Research Article

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Stability analysis, phase plane analysis, and isolated soliton solution to the LGH equation in mathematical physics

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Abstract: In this article, we investigated the Landau–Ginzburg–Higgs (LGH) equation, focusing on the analysis of isolated soliton solutions and their stability. To compute the isolated soliton solutions, we used the advanced auxiliary equation (AAE) approach, which has proven to be a powerful and efficient method for extracting soliton solutions in various nonlinear partial differential equations (NLPDEs). We provided a detailed explanation, both graphically and physically, of the obtained soliton solutions in this article. Furthermore, we used the linear stability technique to conduct a stability analysis of the LGH equation. Additionally, we studied the bifurcation and stability of the equilibria and performed phase plane analysis of the model. We also provided a discussion on the comparisons between the AAE method and two other well-known approaches: the generalized Kudryashov method and the improved Bernoulli sub-equation function method. The application of the AAE approach in this study demonstrates its effectiveness and capability in analysing and extracting soliton solutions in NLPDEs.

Keywords: LGH equation, AAE method, soliton solution, stability analysis

1 Introduction

One of the most important topics for studying nonlinear wave phenomena is the nonlinear evolution equations (NLEEs). It has significant implications for many fields of science and engineering, fluid mechanics, mathematical physics, mathematical biology, hydrodynamics, and many others [1–7]. Since NLEEs are very difficult to unravel, so many powerful analytical and numerical methods are developed and established for solutions such as the sine-Gordon technique [8], the improved F-expansion approach [9], the enhanced (G'/G)-expansion method [10,11], the binary Darboux transformation [12], the variational direct method [13], the extended version of exp $(-\psi(\kappa))$-expansion method [14], the Hirota direct methodology [15], the Lie symmetry approach [16,17], the extended Kudryashov method [18], the extended homoclinic test technique [19], the $(G'/G, 1/G)$ expansion approach [20], the meshless method [21], the Mohand variational transform method [22], the Paul–Painlevé approach method [23], the exact solution method [24], the optimal auxiliary function method [25], the extended simple equation technique [26], the Bernoulli sub-ordinary differential equation approach [27], the (w/g)-expansion method [28], the improved F-expansion and unified methods [29], and the modified version of the new Kudryashov method [30].

In the last few decades, Parkes and Duffy [31] introduced the tanh function method to generate the exact solutions for NLEEs. In the study by Fan [32], the Riccati equation $\varphi' = R + \varphi^2$ is considered and an extended tanh function method was proposed. This concept has been further developed and made clearer and more direct for a class of NLEEs by Yan [33] and Li *et al.* [34]. In addition, the modified

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extended tanh function method was applied to the nonlinear equations and incorporated the travelling wave solutions by Elwakil *et al.* [35]. Lu and Zhang [36] investigated the soliton solutions of the NLEEs using the extended tanh function method. Zhu [37] introduced the extended tanh function method by the generalized Riccati equation mapping method and obtained new non-travelling wave solutions. Recently, Khater *et al.* [38–40] proposed and studied the different types of wave equations. But Zayed disproved some of the solutions of this method, which have been discussed in previously published studies [41–43]. At the same time, El-Ganaini and Zayed gave us the correct form of the solution in ref. [44]. To find a new solution for NLEEs, they have proposed various ansatz approaches based on the Riccati equation. We are aware that when choosing a direct method, the appropriate conversion must be made. But how to find more new solutions for NLEEs under the familiar ansatz seems to be more important. Therefore, in this manuscript, we have further improved the work done in Zhu [37] by introducing advanced auxiliary equation (AAE) method and 27 new solutions. All the corrected solutions made by El-Ganaini and Zayed [44] are in the AAE method. In this article, we consider the nonlinear wave equation of the Landau–Ginzburg–Higgs (LGH) [45,46] equation as:

$$u_{tt} - u_{xx} - m^2u + n^2u^3 = 0, \tag{1.1}$$

where $m, n \in R - \{0\}$. Some researchers have found new and more soliton solutions to the LGH equation with the tanh method [46] and Ansatz method [45]. Using the direct and unified algebraic method, various travelling wave solutions are constructed from the LGH equation [47]. In the study by Kundu [48], the soliton solutions have been explored and graphically analysed both linear and the nonlinear impact of Eq. (1.1) in quantum physics and also constructed the stable soliton solutions from the LGH equation [49,50]. Recently, Ahmad *et al.* [51] and Ali *et al.* [52] have investigated the LGH equation and obtained exact travelling wave solutions.

The objective of this work is to generate isolated closed-form soliton solutions to the LGH equation through the AAE scheme, as inspired by earlier works. We have also initiated the solution with rational, trigonometric, exponential, and hyperbolic function solutions including some free parameters, all of which have applications in a wide range of industries and engineering. The rest of this article is organized as follows: we propose an AAE method for obtaining soliton solutions from NLEEs. In Section 3, we used it to solve the LGH equation and used the results to deduce distinct solutions to the collection of various equations. Section 4 provides a graphic and physical explanation of the solutions that have been found. Section 5 presents an investigation of the stability of the LGH equation and finally, Section 6 discusses phase plane analysis before providing a conclusion.

2 Brief of the AAE method

Consider NLEEs in the following structure:

$$\mathcal{N}(u, u_t, u_x, u_{xx}, u_{tx}, u_{tt}, \ldots\ldots) = 0, \tag{2.1}$$

where $\mathcal{N}$ is a nonlinear polynomial function of wave function $u(x, t)$, including its disparate partial derivatives. We suppose that

$$u(x, t) = u(\Omega), \text{ and } \Omega = x - \sigma t, \tag{2.2}$$

where σ is the speed of the soliton. Eq. (2.2) is converted to Eq. (2.1) into a nonlinear ordinary differential equation (NODE) as:

$$£(u, u', u'', \cdots\cdots) = 0, \tag{2.3}$$

where the prime represents the derivative of Ω. According to the AAE method, the solution of Eq. (2.3) is conjectured to be

$$u(\Omega) = \sum_{i=0}^{Y} M_i q^{ik(\Omega)}, \tag{2.4}$$

where the constants $M_0, M_1, M_2, \ldots\ldots, M_Y$ are unknown and to be calculated later, such that $M_Y \neq 0$. According to the balanced theorem, we obtain the value of Y in Eq. (2.4). $k(\Omega)$ is the solution of the equation:

$$k'(\Omega) = \frac{1}{\ln(q)}\{\mu q^{-k(\Omega)} + \gamma + ƙq^{k(\Omega)}\}. \tag{2.5}$$

In this step, we substitute Eqs. (2.4) and (2.5) into Eq. (2.3) and we obtain an algebraic equation, which is equated left and right sides based on powers of $q^{ik(\Omega)}$, ($i = 0, 1, 2, 3 \ldots\ldots$). As a result, we gain an algebraic equation. Solving these algebraic equations, we find out the values of $M_0, M_1, M_2, \ldots\ldots M_Y$ and σ. The solutions of Eq. (2.5) are obtained as follows:

Case 1: when $\gamma^2 - 4\mu ƙ < 0$ and $ƙ \neq 0$,

$$q^{k(\Omega)} = \frac{-\gamma}{2ƙ} + \frac{\sqrt{4\mu ƙ - \gamma^2}}{2ƙ}\tan\left(\frac{\sqrt{4\mu ƙ - \gamma^2}}{2}\Omega\right), \tag{2.5.1}$$

or

$$q^{k(\Omega)} = \frac{-\gamma}{2ƙ} - \frac{\sqrt{4\mu ƙ - \gamma^2}}{2ƙ}\cot\left(\frac{\sqrt{4\mu ƙ - \gamma^2}}{2}\Omega\right). \tag{2.5.2}$$

Case 2: when $\gamma^2 - 4\mu\kappa > 0$ and $\kappa \neq 0$,

$$q^{k(\Omega)} = \frac{-\gamma}{2\kappa} - \frac{\sqrt{4\mu\kappa - \gamma^2}}{2\kappa}\tanh\left(\frac{\sqrt{4\mu\kappa - \gamma^2}}{2}\Omega\right), \quad (2.5.3)$$

or

$$q^{k(\Omega)} = \frac{-\gamma}{2\kappa} - \frac{\sqrt{4\mu\kappa - \gamma^2}}{2\kappa}\coth\left(\frac{\sqrt{4\mu\kappa - \gamma^2}}{2}\Omega\right). \quad (2.5.4)$$

Case 3: when $\gamma^2 + 4\mu^2 < 0$, $\kappa \neq 0$ and $\kappa = -\mu$,

$$q^{k(\Omega)} = \frac{\gamma}{2\mu} - \frac{\sqrt{-\gamma^2 - 4\mu^2}}{2\mu}\tan\left(\frac{\sqrt{-\gamma^2 - 4\mu^2}}{2}\Omega\right), \quad (2.5.5)$$

or

$$q^{k(\Omega)} = \frac{\gamma}{2\mu} + \frac{\sqrt{-\gamma^2 - 4\mu^2}}{2\mu}\cot\left(\frac{\sqrt{-\gamma^2 - 4\mu^2}}{2}\Omega\right). \quad (2.5.6)$$

Case 4: when $\gamma^2 + 4\mu^2 > 0$, $\kappa \neq 0$ and $\kappa = -\mu$,

$$q^{k(\Omega)} = \frac{\gamma}{2\mu} + \frac{\sqrt{\gamma^2 + 4\mu^2}}{2\mu}\tanh\left(\frac{\sqrt{\gamma^2 + 4\mu^2}}{2}\Omega\right), \quad (2.5.7)$$

or

$$q^{k(\Omega)} = \frac{\gamma}{2\mu} + \frac{\sqrt{\gamma^2 + 4\mu^2}}{2\mu}\coth\left(\frac{\sqrt{\gamma^2 + 4\mu^2}}{2}\Omega\right). \quad (2.5.8)$$

Case 5: when $\gamma^2 - 4\mu^2 < 0$ and $\kappa = \mu$,

$$q^{k(\Omega)} = \frac{-\gamma}{2\mu} + \frac{\sqrt{-\gamma^2 + 4\mu^2}}{2\mu}\tan\left(\frac{\sqrt{-\gamma^2 + 4\mu^2}}{2}\Omega\right), \quad (2.5.9)$$

or

$$q^{k(\Omega)} = \frac{-\gamma}{2\mu} - \frac{\sqrt{-\gamma^2 + 4\mu^2}}{2\mu}\cot\left(\frac{\sqrt{-\gamma^2 + 4\mu^2}}{2}\Omega\right). \quad (2.5.10)$$

Case 6: when $\gamma^2 - 4\mu^2 > 0$ and $\kappa = \mu$,

$$q^{k(\Omega)} = \frac{-\gamma}{2\mu} - \frac{\sqrt{\gamma^2 - 4\mu^2}}{2\mu}\tanh\left(\frac{\sqrt{\gamma^2 - 4\mu^2}}{2}\Omega\right), \quad (2.5.11)$$

or

$$q^{k(\Omega)} = \frac{-\gamma}{2\mu} - \frac{\sqrt{\gamma^2 - 4\mu^2}}{2\mu}\coth\left(\frac{\sqrt{\gamma^2 - 4\mu^2}}{2}\Omega\right). \quad (2.5.12)$$

Case 7: when $\gamma^2 = 4\mu\kappa$,

$$q^{k(\Omega)} = -\frac{2 + \gamma\Omega}{2\kappa\Omega}. \quad (2.5.13)$$

Case 8: when $\mu\kappa < 0$, $\gamma = 0$, and $\kappa \neq 0$,

$$q^{k(\Omega)} = -\sqrt{\frac{-\mu}{\kappa}}\tanh(\sqrt{-\mu\kappa}\,\Omega), \quad (2.5.14)$$

or

$$q^{k(\Omega)} = -\sqrt{\frac{-\mu}{\kappa}}\coth(\sqrt{-\mu\kappa}\,\Omega). \quad (2.5.15)$$

Case 9: when $\gamma = 0$ and $\mu = -\kappa$,

$$q^{k(\Omega)} = \frac{1 + e^{(-2\kappa\ \ \Omega)}}{-1 + e^{(-2\kappa\ \ \Omega)}}. \quad (2.5.16)$$

Case 10: when $\mu = \kappa = 0$,

$$q^{k(\Omega)} = \cosh(\gamma\Omega) + \sinh(\gamma\Omega). \quad (2.5.17)$$

Case 11: when $\mu = \gamma = \varphi$ and $\kappa = 0$,

$$q^{k(\Omega)} = e^{\varphi\Omega} - 1. \quad (2.5.18)$$

Case 12: when $\gamma = \kappa = \varphi$ and $\mu = 0$,

$$q^{k(\Omega)} = \frac{e^{\varphi\Omega}}{1 - e^{\varphi\Omega}}. \quad (2.5.19)$$

Case 13: when $\gamma = (\mu + \kappa)$,

$$q^{k(\Omega)} = -\frac{1 - \mu e^{(\mu-\kappa)\Omega}}{1 - \kappa e^{(\mu-\kappa)\Omega}}. \quad (2.5.20)$$

Case 14: when $\gamma = -(\mu + \kappa)$,

$$q^{k(\Omega)} = \frac{\mu - e^{(\mu-\kappa)\Omega}}{\kappa - e^{(\mu-\kappa)\Omega}}. \quad (2.5.21)$$

Case 15: when $\mu = 0$,

$$q^{k(\Omega)} = \frac{\gamma e^{\gamma\Omega}}{1 - \kappa e^{\gamma\Omega}}. \quad (2.5.22)$$

