Research Article

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Analytical solutions of the extended Kadomtsev–Petviashvili equation in nonlinear media

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Abstract: This manuscript attempts to construct diverse exact traveling wave solutions for an important model called the (3+1)-dimensional Kadomtsev–Petviashvili equation. In order to achieve that, the Jacobi elliptic function technique and the Kudryashov technique are chosen in favor of their noticeable efficacy in dealing with nonlinear dynamical models. As expected, the used approaches lead to a variety of traveling wave solutions of different types. Finally, we have graphically illustrated some of the obtained wave solutions to further make sense of their representation. Also, we provide an overview of the main results at the end.

Keywords: traveling waves, exact solutions, Jacobi functions, exponential solution, KP equation

1 Introduction

Real and complex-valued evolutionary equations are a vital type of nonlinear partial differential equations (NPDEs) that appear in nonlinear sciences. The field of fluid dynamics is an example, where several nonlinear evolution models emerge as a result of modeling diverse physical processes of fluid propagation/transmission, traveling/flow, and other situations in various media. At this junction, it will be appropriate to mention some of the well-known nonlinear dynamical equations that play an imperative role in the evolution of fluid flow, including the Korteweg–de-Vries equations [1–4], the Ablowitz–Kaup–Newell–Segur equation [5], the Davey–Stewartson equation [6], and the Kadomtsev–Petviashvili (KP) equation [7–11] to mention a few. In particular, the equation of great concern in this study is the KP equation which was initiated in 1970 and serves as an important model in nonlinear wave theory [12]. Moreover, the model is also applicable for modeling water waves with insignificant surface tension as well as in modeling waves in thin films with significant surface tension [13]. Also, some of the applications of the model are in plasma physics (see [13]).

The literature on nonlinear evolution equations contains numerous mathematical approaches for the complete treatment of their resulting exact solutions, which are commonly referred to as solitary wave solutions [14,15]. Here, let us recall some of these famous analytical approaches such as the extended auxiliary equation method [16,17], the extended simplest equation method [18], the Jacobi elliptic function method [19], the Kudryashov method [20], and others (see [21–31]). On the other hand, some of the notable numerical approaches are used and developed in the treatment of nonlinear evolution equations and nonlinear fractional differential equations, including the homotopy analysis method [32], the Laplace-homotopy perturbation method [33], and the Adomian decomposition method [34] (see [35,36]).

Exact solutions of NPDEs may shed light on our understanding of many nonlinear phenomena. Moreover, they can be used to verify the accuracy and quantify the errors of different numerical, asymptotic, and approximate analytical methods. This article aims at constructing diverse exact traveling wave solutions of the KP equation [9–11], *via* the application of two promising analytical approaches. The approaches are the Jacobi elliptic function method [19] (also called the modified auxiliary equation method [16,17]) and the Kudryashov technique [20]. The choice of these approaches is motivated by their noticeable efficiency in tackling different classes of NPDEs. Moreover, it is very pertinent to note here that the Jacobi elliptic function technique gives generalized solutions that can be recast to several periodic and hyperbolic function solutions upon playing with the Jacobi elliptic parameter involved.

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Thus, the novelty of this work is to go beyond what is known in the literature by finding exact solutions in terms of Jacobi functions. In addition, the exact traveling wave solutions to be acquired will be systematically examined with regard to their constrain conditions (when existed). We also plot the three-dimensional plots of certain solutions, besides the description of the shapes of the constructed exact profiles. This study takes the following organization. Section 2 contains some details about the adopted methodologies. Section 3 contains the application of the adopted methodologies on the KP equation; however, Sections 4 and 5 give the discussion of results and the conclusion, respectively.

2 Analytical techniques

This section gives the steps for finding a number of exact solutions *via* the application of the Jacobi elliptic function technique [19], also called the modified auxiliary equation technique [16,17] and the Kudryashov technique [20].

