Research Article

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On traveling wave solutions to Manakov model with variable coefficients

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Abstract: We use a general transformation, to find exact solutions for the Manakov system with variable coefficients (depending on the time ε) using an improved tanh–coth method. The solutions obtained in this work are more general compared to those in other works because they involve variable coefficients. The implemented computational method is applied in a direct way on the reduced system, avoid in this way the reduction to only one equation, as occurs in the works respect to exact solutions, made by other authors. Clearly, from the solutions obtained here, new solutions are derived for the standard model (constant coefficients), complementing in this way the results obtained by other authors mentioned here. Finally, we give some discussion on the results and give the respective conclusions.

Keywords: traveling wave solutions, Manakov equation, improved tanh–coth method

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1 Introduction

The soliton theory is actually one of the most important branch of the applied mathematics. In the physical sciences, especially in nonlinear optic and from some year ago, in the communications theory, the study of optical solitons is the great relevance. In recent years, several models have been used to modelling optical solitons, and the nonlinear Schrödinger equation (NLSE) is one of the most important; however, other models such as the Chen-Lee-Liu equation, the Gerdjikov-Ivanov equation, the Ginzburg-Landau equation, the Manakov system, and several other models have been used to generate optical solitons to be applied in process of communication. Our interest in this work is the study, from of point of view of it exact solutions, of the following Manakov system with variable coefficients, given in the following form:

$$\begin{cases} \imath q_{\varepsilon} + \Gamma_1(\varepsilon) q_{xx} + \Theta_1(\varepsilon)(|q|^2 + |r|^2) q = 0 \\ \imath r_{\varepsilon} + \Gamma_2(\varepsilon) r_{xx} + \Theta_2(\varepsilon)(|q|^2 + |r|^2) r = 0, \end{cases} \tag{1.1}$$

where $q = q(x, \varepsilon)$, $r = r(x, \varepsilon)$ represent a complex-value function and are depending on the spacial variable x and the temporal variable ε. The coefficients Γ_1, Γ_2, Θ_1, Θ_2 are functions depending on ε. In the case of constant coefficients, the model (1.1) is known as a system of coupled NLSEs, which are considered as important models in nonlinear optic [1–3]. In this last case, it has been found that the Manakov system is integrable *via* the inverse scattering method [1]; furthermore, this is very important in the design of new technology that require the use of optical pulses [3]. The model presented here, clearly includes the constant coefficients; therefore, model (1.1) is a generalization of the classical Manakov equation presented in references [1–3] and in many others works on the classical Manakov system [4–6]. On the other hand, differential equations with variable coefficients (depending on variable ε) are generalized models that include the standard models (constant coefficients). From the mathematical point of view, the study of that models has a great relevance because it includes a variety of considerations that

can be important in the phenomena described by the model. Furthermore, from the solutions of the generalized model, we can derive solutions for the standard, with the advantage that we can consider new types of solitons that can be used to understand the phenomena described by the respective model and can be used to implement new technology. In recent years, models with variable coefficients have been presented by several authors as equations with practical applications, for instance, the following equation:

$$u_\varepsilon + k_1\varepsilon^n u^2 + k_2\varepsilon^m u_{xxx} = 0,$$

studied in [7], and other equations considered for instance in [8,9] are types of this class of models.

When new equations are appearing to model the pulses transmission, a great variety of computational method have been implemented to solve it. We can found several models and computational methods for solving nonlinear partial differential equation in references [10–37]. Recently, nonlinear partial differential equations with fractional exponents have used for modeling several phenomena of the nature, so that, as in the classical models mentioned in the previous references, new techniques to solve them, specially from the point of view of exact and numerical solutions, have implemented [38–48]. However, one of the methods used in a satisfactory way and that did not appear in the aforementioned references is the improved tanh–coth method (ITCM) presented in refs [49,50], which we will use to solve (1.1). The advantage of this method, compared with others, is that it can be implemented in a mathematical software as Mathematica and Maples, can be applied into systems directly, do not require a special computer, and can be implemented easily, and finally, it is a generalization of the tanh–coth method [51] and the Kudryashov method [52] and is used widely in the literature. The work is organized as follows: In Section 2, we present a brief description of the ITCM to solve a system. In Section 3, we solve (1.1), and we obtain conditions on the coefficients with the aim to derive nontrivial solutions and we present the solutions obtained. Finally, we present a discussion on the results obtained, and we present some final conclusions.

2 Solving Eq. (1.1)

With the aim to solve the Manakov system given by Eq. (1.1), we need, at first, to use the following transformation:

$$\begin{cases} q(x,\varepsilon) = q(\eta)e^{\iota(x+\int \rho(\varepsilon)\mathrm{d}\varepsilon+\eta_0)}, \\ r(x,\varepsilon) = r(\eta)e^{\iota(x+\int \rho(\varepsilon)\mathrm{d}\varepsilon+\eta_0)}, \\ \eta = x + \int \lambda(\varepsilon)\mathrm{d}\varepsilon + \eta_1, \end{cases} \tag{2.1}$$

where, $\rho(\varepsilon)$ and $\lambda(\varepsilon)$ are functions of ε to be determinated latter, and η_0. η_1 arbitrary constants. With the use of (2.1), we have the following system of ordinary differential equations in the unknowns $q(\eta)$, $r(\eta)$ (where by simplicity, we have used the same variables q, r):

$$\begin{cases} -\rho(\varepsilon)q(\eta) - \Gamma_1(\varepsilon)q(\eta) + \Gamma_1(\varepsilon)q''(\eta) \\ \quad + \Theta_1(\varepsilon)(q^2(\eta) + r^2(\eta))u(\eta) \\ \quad + \iota(\lambda(\varepsilon) + 2\Gamma_1(\varepsilon))q'(\eta) = 0, \\ -\rho(\varepsilon)r(\eta) - \Gamma_2(\varepsilon)r(\eta) + \Gamma_2(\varepsilon)r''(\eta) \\ \quad + \Theta_2(\varepsilon)(q^2(\eta) + r^2(\eta))r(\eta) + \iota(\lambda(\varepsilon) + 2\Gamma_2(\varepsilon))r'(\eta) \\ \quad = 0. \end{cases} \tag{2.2}$$