Case 16: when $\kappa = \gamma = \mu \neq 0$,

$$q^{k(\Omega)} = \frac{1}{2}\left\{\sqrt{3}\tan\left(\frac{\sqrt{3}}{2}\mu\Omega\right) - 1\right\}. \quad (2.5.23)$$

Case 17: when $\kappa = \gamma = 0$,

$$q^{k(\Omega)} = \mu\Omega. \quad (2.5.24)$$

Case 18: when $\nu = \gamma = 0$,

$$q^{k(\Omega)} = \frac{-1}{\kappa\Omega}. \quad (2.5.25)$$

Case 19: when $\kappa = \mu$ and $\gamma = 0$,

$$q^{k(\Omega)} = \tan(\mu\Omega). \quad (2.5.26)$$

Case 20: when $\kappa = 0$,

$$q^{k(\Omega)} = e^{\gamma\Omega} - \frac{a}{b}. \quad (2.5.27)$$

Substituting these values of $M_i (i = 0, 1, 2, ..., Y)$, μ, γ, k, and function $k(\Omega)$ into Eq. (2.4) produces numerous soliton solutions to Eq. (2.1) [53].

3 Solutions of the LGH equation

The general and broad-ranging closed-form steady soliton solutions to the LGH equation has been established, and implementation of the new auxiliary equation method is presented to the LGH equation in this section. We will explore the gigantic amount of soliton solution of the LGH equation, and all-wave phenomena play a significant role in the modern science and engineering.

By means of the wave renovation $u(x, t) = u(\Omega)$ and $\Omega = x - \sigma t$, then Eq. (1.1) is converted to the NODE as assumed:

$$(\sigma^2 - 1)u'' - m^2 u + n^2 u^3 = 0, \tag{3.1}$$

where the notation denotes the derivatives comprising the linear and nonlinear terms. Balancing u'' and u^3 yields $Y = 1$. The general solution of Eq. (3.1) is as follows:

$$U(x, t) = M_0 + M_1 a^{k(\Omega)}, \tag{3.2}$$

where $M_1 \neq 0$ and the solution of the nonlinear Eq. (2.5) is $k(\Omega)$. Substitute Eq. (3.2) in place of Eq. (3.1), and the coefficients of like terms $q^{ik(\Omega)} (i = 0, 1, 2, 3, 4)$ are equal to zero. Take the algebraic equations below:

$$\gamma\sigma^2\mu M_1 + n^2 M_0^3 - \gamma\mu M_1 - m^2 M_0 = 0,$$

$$\sigma^2 \gamma^2 M_1 + 2\sigma^2 k\mu M_1 + 3n^2 M_0^2 M_1 - \gamma^2 M_1 - 2k\mu M_1 - m^2 M_1 = 0,$$

$$3\sigma^2\gamma k M_1 + 3n^2 M_0 M_1^2 - 3\gamma k M_1 = 0,$$

$$2\sigma^2 k^2 M_1 + n^2 M_1^3 - 2k^2 M_1 = 0.$$

To solve the aforementioned algebraic system, we attain the solution set as:

$$\sigma = \sqrt{\frac{\gamma^2 - 4\mu k - 2m^2}{\gamma^2 - 4\mu k}}, M_0 = \pm\frac{\gamma m}{\sqrt{\gamma^2 - 4\mu k}\, n}, \text{and} M_1 = \pm\frac{2km}{\sqrt{\gamma^2 - 4\mu k}\, n}. \tag{3.3}$$

Substituting Eq. (3.3) into Eq. (3.2) and along with Eqs. (2.5.1)–(2.5.27), we obtain the soliton solutions from the LGH equation and the resulting solutions are listed below in different clusters.

Cluster 1: when $\gamma^2 - 4\mu k < 0$ and $k \neq 0$, we acquire

$$u_{1,2}(\Omega) = -\frac{m\sqrt{4\mu k - \gamma^2}}{n\sqrt{\gamma^2 - 4\mu k}} \times \tan\left(\frac{\sqrt{4\mu k - \gamma^2}}{2}\Omega\right), \tag{3.4}$$

$$u_{3,4}(\Omega) = -\frac{m\sqrt{4\mu k - \gamma^2}}{n\sqrt{\gamma^2 - 4\mu k}} \times \cot\left(\frac{\sqrt{4\mu k - \gamma^2}}{2}\Omega\right), \tag{3.5}$$

where $\Omega = \sqrt{\frac{\gamma^2 - 4\mu k - 2m^2}{\gamma^2 - 4\mu k}}\, t \pm x$.

Cluster 2: when $\gamma^2 - 4\mu k > 0$ and $k \neq 0$, we have

$$u_{5,6}(\Omega) = \frac{m}{n} \times \tanh\left(\frac{\sqrt{\gamma^2 - 4\mu k}}{2}\Omega\right), \tag{3.6}$$

$$u_{7,8}(\Omega) = \frac{m}{n} \times \coth\left(\frac{\sqrt{\gamma^2 - 4\mu k}}{2}\Omega\right), \tag{3.7}$$

where $\Omega = \sqrt{\frac{\gamma^2 - 4\mu k - 2m^2}{\gamma^2 - 4\mu k}}\, t \pm x$.

Cluster 3: when $\gamma^2 + 4\mu^2 < 0, k \neq 0$ and $k = -\mu$, we reach

$$u_{9,10}(\Omega) = -\frac{m\sqrt{-\gamma^2 - 4\mu^2}}{n\sqrt{\gamma^2 + 4\mu^2}} \times \tan\left(\frac{\sqrt{-\gamma^2 - 4\mu^2}}{2}\Omega\right), \tag{3.8}$$

$$u_{11,12}(\Omega) = \frac{m\sqrt{-\gamma^2 - 4\mu^2}}{n\sqrt{\gamma^2 + 4\mu^2}} \times \cot\left(\frac{\sqrt{-\gamma^2 - 4\mu^2}}{2}\Omega\right), \tag{3.9}$$

where $\Omega = \sqrt{\frac{\gamma^2 + 4\mu^2 - 2m^2}{\gamma^2 + 4\mu^2}}\, t \pm x$.

Cluster 4: when $\gamma^2 + 4\mu^2 > 0, k \neq 0$ and $k = -\mu$, we attain

$$u_{13,14}(\Omega) = \frac{m}{n} \times \tanh\left(\frac{\sqrt{\gamma^2 + 4\mu^2}}{2}\Omega\right), \tag{3.10}$$

$$u_{15,16}(\Omega) = \frac{m}{n} \times \coth\left(\frac{\sqrt{\gamma^2 + 4\mu^2}}{2}\Omega\right), \tag{3.11}$$

where $\Omega = \sqrt{\frac{\gamma^2 + 4\mu^2 - 2m^2}{\gamma^2 + 4\mu^2}}\, t \pm x$.

