Research Article

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The soliton solutions for stochastic Calogero–Bogoyavlenskii Schiff equation in plasma physics/fluid mechanics

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Abstract: The generalized (2+1)-dimensional stochastic Calogero–Bogoyavlenskii Schiff equation (SCBSE) driven by a multiplicative Brownian motion is taken into consideration. The Riccati equation mapping and He's semi-inverse methods are utilized to obtain the rational function, hyperbolic function, and trigonometric function for SCBSE. We expand some solution from previous studies. The acquired solutions of SCBSE may explain many exciting physical phenomena because it is widely used in plasma physics and fluid dynamics. Also, it explains the relationship between the Riemann *y*-axis propagating wave and the long *x*-axis propagating wave. Using a variety of 2D and 3D graphs, we illustrate how the Brownian motion influences the exact solutions of SCBSE.

Keywords: stochastic Calogero–Bogoyavlenskii Schiff equation, exact stochastic solutions, Riccati equation mapping method

1 Introduction

In many scientific fields, such as fluid dynamics, chemical physics, plasma physics, and optical fibers, nonlinear wave phenomena can be observed. Partial differential equations (PDEs) are essential for clarifying these wave phenomena. As a result, finding the solutions of these PDEs is necessary. Numerous methods for solving PDEs, such as (G'/G)-expansion method [1,2], Kudryashov method [3], first-integral method [4], sine–cosine method [5,6], $\exp(-\phi(\varsigma))$-expansion method [7], direct algebraic method [8], perturbation method [9,10], tanh function method [11], sine–Gordon expansion method [12], and Jacobi elliptic function [13] have been presented. Recently, from matrix spectral problems, integrable equations are generated. We can obtain reduced integrable equations through specific symmetric reductions on potentials such as nonlinear Schrödinger equation [14] and modified Korteweg–de Vries equations [15,16]. Moreover, many integrable equations have been investigated by formulating and analyzing their Riemann–Hilbert problems derived from the associated given matrix spectral problems (see for instance [17–19] and references therein).

In general, stochastic PDEs are used to handle systems that face random impacts in many fields such as materials sciences, finance, information systems, biophysics, electrical engineering, condensed matter climate, and physics system modeling [20,21]. The importance of including stochastic term in complex system models has been recognized. Recently, exact solutions for several SPDEs, for example [22–25], have been found.

Therefore, stochastic effects must be taken into account in PDEs. Here, we consider the generalized (2 + 1)-dimensional stochastic Calogero–Bogoyavlenskii Schiff equation (SCBSE) driven in the Itô sense by a multiplicative Brownian motion:

$$\mathcal{Y}_{xt} + \mathcal{Y}_{xxxy} + a\mathcal{Y}_{xx}\mathcal{Y}_y + b\mathcal{Y}_x\mathcal{Y}_{xy} = \delta\mathcal{Y}_x\mathcal{B}_t, \tag{1}$$

where $\mathcal{Y}(x, y, t)$ denotes the wave profile. a and b are nonzero constants. $\mathcal{B}$ is a Brownian motion (BM), ρ is the noise strength. When $a = -2$, $b = -4$, and $\delta = 0$, we obtain the following (2 + 1)-dimensional breaking soliton equation:

$$\mathcal{Y}_{xt} + \mathcal{Y}_{xxxy} - 2\mathcal{Y}_{xx}\mathcal{Y}_y - 4\mathcal{Y}_x\mathcal{Y}_{xy} = 0.$$

While for $a = 4$, $b = 4$, and $\delta = 0$, we obtain the (2 + 1)-dimensional Bogoyavlenskii's breaking soliton equation:

$$\mathcal{Y}_{xt} + \mathcal{Y}_{xxxy} + 4\mathcal{Y}_{xx}\mathcal{Y}_y + 4\mathcal{Y}_x\mathcal{Y}_{xy} = 0.$$

If we put $\delta = 0$ in Eq. (1), then we obtain the deterministic Calogero–Bogoyavlenskii Schiff equation:

$$\mathcal{Y}_{xt} + \mathcal{Y}_{xxxy} + a\mathcal{Y}_{xx}\mathcal{Y}_y + b\mathcal{Y}_x\mathcal{Y}_{xy} = 0, \tag{2}$$

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which explains the relationship between the Riemann y-axis propagating wave and the long x-axis propagating wave. Also, it is widely used in plasma physics and fluid dynamics. As a result, a number of authors have investigated a wide range of analytical solutions of Eq. (2), including direct integration and Lie symmetries [26], multiple exp-function method [27], $(\frac{G'}{G'+G+A})$-expansion method [28], extended tanh methods and improved (G'/G)-expansion method [29], sine–cosine method [30], and (G'/G)-expansion method [31]. While the stochastic exact solutions to Eq. (1) are not examined at this time.