Here, $q'(\varepsilon) = \frac{\mathrm{d}q}{\mathrm{d}\varepsilon}$, $r'(\varepsilon) = \frac{\mathrm{d}r}{\mathrm{d}\varepsilon}$. As we mentioned previously, one of the functions that we need to obtain is $\lambda(\varepsilon)$, so that, in the first equation of (2.2), we consider:

$$\lambda(\varepsilon) = -2\Gamma_1(\varepsilon). \tag{2.3}$$

With this selection, the imaginary part of the first equation of (2.2) is eliminated. Now, replacing (2.3), in the second equation of (2.2) and considering the following condition on system (1.1)

$$\Gamma_1(\varepsilon) = \Gamma_2(\varepsilon), \tag{2.4}$$

we can eliminate the imaginary part of the second equation of (2.2), so that, finally, (2.2) converts to

$$\begin{cases} -\rho(\varepsilon)q(\eta) - \Gamma_1(\varepsilon)q(\eta) + \Gamma_1(\varepsilon)q''(\eta) \\ \quad + \Theta_1(\varepsilon)(q^2(\eta) + r^2(\eta))q(\eta) = 0, \\ -\rho(\varepsilon)r(\eta) - \Gamma_1(\varepsilon)r(\eta) + \Gamma_1(\varepsilon)r''(\eta) \\ \quad + \Theta_2(\varepsilon)(q^2(\eta) + r^2(\eta))r(\eta) = 0. \end{cases} \tag{2.5}$$

2.1 The ITCM method

A full description of the ITCM can be found in reference [49]. However, similar to all computational methods, the first step is transformation of the nonlinear partial differential equation (or system of nonlinear partial differential equations) to an ordinary differential equation (or system of ordinary differential equations), by means of adequate transformation. In our case, we have used (2.1) to reduce (1.1) into (2.2). In the following step of the ITCM method,

we illustrate them by applying directly to our system. We consider solutions of (2.5) in the following form:

$$\begin{cases} q(\eta) = \sum_{i=0}^{M} a_i(\varepsilon)\phi(\eta)^i + \sum_{i=M+1}^{2M} a_i(\varepsilon)\phi(\eta)^{M-i}, \\ r(\eta) = \sum_{i=0}^{N} b_i(\varepsilon)\phi(\eta)^i + \sum_{i=N+1}^{2N} b_i(\varepsilon)\phi(\eta)^{N-i}, \end{cases} \tag{2.6}$$

where $\phi(\eta)$ is solution of the Riccati equation [50]:

$$\phi'(\eta) = \sigma(\varepsilon)\phi^2(\eta) + \varrho(\varepsilon)\phi(\eta) + \chi(\varepsilon). \tag{2.7}$$

By substituting (2.6) into (2.5) and applying the balancing technique, we have $M + 2 = 3M$ and $N + 2 = 3N$, respectively, thus obtaining $M = 1$, $N = 1$. With this values, (2.6) reduces to

$$\begin{cases} q(\eta) = a_0(\varepsilon) + a_1(\varepsilon)\phi(\eta) + a_2(\varepsilon)\phi(\eta)^{-1}, \\ r(\eta) = b_0(\varepsilon) + b_1(\varepsilon)\phi(\eta) + b_2(\varepsilon)\phi(\eta)^{-1}. \end{cases} \tag{2.8}$$

By substituting (2.8) into (2.5) and considering (2.7), we have an algebraic system in the unknowns $\chi(\varepsilon)$, $\varrho(\varepsilon)$, $\sigma(\varepsilon)$, $\rho(\varepsilon)$, $a_0(\varepsilon)$, $a_1(\varepsilon)$, $a_2(\varepsilon)$, $b_0(\varepsilon)$, $b_1(\varepsilon)$, $b_2(\varepsilon)$:

$$\begin{cases}
3a_1\Gamma_1(\varepsilon)\varrho(\varepsilon)\sigma(\varepsilon) + 2a_1b_0b_1\Theta_1(\varepsilon) + a_0b_1^2\Theta_1(\varepsilon) + 3a_0a_1^2\Theta_1(\varepsilon) = 0, \\
2a_1\Gamma_1(\varepsilon)\sigma^2(\varepsilon) + a_1b_1^2\Theta_1(\varepsilon) + a_1^3\Theta_1(\varepsilon) = 0, \\
a_1^2b_0\Theta_2(\varepsilon) + 2a_0a_1b_1\Theta_2(\varepsilon) + 3\Gamma_1(\varepsilon)\varrho(\varepsilon)b_1\sigma(\varepsilon) + 3b_0b_1^2\Theta_2(\varepsilon) = 0, \\
a_1^2b_1\Theta_2(\varepsilon) + 2\Gamma_1(\varepsilon)b_1\sigma^2(\varepsilon) + b_1^3\Theta_2(\varepsilon) = 0, \\
3\chi(\varepsilon)a_2\Gamma_1(\varepsilon)\varrho(\varepsilon) + 2a_2b_0b_2\Theta_1(\varepsilon) + a_0b_2^2\Theta_1(\varepsilon) + 3a_0a_2^2\Theta_1(\varepsilon) = 0, \\
2\chi^2(\varepsilon)a_2\Gamma_1(\varepsilon) + a_2b_2^2\Theta_1(\varepsilon) + a_2^3\Theta_1(\varepsilon) = 0, \\
a_2^2b_0\Theta_2(\varepsilon) + 2a_0a_2b_2\Theta_2(\varepsilon) + 3\chi(\varepsilon)\Gamma_1(\varepsilon)\varrho(\varepsilon)b_2 + 3b_0b_2^2\Theta_2(\varepsilon) = 0, \\
a_2^2b_2\Theta_2(\varepsilon) + 2\chi(\varepsilon)^2\Gamma_1(\varepsilon)b_2 + b_2^3\Theta_2(\varepsilon) = 0, \\
\chi(\varepsilon)a_1\Gamma_1(\varepsilon)\varrho(\varepsilon) + a_2\Gamma_1(\varepsilon)\varrho(\varepsilon)\sigma(\varepsilon) - a_0\Gamma_1(\varepsilon) + a_0b_0^2\Theta_1(\varepsilon) + 2a_0b_1b_2\Theta_1(\varepsilon) \\
+ 2a_2b_0b_1\Theta_1(\varepsilon) + 2a_1b_0b_2\Theta_1(\varepsilon) + a_0^3\Theta_1(\varepsilon) + 6a_1a_2a_0\Theta_1(\varepsilon) - a_0\rho(\varepsilon) = 0, \\
2\chi(\varepsilon)a_1\Gamma_1(\varepsilon)\sigma(\varepsilon) + a_1\Gamma_1(\varepsilon)\varrho^2(\varepsilon) - a_1\Gamma_1(\varepsilon) + a_1b_0^2\Theta_1(\varepsilon) + a_2b_1^2\Theta_1(\varepsilon) \\
+ 2a_0b_0b_1\Theta_1(\varepsilon) + 2a_1b_1b_2\Theta_1(\varepsilon) + 3a_0^2a_1\Theta_1(\varepsilon) + 3a_1^2a_2\Theta_1(\varepsilon) - a_1\rho(\varepsilon) = 0, \\
2\chi(\varepsilon)a_2\Gamma_1(\varepsilon)\sigma(\varepsilon) + a_2\Gamma_1(\varepsilon)\varrho^2(\varepsilon) - a_2\Gamma_1(\varepsilon) + a_2b_0^2\Theta_1(\varepsilon) + a_1b_2^2\Theta_1(\varepsilon) \\
+ 2a_0b_0b_2\Theta_1(\varepsilon) + 2a_2b_1b_2\Theta_1(\varepsilon) + 3a_1a_2^2\Theta_1(\varepsilon) + 3a_0^2a_2\Theta_1(\varepsilon) - a_2\rho(\varepsilon) = 0, \\
a_0^2b_0\Theta_2(\varepsilon) + 2a_1a_2b_0\Theta_2(\varepsilon) + 2a_0a_2b_1\Theta_2(\varepsilon) + 2a_0a_1b_2\Theta_2(\varepsilon) \\
+ \chi(\varepsilon)\Gamma_1(\varepsilon)\varrho(\varepsilon)b_1 + \Gamma_1(\varepsilon)\varrho(\varepsilon)b_2\sigma(\varepsilon) - \Gamma_1(\varepsilon)b_0 + b_0^3\Theta_2(\varepsilon) + 6b_1b_2b_0\Theta_2(\varepsilon) - b_0\rho(\varepsilon) = 0, \\
2a_0a_1b_0\Theta_2(\varepsilon) + a_0^2b_1\Theta_2(\varepsilon) + 2a_1a_2b_1\Theta_2(\varepsilon) + a_1^2b_2\Theta_2(\varepsilon) \\
+ 2\chi(\varepsilon)\Gamma_1(\varepsilon)b_1\sigma(\varepsilon) + \Gamma_1(\varepsilon)b_1\varrho^2(\varepsilon) - \Gamma_1(\varepsilon)b_1 + 3b_0^2b_1\Theta_2(\varepsilon) + 3b_1^2b_2\Theta_2(\varepsilon) - b_1\rho(\varepsilon) = 0, \\
2a_0a_2b_0\Theta_2(\varepsilon) + a_2^2b_1\Theta_2(\varepsilon) + a_0^2b_2\Theta_2(\varepsilon) + 2a_1a_2b_2\Theta_2(\varepsilon) \\
+ 2\chi(\varepsilon)\Gamma_1(\varepsilon)b_2\sigma(\varepsilon) + \Gamma_1(\varepsilon)b_2\varrho(\varepsilon)^2 - \Gamma_1(\varepsilon)b_2 + 3b_1b_2^2\Theta_2(\varepsilon) + 3b_0^2b_2\Theta_2(\varepsilon) - b_2\rho(\varepsilon) = 0.
\end{cases} \tag{2.9}$$

$\Gamma_1(\varepsilon)$, $\Theta_1(\varepsilon)$, and $\Theta_2(\varepsilon)$ are the coefficients of Eq. (1.1). The rest of parameters in the previous system depend on variable ε ($a_1 = a_1(\varepsilon), \dots$). By using a mathematical software such as Maple or Mathematica, we obtain the following solution of the system (2.9), from which, we can obtain nontrivial solutions for (1.1):

$$\begin{cases} a_0(\varepsilon) = b_0(\varepsilon) = \chi(\varepsilon) = \sigma(\varepsilon) = 0, \\ a_1(\varepsilon) = \pm\imath b_1(\varepsilon), \\ a_2(\varepsilon) = \pm\imath b_2(\varepsilon), \\ \rho(\varepsilon) = \Gamma(\varepsilon)(-1 + \varrho^2(\varepsilon)). \end{cases} \tag{2.10}$$

With these values, the solution of (2.7) is expressed as follows:

$$\phi(\eta) = e^{\varrho(\varepsilon)\eta}. \tag{2.11}$$

Here $\varrho(\varepsilon)$ is the arbitrary function. According to (2.8), and (2.10), we have