Cluster 5: when $\gamma^2 - 4\mu^2 < 0$ and $k = \mu$, we gain

$$u_{17,18}(\Omega) = -\frac{m\sqrt{4\mu^2 - \gamma^2}}{n\sqrt{\gamma^2 - 4\mu^2}} \times \tan\left(\frac{\sqrt{4\mu^2 - \gamma^2}}{2}\Omega\right), \tag{3.12}$$

$$u_{19,20}(\Omega) = \frac{m\sqrt{4\mu^2 - \gamma^2}}{n\sqrt{\gamma^2 - 4\mu^2}} \times \cot\left(\frac{\sqrt{4\mu^2 - \gamma^2}}{2}\Omega\right), \tag{3.13}$$

where $\Omega = \sqrt{\frac{\gamma^2 - 4\mu^2 - 2m^2}{\gamma^2 - 4\mu^2}}\, t \pm x$.

Cluster 6: when $\gamma^2 - 4\mu^2 > 0$ and $k = \mu$, we attain

$$u_{21,22}(\Omega) = \frac{m}{n} \times \tanh\left(\frac{\sqrt{\gamma^2 - 4\mu^2}}{2}\Omega\right), \tag{3.14}$$

$$u_{23,24}(\Omega) = \frac{m}{n} \times \coth\left(\frac{\sqrt{\gamma^2 - 4\mu^2}}{2}\Omega\right), \tag{3.15}$$

where $\Omega = \sqrt{\frac{\gamma^2 - 4\mu^2 - 2m^2}{\gamma^2 - 4\mu^2}}\, t \pm x$.

Cluster 7: when $\mu k < 0$, $\gamma = 0$ and $k \neq 0$, we gain

$$u_{25,26}(\Omega) = \frac{m}{n} \times \tanh\left(\frac{\sqrt{-\mu k}}{2}\Omega\right), \tag{3.16}$$

$$u_{27,28}(\Omega) = \frac{m}{n} \times \coth\left(\frac{\sqrt{-\mu k}}{2}\Omega\right), \tag{3.17}$$

where $\Omega = \sqrt{\frac{4\mu k + 2m^2}{\mu k}}\, t \pm 2x$.

Cluster 8: when $\gamma = 0$ and $\mu = -k$, the solution is

$$u_{29,30}(\Omega) = \frac{m}{n} \times \frac{1 + e^{k\Omega}}{-1 + e^{k\Omega}}, \tag{3.18}$$

where $\Omega = \pm\sqrt{\frac{4k^2 - 2m^2}{k^2}}\, t - 2x$.

Cluster 9: when $\gamma = k = \varphi$ and $\mu = 0$,

$$u_{31,32}(\Omega) = \frac{m}{n} \times \frac{e^{\varphi\Omega} + 1}{e^{\varphi\Omega} - 1}, \tag{3.19}$$

where $\Omega = x \pm \sqrt{\frac{\varphi^2 - 2m^2}{\varphi^2}}\, t$.

Cluster 10: when $\gamma = (\mu + k)$, we acquire

$$u_{33,34}(\Omega) = \frac{m}{n} \times \frac{ke^{(k-\mu)\Omega} + 1}{ke^{(k-\mu)\Omega} - 1}, \tag{3.20}$$

where $\Omega = \pm\sqrt{\frac{k^2 - 2\mu k - 2m^2 + \mu^2}{(k - \mu)^2}}\, t - x$.

Cluster 11: when $\gamma = -(\mu + k)$, we acquire

$$u_{35,36}(\Omega) = \frac{m}{n} \times \frac{k + e^{(k-\mu)\Omega}}{-k + e^{(k-\mu)\Omega}}, \tag{3.21}$$

where $\Omega = \pm\sqrt{\frac{k^2 - 2\mu k - 2m^2 + \mu^2}{(k - \mu)^2}}\, t - x$.

Cluster 12: when $\mu = 0$, we obtain

$$u_{37,38}(\Omega) = \frac{m}{n} \times \frac{ke^{\Omega} + 1}{ke^{\Omega} - 1}, \tag{3.22}$$

where $\Omega = \pm\sqrt{\gamma^2 - 2m^2}\quad t + x\gamma$.

Cluster 13: when $k = \gamma = \mu \neq 0$, we become

$$u_{39,40}(\Omega) = -\frac{\mu m}{\sqrt{-\mu^2}\, n} \times \tan\left(\frac{\sqrt{3}\mu}{6}\Omega\right), \tag{3.23}$$

where $\Omega = \sqrt{\frac{6m^2 + 9\mu^2}{\mu^2}}\, t \pm 3x$.

Cluster 14: when $k = \Upsilon$ and $\gamma = 0$, we become

$$u_{41,42}(\Omega) = -\frac{\mu m}{\sqrt{-\mu^2}\, n} \times \tan\left(\frac{\mu}{2}\Omega\right), \tag{3.24}$$

where $\Omega = \sqrt{\frac{2m^2 + 4\mu^2}{\mu^2}}\, t \pm 2x$.

When applying these three conditions, namely, $=k = 0$, $\mu = \gamma = \varphi$, and $k = 0$ and $k = 0$, we have found a constant function solution, which is not written in this literature because it has no physical sense.

Comparison: Now, we will compare the AAE method, the generalized Kudryashov method [56], and the improved Bernoulli sub-equation function method [49], through the solutions of LGH equation. The generalized Kudryashov method and the improved Bernoulli sub-equation function method are two distinct approaches used to analyse equations in mathematical physics, each with its characteristics and advantages.

i. Barman *et al.* investigated the LGH equation using the generalized Kudryashov method in their research [56]. They identified three solutions and derived 11 additional solutions for specific parameter values. Nevertheless, in this article, we used the AAE method and discovered 21 unique solutions expressed in hyperbolic, trigonometric, and exponential functions. The solutions we obtained in our study differ from those presented by Barman *et al.* [56].
ii. Islam and Akbar [49], on the other hand, used the improved Bernoulli sub-equation function method to analyse the LGH equation in their study. They were able to find only two hyperbolic solutions. By introducing the parameter $\sigma = \frac{\sqrt{(\gamma^2 - 4\mu k)}}{2}$, the solutions (Eqs. (19) or (22) and Eqs. (20) or (23)) are in [49] align with our solutions (3.7) and (3.6). In addition, using the AAE method, we derived a total of 21 solutions expressed in hyperbolic, trigonometric, and exponential functions. Consequently, our AAE method provided 19 novel solutions for the LGH equation when compared to the improved Bernoulli sub-equation function method.

Remark: We were able to simplify the obtained solutions and verified the accuracy with the help of Maple. All solutions are corrected.

