(\varphi)) \\
w_1(\varphi, t) &= -\frac{9t^{\mu}}{\Gamma(1+\mu)} \tanh(\varphi)\left(\frac{7}{8} - 2\tanh^2(\varphi)\right)^6 \mathrm{sech}^2 \\
&\quad (\varphi) \\
&\quad \times \left(c5 - 2c6\tanh^2(\varphi) + \frac{7c6}{8}\right).
\end{aligned}
\tag{21}
$$

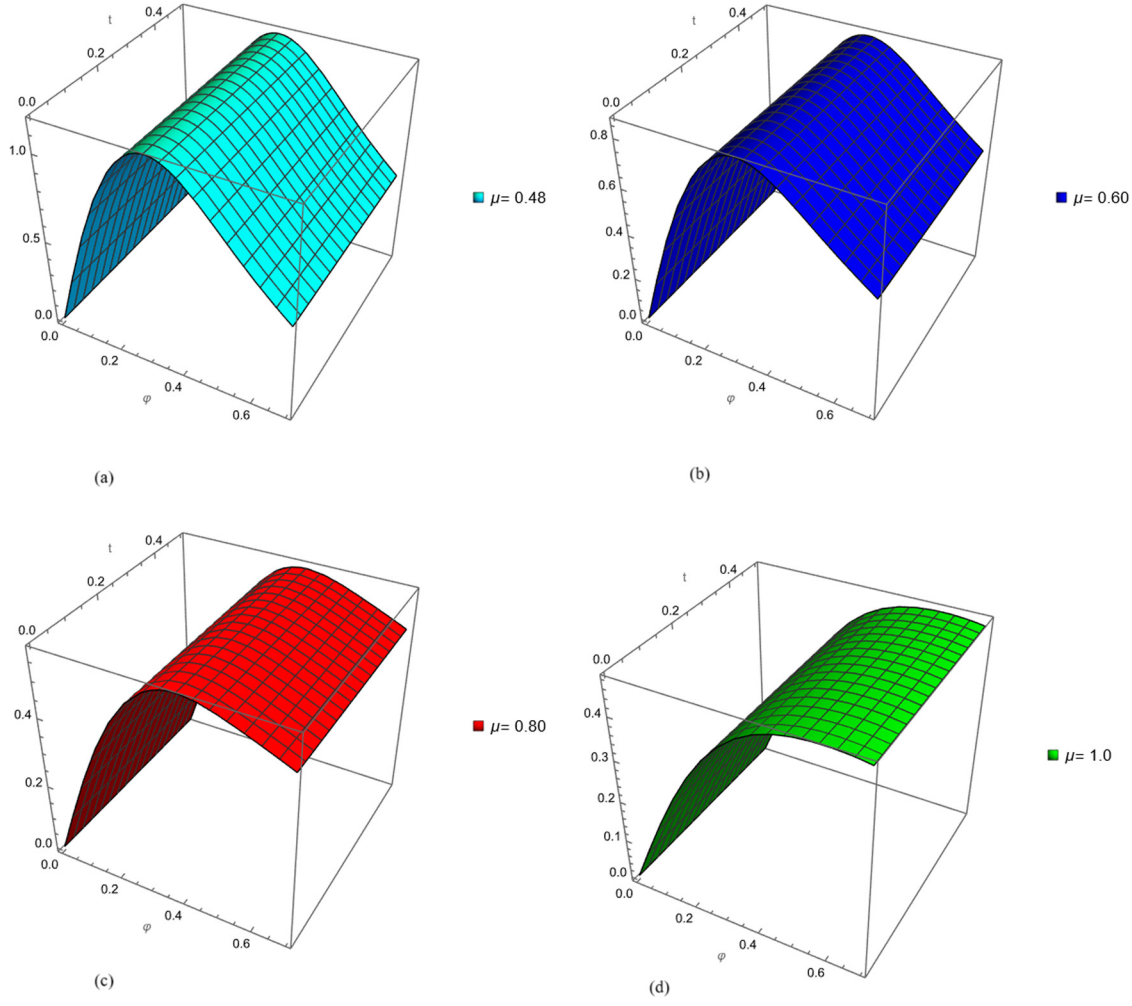

Figure 1: The fractional order of (a) $\mu = 0.4$, (b) OAFM $\mu = 0.6$, (c) $\mu = 0.8$, and (d) $\mu = 1$ of $u(\varphi, t)$ at $t = 0.004$.