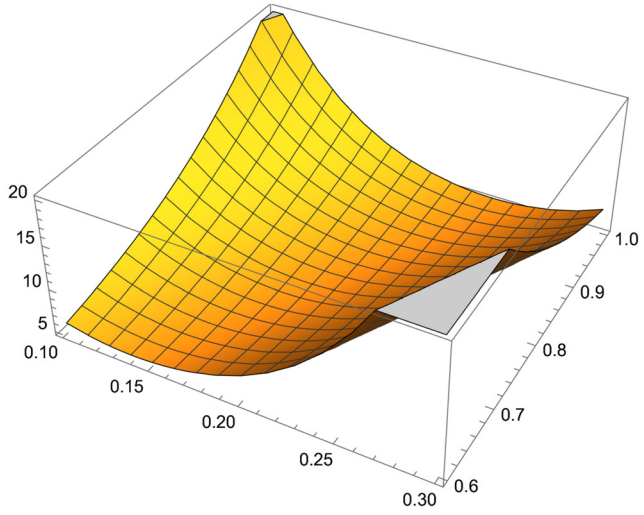
$$\begin{cases} q(\eta) = \pm\imath b_1(\varepsilon)\phi(\eta) \pm \imath b_2(\varepsilon)\phi(\eta)^{-1}, \\ r(\eta) = b_1(\varepsilon)\phi(\eta) + b_2(\varepsilon)\phi(\eta)^{-1}, \end{cases} \tag{2.12}$$

where $\phi(\eta)$ is earlier given in (2.11) and $b_1(\varepsilon)$, $b_2(\varepsilon)$ arbitrary functions. Finally, taking into account (2.1), (2.3),

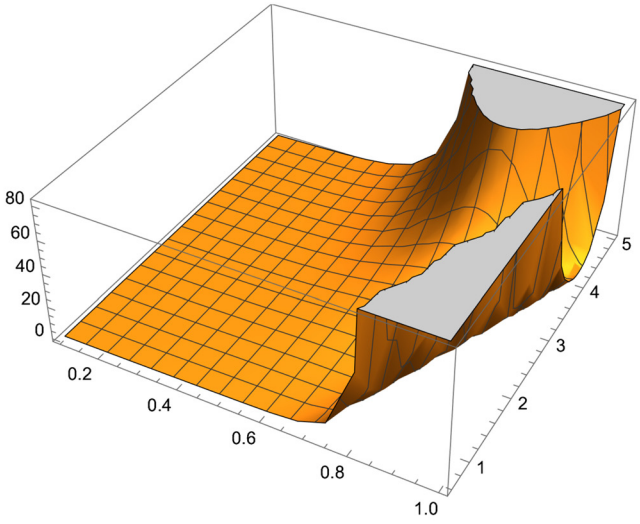
and (2.12), under condition (2.4), we have the following solution of system (1.1):

$$\begin{cases} q(x,\varepsilon) = \pm \iota r(\eta) e^{\iota\left(x + \int \Gamma_1(\varepsilon)(-1+\varrho^2(\varepsilon))\mathrm{d}t + \eta_0\right)} \\ r(x,\varepsilon) = r(\eta) e^{\iota\left(x + \int \Gamma_1(\varepsilon)(-1+\varrho^2(\varepsilon))\mathrm{d}t + \eta_0\right)} \\ \eta = x + \int -2\Gamma_1(\varepsilon)\mathrm{d}t + \eta_1, \end{cases} \tag{2.13}$$

where $\varrho(\varepsilon)$ is arbitrary function, η_0 and η_1 arbitrary constants, and $r(\eta)$ is given in (2.12).



(a) r_c



(b) r_v

r_c corresponds to graph of $r(\eta)$ given in (2.12), for the following constant values: $b_1(\varepsilon) = 2$, $b_2(\varepsilon) = 3$, $\varrho(\varepsilon) = 4$, and $\Gamma_1(\varepsilon) = 2$, for $[x, t] \in [0.6, 1] \times [0.1, 0.3]$ and $\eta_1 = 0$.

r_v corresponds to graph of $r(\eta)$ given in (2.12), for the following variable values: $b_1(\varepsilon) = 2t, b_2(\varepsilon) = 3t^2, \varrho(\varepsilon) = 4t^3$, and $\Gamma_1(\varepsilon) = 2t^2$, for $[x, t] \in [0.6, 5] \times [0.1, 1]$ and $\eta_1 = 0$.

2.2 A second case

In Section 2.1, we have obtained solutions for system (1.1), for the case given by (2.4). In this section, we obtain new solutions, taking into account the following additionally condition:

$$\Theta_1(\varepsilon) = \Theta_2(\varepsilon). \tag{2.14}$$

With (2.14), the system (2.5) reduces to

$$\begin{cases} -\rho(\varepsilon)q(\eta) - \Gamma_1(\varepsilon)q(\eta) + \Gamma_1(\varepsilon)q''(\eta) \\ \quad + \Theta_1(\varepsilon)(q^2(\eta) + r^2(\eta))q(\eta) = 0, \\ -\rho(\varepsilon)r(\eta) - \Gamma_1(\varepsilon)r(\eta) + \Gamma_1(\varepsilon)r''(\eta) \\ \quad + \Theta_1(\varepsilon)(q^2(\eta) + r^2(\eta))r(\eta) = 0. \end{cases} \tag{2.15}$$

Under these new conditions, the following are new solutions of the system (2.9):

$$\begin{cases} \rho(\varepsilon) = \pm \dfrac{2\Theta_1(\varepsilon)a_2^2(\varepsilon)b_1(\varepsilon) - \Gamma_1(\varepsilon)b_2(\varepsilon) + 2\Theta_1(\varepsilon)b_1(\varepsilon)b_2^2(\varepsilon)}{b_2(\varepsilon)}, \\ \chi(\varepsilon) = \pm \dfrac{\iota\sqrt{\Theta_1(\varepsilon)}\sqrt{a_2^2(\varepsilon) + b_2^2(\varepsilon)}}{\sqrt{2\Gamma_1(\varepsilon)}}, \\ \sigma(\varepsilon) = \pm \dfrac{\iota\sqrt{\Theta_1(\varepsilon)}\, b_1(\varepsilon)\sqrt{a_2^2(\varepsilon) + b_2^2(\varepsilon)}}{\sqrt{2\Gamma_1(\varepsilon)}\, b_2(\varepsilon)}, \\ a_1(\varepsilon) = \dfrac{a_2(\varepsilon)b_1(\varepsilon)}{b_2(\varepsilon)}, \\ \varrho(\varepsilon) = a_0(\varepsilon) = b_0(\varepsilon) = 0, \end{cases} \tag{2.16}$$