Therefore, we first consider the following NPDE:

$$P_1(u, u_x, u_y, u_z, u_t, u_{xt}, u_{yt}, u_{zt}, u_{xx}, \ldots) = 0, \tag{1}$$

the steps of the techniques are presented in what follows:

Step 1. We start by assuming that:

$$u(x, y, z, t) = U(\zeta), \quad \zeta = x + ay + bz + ct, \tag{2}$$

where a, b, and c are the constants. Therefore, on using the transformation given in Eq. (3) into Eq. (1), the following nonlinear ordinary differential equation is obtained:

$$P_2(U, U', U'', U''' \ldots) = 0. \tag{3}$$

Step 2. Jacobi elliptic function technique

The Jacobi elliptic function technique assumes that the solution of Eq. (3) has the form:

$$U(\zeta) = \sum_{i=-n}^{n} \mu_i \ \psi^i(\zeta), \tag{4}$$

where μ_i's are the arbitrary undetermined constants, and n is a positive integer. Also, $\psi(\zeta)$ is said to be satisfied by the following equation:

$$\psi'^2(\zeta) = \alpha_0 + \alpha_1\psi^2(\zeta) + \alpha_2\psi^4(\zeta), \tag{5}$$

where α_0, α_1, and α_2 are the constants. Also, Eq. (5) satisfies the following solution cases:

Case 1. When $\alpha_0 = 1$, $\alpha_1 = -(1 + k^2)$, $\alpha_2 = k^2$, then Eq. (5) admits the following solution:

$$\psi(\zeta) = \mathrm{sn}(\zeta, k), \tag{6}$$

where $\mathrm{sn}(\zeta, k)$ is the Jacobi function and k represents the elliptic modulus such that $0 < k < 1$.

Case 2. When $\alpha_0 = 1 - k^2$, $\alpha_1 = 2k^2 - 1$, and $\alpha_2 = -k^2$, then Eq. (5) admits the following solution:

$$\psi(\zeta) = \mathrm{cn}(\zeta, k), \tag{7}$$

and $\mathrm{cn}(\zeta, k)$ is the Jacobi function, and k is as stated earlier.

Case 3. When $\alpha_0 = k^2 - 1$, $\alpha_1 = 2 - k^2$, and $\alpha_2 = -1$, then Eq. (5) admits the following solution:

$$\psi(\zeta) = \mathrm{dn}(\zeta, k), \tag{8}$$

where $\mathrm{dn}(\zeta, k)$ is the Jacobi function.

Case 4. When $\alpha_0 = k^2$, $\alpha_1 = -(1 + k^2)$, and $\alpha_2 = 1$, then Eq. (5) admits the following solution:

$$\psi(\zeta) = \mathrm{ns}(\zeta, k), \tag{9}$$

where $\mathrm{ns}(\zeta, k)$ is also the Jacobi function.

Case 5. When $\alpha_0 = 1 - k^2$, $\alpha_1 = 2 - k^2$, and $\alpha_2 = 1$, then Eq. (5) admits the solution of the form:

$$\psi(\zeta) = \mathrm{cs}(\zeta, k), \tag{10}$$

where $\mathrm{cs}(\zeta, k)$ is the Jacobi function *cs*.

Case 6. When $\alpha_0 = 1$, $\alpha_1 = 2k^2 - 1$, and $\alpha_2 = k^2(k^2 - 1)$, then Eq. (5) admits the solution of the form:

$$\psi(\zeta) = \mathrm{sd}(\zeta, k), \tag{11}$$

where $\mathrm{sd}(\xi, k)$ is the Jacobi function *sd*.

Step 3: Kudryashov technique

The Kudryashov technique expresses the solution of Eq. (3) in the form:

$$U(\zeta) = \sum_{i=0}^{n} \mu_i \Phi^i(\zeta), \tag{12}$$

where μ_i's are non-zero constants, and n is a positive integer. Moreover, $\Phi(\zeta)$ is said to be satisfied by the following equation:

$$\Phi'(\zeta) = \Phi^2(\zeta) - \Phi(\zeta), \tag{13}$$

In addition, Eq. (13) admits the following solution:

$$\Phi(\zeta) = \frac{1}{he^{\zeta} + 1}, \tag{14}$$

where h is an arbitrary constant.

Step 4. The value of n in both Eqs (4) and (12) is carried out by the use of homogeneous balancing principle.