According to the OAFM procedure,

$$\begin{aligned} u(\varphi, t) &= u_0(\varphi, t) + u_1(\varphi, t), \\ v(\varphi, t) &= v_0(\varphi, t) + v_1(\varphi, t), \\ w(\varphi, t) &= w_0(\varphi, t) + w_1(\varphi, t). \end{aligned} \tag{22}$$

Using Eqs (16) and (21), we have

$$\begin{aligned} u(\zeta, t) &= \cos(\varphi)\tanh(\varphi) + \frac{t^{\mu}}{8\Gamma(1+\mu)}\cos(\varphi)\tanh(\varphi) \\ &\quad \times (c1 + c2\cos^2(\varphi)\tanh^2(\varphi)) \\ &\quad \times (16\cos(\varphi)\,\mathrm{sech}^2(x) - 17\sin(\varphi)\tanh(\varphi)), \\ v(\zeta, t) &= \sin(\varphi)\tanh(\varphi) + \frac{t^{\mu}}{8\Gamma(1+\mu)}\sin^4(\varphi)\tanh^4(\varphi) \\ &\quad \times (c3 + c4\sin(\varphi)\tanh(\varphi))(17\cos(\varphi)\tanh(\varphi) \\ &\quad + 16\sin(\varphi)\,\mathrm{sech}^2(\varphi)), \\ w(\zeta, t) &= \frac{7}{8} - 2\tanh^2(\varphi) - \frac{9t^{\mu}}{\Gamma(1+\mu)}\tanh(\varphi) \\ &\quad \times \left(\frac{7}{8} - 2\tanh^2(\varphi)\right)^6 \mathrm{sech}^2(\varphi) \\ &\quad \times \left(c5 - 2c6\tanh^2(\varphi) + \frac{7c6}{8}\right). \end{aligned} \tag{23}$$

The exact result is given as (Tables 1, 2, and 3).

4 Numerical and graphical results

The graphical analysis in this section provides insights into the behavior of the solutions to the nonlinear system of coupled Schrodinger–KdV equations with varying fractional orders (μ) at a specific time point ($t = 0.004$). Figure 1 shows the set of graphs that depicts the behavior of the function $u(\varphi, t)$ at $t = 0.004$ for different fractional orders: (a) $\mu = 0.4$, (b) $\mu = 0.6$, (c) $\mu = 0.8$, and (d) $\mu = 1$. These visualizations illustrate how changing the fractional order affects the behavior of u at this specific time instance, shedding light on the impact of fractional order on the solution. Similarly, Figure 2 presents the behavior of the function $v(\varphi, t)$ at $t = 0.004$ for various fractional orders: (a) $\mu = 0.6$, (b) $\mu = 0.7$, (c) $\mu = 0.8$, and (d) $\mu = 1$. These graphs allow us to observe how altering the fractional order influences the characteristics of v at this particular time. Figure 3 focuses on the function $w(\varphi, t)$ at $t = 0.004$ under different