$a_2(\varepsilon)$, $b_1(\varepsilon)$, and $b_2(\varepsilon)$ arbitrary functions (depending on variable ε).

$$\begin{cases} \rho(\varepsilon) = \dfrac{4\Theta_1(\varepsilon)a_2^2(\varepsilon)b_1(\varepsilon) - \Gamma_1(\varepsilon)b_2(\varepsilon) + 4\Theta_1(\varepsilon)b_1(\varepsilon)b_2^2(\varepsilon)}{b_2(\varepsilon)}, \\ \chi(\varepsilon) = \pm\dfrac{\iota\sqrt{\Theta_1}\sqrt{a_2^2(\varepsilon) + b_2^2(\varepsilon)}}{\sqrt{2\Gamma_1(\varepsilon)}}, \\ \sigma(\varepsilon) = \mp\dfrac{\iota\sqrt{\Theta_1(\varepsilon)}\,b_1(\varepsilon)\sqrt{a_2^2(\varepsilon) + b_2^2(\varepsilon)}}{\sqrt{2\Gamma_1(\varepsilon)}\,b_2(\varepsilon)}, \\ a_1(\varepsilon) = \dfrac{a_2(\varepsilon)b_1(\varepsilon)}{b_2(\varepsilon)}, \\ \varrho(\varepsilon) = a_0(\varepsilon) = b_0(\varepsilon) = 0, \end{cases} \tag{2.17}$$

$a_2(\varepsilon)$, $b_1(\varepsilon)$, and $b_2(\varepsilon)$ arbitrary functions (depending on variable ε).

$$\begin{cases} \rho(\varepsilon) = \Gamma(\varepsilon)(-1 + \varrho^2(\varepsilon)), \\ a_1(\varepsilon) = \pm\iota b_1(\varepsilon), \\ a_2(\varepsilon) = \pm\iota b_2(\varepsilon), \chi(\varepsilon) = \sigma(\varepsilon) = a_0(\varepsilon) = b_0(\varepsilon) \\ \qquad = 0, \end{cases} \tag{2.18}$$

$b_1(\varepsilon)$ and $b_2(\varepsilon)$ arbitrary functions (depending on variable ε).

$$\begin{cases} \rho(\varepsilon) = -\Gamma(\varepsilon) + \Gamma(\varepsilon)\varrho^2(\varepsilon) + 4\Theta_1(\varepsilon)b_1(\varepsilon)b_2(\varepsilon), \\ a_1(\varepsilon) = \iota b_1(\varepsilon), \\ a_2(\varepsilon) = -\iota b_2(\varepsilon), \chi(\varepsilon) = \sigma(\varepsilon) = a_0(\varepsilon) = b_0(\varepsilon) \\ \qquad = 0, \end{cases} \tag{2.19}$$

$b_1(\varepsilon)$ and $b_2(\varepsilon)$ arbitrary functions (depending on variable ε).

We can obtain several expressions, which include periodic and hyperbolic functions, for solutions of (2.15), using the following classifications of solutions of Eq. (2.7) [50] and the previous set of solutions of (2.9), given from (2.16) until (2.19):

(1) If $\chi(\varepsilon) \neq 0$, $\sigma(\varepsilon) \neq 0$, and $\varrho(\varepsilon) \neq 0$:

$$\phi(\eta) = \begin{cases} \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{1}{\eta} - \dfrac{\varrho(\varepsilon)}{2}\right), & \varrho^2(\varepsilon) = 4\sigma(\varepsilon)\chi(\varepsilon) \\ \dfrac{1}{\sigma(\varepsilon)}\left(\dfrac{\sqrt{4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon)}}{2}\tan\left[\dfrac{\sqrt{4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon)}}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & 4\chi(\varepsilon)\sigma(\varepsilon) - \varrho(\varepsilon)^2 > 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{\sqrt{4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon)}}{2}\cot\left[\dfrac{\sqrt{4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon)}}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & 4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon) > 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{\sqrt{\varrho^2(\varepsilon) - 4\chi(\varepsilon)\sigma(\varepsilon)}}{2}\tanh\left[\dfrac{\sqrt{\varrho^2(\varepsilon) - 4\chi(\varepsilon)\sigma(\varepsilon)}}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & 4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon) < 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{\sqrt{\varrho^2(\varepsilon) - 4\chi(\varepsilon)\sigma(\varepsilon)}}{2}\coth\left[\dfrac{\sqrt{\varrho^2(\varepsilon) - 4\chi(\varepsilon)\sigma(\varepsilon)}}{2}\eta\right] - \dfrac{\varrho}{2}\right), & 4\chi(\varepsilon)\sigma(\varepsilon) - \varrho^2(\varepsilon) < 0 \end{cases} \tag{2.20}$$

(2) If $\chi(\varepsilon) = 0$, $\sigma(\varepsilon) \neq 0$:

$$\phi(\eta) = \begin{cases} \dfrac{\varrho(\varepsilon)}{-\sigma(\varepsilon) + \varrho(\varepsilon)e^{-\varrho(\varepsilon)\eta}}, & \varrho(\varepsilon) \neq 0 \\ -\dfrac{1}{\sigma(\varepsilon)\eta}, & \varrho(\varepsilon) = 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(\dfrac{\sqrt{-\varrho^2(\varepsilon)}}{2}\tan\left[\dfrac{\sqrt{-\varrho^2(\varepsilon)}}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & \varrho^2(\varepsilon) < 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{\sqrt{-\varrho^2(\varepsilon)}}{2}\cot\left[\dfrac{\sqrt{-\varrho^2(\varepsilon)}}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & \varrho^2(\varepsilon) < 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{\varrho(\varepsilon)}{2}\tanh\left[\dfrac{\varrho(\varepsilon)}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & \varrho^2(\varepsilon) > 0 \\ \dfrac{1}{\sigma(\varepsilon)}\left(-\dfrac{\varrho(\varepsilon)}{2}\coth\left[\dfrac{\varrho(\varepsilon)}{2}\eta\right] - \dfrac{\varrho(\varepsilon)}{2}\right), & \varrho^2(\varepsilon) > 0 \end{cases} \tag{2.21}$$