Step 5. Finally, upon substituting Eqs (4) or (12), as the case may be using the Jacobi elliptic function technique or the Kudryashov technique, together with Eq. (5) or Eq. (13) into Eq. (3) and equating all the coefficients of different powers of $\psi(\zeta)$ or $\Phi(\zeta)$ to zero, we find a system of algebraic equations for μ_i. Accordingly, a solution of Eq. (1) is therefore determined.

3 Exact solutions for the extended KP equation

Consider the extended (3+1)-dimensional KP given by [9–11]:

$$(u_t + 6uu_x + u_{xxx})_x + \beta(u_{yy} + u_{zz}) = 0, \tag{15}$$

where β is a real constant.

Therefore, upon using the transformation given in Eq. (3) that reads

$$u(x, y, z, t) = U(\zeta), \quad \zeta = x + ay + bz + ct,$$

where a, b, and c are the constants to be evaluated; thereafter, Eq. (15) thus transforms into the following:

$$cU'' + 6(UU'' + U'^2) + U'''' + \beta(a^2U'' + b^2U'') = 0. \tag{16}$$

Next, upon making use of the homogeneous balancing principle on Eq. (16), we find $n = 2$.

3.1 Exact solutions *via* Jacobi elliptic function technique

Therefore, the solution of Eq. (16) is expressed *via* the application of the Jacobi elliptic function technique as follows:

$$U(\zeta) = \mu_0 + \mu_1\psi(\zeta) + \mu_2\psi^2(\zeta) + \frac{\mu_{-1}}{\psi(\zeta)} + \frac{\mu_{-2}}{\psi^2(\zeta)}, \tag{17}$$

where $\psi(\zeta)$ is said to satisfy Eq. (5). Then, on putting Eq. (17) with the use of Eq. (5) into Eq. (16) and thereafter putting the coefficients of $\psi(\zeta)$ to zero, we obtain a system of algebraic equations. Solving this system for $\mu_0, \mu_1, \mu_2, \mu_{-1}$, and μ_{-2}, the following solution sets are acquired:

Set 1.

$$\mu_0 = \frac{1}{6}(a^2(-\beta) - 4\alpha_1 - b^2\beta - c), \mu_1 = 0, \mu_2 = 0, \\ \mu_{-1} = 0, \mu_{-2} = -2\alpha_0. \tag{18}$$

Using the result in Eq. (18), we obtain the following.

Case 1. If $\alpha_0 = 1, \alpha_1 = -(1 + k^2)$, and $\alpha_2 = k^2$, then the KP equation in Eq. (15) has a solution in the form:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12}{\mathrm{sn}(\zeta, k)^2} - c + 4k^2 + 4\right). \tag{19}$$

Therefore, the aforementioned solution becomes the following:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12\coth^2(ay + bz + ct \\ + x) - c + 8) \tag{20}$$

and

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12\csc^2(ay + bz + ct \\ + x) - c + 4), \tag{21}$$

when $k \to 1$ and $k \to 0$, respectively. In addition, the three-dimensional (3D) plots are depicted in Figure 1. The figure describes the solutions determined in Eqs (20) and (21), for graphical visualization.

Case 2. If $\alpha_0 = 1 - k^2, \alpha_1 = 2k^2 - 1$, and $\alpha_2 = -k^2$, then Eq. (15) admits the following solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) + \frac{12(k^2 - 1)}{\mathrm{cn}(\zeta, k)^2} - c - 8k^2 + 4\right). \tag{22}$$

Moreover, the aforementioned solution transforms into the following:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12\sec^2(ay + bz + ct \\ + x) - c + 4), \tag{23}$$

when $k \to 0$.

Case 3. If $\alpha_0 = k^2 - 1, \alpha_1 = 2 - k^2$, and $\alpha_2 = -1$, in this case Eq. (15) has a solution in the following form:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12(k^2 - 1)}{\mathrm{dn}(\zeta, k)^2} - c + 4k^2 - 8\right). \tag{24}$$

Case 4. If $\alpha_0 = k^2, \alpha_1 = -(1 + k^2)$, and $\alpha_2 = 1$, then Eq. (15) satisfies the following solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12k^2}{\mathrm{ns}(\zeta, k)^2} - c + 4k^2 + 4\right). \tag{25}$$

This solution reduces to:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12\tanh^2(ay + bz + ct \\ + x) - c + 8), \tag{26}$$

when $k \to 1$. Figure 2 represents the solution given in Eq. (26).