4 Graphical and physical interpretation of the LGH equation

In this segment, we will transform exactly the properties of the obtained solution and graphically display the results. It can be noted that we have found unambiguous and steady

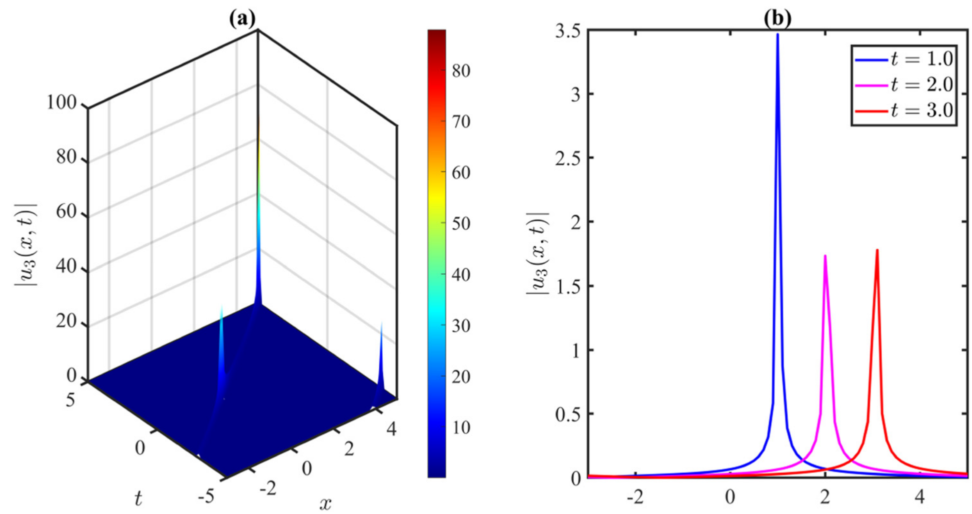

Figure 1: 3D wave profile and 2D combined chart of the solution $u_3(x, t)$.

wave solutions in different forms from the LGH equations. The LGH equation gives us various types of solutions as rational function solutions, hyperbolic function solutions, trigonometric function solutions, and exponential function solutions. Using the many precise values of the applicable parameters for each solution, we generate the soliton profile. The features of the computed results are drawn in 3D and 2D wave profiles. All solutions are given physical meaning and applied to the different branches such as plasma physics, optical fibres, and nonlinear optic and mathematical physics. Graphical and physical discussions are given below:

The solution (3.5) mollifies the ailment $\gamma^2 - 4\mu k < 0$ and $k \neq 0$. We have drawn the 3D and 2D wave phenomena of the solution (3.5), which is a trigonometric (cot) function. The 3D wave phenomena exhibit the wave propagation and periodic shape along the x and y axes, as shown in Figure 1(a) because of the values of $m = 0.1, n = 2, \gamma = 0.1, \mu = 0.25$, and $k = 0.5$ within the displacement $-5 \leq x, t \leq 5$. We noted that the phase component of the solution (3.5) is frequently different for different soliton values of t, as illustrated in Figure 1(b). In addition, the constant values of the parameters are selected, and the soliton is stable, which is shown in Figure 1(b).

In the circumstance of $\gamma^2 - 4\mu k > 0$ and $k \neq 0$, the solution of Eq. (3.6) represents the kink-type wave shape and wave propagation along with x and y axes conforming to the static parameters $m = 1, n = 2, \gamma = 8, \mu = 0.25$, and $k = 0.25$ within the boundary $-5 \leq x, t \leq 5$, as depicted in Figure 2(a). The wave shape comes from the hyperbolic trigonometric function. We observed that the phase component of the solution (3.6) remains unchanged and changed for the different parameters. In addition, the constant values of

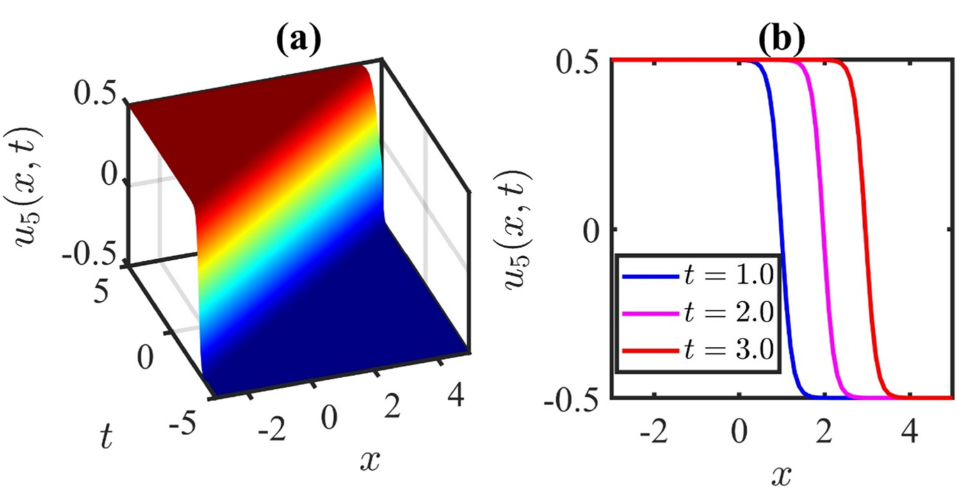

Figure 2: 3D wave profile and 2D combined chart of the solution $u_5(x, t)$.

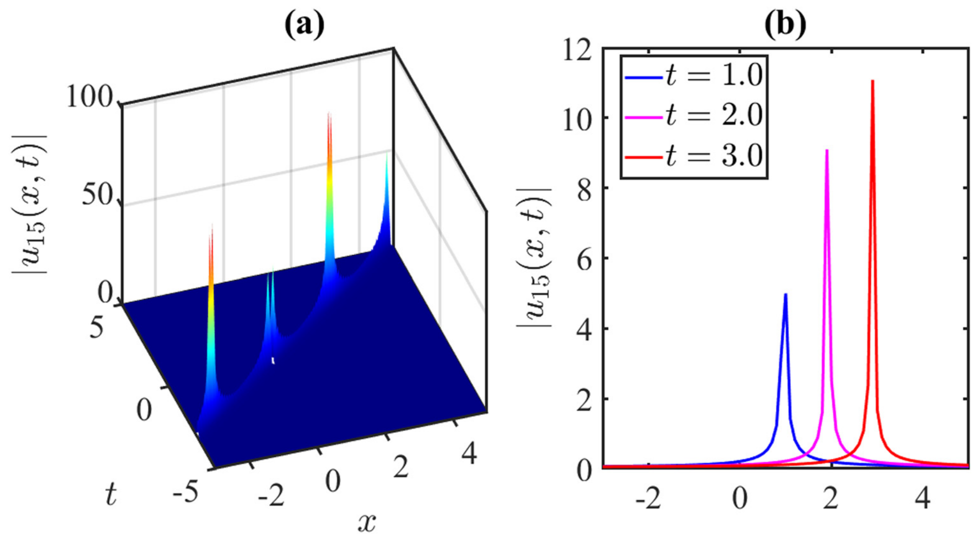

Figure 3: 3D wave profile and 2D combined chart of the solution $u_{15}(x, t)$.

the parameters are selected, and the soliton is stable, which is shown in Figure 2(b). Figure 2(b) displays the 2D wave profile of the same solution as Figure 2(a) in $-5 \le t \le 5$.