Our purpose of this article is to achieve the exact stochastic solutions of SCBSE (1). To obtain these solutions, we utilize two various methods including Riccati equation mapping method and He's semi-inverse method. We expand some solution from previous studies such as the solutions stated in previous studies [29–31]. The stochastic term in Eq. (1) makes the solutions extremely useful for identifying numerous crucial physical phenomena, and physicists would be advised to take them into account. In addition, we provide a large number of diagrams by using MATLAB to investigate the effect of noise on the SCBSE solution (1).

A brief summary of the contents of this article is as follows: The wave equation of SCBSE (1) is derived in Section 2. Achieving exact solutions for the SCBSE is the focus of Section 3. In Section 4, we examine how the Brownian motion effects the solutions of SCBSE. Finally, the paper's conclusions are laid out.

2 Wave equation for SCBSE

The accompanying wave transformation is employed to derive the SCBSE (1) wave equation:

$$\mathcal{Y}(x,y,t)=\mathcal{P}(\ell)e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)},\quad \ell=\ell_1 x+\ell_2 y+\ell_3 t, \tag{3}$$

where the function $\mathcal{P}$ is a deterministic, ℓ_1, ℓ_2, and ℓ_3 are undefined constants. We observe that

$$\begin{aligned}\mathcal{Y}_x&=\ell_1\mathcal{P}'e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)},\quad \mathcal{Y}_y=\ell_2\mathcal{P}'e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)},\\ \mathcal{Y}_{xy}&=\ell_1\ell_2\mathcal{P}''e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)},\ \mathcal{Y}_{xx}=\ell_1^2\mathcal{P}''e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)},\\ \mathcal{Y}_{xxxy}&=\ell_2\ell_1^3\mathcal{P}''''e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)},\\ \mathcal{Y}_{xt}&=[\ell_1\ell_3\mathcal{P}''+\delta\ell_1\mathcal{P}'\mathcal{B}_t]e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)}.\end{aligned} \tag{4}$$

Inserting Eq. (4) into Eq. (1) yields

$$(\ell_1\ell_3)\mathcal{P}''+\ell_2\ell_1^3\mathcal{P}''''+\ell_2\ell_1^2(a+b)\mathcal{P}'\mathcal{P}''e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)}=0. \tag{5}$$

When we take into account the expectations of both parties, we obtain

$$(\ell_1\ell_3)\mathcal{P}''+\ell_2\ell_1^3\mathcal{P}''''+\ell_2\ell_1^2(a+b)\mathcal{P}'\mathcal{P}''e^{-\frac{1}{2}\delta^2 t}\mathbb{E}e^{(\delta\mathcal{B}(t))}=0. \tag{6}$$

Since $\mathcal{B}(t)$ is the Brownian motion, then $\mathbb{E}e^{\delta\mathcal{B}(t)}=e^{(\frac{1}{2}\delta^2 t)}$, Eq. (6) turns into

$$\mathcal{P}''''+\hbar_1\mathcal{P}''+2\hbar_2\mathcal{P}'\mathcal{P}''=0, \tag{7}$$

where

$$\hbar_1=\frac{\ell_3}{\ell_2\ell_1^2}\quad\text{and}\quad \hbar_2=\frac{(a+b)}{2\ell_1}. \tag{8}$$

Integrating Eq. (7) yields

$$\mathcal{P}'''+\hbar_1\mathcal{P}'+\hbar_2(\mathcal{P}')^2=0, \tag{9}$$

where integral constant was not considered.

3 Exact solutions of SCBSE

Two various methods such as Riccati equation mapping (REM) [32] and He's semi-inverse are used to obtain the solutions of Eq. (9). After that, the solutions to the SCBSE (1) are found.