Case 5. If $\alpha_0 = 1 - k^2, \alpha_1 = 2 - k^2$, and $\alpha_2 = 1$, then Eq. (15) has a solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12(1 - k^2)}{\mathrm{cs}(\zeta, k)^2} - c + 4k^2 - 8\right). \tag{27}$$

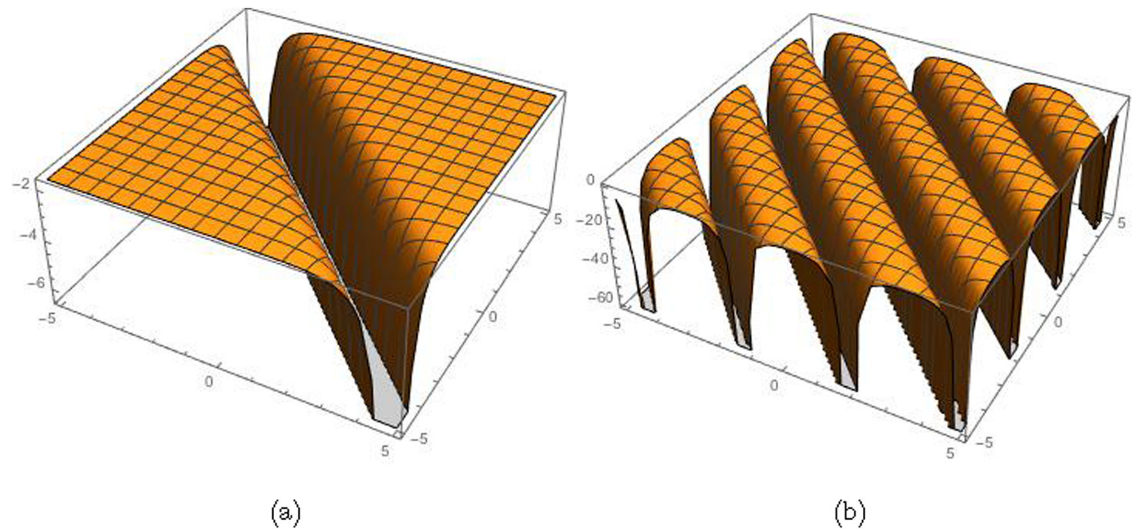


Figure 1: 3D plots (a) and (b) for solutions reported in Eqs (20) and (21), respectively, when $t = z = 0$ and $a = b = c = \beta = 1$.