The 3D wave structure displays the physical appearance of wave phenomena of the NLEEs. The solution of Eq. (3.11) is a hyperbolic trigonometric function solution, which is embraced on condition $\gamma^2 + 4\mu^2 > 0, k \neq 0$, and $k = -\mu$. Figure 3(a) represents the 3D wave structure called the singular periodic wave profile and the wave propagation along x and y axes, for the parameters $m = 0.1, n = 2, \gamma = 0.1, \mu = 0.25$, and the limit $-5 \le x, t \le 5$. Figure 3(b) shows a 2D line diagram that illustrates the effects of the phase and amplitude component for various values of the soliton parameters.

Finally, in the case of $\mu k < 0, \gamma = 0$, and $k \neq 0$, the exact solution of Eq. (3.16) represents the wave performance. Figure 4(a) indicates the behaviour of Solution (3.16) that affords the 3D wave profile named kink shape and wave propagation along with x and y axes, for the parameters $m = 0.01, n = 0.02, k = 0.01$, and $\mu = -19.4$ and the range $-5 \le x, t \le 5$. The 2D line diagram displays the effects of the phase component for the different values of soliton parameters, which is portrayed in Figure 4(b). In addition, the constant values of the parameters are selected, and the soliton is stable, which is shown in Figure 4(b).

5 Stability analysis

This paragraph will analyse the stability [54,55] of the leading Eq. (1.1) along with the perturbed solution of the following form [54]:

$$u(x, t) = \alpha v(x, t) + V_0. \tag{5.1}$$

It is clear that any constant V_0 is a steady-state solution for (1.1). Persisting (5.1) into (1.1), one reaches at

$$\begin{aligned} &\alpha v_{tt} - \alpha v_{xx} - m^2\alpha v - m^2 V_0 + n^2\alpha^2 v^2 + 3n^2\alpha^2 v \\ &+ 3n^2\alpha v V_0 + n^2 V_0^3 = 0. \end{aligned} \tag{5.2}$$

Linearizing the aforementioned equation in α

$$\alpha v_{tt} - \alpha v_{xx} - m^2\alpha + 3n^2\alpha V_0 = 0, \tag{5.3}$$

and let that Eq. (5.3) has a solution as

$$v(x, t) = e^{i(Qx+\omega t)}, \tag{5.4}$$

where Q is the normalized wave number, plugging (5.4) into (5.3) and solving for ω, we obtain

$$\omega = -\sqrt{Q^2 - m^2 + 3n^2 V_0}. \tag{5.5}$$

The sign of the aforementioned relation is always negative for all values of $Q^2 - m^2 + 3n^2 V_0 > 0$, as can be seen from Eq. (5.5). Thus, the dispersion is stable.

6 Phase plane analysis

To begin the phase plane analysis for the LGH equation, we offer $u = \phi$ and $\phi' = \psi$ and can rewrite Eq. (3.2) as a first-order dynamical system of the following form:

$$\begin{cases} \phi' = \psi = f(\phi, \psi), \\ \psi' = \dfrac{m^2}{\sigma^2 - 1}\phi - \dfrac{n^2}{\sigma^2 - 1}\phi^3 = g(x, y). \end{cases} \tag{6.1}$$

The Hamiltonian function of the planar system Eq. (6.1) is

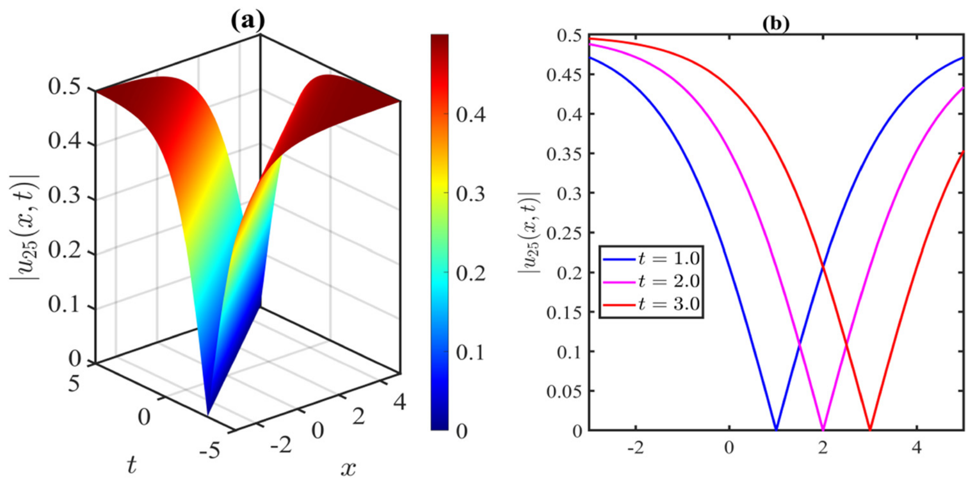

Figure 4: 3D wave profile and 2D combined chart of the solution $u_{25}(x, t)$.