3.1 REM method

The Riccati–Bernoulli equation has the form:

$$\mathcal{P}'=\alpha\mathcal{P}^2+\beta\mathcal{P}+\gamma, \tag{10}$$

where α, β, and γ are constants. Utilizing Eq. (10), we have

$$\begin{aligned}\mathcal{P}'''&=6\alpha^3\mathcal{P}^4+12\beta\alpha^2\mathcal{P}^3+(8\gamma\alpha^2+7\alpha\beta^2)\mathcal{P}^2\\&\quad+(\beta^3+8\beta\alpha\gamma)\mathcal{P}+(\beta^2+2\alpha\gamma^2).\end{aligned} \tag{11}$$

Plugging Eqs (10) and (11) into Eq. (9), we obtain

$$\begin{aligned}&(6\alpha^3+\alpha^2\hbar_2)\mathcal{P}^4+(12\beta\alpha^2+2\beta\alpha\hbar_2)\mathcal{P}^3\\&+(8\gamma\alpha^2+7\alpha\beta^2+\alpha\hbar_1+2\gamma\alpha\hbar_2+\beta^2\hbar_2)\mathcal{P}^2\\&+(\beta^3+8\beta\alpha\gamma+\beta\hbar_1+2\gamma\beta\hbar_2)\mathcal{P}\\&+(\beta^2+2\alpha\gamma^2+\gamma\hbar_1+\gamma^2\hbar_2)=0.\end{aligned}$$

We obtain by assigning each coefficient of $\mathcal{P}^k$ to zero

$$6\alpha^3+\alpha^2\hbar_2=0,$$

$$12\beta\alpha^2+2\beta\alpha\hbar_2=0,$$

$$8\gamma\alpha^2+7\alpha\beta^2+\alpha\hbar_1+2\gamma\alpha\hbar_2+\beta^2\hbar_2=0,$$

$$\beta^3+8\beta\alpha\gamma+\beta\hbar_1+2\gamma\beta\hbar_2,$$

and

$$\beta^2 + 2\alpha\gamma^2 + \gamma\hbar_1 + \gamma^2\hbar_2 = 0.$$

The result of solving these equations is

$$\alpha = \frac{-\hbar_2}{6}, \beta = 0, \text{ and } \gamma = \frac{-3\hbar_1}{2\hbar_2}. \tag{12}$$

Now, we can rewrite Eq. (10) as

$$\frac{d\mathcal{P}}{\mathcal{P}^2 + \left(\frac{\gamma}{\alpha}\right)} = \alpha d\ell. \tag{13}$$

There are different sets relying on γ and α as follows:

Family I: When $\gamma\alpha > 0$, thus the solutions of Eq. (10) are as follows:

$$\mathcal{P}_1(\ell) = \sqrt{\frac{\gamma}{\alpha}} \tan(\sqrt{\gamma\alpha}\,\ell),$$

$$\mathcal{P}_2(\ell) = -\sqrt{\frac{\gamma}{\alpha}} \cot(\sqrt{\gamma\alpha}\,\ell),$$

$$\mathcal{P}_3(\ell) = \sqrt{\frac{\gamma}{\alpha}} (\tan(\sqrt{4\gamma\alpha}\,\ell) \pm \sec(\sqrt{4\gamma\alpha}\,\ell)),$$

$$\mathcal{P}_4(\ell) = -\sqrt{\frac{\gamma}{\alpha}} (\cot(\sqrt{4\gamma\alpha}\,\ell) \pm \csc(\sqrt{4\gamma\alpha}\,\ell)),$$

$$\mathcal{P}_5(\ell) = \frac{1}{2}\sqrt{\frac{\gamma}{\alpha}} \left(\tan\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right) - \cot\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right)\right),$$

$$\mathcal{P}_6(\ell) = \sqrt{\frac{\gamma}{\alpha}} \left(\frac{\sin(\sqrt{4\gamma\alpha}\,\ell)}{\sin(\sqrt{4\gamma\alpha}\,\ell) \pm 1}\right),$$

$$\mathcal{P}_7(\ell) = 2\sqrt{\frac{\gamma}{\alpha}} \left(\frac{\sin(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell)\cos\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right)}{2\cos^2\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right) - 1}\right).$$

Then, SCBSE (1) has the trigonometric function solutions:

$$\mathcal{Y}_1(x, y, t) = \sqrt{\frac{\gamma}{\alpha}} \tan(\sqrt{\gamma\alpha}\,\ell) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{14}$$

$$\mathcal{Y}_2(x, y, t) = -\sqrt{\frac{\gamma}{\alpha}} \cot(\sqrt{\gamma\alpha}\,\ell) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{15}$$

$$\mathcal{Y}_3(x, y, t) = \sqrt{\frac{\gamma}{\alpha}} (\tan(\sqrt{4\gamma\alpha}\,\ell) \pm \sec(\sqrt{4\gamma\alpha}\,\ell)), e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{16}$$