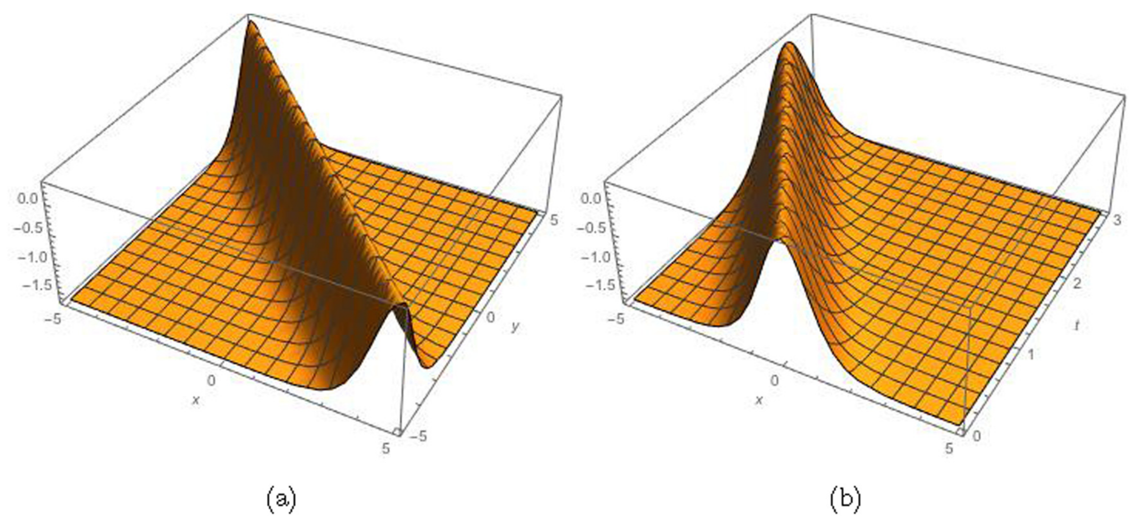


Figure 2: 3D plots for solutions reported in Eq. (26) when $z = 0$ and $a = b = c = \beta = 1$, at (a) $t = 0$ and (b) $y = 0$.

This solution reduces to:

$$u(x,y,z,t) = \frac{1}{6}(-\beta(a^2+b^2) - 12\tan^2(ay + bz + ct + x) - c - 8), \tag{28}$$

when $k \to 0$.

Case 6. If $\alpha_0 = 1$, $\alpha_1 = 2k^2 - 1$, and $\alpha_2 = k^2(k^2 - 1)$, then Eq. (15) satisfies the following solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2+b^2) - \frac{12}{\mathrm{sd}(\zeta,k)^2} - c - 8k^2 + 4\right). \tag{29}$$

Therefore, the aforementioned solution reduces to:

$$u(x,y,z,t) = \frac{1}{6}(-\beta(a^2+b^2) - 12\,\mathrm{csch}^2(ay + bz + ct + x) - c - 4), \tag{30}$$

when $k \to 1$.

Set 2.

$$\mu_0 = \frac{1}{6}(a^2(-\beta) - 4\alpha_1 - b^2\beta - c),\ \mu_1 = 0,\ \mu_{-2} = 0,\ \mu_{-1} = 0,\ \mu_2 = -2\alpha_2. \tag{31}$$

Using the result in Eq. (31), we obtain the following

Case 1. If $\alpha_0 = 1, \alpha_1 = -(1 + k^2)$, and $\alpha_2 = k^2$, then the KP equation in Eq. (15) has a solution in the form:

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) - 12k^2 \operatorname{sn}(\zeta, k)^2 - c + 4k^2 + 4). \quad (32)$$

Case 2. If $\alpha_0 = 1 - k^2, \alpha_1 = 2k^2 - 1$, and $\alpha_2 = -k^2$, then Eq. (15) admits the following solution:

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) + 12k^2 \operatorname{cn}(\zeta, k)^2 - c - 8k^2 + 4). \quad (33)$$

Then, we obtain the following explicit solution:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) + 12 \operatorname{sech}^2(ay + bz + ct + x) - c - 4), \quad (34)$$

when $k \to 1$.

Case 3. If $\alpha_0 = k^2 - 1, \alpha_1 = 2 - k^2$, and $\alpha_2 = -1$, then Eq. (15) has a solution in the following form:

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) + 12 \operatorname{dn}(\zeta, k)^2 - c + 4k^2 - 8). \quad (35)$$

Case 4. If $\alpha_0 = k^2, \alpha_1 = -(1 + k^2)$, and $\alpha_2 = 1$, then Eq. (15) satisfies the following solution:

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) - 12 \operatorname{ns}(\zeta, k)^2 - c + 4k^2 + 4). \quad (36)$$

Case 5. If $\alpha_0 = 1 - k^2, \alpha_1 = 2 - k^2$, and $\alpha_2 = 1$, then Eq. (15) has a solution:

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) - 12 \operatorname{cs}(\zeta, k)^2 - c + 4k^2 - 8). \quad (37)$$

Hence, the aforementioned solution transforms to the following:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12 \cot^2(ay + bz + ct + x) - c - 8), \quad (38)$$

when $k \to 0$.