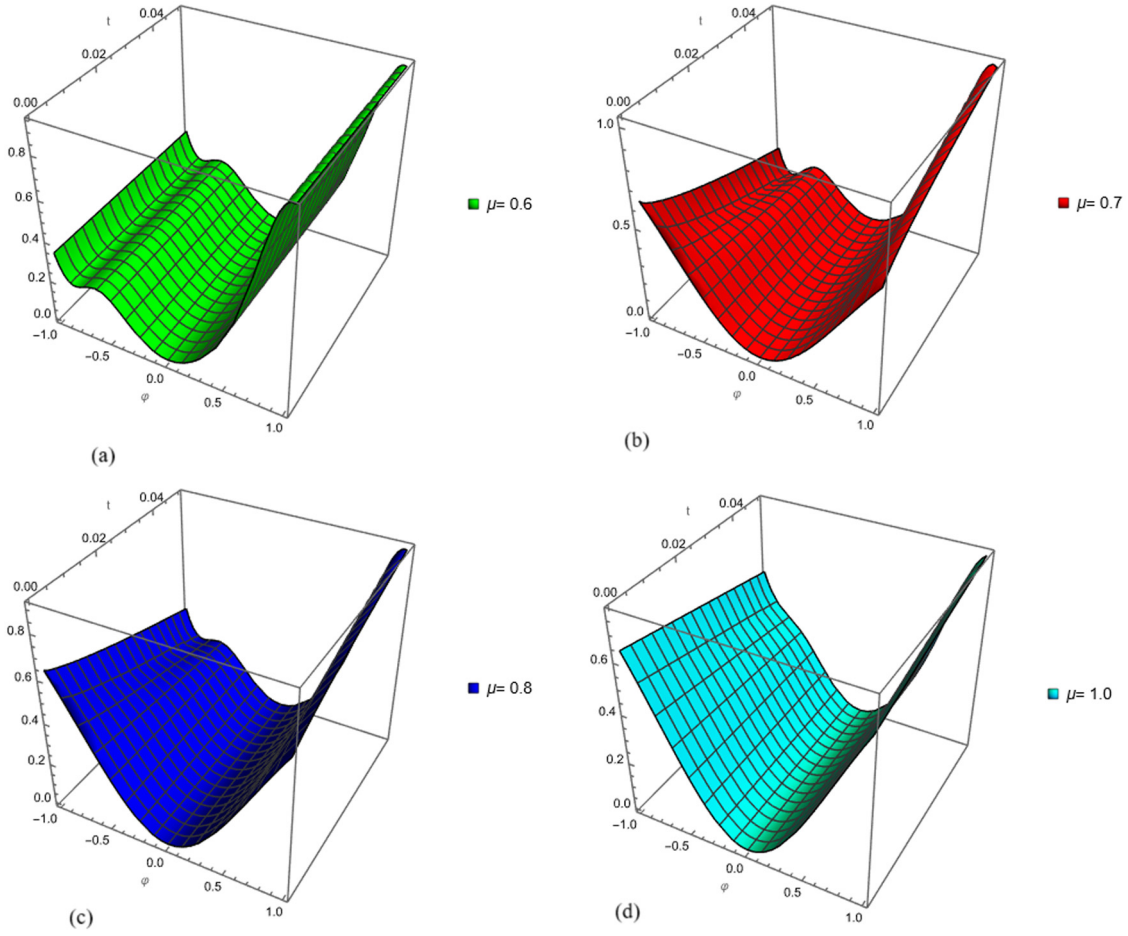

Figure 2: The fractional order of (a) $\mu = 0.6$, (b) $\mu = 0.7$, (c) $\mu = 0.8$, and (d) $\mu = 1$ of $v(\varphi, t)$ at $t = 0.004$.