(3) If $\varrho(\varepsilon) = 0$, $\sigma(\varepsilon) \neq 0$

$$\phi(\eta) = \begin{cases} -\dfrac{1}{\sigma(\varepsilon)\eta}, & \chi(\varepsilon) = 0 \\ \dfrac{1}{\sigma(\varepsilon)}(\sqrt{\chi(\varepsilon)\sigma(\varepsilon)}\tan[\sqrt{\chi(\varepsilon)\sigma(\varepsilon)}\eta]), & \chi(\varepsilon)\sigma(\varepsilon) > 0 \\ \dfrac{1}{\sigma(\varepsilon)}(-\sqrt{\chi(\varepsilon)\sigma(\varepsilon)}\cot[\sqrt{\chi(\varepsilon)\sigma(\varepsilon)}\eta]), & \chi(\varepsilon)\sigma(\varepsilon) > 0 \\ \dfrac{1}{\sigma(\varepsilon)}(-\sqrt{-\chi(\varepsilon)\sigma(\varepsilon)}\tanh[\sqrt{-\chi(\varepsilon)\sigma(\varepsilon)}\eta]), & \chi(\varepsilon)\sigma(\varepsilon) < 0 \\ \dfrac{1}{\sigma(\varepsilon)}(-\sqrt{-\chi(\varepsilon)\sigma(\varepsilon)}\coth[\sqrt{-\chi(\varepsilon)\sigma(\varepsilon)}\eta]), & \chi(\varepsilon)\sigma(\varepsilon) < 0. \end{cases} \tag{2.22}$$

(4) If $\sigma(\varepsilon) = 0$, $\varrho(\varepsilon) \neq 0$

$$\phi(\eta) = \frac{-\chi(\varepsilon) + \varrho(\varepsilon)e^{\varrho(\varepsilon)\eta}}{\varrho(\varepsilon)}. \tag{2.23}$$

We illustrate solutions of (2.7), using the values given by (2.16) and (2.22) (taking the first sign in each case): First, we note that $(\chi(\varepsilon))(\sigma(\varepsilon)) = -\frac{b_1(\varepsilon)\Theta_1(\varepsilon)(a_2(\varepsilon)^2 + b_2(\varepsilon)^2)}{2\Gamma_1(\varepsilon)b_2(\varepsilon)}$, so that, after simplifications, we have:

$$\phi(\eta) = \begin{cases} -\dfrac{\sqrt{2\Gamma_1(\varepsilon)}\,b_2(\varepsilon)}{(\sqrt{-\Theta_1(\varepsilon)}\,b_1(\varepsilon)\sqrt{a_2^2(\varepsilon) + b_2^2(\varepsilon)})\eta}, & \text{if } \chi(\varepsilon) = 0, \\ \sqrt{\dfrac{b_2(\varepsilon)}{b_1(\varepsilon)}}\tan[\sqrt{-\dfrac{b_1(\varepsilon)\Theta_1(\varepsilon)(a_2(\varepsilon)^2 + b_2(\varepsilon)^2)}{2\Gamma_1(\varepsilon)b_2(\varepsilon)}}\eta] & \text{if } \chi(\varepsilon)\sigma(\varepsilon) > 0 \\ -\sqrt{\dfrac{b_2(\varepsilon)}{b_1(\varepsilon)}}\cot[\sqrt{-\dfrac{b_1(\varepsilon)\Theta_1(\varepsilon)(a_2(\varepsilon)^2 + b_2(\varepsilon)^2)}{2\Gamma_1(\varepsilon)b_2(\varepsilon)}}\eta] & \text{if } \chi(\varepsilon)\sigma(\varepsilon) > 0 \\ -\sqrt{-\dfrac{b_2(\varepsilon)}{b_1(\varepsilon)}}\tanh[\sqrt{\dfrac{b_1(\varepsilon)\Theta_1(\varepsilon)(a_2(\varepsilon)^2 + b_2(\varepsilon)^2)}{2\Gamma_1(\varepsilon)b_2(\varepsilon)}}\eta] & \text{if } \chi(\varepsilon)\sigma(\varepsilon) < 0 \\ -\sqrt{-\dfrac{b_2(\varepsilon)}{b_1(\varepsilon)}}\coth[\sqrt{\dfrac{b_1(\varepsilon)\Theta_1(\varepsilon)(a_2(\varepsilon)^2 + b_2(\varepsilon)^2)}{2\Gamma_1(\varepsilon)b_2(\varepsilon)}}\eta] & \text{if } \chi(\varepsilon)\sigma(\varepsilon) < 0 \end{cases} \tag{2.24}$$

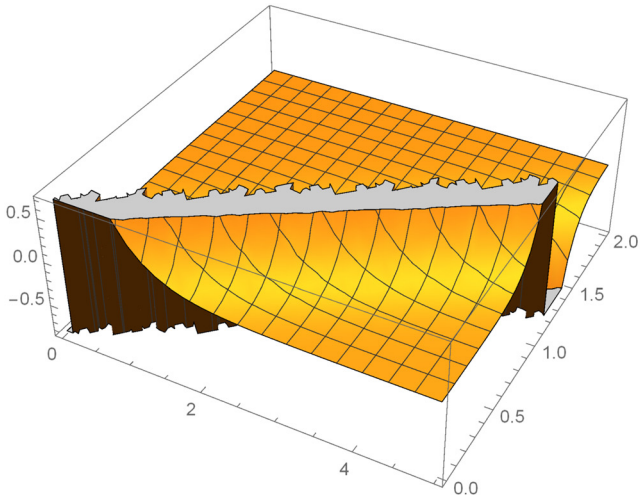
Therefore, according to (2.1), (2.8), and (2.16), we have the following solution for (1.1) (under two conditions $\Gamma_1(\varepsilon) = \Gamma_2(\varepsilon)$ and $\Theta_1(\varepsilon) = \Theta_2(\varepsilon)$):

$$\begin{cases} q(x,\varepsilon) = \left(\dfrac{a_2(\varepsilon)b_1(\varepsilon)}{b_2(\varepsilon)}\phi(\eta) + a_2(\varepsilon)\phi(\eta)^{-1}\right)e^{\iota(x+\int \rho(\varepsilon)\mathrm{d}\varepsilon+\eta_0)}, \\ r(x,\varepsilon) = (b_1(\varepsilon)\phi(\eta) + b_2(\varepsilon)\phi(\eta)^{-1})e^{\iota(x+\int \rho(\varepsilon)\mathrm{d}\varepsilon+\eta_0)}, \end{cases} \tag{2.25}$$