Case 6. If $\alpha_0 = 1, \alpha_1 = 2k^2 - 1$, and $\alpha_2 = k^2(k^2 - 1)$, then Eq. (15) satisfies the following solution:

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) - 12k^2(k^2 - 1) \operatorname{sd}(\zeta, k)^2 - c - 8k^2 + 4). \quad (39)$$

Set 3.

$$\mu_0 = \frac{1}{6}(a^2(-\beta) - 4\alpha_1 - b^2\beta - c), \mu_1 = 0, \quad \mu_{-2} = -2\alpha_0, \mu_{-1} = 0, \mu_2 = -2\alpha_2. \quad (40)$$

Using the result in Eq. (40), we obtain the following.

Case 1. If $\alpha_0 = 1, \alpha_1 = -(1 + k^2)$, and $\alpha_2 = k^2$, then the KP equation in Eq. (15) has a solution in the form:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12}{\operatorname{sn}(\zeta, k)^2} - 12k^2 \operatorname{sn}(\zeta, k)^2 - c + 4k^2 + 4\right). \quad (41)$$

Therefore, from the aforementioned transformed solution, the following explicit solution is attained:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12 \coth^2(ay + bz + ct + x) - 12 \tanh^2(ay + bz + ct + x) - c + 8), \quad (42)$$

when $km \to 1$.

Case 2. If $\alpha_0 = 1 - k^2, \alpha_1 = 2k^2 - 1$, and $\alpha_2 = -k^2$, then Eq. (15) admits the following solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12(1 - k^2)}{\operatorname{cn}(\zeta, k)^2} + 12k^2 \operatorname{cn}(\zeta, k)^2 - c - 8k^2 + 4\right). \quad (43)$$

Case 3. If $\alpha_0 = k^2 - 1, \alpha_1 = 2 - k^2$, and $\alpha_2 = -1$, then Eq. (15) has a solution in the following form:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12(k^2 - 1)}{\operatorname{dn}(\zeta, k)^2} + 12 \operatorname{dn}(\zeta, k)^2 - c + 4k^2 - 8\right). \quad (44)$$

Case 4. If $\alpha_0 = m^2, \alpha_1 = -(1 + k^2)$, and $\alpha_2 = 1$, then Eq. (15) satisfies the following solution:

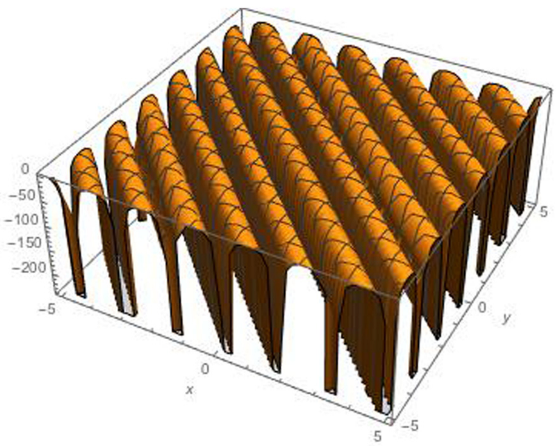
$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12k^2}{\operatorname{ns}(\zeta, k)^2} - 12 \operatorname{ns}(\zeta, k)^2 - c + 4k^2 + 4\right). \quad (45)$$

Case 5. If $\alpha_0 = 1 - k^2, \alpha_1 = 2 - k^2$, and $\alpha_2 = 1$, then Eq. (15) has a solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12(1 - k^2)}{\operatorname{cs}(\zeta, k)^2} - 12 \operatorname{cs}(\zeta, k)^2 - c + 4k^2 - 8\right). \quad (46)$$

Finally, the aforementioned transformed solution reduces to the following solution for the governing model:

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - 12 \cot^2(ay + bz + ct + x) - 12 \tan^2(ay + bz + ct + x) - c - 8), \quad (47)$$



(a)

Figure 3: 3D plot for solutions reported in Eq. (47) when $t = z = 0$ and $a = b = c = \beta = 1$.

when $k \to 0$. Figure 3 represents the solutions given in Eq. (47).