$$\mathcal{Y}_4(x, y, t) = -\sqrt{\frac{\gamma}{\alpha}} (\cot(\sqrt{4\gamma\alpha}\,\ell) \pm \csc(\sqrt{4\gamma\alpha}\,\ell)) e^{\left(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t\right)}, \tag{17}$$

$$\mathcal{Y}_5(x, y, t) = \frac{1}{2}\sqrt{\frac{\gamma}{\alpha}} \left(\tan(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell) - \cot\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right)\right) e^{\left(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t\right)}, \tag{18}$$

$$\mathcal{Y}_6(x, y, t) = \sqrt{\frac{\gamma}{\alpha}} \left(\frac{\sin(\sqrt{4\gamma\alpha}\,\ell)}{\sin(\sqrt{4\gamma\alpha}\,\ell) \pm 1}\right) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{19}$$

$$\mathcal{Y}_7(x, y, t) = 2\sqrt{\frac{\gamma}{\alpha}} \left(\frac{\sin\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right)\cos\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right)}{2\cos^2\left(\frac{1}{2}\sqrt{\gamma\alpha}\,\ell\right) - 1}\right) \times e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{20}$$

where $\ell = \ell_1 x + \ell_2 y + \ell_3 t$.

Family II: When $\gamma\alpha < 0$, thus the solutions of Eq. (10) are as follows:

$$\mathcal{P}_8(\ell) = -\sqrt{\frac{-\gamma}{\alpha}} \tanh(\sqrt{-\gamma\alpha}\,\ell),$$

$$\mathcal{P}_9(\ell) = -\sqrt{\frac{-\gamma}{\alpha}} \coth(\sqrt{-\gamma\alpha}\,\ell),$$

$$\mathcal{P}_{10}(\ell) = -\sqrt{\frac{-\gamma}{\alpha}} (\tanh(\sqrt{-4\gamma\alpha}\,\ell) \pm i\,\mathrm{sech}(\sqrt{-4\gamma\alpha}\,\ell)),$$

$$\mathcal{P}_{11}(\ell) = -\sqrt{\frac{-\gamma}{\alpha}} (\coth(\sqrt{-4\gamma\alpha}\,\ell) \pm \mathrm{csch}(\sqrt{-4\gamma\alpha}\,\ell)),$$

$$\mathcal{P}_{12}(\ell) = \frac{-1}{2}\sqrt{\frac{-\gamma}{\alpha}} \left(\tanh\left(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell\right) + \coth\left(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell\right)\right),$$

$$\mathcal{P}_{13}(\ell) = \sqrt{\frac{-\gamma}{\alpha}} \left(\frac{\sinh(\sqrt{-4\gamma\alpha}\,\ell)}{\cosh(\sqrt{-4\gamma\alpha}\,\ell) \pm 1}\right),$$

$$\mathcal{P}_{14}(\ell) = 2\sqrt{\frac{-\gamma}{\alpha}} \left(\frac{\sinh(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell)\cosh(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell)}{2\cosh^2\left(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell\right) - 1}\right).$$

Then, SCBSE (1) has the hyperbolic function solutions:

$$\mathcal{Y}_8(x, y, t) = -\sqrt{\frac{-\gamma}{\alpha}} \tanh(\sqrt{-\gamma\alpha}\,\ell) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{21}$$

$$\mathcal{Y}_9(x, y, t) = -\sqrt{\frac{-\gamma}{\alpha}} \coth(\sqrt{-\gamma\alpha}\,\ell) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{22}$$

$$\mathcal{Y}_{10}(x, y, t) = -\sqrt{\frac{-\gamma}{\alpha}} (\tanh(\sqrt{-4\gamma\alpha}\,\ell) \pm i\,\mathrm{sech}(\sqrt{-4\gamma\alpha}\,\ell)) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{23}$$

$$\mathcal{Y}_{11}(x, y, t) = -\sqrt{\frac{-\gamma}{\alpha}} (\coth(\sqrt{-4\gamma\alpha}\,\ell) \pm \mathrm{csch}(\sqrt{-4\gamma\alpha}\,\ell)) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{24}$$

$$\mathcal{Y}_{12}(x, y, t) = \frac{-1}{2}\sqrt{\frac{-\gamma}{\alpha}} \left(\tanh\left(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell\right) + \coth(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell)\right) e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{25}$$

$$\mathcal{Y}_{13}(x,y,t) = \sqrt{\frac{-\gamma}{\alpha}}\left(\frac{\sinh(\sqrt{-4\gamma\alpha}\,\ell)}{\cosh(\sqrt{-4\gamma\alpha}\,\ell) \pm 1}\right)e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)}, \tag{26}$$

$$\mathcal{Y}_{14}(x,y,t) = 2\sqrt{\frac{-\gamma}{\alpha}}\left(\frac{\sinh(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell)\cosh(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell)}{2\cosh^2\left(\frac{1}{2}\sqrt{-\gamma\alpha}\,\ell\right) - 1}\right) \times e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)}, \tag{27}$$

where $\ell = \ell_1 x + \ell_2 y + \ell_3 t$.