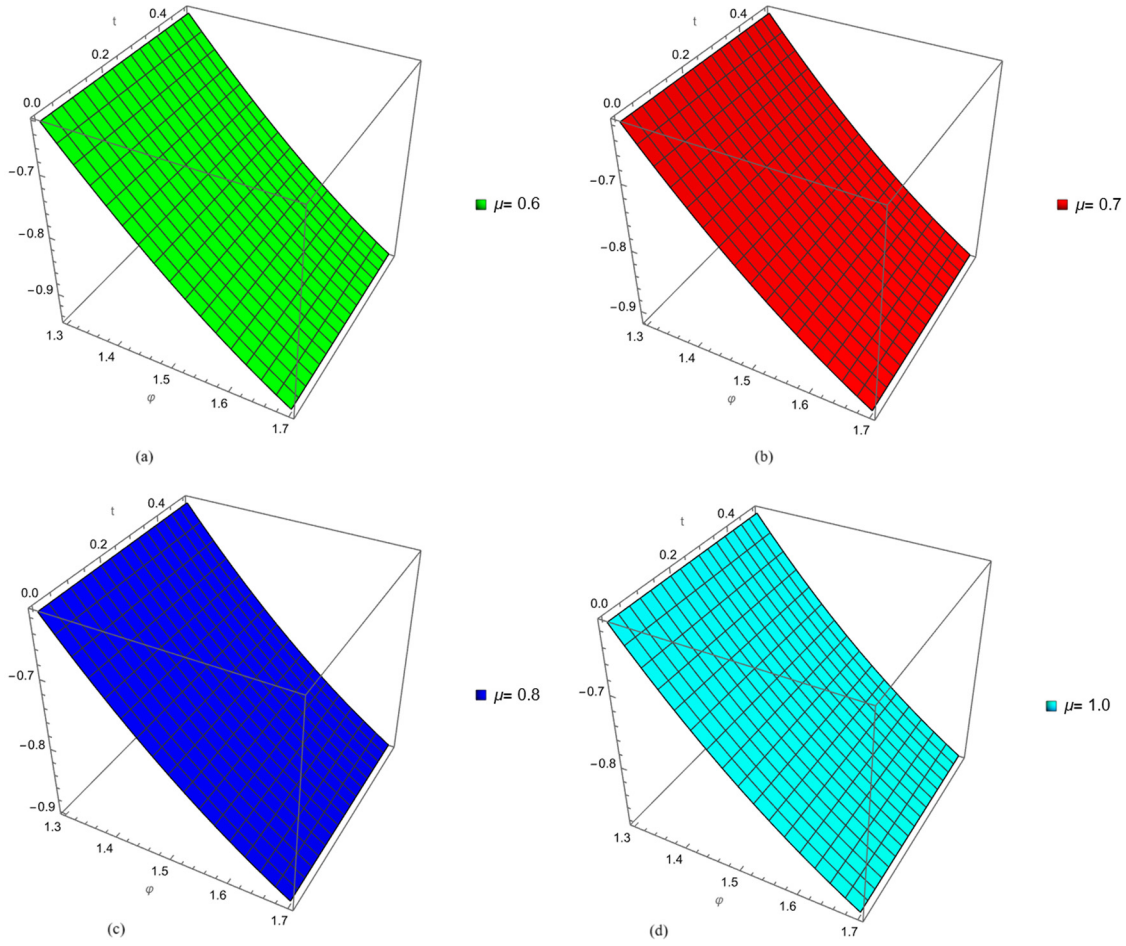

Figure 3: The fractional order of (a) μ = 0.6, (b) μ = 0.7, (c) μ = 0.8, and (d) μ = 1 of $w(\varphi, t)$ at t = 0.004.

fractional order conditions: (a) μ = 0.6, (b) μ = 0.7, (c) μ = 0.8, and (d) μ = 1. These visual representations enable us to explore the variations in the behavior of w as the fractional order μ changes. By examining these figures, one can gain a better understanding of how the fractional order parameter (μ) influences the solutions u, v, and w in the nonlinear system of coupled Schrodinger–KdV equations. These graphical representations provide valuable insights into the dynamics of the system at a specific time point and the role of the fractional order in shaping these dynamics.

5 Conclusion

Finally, this research ventured into the domain of nonlinear systems by using the OAFM to evaluate a coupled system of Schrödinger–KdV equations using the Caputo operator. The OAFM demonstrated its effectiveness in handling difficult nonlinear issues by approximating solutions to the intricate dynamics of the coupled equations. The reported numerical and graphical assessments proved the method's correctness and efficiency, demonstrating its potential for dealing with difficult mathematical physics problems. This research adds to a greater knowledge of coupled Schrödinger–KdV equations and their ramifications across numerous scientific areas by effectively revealing insights into the behavior of the nonlinear system. The OAFM's relevance as a useful instrument in the arsenal of mathematical analysis is reinforced by its capacity to offer trustworthy solutions to complex nonlinear systems.

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