Case 6. If $\alpha_0 = 1$, $\alpha_1 = 2k^2 - 1$, and $\alpha_2 = k^2(k^2 - 1)$, then Eq. (15) satisfies the following solution:

$$u(\zeta) = \frac{1}{6}\left(-\beta(a^2 + b^2) - \frac{12}{\operatorname{sd}(\zeta, k)^2} - 12k^2(k^2 - 1)\operatorname{sd}(\zeta, k)^2 - c - 8k^2 + 4\right). \tag{48}$$

3.2 Exact solutions *via* Kudryashov technique

In the same manner, the KP equation admits the following solution form, *via* the application of the Kudryashov technique as follows:

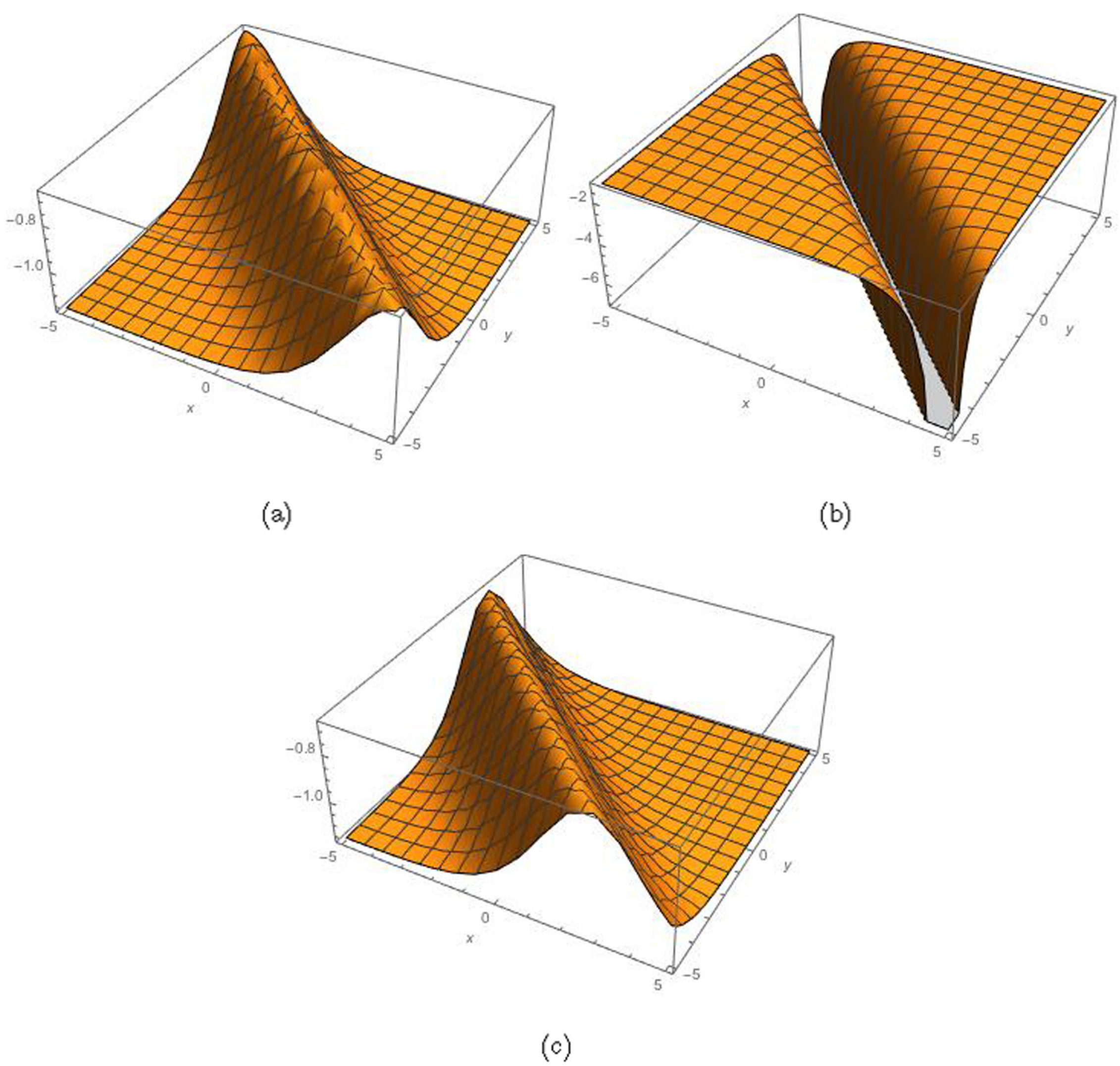


Figure 4: 3D plots for solutions reported in Eq. (52) when $t = z = 0$ and $a = b = c = \beta = 1$, at (a) $h = 1$, (b) $h = 1$, and (c) $h = 10$.