Family III: When $\gamma = 0, \alpha \neq 0$, then the solution of Eq. (10) is

$$\mathcal{P}(\ell) = \frac{-1}{\alpha\ell}.$$

Then, we obtain the rational function solution of SCBSE (1) as

$$\mathcal{Y}_{15}(x,y,t) = \left(\frac{-1}{\alpha(\ell_1 x + \ell_2 y + \ell_3 t)}\right)e^{(\delta\mathcal{B}(t)-\frac{1}{2}\delta^2 t)}. \tag{28}$$

Remark 1. Putting $\alpha = 4, b = 2$, and $\delta = 0$ in Eqs (14), (15), (18), (21), (22), and (25), the identical solutions (37), (40), (43), and (46) are given in the study by Shakeel and Mohyud-Din [29].

Remark 2. Putting $\delta = 0$ in Eqs (14) and (15), the identical solutions (24) are given inthe study by Najafi and Arbabi [30].

Remark 3. Putting $\delta = 0$ in Eqs (14), (15), (21), and (22), the identical answers (27)–(30) are given in the study by Najafi and Arbabi [31].

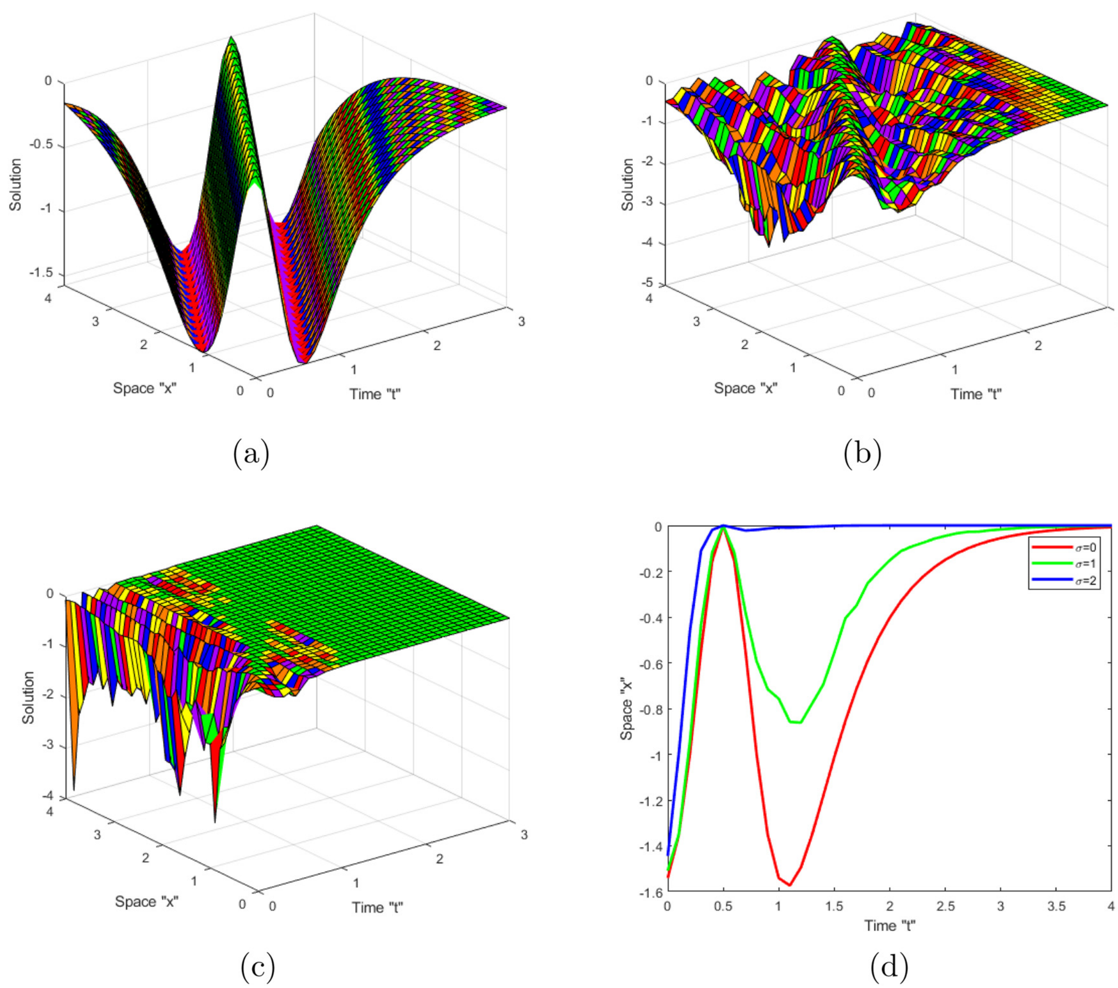

Figure 1: (a)–(c) 3D-shape of solution given in Eq. (33) for various $\delta = 0, 1, 2$. (d) 2D-shape for these values of δ. (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 2$, and (d) $\delta = 0, 1, 2$.

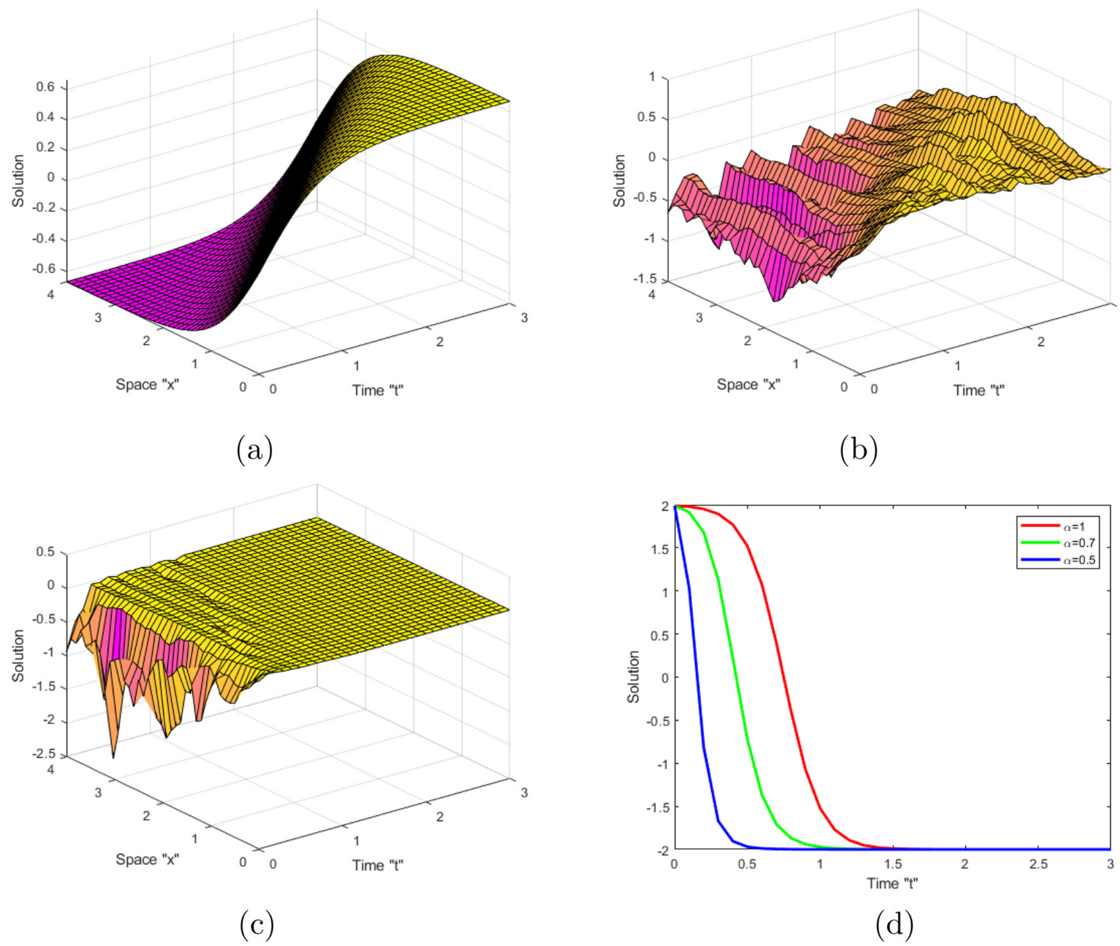

Figure 2: (a)–(c) 3D-shape of solution given in Eq. (21) for various $\delta = 0, 1, 2$. (d) 2D-shape for these values of δ. (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 2$, and (d) $\delta = 0, 1, 2$.