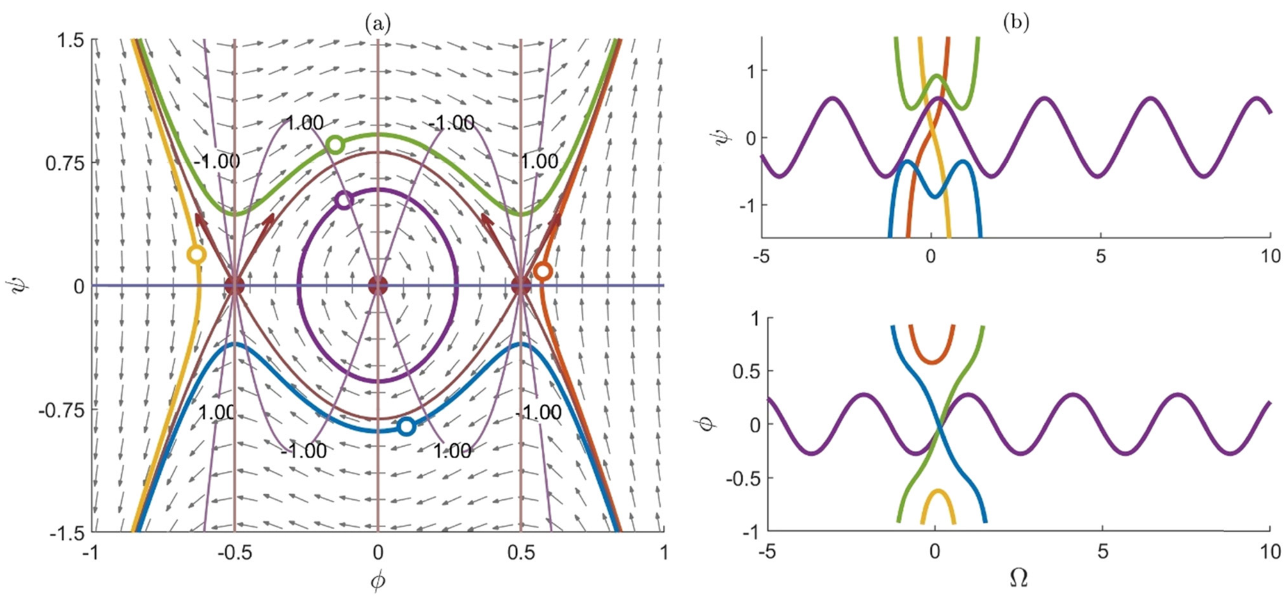

Figure 5: Phase portrait and corresponding solution of the planar system (3.25) for the values of $m = 1$, $n = 2$, $\sigma = 0.9$. The equilibria $(\pm 0.50, 0)$ are unstable saddle and $(0, 0)$ is a centre.

$$H(\phi, \psi) = \frac{\psi^2}{2} - \frac{m^2}{2(\sigma^2 - 1)}\phi^2 + \frac{n^2}{4(\sigma^2 - 1)}\phi^4 (=h). \quad (6.2)$$

The equilibria of the system Eq. (6.1) are $(0, 0)$ and $(\pm\frac{m}{n}, 0)$.

If either $m = 0$ or $n = 0$, then the planar system (6.1) provides only one equilibrium point $(0, 0)$.

Jacobian matrix of the system (6.1) is $J(\phi, \psi) = \begin{pmatrix} 0 & 1 \\ \frac{m^2}{\sigma^2-1} - \frac{3n^2}{\sigma^2-1}\phi^2 & 0 \end{pmatrix}$. The eigenvalues of J are given by $\det(J - \lambda I_{2\times 2}) = 0$, which implies

$$\lambda^2 - tr(J)\lambda + \det(J) = 0,$$

where $\mathrm{tr}(J) = 0$ and $\det(J) = \frac{3n^2}{\sigma^2-1}\phi^2 - \frac{m^2}{\sigma^2-1}$.

Case i. Stability of (0, 0): for this case if $\frac{m^2-3n^2}{\sigma^2-1} > 0$ and $\sigma \neq 1$, then the eigenvalues $\lambda_1 = \sqrt{\frac{m^2-3n^2}{\sigma^2-1}}$ and $\lambda_1 = -\sqrt{\frac{m^2-3n^2}{\sigma^2-1}}$ are the real, opposite sign. Hence, the equilibrium point $(0, 0)$ is an unstable saddle. If $\frac{m^2-3n^2}{\sigma^2-1} < 0$ and $\sigma \neq 1$, then the eigenvalues $\lambda_1 = i\sqrt{\frac{m^2-3n^2}{\sigma^2-1}}$ and $\lambda_1 = -i\sqrt{\frac{m^2-3n^2}{\sigma^2-1}}$ are pure imaginary. And so the equilibrium

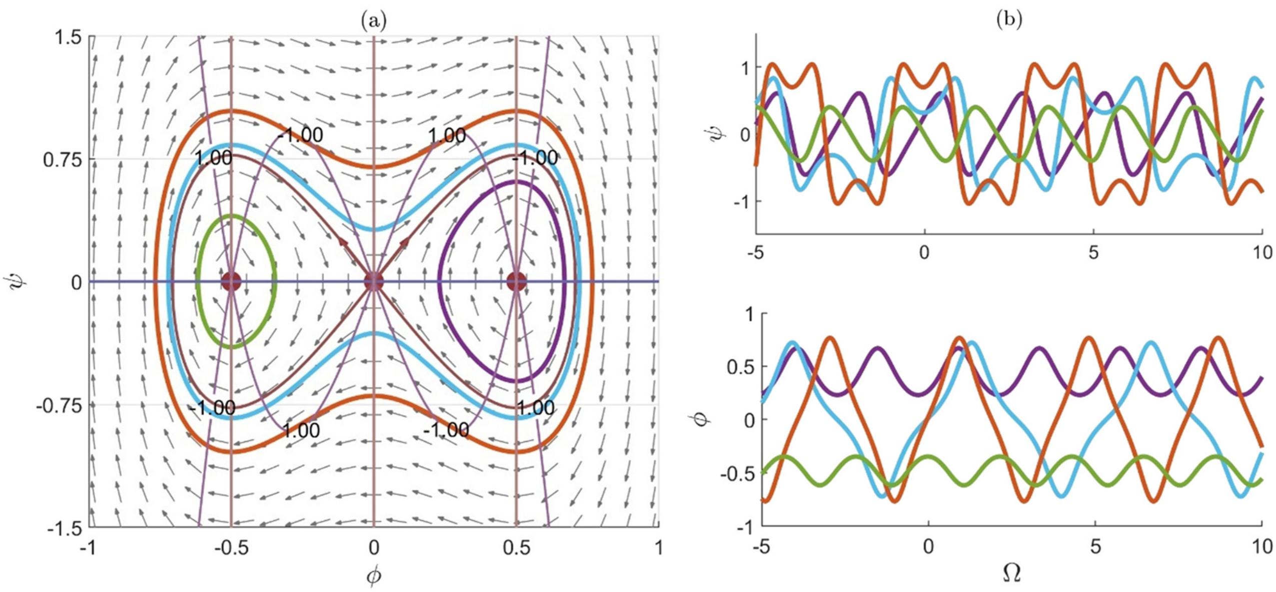

Figure 6: Phase portrait and corresponding solution of the planar system (3.25) for the values of $m = 1, n = 2, \sigma = 1.1$. The equilibria (±0.50,0) are centre and (0, 0) is an unstable saddle.

point (0, 0) is a stable centre. We may conclude that the stability of the equilibrium point (0, 0) can alter due to the change in the values of the parameters (Figures 5–8).