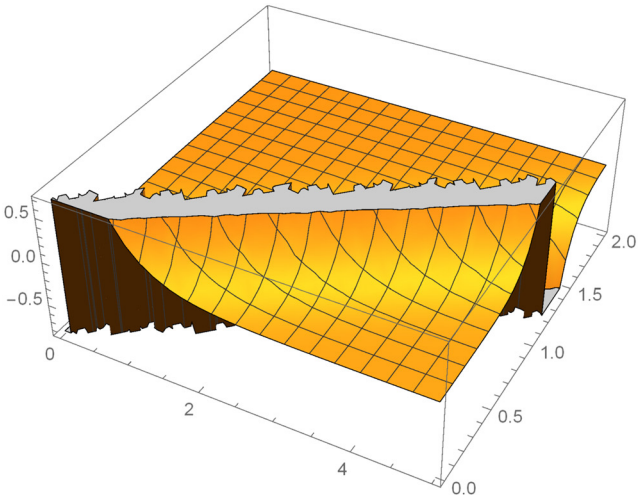
where $\rho(\varepsilon) = \frac{2\Theta_1(\varepsilon)a_2^2(\varepsilon)b_1(\varepsilon) - \Gamma_1(\varepsilon)b_2(\varepsilon) + 2\Theta_1(\varepsilon)b_1(\varepsilon)b_2^2(\varepsilon)}{b_2(\varepsilon)}$, and $\eta = x + \int -2\Gamma_1(\varepsilon)\mathrm{d}\varepsilon + \eta_1$, and $a_2(\varepsilon)$, $b_1(\varepsilon)$, $b_2(\varepsilon)$ are arbitrary functions depending on the variable ε, and $\phi(\eta)$ is given by (2.24). In the previous expressions,

$$\begin{cases} q(\eta) = \dfrac{a_2(\varepsilon)b_1(\varepsilon)}{b_2(\varepsilon)}\phi(\eta) + a_2(\varepsilon)\phi(\eta)^{-1} \\ r(\eta) = b_1(\varepsilon)\phi(\eta) + b_2(\varepsilon)\phi(\eta)^{-1} \end{cases} \tag{2.26}$$

are solutions of (2.8). By using the expression tanh in (2.24), the following are graphics corresponding to (2.26), for adequate values of the respective variables:

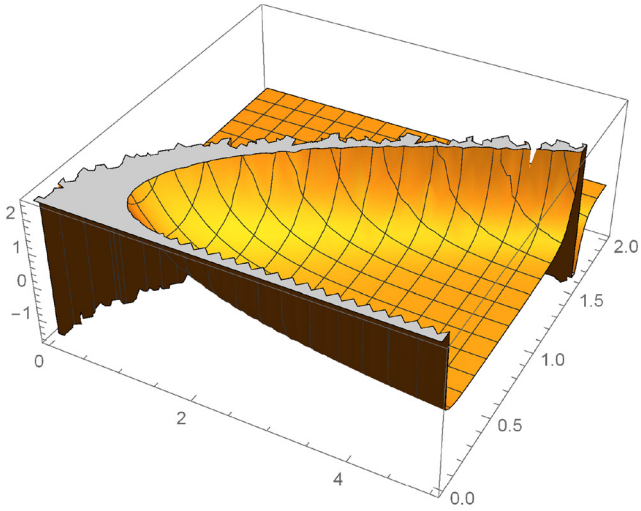


(c) q_1

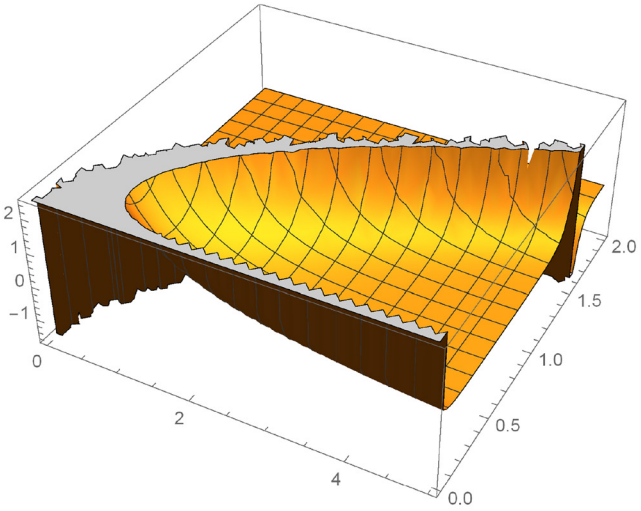


(d) r_1

q_1 and r_1 correspond to values: $\Gamma(\varepsilon) = 2$, $\Theta(\varepsilon) = -2$, $a_2(\varepsilon) = 1$, $b_1(\varepsilon) = -1$, $b_2(\varepsilon) = 1$, and $(x, \varepsilon) \in [0, 5] \times [0, 2]$.