3.2 He's semi-inverse method

We derive the next variational formulations by using He's semi-inverse approach, which is described in previous studies [33–35]:

$$\mathbb{J}(\mathcal{P}) = \int_0^\infty \left[\frac{1}{2}(\mathcal{P}'')^2 - \frac{1}{2}\hbar_1(\mathcal{P}')^2 + \frac{1}{3}\hbar_2(\mathcal{P}')^3\right]\mathrm{d}\ell. \tag{29}$$

Following the form given by Ye and Mo [36], we assume the solution to (7) as

$$\mathcal{P}(\ell) = \mathcal{K}\operatorname{sech}(\ell), \tag{30}$$

where $\mathcal{K}$ is an unidentified constant. Plugging Eq. (30) into Eq. (29), we attain

$$\begin{aligned}\mathbb{J} &= \frac{1}{2}\mathcal{K}^2\int_0^\infty [\operatorname{sech}^2(\ell)\tanh^4(\ell) + \operatorname{sech}^4(\ell)\tanh^2(\ell) \\ &\quad + \operatorname{sech}^6(\ell) - \hbar_1\operatorname{sech}^2(\ell)\tanh^2(\ell) \\ &\quad + \frac{2}{3}\hbar_2\mathcal{K}\operatorname{sech}^3(\ell)\tanh^3(\ell)\Big]\mathrm{d}\ell \\ &= \frac{1}{2}\mathcal{K}^2\int_0^\infty [\operatorname{sech}^2(\ell) - \hbar_1\operatorname{sech}^2(\ell)\tanh^2(\ell) \\ &\quad + \frac{2}{3}\hbar_2\mathcal{K}\operatorname{sech}^3(\ell)\tanh^3(\ell)\Big]\mathrm{d}\ell \\ &= \frac{\mathcal{K}^2}{2} - \hbar_1\frac{\mathcal{K}^2}{6} - \frac{2}{45}\hbar_2\mathcal{K}^3.\end{aligned}$$

Making $\mathbb{J}$ stationary related to $\mathcal{K}$ as follows:

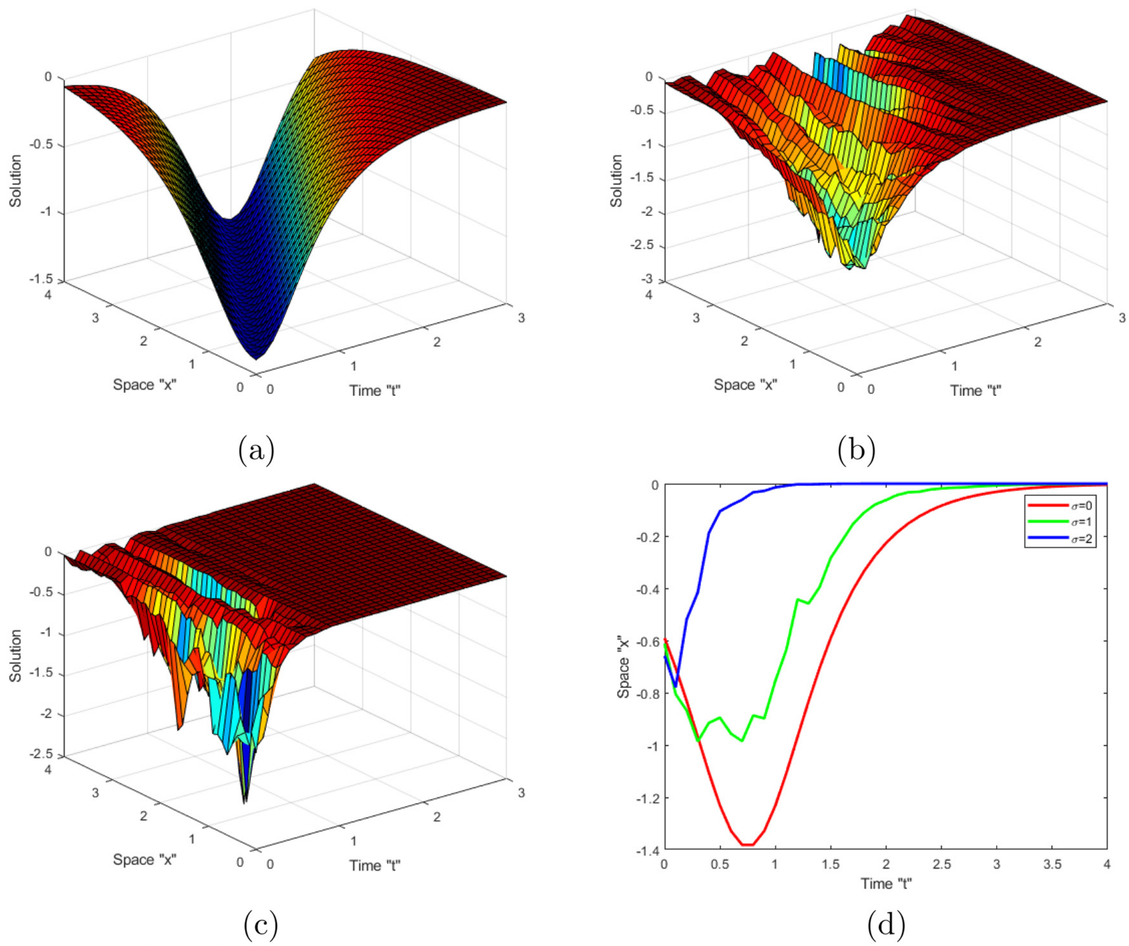

Figure 3: (a)–(c) 3D-shape of solution given in Eq. (32) for several $\delta = 0, 1, 2$. (d) 2D-shape for these values of δ. (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 2$, and (d) $\delta = 0, 1, 2$.

$$\frac{\partial \mathbb{J}}{\partial \mathcal{K}} = \left(1 - \frac{1}{3}\hbar_1\right)\mathcal{K} - \frac{2}{15}\hbar_2\mathcal{K}^2 = 0. \tag{31}$$

Solving Eq. (31) yields

$$\mathcal{K} = \frac{15 - 5\hbar_1}{2\hbar_2}.$$

Therefore, the solution of Eq. (7) is

$$\mathcal{P}(\ell) = \frac{15 - 5\hbar_1}{6\hbar_2}\operatorname{sech}(\ell).$$

Now, the solution of SCBSE (1) is

$$\mathcal{Y}(x, y, t) = \frac{15 - 5\hbar_1}{6\hbar_2}\operatorname{sech}(\ell_1 x + \ell_2 y + \ell_3 t)e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}. \tag{32}$$

We may do the same with the solution to Eq. (7) as

$$\mathcal{P}(\ell) = \mathcal{N}\operatorname{sech}(\ell)\tanh^2(\ell).$$

We obtain by repeating the previous techniques

$$\mathcal{N} = \frac{11(1{,}199 - 213\hbar_1)}{1456\hbar_2}.$$

So, the solution of SCBSE (1) is

$$\mathcal{Y}(x, y, t) = \frac{11(1{,}199 - 213\hbar_1)}{1{,}456\hbar_2}\operatorname{sech}(\ell)\tanh^2(\ell)e^{(\delta\mathcal{B}(t) - \frac{1}{2}\delta^2 t)}, \tag{33}$$

where $\ell = \ell_1 x + \ell_2 y + \ell_3 t$.

4 Impacts of Wiener process

We now investigate the impact of WP on the obtained solution of the SCBSE (1). Many graphs illustrating the performance of different solutions are given. Let us fix the parameters $\ell_1 = 1$, $\ell_2 = 1$, $\ell_3 = -2$, $y = 0$, $x \in [0, 4]$ and $t \in [0, 4]$, for some solutions that have been found, such as (21), (32), and (33), so that we can study them further. In the following figures, we can see the impact of noise on the solutions.

It can be seen from Figures 1–3 that there exist several solutions, such as dark, bright, periodic, kink, and others, when the noise disappeared (*i.e.*, at $\delta = 0$). After a few modest transit patterns, the surface obtains much flatter when noise appeared and the intensity is increased. This was confirmed using a 2D graph. This implies that the SCBSE solutions are affected by the Wiener process and are stabilized at zero.

5 Conclusion

We considered here the generalized (2+1)-dimensional SCBSE forced by multiplicative Brownian motion. The Riccati equation mapping and He's semi-inverse methods are used to obtain the solutions of the SCBSE in the form of rational, hyperbolic, and trigonometric functions. We expanded some solution from previous studies such as the solutions stated in previous studies [29–31]. The obtained solutions may be used to explain a wide variety of exciting physical phenomena because it is widely used in plasma physics and fluid dynamics. Finally, we created a huge number of 2D and 3D graphics to show the effect of the Wiener process on the analytical solutions of the SCBSE.

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