Case ii. Stability of $\left(\pm\frac{m}{n}, 0\right)$: for this case if $\frac{2m^2}{\sigma^2-1} > 0$ and $\sigma \neq 1$, then the eigenvalues $\lambda_1 = i\sqrt{\frac{2m^2}{\sigma^2-1}}$ and $\lambda_1 = -i\sqrt{\frac{2m^2}{\sigma^2-1}}$ are purely imaginary and provide a stable centre at $\left(\pm\frac{m}{n}, 0\right)$. If $\frac{2m^2}{\sigma^2-1} < 0$ and $\sigma \neq 1$, then the eigenvalues are $\lambda_1 = \sqrt{\frac{2m^2}{\sigma^2-1}}$ and $\lambda_1 = -\sqrt{\frac{2m^2}{\sigma^2-1}}$ (real, opposite sign). So the equilibrium point $\left(\pm\frac{m}{n}, 0\right)$ is an unstable saddle. This analysis concludes that the stability of the equilibrium point $\left(\pm\frac{m}{n}, 0\right)$ can alter from a stable centre to an unstable

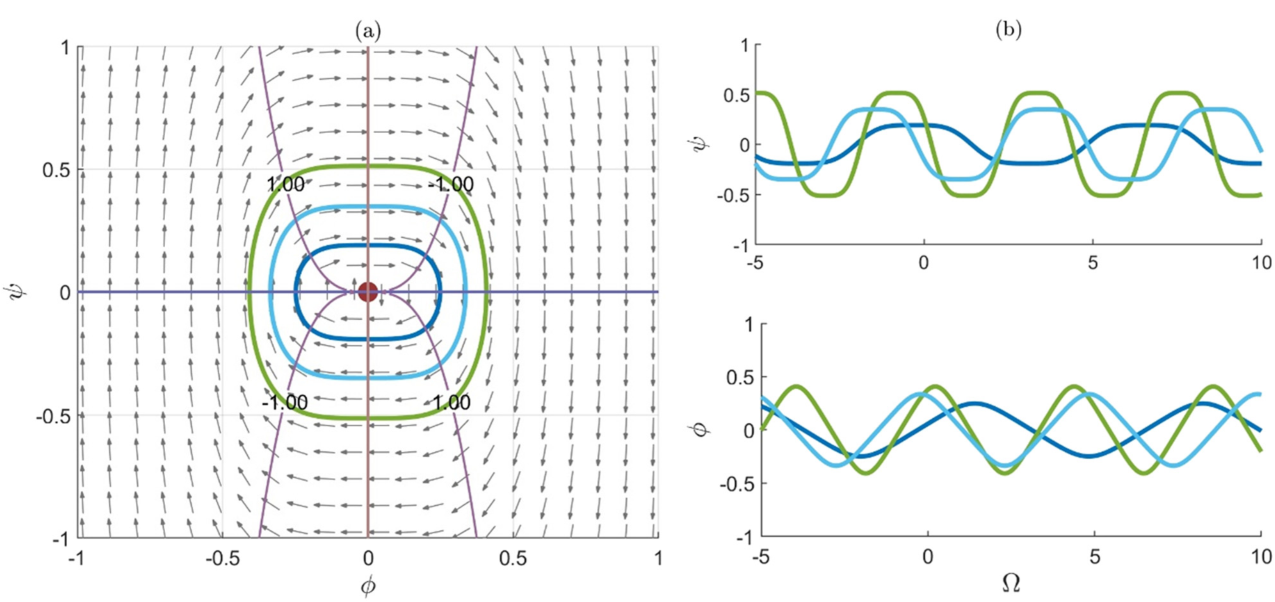

Figure 7: Phase portrait and corresponding solution of the planar system (3.25) for the values of $m = 0, n = 2, \sigma = 1.1$. (0, 0) is a degenerate equilibrium.

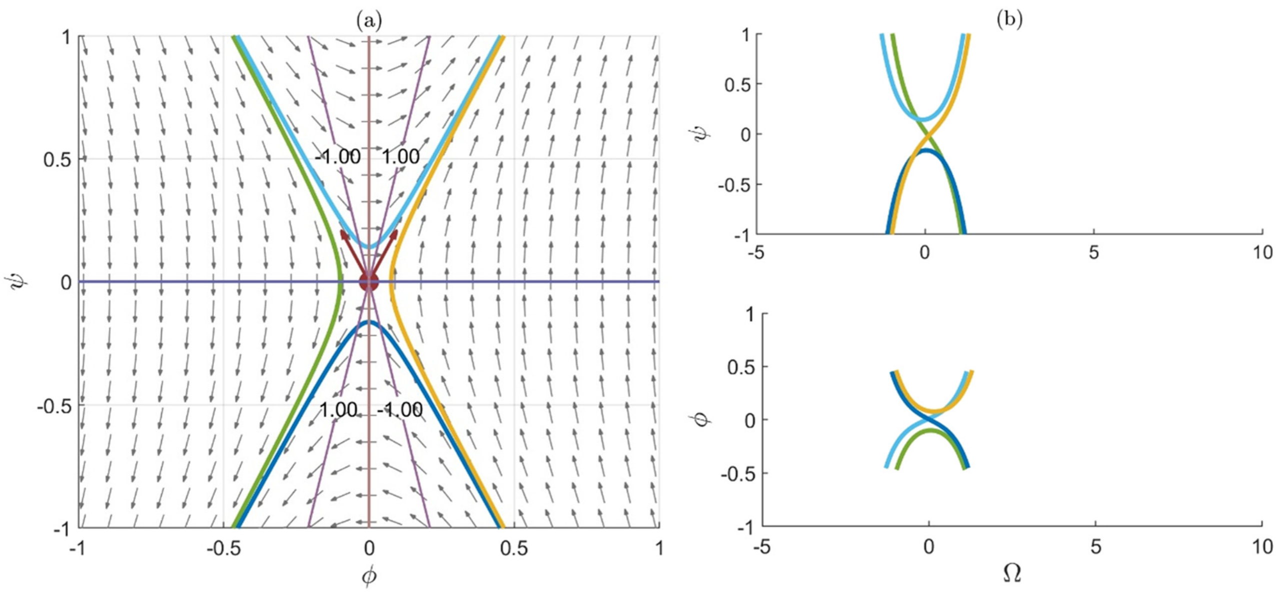

Figure 8: Phase portrait and corresponding solution of the planar system (3.25) for the values of $m = 1$, $n = 0$, $\sigma = 1.1$. The equilibrium point (0, 0) is an unstable saddle.

saddle due to the change in the values of the parameters (Figures 5 and 6).

7 Conclusion

Using the AAE method, we have successfully acquired a range of soliton solutions, including solitons with kink-shaped profiles, singular periodic profiles, singular bell-shaped profiles, and V-shaped soliton solutions of the LGH equation. We have provided graphical and physical explanations by creating 2D and 3D diagrams, illustrating how the dynamic behaviour of the solutions changes as the values are altered. The utilization of these diagrams showcases the simplicity, effectiveness, and user-friendliness of the suggested method. Furthermore, we have examined the bifurcation and stability of the system in proximity to the equilibrium points. The system's dynamics underwent modifications as a result of variations in the parameter values (Figures 5–8). We also provided a discussion on the comparisons between the AAE method and two other well-known approaches: the generalized Kudryashov method and the improved Bernoulli sub-equation function method. By discussing these comparisons, we provided insights into the advantages and characteristics of the AAE method in relation to the generalized Kudryashov method and the improved Bernoulli sub-equation function method. This analysis aids in understanding the strengths and limitations of each method and assists researchers in choosing the most appropriate approach for their specific problem. In addition, we will use an alternative approach in future research to solve the LGH equation, which promises to be highly advantageous in obtaining precise solutions for NLEEs.

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