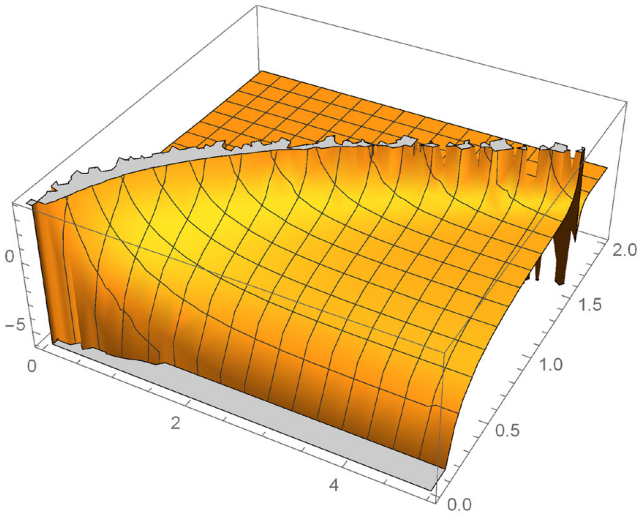


(e) q_2

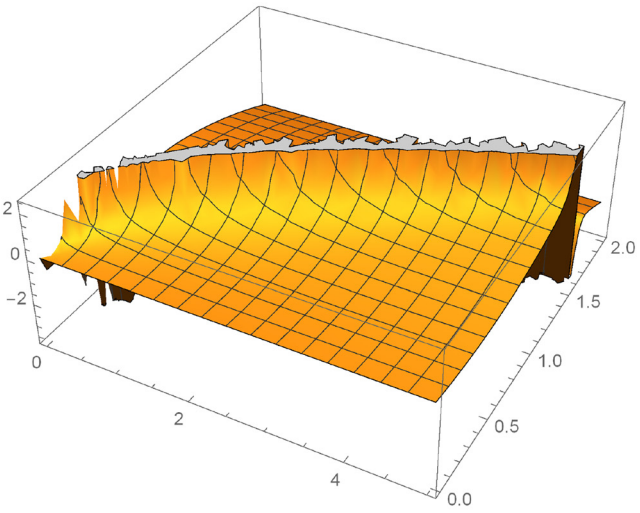


(f) r_2

q_2 and r_2 correspond to values of the parameters: $\Gamma(\varepsilon) = 2\varepsilon$, $\Theta(\varepsilon) = -2\varepsilon^2$, $a_2(\varepsilon) = 1$, $b_1(\varepsilon) = -1$, $b_2(\varepsilon) = 1$, and $(x, \varepsilon) \in [0, 5] \times [0, 2]$. Here, we have taken variable coefficients for Γ and Θ.



(g) q_3



(h) r_3

Finally, q_3 and r_3 are the graphs of (2.26) corresponding to values $\Gamma(\varepsilon) = 2\varepsilon$, $\Theta(\varepsilon) = -2\varepsilon^2$, $a_2(\varepsilon) = 1$, $b_1(\varepsilon) = -\varepsilon$, $b_2(\varepsilon) = \varepsilon^2$, and $(x, \varepsilon) \in [0, 5] \times [0, 2]$. We have taken variable coefficients for all free parameters.

For all previous graphics, we have used $\eta_1 = 0$.

3 Results and discussion

In this work, we have used the ITCM to obtain solutions for a system with variable coefficients, applying directly the method to the respective system (after reduction to ordinary equations), thus avoiding the reduction to only one equation, as presented in the previous study. The first observation deals with the effectiveness of the method. A second observation, all solutions obtained here, is in terms of variable coefficients, which is new in the literature for the model studied in this work. The model considered here is known as the Manakov system with variable coefficients (1.1). We have obtained solutions for it in two particular cases: In the first case, we have considered (2.4) to obtain the solution given by (2.13). As we have mentioned early, the solutions are new in the literature due to existence of variable coefficients; however, if we take the coefficients as constants, as a particular case, the solutions given by (2.13) are again new. The graphs showed by r_c and r_v illustrate the two cases, taking constant coefficients (in the case of r_c) where we have obtained a solitary wave with smooth perfil, and by taking variable coefficients (in the case of r_v), we can obtain a new structure of the solution, where we can note the evolution of the previous wave to a wave with a truncate perfil. We think that this fact can be taken into account in a real application of the model. In a second case, we have assumed additionally the condition (2.14), and we have obtained solutions for (1.1). In this last case, the solutions obtained are new in the literature due to variable coefficients. However, if we take constant coefficients, the solutions are new, compared with those obtained in references [3–5]. The solutions obtained and expressed by (q_1, r_1) have the structure of a dark soliton, but in the case were we have used some constant coefficients combined with variable coefficient, the wave have a relevant change, and the new wave have a part smooth as r_c, r_v, with special evolutions. In the third case, where we have taken all variable coefficients, q_3, v_3, the waves have the structure similar to dark solitons; however in this case, it is cear that q_3 is different to r_3. The method used here gives us solutions for (1.1), with a more general structure compared, for instance, with those obtained in ref. [3], which can be seen in the solutions obtained for (2.15), which include a sum of tanh and coth functions, using the solutions mentioned in Eq. (2.7). The results obtained in this study are complementary to those found by other researchers. These results contribute to the development of the solitons theory, especially since the use of variable coefficients has become more important recently.

4 Conclusion

We have obtained exact solutions for the Manakov system with variable coefficients (1.1), which, due to its variable coefficients, are new in the literature. The ITCM has been applied directly on the ordinary differential system (2.2), (2.15), for obtaining exact solutions of (1.1) in a satisfactory way. The case in which the system has constant coefficients can be derived as a particular case. New solutions for the classical Manakov model (constant coefficients) are derived from the solutions obtained here. From the graphs r_c and r_v, we can see that with the use of variable coefficients, we can have different structures of the solutions, compared with the solutions in the case of constants coefficients. We have found new solutions for the standard model by obtaining optical solutions for the model considered here under two conditions: firstly, when $\Gamma_1(\varepsilon) = \Gamma_2(\varepsilon)$, and secondly, when $\Theta_1(\varepsilon) = \Theta_2(\varepsilon)$. Comparing that solutions with those obtained by others authors [3–5], we can conclude that our results are complementary to those obtained in the mentioned previously mentioned references. By using (2.20), (2.21), (2.22), and (2.23), new type of solutions can be derived to complement our work, and to keep tis short, we have omitted here.

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