$$U(\zeta) = \mu_0 + \mu_1\Phi(\zeta) + \mu_2\Phi^2(\zeta), \tag{49}$$

where $\Phi(\zeta)$ is said to satisfy the differential equation earlier mentioned in Eq. (13). Then, on substituting Eq. (49) with the help of Eq. (13) into Eq. (16), and further equating the resulting coefficients of $\Phi(\zeta)$ to zero, we obtain a system of algebraic equations. Furthermore, solving the resultant system for μ_0, μ_1, and μ_2, we obtain the following solution case:

$$\mu_0 = \frac{1}{6}(-\beta(a^2 + b^2) - c - 1),\ \ \mu_1 = 2,\ \ \mu_2 = -2. \tag{50}$$

Using the aforementioned values into Eq. (49), we obtain

$$u(\zeta) = \frac{1}{6}(-\beta(a^2 + b^2) - c - 1) + \frac{2}{he^{\zeta} + 1} - \frac{2}{(he^{\zeta} + 1)^2}, \tag{51}$$

or more explicitly,

$$u(x, y, z, t) = \frac{1}{6}(-\beta(a^2 + b^2) - c - 1) + \frac{2}{he^{ay+bz+ct+x} + 1} - \frac{2}{(he^{ay+bz+ct+x} + 1)^2}, \tag{52}$$

where h is an arbitrary constant. Moreover, the aforementioned exponential solution (Eq. (52)) obtained *via* the application Kudryashov approach is shown in Figure 4 for different values of h.

4 Discussion

This study examines the extended KP equation by constructing diverse exact solutions with the use of two analytical approaches. The approaches of interest in this study are the Jacobi elliptic function method and the Kudryashov method, which are chosen owing to their successful applicability in treating classes of both real and complex-valued evolution equations – arising from dissimilar nonlinear processes, and the general nonlinear sciences. As expected, the approaches of choice revealed a variety of traveling wave solutions of different types. Starting with the Jacobi elliptic function technique, a lot of Jacobi elliptic function solutions generalizing related exact traveling wave solutions in the literature are obtained; moreover, these functions/solutions are recast to a series of periodic and hyperbolic structures upon playing with the generalized Jacobi parameter k. In addition, the Kudryashov technique gives only one exact traveling wave solution featuring exponential function; this is, however, known for the Kudryashov technique in disclosing pretty few exact solutions – see the application of the method in refs [15] and [20], among others, where few solutions are similarly disclosed by the approach. Finally, the current study also gives the 3D plots of some of the acquired solutions in Figures 1–4. Figure 1 shows the dark solution as in (a) and the periodic soliton solution as in (b), while Figure 2 represents the bright soliton solution. Figure 3 depicts the shape of periodic solution for Eq. (47). The solution in Eq. (52) is plotted for different values of h.

5 Conclusion

As a concluding note, this study has examined a universal nonlinear wave model called the extended KP equation, through the construction of diverse exact traveling wave solutions. The obtained exact solution was carried out *via* the application of two promising analytical approaches by the names the Jacobi elliptic function technique (also called the modified auxiliary equation technique) and the Kudryashov technique. The choice of these approaches was motivated by their noticeable efficacy in dealing with diverse nonlinear evolution and Schrodinger equations. As expected, the used approaches revealed different traveling wave solution cases involving a range of Jacobi functions using Jacobi elliptic techniques; however, an exponential traveling wave solution was obtained using the Kudryashov technique. It is important to state here that Jacobi elliptic function solutions reduce to hyperbolic and periodic solutions by varying the parameter k. Finally, we have graphically illustrated some of the obtained solutions to further make sense of their depictions. The used techniques are recommended to solve nonlinear equations.

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